

Image Super-resolution Based on MCA and Wavelet-Domain HMT

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ABSTRACT: In this paper we propose an image super-resolution algorithm using The Morphological Component Analysis(MCA) and wavelet-domain Hidden Markov Tree(HMT) model. The MCA is a useful method for signal decomposing, using proper basis, we could separate features contained in a signal when these features present different morphological aspects. Wavelet-domain HMT models the dependencies of multiscale wavelet coefficients through the state probabilities of wavelet coefficients. In this paper, we first decompose an image into texture and piecewise smooth (cartoon) parts, then enlarge the cartoon part with interpolation, because wavelet-domain HMT accurately characterizes the statistics of real-world images, we specify it as the prior distribution and then formulate the image super-resolution problem as a constrained optimization problem to acquire the enlarged texture part, finally we get a fine result.

KEYWORDS: Image Super-resolution; image Decompose; MCA; Wavelet-Domain HMT

I. INTRODUCTION

The problem of obtaining a super-resolution image from one or several low-resolution images has been studied by many researchers in recent years. Generally speaking, a low-resolution image can be considered as the degraded version of the high-resolution image and the degradations are characterized by blurring and down-sampling. We know that when blurring or down-sampling a signal, its details, or high-frequency component will be cut down, so essentially the task of image super-resolution is to extract the “lost” high-frequency component of the high-resolution image from one or several low-resolution images. In this paper, we will introduce a new image super-resolution algorithm; we use the MCA to decompose a low-resolution image to its texture and piecewise smooth parts, then use the Wavelet-Domain HMT model to estimate the texture part of high-resolution image. And our experiments proved that this is an effective method.

II. THE MORPHOLOGICAL COMPONENT ANALYSIS

The task of decomposing signals into their building atoms^[1-3] is of great interest for many applications. In such problems a typical assumption is made that the given signal is a linear mixture of several source signals of more coherent origin. The Morphological Component Analysis (MCA)^[4] is an effective method which allows us to separate features contained in an image when these features present different morphological aspects. In this paper, we will decompose an image to texture and piece-wise-smooth (cartoon) parts using the MCA.

A. Model Assumption of MCA

Assume that a signal S is a linear combination of its different morphological parts:

$$S = \sum_{k=1}^K S_k, \quad (1)$$

where each s_k represents a different part of the signal to be decomposed.

In The Morphological Component Analysis, there are some assumptions:

- For every s_k , there will be a “dictionary” Φ_k . When solving

$$\alpha_k^{opt} = \text{Arg min}_{\alpha} \|\alpha\|_0 \text{ Subject to: } S_A = \Phi_A \alpha, \quad (2)$$

It leads to a very sparse solution: most coefficients are small, and the relatively few large coefficients capture most of the information.

- For another component S_l ($l \neq k$), when using Φ_k to solve the above problem, it leads to a very non-sparse solution.

The assumptions suggested that the dictionary Φ_k is distinguishing between the different types of signals to be separated. Dictionaries Φ_k have a fast transformation T_k ($\alpha_k = T_k S_k$) and reconstruction R_k ($s_k = R_k \alpha_k$).

B. The MCA concept

The MCA needs to solve an optimization task:

$$\{\alpha_1^{opt}, \dots, \alpha_k^{opt}\} = \text{Arg min}_{\{\alpha_1, \dots, \alpha_k\}} \sum_{k=1}^K \|\alpha_k\|_0 \quad (3)$$

$$\text{Subject to: } S = \sum_{k=1}^K \Phi_k \alpha_k.$$

Based on the assumptions, it will get a sparse solution, where each sparse coefficient α_k^{opt} represents different part of the signal. Account of the problem formulated in (3) is non-convex and hard to solve, the MCA algorithm replaces the 0-norm with 1-norm, and it will be a solvable optimization problem (Linear Programming):

$$\{\alpha_1^{opt}, \dots, \alpha_k^{opt}\} = \text{Arg min}_{\{\alpha_1, \dots, \alpha_k\}} \sum_{k=1}^K \|\alpha_k\|_1 \quad (4)$$

