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Data-driven controller design for general MIMO nonlinear systems via virtual reference feedback tuning and neural networks

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ABSTRACT

In this paper, we develop a novel data-driven multivariate nonlinear controller design method for multi-input–multi-output (MIMO) nonlinear systems via virtual reference feedback tuning (VRFT) and neural networks. To the best of authors' knowledge, it is the first time to introduce VRFT to MIMO nonlinear systems in theory. Unlike the standard VRFT for linear systems, we restate the model reference control problem with time-domain model in the absence of transfer functions and simplify the objective function of VRFT without a linear filter. Then, we prove that the objective function of VRFT reaches the minimum at the same point as the optimization problem of model reference control and give the relationship between the bounds of the two optimization problems of model reference control and VRFT. A three-layer neural network is used to implement the developed method. Finally, two simulations are conducted to verify the validity of our method.

1. Introduction

Traditionally, a suitable controller is designed by the mathematical model of the plant which is identified from the input and output data. However, with the rapid development of science and technology, the industrial process and production equipment become more and more complex, which makes establishing accurate mathematical models costly and even unattainable. Imprecise models will bring about the model error into the controller, which implies that the system cannot reach the expected goal. Fortunately, with the development of information technology, especially the accurate sensor technology and data storage technology, huge amounts of data are recorded and stored in the daily production. To make full use of data and solve the direct control design problem, data-driven control is proposed and gets the attention of more and more researchers.

Compared with model-based control, data-driven control designs the controller directly without mathematical models. Progress has been made to show the advantages of data-driven control over traditional model-based controls [1–3]. In the past few decades, various data-driven methods have been proposed under some system hypotheses in different environments. Tuning the controller online by estimating the gradient of the goal function is an effective idea of the data-driven control. For instance, simultaneous perturbation stochastic approximation (SPSA) introduced by Spall estimates the gradient by stochastic approximation [4,5] and model free adaptive control (MFAC) proposed by Hou replaces the gradient with pseudo-partial derivative [6–8]. The idea of iterations also has good applications in data-driven method. For instance, iterative learning control (ILC) [9–14] suits for the systems when the off-line data can be obtained repeatedly or periodically and iterative feedback tuning (IFT) developed by Hjalmarsson [15–17] is based on an iterative gradient descent approach. Additionally, in the field of optimal control, adaptive dynamic programming (ADP) [18–21] is a significant and hot topic. Many data-driven and model-free methods based on ADP have been established [22–32]. Different from the above methods, virtual reference feedback tuning (VRFT), which is originally proposed by Guardabassi and Savaresi [33], provides a global solution to a model reference control problem with one-shot off-line data. VRFT has the advantages of less calculation than iterative methods, global optimal solution compared to local optimal solutions of gradient methods and just one-shot off-line data with no need of detected signal. Until now, VRFT has been developed for single-input single-output (SISO) linear systems [34,35], multi-input multi-output (MIMO) linear systems [36,37], and SISO nonlinear systems [38,39].

In the aspect of applications, more and more data-driven control methods are designed to solve practical problems in recent years. In [40], a data-driven approach was designed to control

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batch processes with applications to a gravimetric blender. Marcel et al. proposed a robust data-driven control for solving synchronization problem [41]. An iterative data-driven tuning method of controllers was developed for nonlinear systems with applications to angular position control of an aerodynamic system [42]. A data-driven self-tuning control was designed by iterative learning control to optimize the control parameters of turbocharged engines [43]. In [44], Chi et al. presented a unified data-driven design framework of optimality-based generalized iterative learning control. VRFT was also applied to nonlinear systems by neural controllers [45] and MIMO linear systems [46,37].