Subject to: $S = \sum_{k=1}^K \Phi_k \alpha_k$,

Relaxing the constraint in (4), an approximate form will be:

$$\{\alpha_1^{opt}, \dots, \alpha_K^{opt}\} = \underset{\{\alpha_1, \dots, \alpha_K\}}{\text{Arg min}} \sum_{k=1}^K \|\alpha_k\|_1 + \lambda \left\| S - \sum_{k=1}^K \Phi_k \alpha_k \right\|_2^2, \quad (5)$$

Replace the coefficients α_k^{opt} with s_k :

$$\{s_1^{opt}, \dots, s_K^{opt}\} = \underset{\{s_1, \dots, s_K\}}{\text{Arg min}} \sum_{k=1}^K \|T_k s_k\|_1 + \lambda \left\| S - \sum_{k=1}^K s_k \right\|_2^2, \quad (6)$$

Add constraints on each individual signal:

$$\{s_1^{opt}, \dots, s_K^{opt}\} = \underset{\{s_1, \dots, s_K\}}{\text{Arg min}} \sum_{k=1}^K \|T_k s_k\|_1 + \lambda \left\| S - \sum_{k=1}^K s_k \right\|_2^2 + \sum_{k=1}^K \gamma_k \zeta_k(s_k), \quad (7)$$

where \square_k implements constraints on component s_k . By solving problem (7), we can get every different component s_k of the signal S .

III. WAVELET-DOMAIN HMT

It is well known that wavelet coefficients are statistically dependent due to two properties of wavelet transform [5]:

- Clustering: If a wavelet coefficient is large/small, the adjacent coefficients are likely to be large/small.
- Persistence: Large/small coefficients tend to propagate across the scales.

The wavelet domain hidden markov tree model (HMT) proposed by Crouse et al [5] captures this statistical dependence. Each coefficient is modeled as a mixture with a state variable. The state of a coefficient is only determined by its parent state. Thus coefficients across scale yield markov chain model. It also assumes that each coefficient in the same scale is Gaussian distributed when conditioned on its state models the marginal distribution of each coefficient is a Gaussian mixture.

In this paper we will improve the HMT model to reconstruct the texture part of the High-resolution image.

A. The Discrete Wavelet Transform

The wavelet transform is an atomic decomposition that represents a one-dimensional (1-D) signal $z(t)$ in terms of shifted and dilated versions of a prototype bandpass wavelet function $\phi(t)$, and shifted versions of a lowpass scaling function $\varphi(t)$ [6, 7]. For special choices of the wavelet and scaling functions, the atoms:

$$\begin{aligned} \phi_{j,k}(t) &\equiv 2^{-j/2} \phi(2^{-j}t - k) \\ \varphi_{j,k}(t) &\equiv 2^{-j/2} \varphi(2^{-j}t - k) \end{aligned} \quad (8)$$

form an orthonormal basis, and we have the signal representation:

$$z(t) = \sum_k u_k \phi_{j_0,k}(t) + \sum_{j=-\infty}^{j_0} \sum_k w_{j,k} \varphi_{j,k}(t), \quad (9)$$

Where $w_{j,k} = \int z(t) \varphi_{j,k}^*(t) dt$, $u_k = \int z(t) \phi_{j_0,k}^*(t) dt$.

In this representation, j indexes the scale or resolution of analysis—smaller j corresponds to higher resolution analysis. j_0 indicates the coarsest scale or lowest resolution of analysis. k indexes the spatial location of analysis. For a wavelet $\varphi(t)$ centered at time zero and frequency f_0 , the wavelet coefficient $w_{j,k}$ measures the signal content around time $2^j k$ and frequency $2^j f_0$. The scaling coefficient measures the local mean around time $2^j k$.

For 2-D signal $f(x, y)$, the wavelet transform is:

$$f(x, y) = \sum_{m,n} u_{m,n} \phi_{j_0,m,n}^{LL}(x, y) + \sum_{B} \sum_{j=-\infty}^{j_0} \sum_{m,n} w_{j,m,n}^B \varphi_{j,m,n}^B(x, y), \quad (10)$$

$$\begin{aligned} u_{m,n} &= \iint f(x, y) \phi_{j_0,m,n}^{LL*}(x, y) dx dy \\ \text{where} \quad w_{j,m,n}^B &= \iint f(x, y) \varphi_{j,m,n}^{B*}(x, y) dx dy \end{aligned}$$

$\{\phi_{j_0,m,n}^{LL}, \varphi_{j,m,n}^B, j, m, n \in \mathbb{Z}\}$ is the basis of 2-D wavelet transform, $j_0 \in \mathbb{Z}$, $B \in \{LH, HL, HH\}$. LL , LH , HL , HH correspond to the approximation coefficients and details coefficients (horizontal, vertical, and diagonal, respectively).