However, as a well-known data-driven method, there are very few results of VRFT for MIMO nonlinear systems in both theory and applications. Different from SISO nonlinear systems and linear systems, MIMO nonlinear systems are much more complex, and it is a much tougher task to demonstrate the validity of VRFT in this case. Nevertheless, data-driven control aims to solve the control problem of complex and highly nonlinear plant, and the theory of linear systems and SISO systems is not sufficient. Therefore, it is of great importance to investigate VRFT for MIMO nonlinear systems. To the best of our knowledge, our work is the first to present the theoretical analysis of VRFT for MIMO nonlinear systems.

This paper studies the problem of model reference controller design of general MIMO nonlinear systems by using VRFT and proves the validity of the established method. First, to avoid the difficulty of solving nonlinear transfer function, we recall the optimization problem of model reference control with time-domain model. Second, we prove that the time-domain model optimization problems of VRFT and the model reference control have the same solution. We also obtain the relationship of the bounds of the two optimization problems. Finally, we provide the implementation of VRFT in MIMO nonlinear systems by neural networks.

The rest of this paper is organized as follows. Section 2 gives the basic assumptions of the system and presents the optimization problem of model reference control in MIMO nonlinear systems. Section 3 describes the VRFT approach in MIMO nonlinear systems and proves the equivalence of the optimization problems of model reference control and VRFT. Furthermore, the relationship between the bounds of the two problems is also discussed in this section. Section 4 introduces a three-layer neural network to approximate the controller with the aid of VRFT. Section 5 illustrates the simulation results in noiseless and noisy environments which show the effectiveness of our method, respectively. Section 6 gives the conclusion.

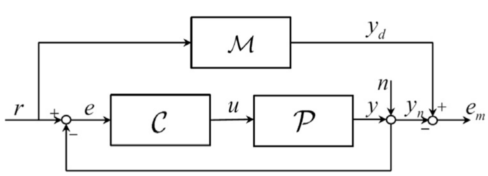

Fig. 1. Model reference control.

2. Optimization problem of model reference control

The control system is shown in Fig. 1. u is the control input and y is the output of the plant. y_n is the plant output corrupted by noise n and r is the reference signal. It is a classical closed-loop control system where the controller $\mathcal{C}$ processes the error signal e so as to generate the control input u to the plant $\mathcal{P}$. The plant $\mathcal{P}$ and the controller $\mathcal{C}$ are nonlinear and multivariate. We assume that there is a reference model $\mathcal{M}$ which describes the relationship between the reference input r and the desired output y_d. Our goal is to design the controller $\mathcal{C}$ to make the performance of the closed-loop control system as close as possible to $\mathcal{M}$, which means that the error e_m between the output of control system and reference model with the same reference input is as small as possible.

For linear systems, the transfer function model is used to describe the problem of model reference control [33,36]. However, it is well known that the transfer function is not suitable for the

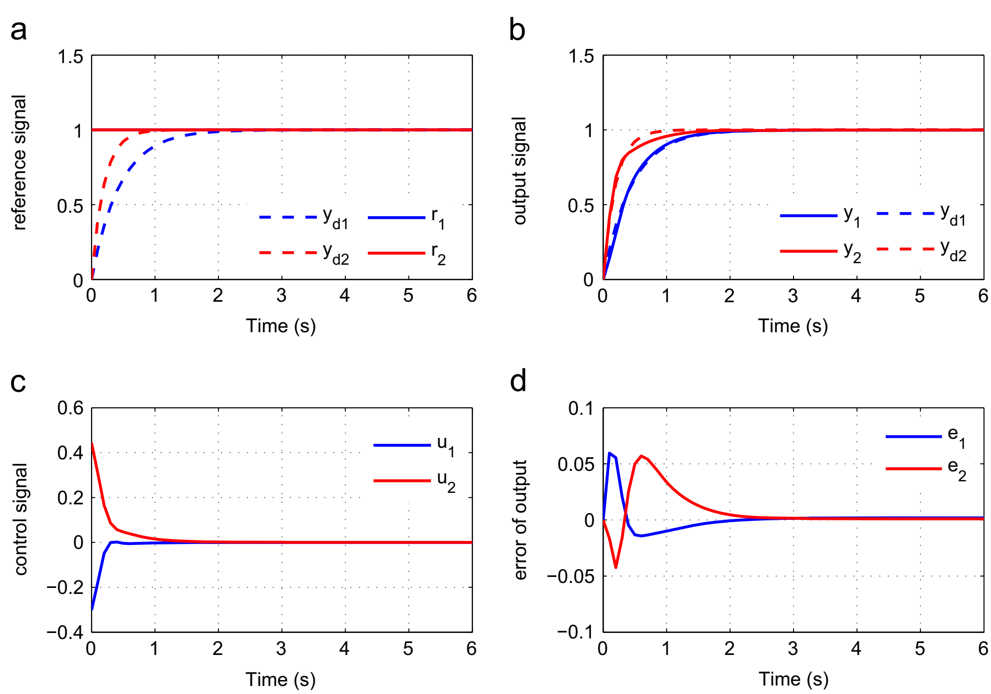

Fig. 2. Responses of the system with reference to unit step signal in noiseless environment. (a) Reference input r and desired output y_d. (b) Desired output y_d and actual output y. (c) Control signal u_1 and u_2. (d) Error of output e_1 and e_2.

analysis of nonlinear systems. In [38], the authors restated the problem of model reference control in SISO nonlinear systems, but it is difficult to extend this method to MIMO nonlinear systems. Hence, in this section, we redefine the problem of model reference control in MIMO nonlinear systems and make the basic assumptions of the system as follows.

First, the plant $\mathcal{P}$ is a discrete-time MIMO nonlinear system, which is described by

$$y(k) = p(u(k-1), \ldots, u(k-n_{p_u}), y(k-1), \ldots, y(k-n_{p_y})), \tag{1}$$

where $u(k) \in \mathcal{U} \subset \mathbb{R}^m$ is the input of the plant, $y(k) \in \mathcal{Y} \subset \mathbb{R}^n$ is the output of the plant. $\mathcal{U}$ and $\mathcal{Y}$ are bounded closed convex sets. n_{p_y} and n_{p_u} are the orders of output y and input u in the plant, respectively. To simplify the equation, we let

$$IC_p(k-1) = \{u(k-2), \ldots, u(k-n_{p_u}),\\ y(k-1), \ldots, y(k-n_{p_y})\}.$$

Eq. (1) can be rewritten as

$$y(k) = p(u(k-1), IC_p(k-1)). \tag{2}$$

We assume that the plant satisfies the following conditions:

(1) Function $p(\cdot)$ is continuous with all variables.
(2) System (1) is controllable and bounded input bounded output stable.
(3) The initial condition is known and denoted by

$$IC_p(0) = \{u(-1), \ldots, u(1-n_{p_u}), y(0), \ldots, y(1-n_{p_y})\}.$$

(4) $p(\cdot)$ is invertible with respect to $u(k)$, i.e., $\partial p(\cdot)/\partial u(k) \neq 0$, which means that for any $y(k) \in \mathbb{R}^n$, there is a unique $u(k) \in \mathbb{R}^m$ satisfying (1) with any fixed initial condition.

Remark 1. The controllability is a basic assumption of the system and necessary for controller design. However, it is difficult to analyze controllability and observability by data-based methods. Some researchers presented several data-based methods to analyze the controllability and stability of unknown systems [47,48]. The initial condition is usually assigned to zero. Actually, the initial condition has little effect when the running time of the plant is sufficiently long. The reference input r is sufficiently excited. The continuity condition is the basic assumption of the plant and additional conditions will be given for analyzing the properties of VRFT in the sequel. The invertibility is certainly true for linear systems and also for a large class of nonlinear systems. It is necessary for the implementation of VRFT which will be discussed in the sequel.