The 2-D wavelet transform leads to a natural quad-tree organization of the wavelet coefficients in each sub band (Fig. 1):

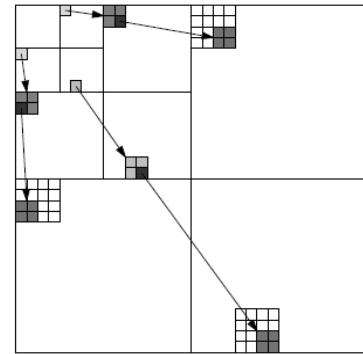


Figure 1. The quad-tree organization of the wavelet coefficients

B. Wavelet-Domain HMT

The HMT model associates a hidden state (white circle) with each wavelet coefficient (black circle) (Fig. 2):

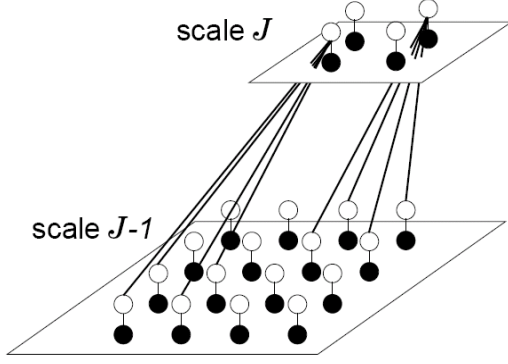


Figure 2. The Wavelet-Domain HMT

The dependencies between the wavelet coefficients are modeled as dependencies between the hidden states:

- The HMT models the marginal distribution of each real DWT coefficient as a Gaussian mixture. To each coefficient w_i , we associate a discrete hidden state S_i that takes on values m with probability mass function $p_{s_i}(m)$ (pmf). Conditioned on $S_i = m$, w_i is Gaussian with mean $\mu_{i,m}$ and variance $\sigma_{i,m}$. Thus, its overall marginal probability density function(pdf) is given by:

$$f_{w_i}(w_i) = \sum_{m=1}^M p_{s_i}(m) f_{w_i|S_i}(w_i|S_i = m), \quad (11)$$

where

$$p_{s_i}(m) = p(s_i = m|W, \theta),$$

$$f_{w_i|S_i}(w_i|S_i = m) = g(w_i; \mu_{i,m}, \sigma_{i,m}^2).$$

- To match the Persistence property of large and small coefficients, the HMT applies a Markov chain dependency structure to the hidden states across scale. The state transition probabilities between the connected states model the persistence of large/small coefficient magnitudes across scale. Using $\rho(i)$ to denote the index of the parent of node i , the parameter

$$\epsilon_{i,\rho(i)}^{mr} = p_{S_i|S_{\rho(i)}}(m|S_{\rho(i)} = r) \quad (12)$$

gives the probability that a child coefficient w_i has hidden state m when its parent $\rho(i)$ has state r .

Group these parameters into a vector, the HMT model can be describe as:

$$\theta = \{p_{s_i}(m), \mu_{i,m}, \sigma_{i,m}^2, \epsilon_{i,\rho(i)}^{mr}\}. \quad (13)$$

The HMT can be trained to capture the wavelet-domain features of the image of interest using the iterative

expectation-maximization (EM) algorithm^[5]. For a given set of training data, the trained model $f_{w_i}(w_i|\theta)$ approximates the joint probability density function (pdf) of the wavelet coefficients.

IV. AGROITHM DESCRIPTION

We first decompose an image to texture and piece-wise-smooth (cartoon) parts using the MCA. In this step, we use the DWT for the texture - denoted D , the curvelet transform for the cartoon part, denote C . Returning to the separation process as posed earlier, we have two unknowns: s_D and s_C . The optimization problem to be solved is:

$$\{s_D^{opt}, s_C^{opt}\} = \underset{\{s_D, s_C\}}{\text{Argmin}} \|Ds_D\|_1 + \|Cs_C\|_1 + \lambda \|s - s_D - s_C\|_2^2 + \gamma TV_{\xi}, \quad (14)$$

where s_D is the texture part and s_C is the piecewise smooth part.

We enlarged the cartoon part of the low-resolution image with cubic spline interpolation for the cartoon part of the High-resolution image.

For the texture part, we found that although it just a component of the image, not the wavelet coefficients, but it also has a Generalize Gaussian Distribution and has Persistence property cross scales, so we used the HMT model to estimate the texture part of the High- resolution image.

A. Problem formulation

We first formulate the high-to-low image formation process:

$$Lo = DCSu + \eta, \quad (15)$$

where Su is the High- resolution image, Lo is the Low-resolution image, η is the noise in the low-resolution image, D corresponds to the Down-sample process and C corresponds to the blurring process.

We take a Bayesian approach to this problem:

$$\hat{Su} = \arg \max_{Su} \Pr(Su|Lo). \quad (16)$$

The posterior probability:

$$\Pr(Su | Lo) = \frac{\Pr(Lo|Su) \Pr(Su)}{\Pr(Lo)},$$

where $\Pr(Lo)$ is the probability of we get the Low-resolution image, it is irrelevant to Su , so we can simplify (16):

$$\hat{Su} = \arg \max_{Su} \Pr(Lo|Su) \Pr(Su).$$

$\Pr(Su|Lo)$ is the conditional probability, determined by the

distribution of the noise η . Assume that η is an independent and identically distributed white noise, and then the conditional probability will be

$$\Pr(Lo|Su) = \frac{1}{(\sqrt{2\pi}\sigma)^M} e^{-\frac{\|DCSu - Lo\|_2^2}{2\sigma^2}}. \quad (17)$$

Based on the Wavelet-Domain HMT, the prior model of the wavelet coefficients can be formulated as:

$$\Pr(W_{Su}) = \Pr(W|S) = \prod_{1 \leq i \leq N} \sum_{m=1}^M p_{s_i}(m) f_{W_{Su}|S}(w_i|S_i = m), \quad (18)$$

In this paper, we need estimate the texture part of the High-resolution image, so replace W_{Su} with Su_T :

$$\Pr(Su_T) = \Pr(Su_T|S) = \prod_{1 \leq i \leq N} \sum_{m=1}^M p_{s_i}(m) f_{Su_T|S}(Su_{T,i}|S_i = m). \quad (19)$$

From equation (16), (17) and (19), we can get:

$$\hat{Su}_T = \arg \max_{Su_T} \frac{1}{(\sqrt{2\pi}\sigma)^M} e^{-\frac{\|DC(Su_T + Su_D) - Lo\|_2^2}{2\sigma^2}} \cdot \prod_{1 \leq i \leq N} \sum_{m=1}^M p_{s_i}(m) f_{Su_T|S}(Su_{T,i}|S_i = m). \quad (20)$$

As earlier mentioned in this paper, we already have an estimation of the cartoon part of the High-resolution image. So we can replace Su with $Su_T + Su_D$ in (20), where Su_D is the cartoon part, Su_T is the texture part. Then we get:

$$\hat{Su}_T = \arg \max_{Su_T} \frac{1}{(\sqrt{2\pi}\sigma)^M} e^{-\frac{\|DC(Su_T + Su_D) - Lo\|_2^2}{2\sigma^2}} \cdot \prod_{1 \leq i \leq N} \sum_{m=1}^M p_{s_i}(m) f_{Su_T|S}(Su_{T,i}|S_i = m), \quad (21)$$

or a simple mode:

$$\hat{Su}_T = \arg \min_{Su_T} -\log \left[\frac{1}{(\sqrt{2\pi}\sigma)^M} e^{-\frac{\|DC(Su_T + Su_D) - Lo\|_2^2}{2\sigma^2}} \cdot \prod_{1 \leq i \leq N} \sum_{m=1}^M p_{s_i}(m) f_{Su_T|S}(Su_{T,i}|S_i = m) \right], \quad (22)$$

where $f_{Su_T|S}(Su_{T,i}|S_i = m)$ is a Gaussian function, it makes the optimization problem too complicated to be solved. In Zhao's estimate algorithm^[8], they considered a special case, if $p_{s_i}(m) = 1$, then:

$$-\log \prod_{1 \leq i \leq N} \sum_{m=1}^M p_{s_i}(m) f_{Su_T|S}(Su_{T,i}|S_i = m) = \frac{Su_{T,i}^2}{2\sigma_{i,m}^2},$$

If $m = 1$ denote the coefficient is "small", then $\sigma_{i,1}^2 \ll \sigma_{i,2}^2$, so:

$-\log \prod_{1 \leq i \leq N} \sum_{m=1}^M p_{s_i}(m) f_{Su_T|S}(Su_{T,i}|S_i = m)$ acts as a penalty function, if $p_{s_i}(1) = 1$, the coefficient is "suppressed"; $p_{s_i}(2) = 1$, the coefficient is "encouraged". So it can be simplified to:

$$\sum_{i=1}^N \frac{Su_{T,i}}{p_{s_i}(1)\sigma_{i,1}^2 + p_{s_i}(2)\sigma_{i,2}^2}. \quad (23)$$

Then equation (21) will be:

$$\hat{Su}_T = \arg \min_{Su_T} \sum_{i=1}^N \frac{Su_{T,i}}{p_{s_i}(1)\sigma_{i,1}^2 + p_{s_i}(2)\sigma_{i,2}^2} + \frac{1}{2\sigma^2} \|DC(Su_T + Su_D) - Lo\|_2^2, \quad (24)$$

Solve this optimization problem, we can get the estimation of the texture part, then finally the estimated high-resolution image is:

$$\hat{Su} = \hat{Su}_T + \hat{Su}_S.$$

B. Solving problem

Note that, because Su_T is coupled with $p_{s,i}(m)$, so the optimization problem needs alternate: After getting an Su_T , an Upward-Downward algorithm is needed to update $p_{s,i}(m)$, the iteration ended when $p_{s,i}(m)$ changes very slightly.

We solve this problem as follows:

Step 1: decompose the low-resolution image Lo into its texture part Lo_T and cartoon part Lo_S , enlarge Lo_S with cubic spline interpolation as Su_S ;

Step 2: set counter $k = 0$, initialize the posterior state probabilities $p_{s,i}^k(1) = 1, 1 \leq i \leq N$;

Step 3: solve problem (24) and obtain the solution Su_T^k using conjugate-gradient method;

Step 4: compute $p_{s,i}^{k+1}(m) = p(s_i = m | Su_T^k, \theta)$ with Upward-Downward algorithm;

Step 5: if $\sum_{i=1}^N |p_{s_i}^{k+1} - p_{s_i}^k| < \epsilon$, go to step 6, else set $k = k + 1$ and go to step 3;

Step 6: get the high-resolution image $Su = Su_T^k + Su_S$.

V. EXPERIMENTS

In our experiments, we used three standard images: Lena, Bridge and Baboon with 512×512 pixels.

We used the fuzzy kernel

$$C = \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

to blur the high-resolution image (512×512 pixels), and then proceeded a down-sample step to get a low-resolution image (256×256 pixels). The HMT model trained from the texture parts of the low-resolution image in each scales with Xiao's estimate algorithm^[9]. And we used SSIM^[10] to assess the quality of estimated high-resolution image.

The experimental results are shown in the following figures:

A. *Lena*



Figure 3. High-resolution image with cubic spline interpolation (512x512 pixels).



Figure 4. High-resolution image with cubic spline interpolation (512x512 pixels).

B. *Bridge*

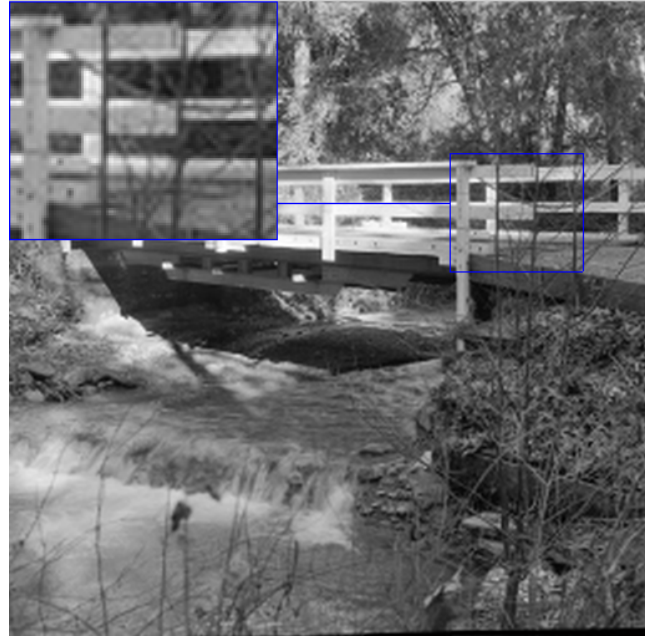


Figure 5. High-resolution image with cubic spline interpolation (512x512 pixels).



Figure 6. High-resolution image with the MCA-HMT algorithm (512x512 pixels).

C. Baboon

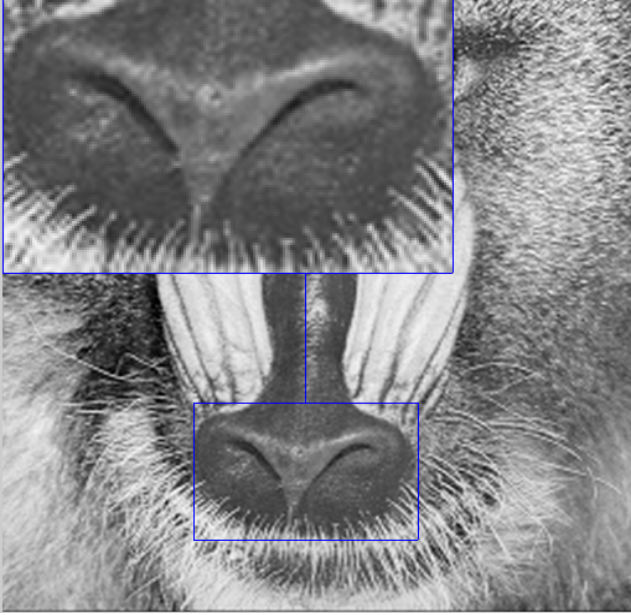


Figure 7. High-resolution image with cubic spline interpolation (512x512 pixels).

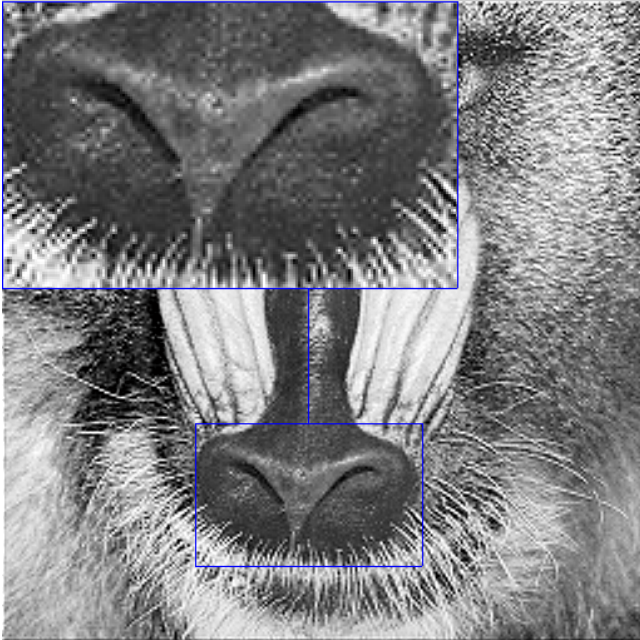


Figure 8. High-resolution image with the MCA-HMT algorithm (512x512 pixels).

TABLE I. EXPERIMENTAL RESULTS (SSIM)

Image	Lena	Bridge	Baboon
the MCA-HMT algorithm	0.9824	0.9598	0.9269
cubic spline interpolation	0.9637	0.9372	0.9200

We found that, with the increase of the textures in an image, the SSIM of the estimated high-resolution image appears a distinct decrease: i.e. the SSIM of Lena is 0.9824, and the SSIM of Baboon is 0.9269. However, although Baboon has a low SSIM, its visual effect is fine.

VI. CONCLUSIONS

In this paper, we decomposed an image into two parts and used different method for the estimation of each part, the experiment shows that this algorithm is very effective; maybe we could decompose an image into more parts, and by using proper estimate method we could acquire an exciting result. This will be confirmed in our future works.

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