Second, the controller is assumed to be a nonlinear function as follows:

$$u(k) = c(e(k), \ldots, e(k-n_{c_e}), u(k-1), \ldots, u(k-n_{c_u})), \tag{3}$$

where $u(k) \in \mathbb{R}^m$ is the control signal and $e(k) = r(k) - y(k)$ is the error signal. n_{c_u} and n_{c_e} are the orders of control u and error e in the controller, respectively. For simplification of discussion, we let

$$IC_c(k) = \{e(k-1), \ldots, e(k-n_{c_e}), u(k-1), \ldots, u(k-n_{c_u})\},$$

then Eq. (3) can be rewritten as

$$u(k) = c(e(k), IC_c(k)). \tag{4}$$

We assume that the function $c(\cdot)$ is continuous with all variables and the initial condition is known and is denoted by

$$IC_c(0) = \{e(-1), \ldots, e(1-n_{c_e}), u(-1), \ldots, u(1-n_{c_u})\}.$$

As $e(k) = r(k) - y(k)$ and $r(k)$ is known in advance, the initial condition is rewritten as

$$IC_c(0) = \{y(-1), \ldots, y(1-n_{c_y}), u(-1), \ldots, u(1-n_{c_u})\}.$$

It is obvious that the variables appeared in both $IC_c(0)$ and $IC_p(0)$ should be the same.

According to (2) and (4), the closed-loop control system can be represented as

$$y(k) = p(u(k-1), IC_p(k-1))\\ = p(c(e(k-1), IC_c(k-1)), IC_p(k-1)). \tag{5}$$

We choose a group of nonlinear functions with fixed structure and undetermined parameters as the candidate controllers

$$u_\theta(k) = c(e(k), IC_c(k); \theta), \tag{6}$$

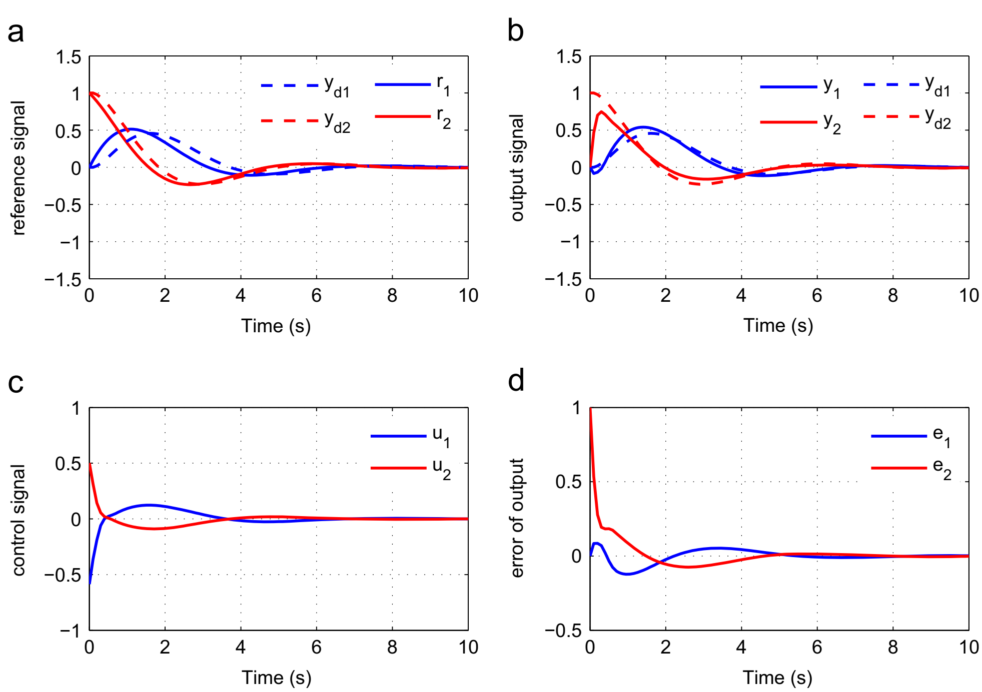

Fig. 3. Responses of the system with reference to damping sine and cosine signals in noiseless environment. (a) Reference input r and desired output y_d. (b) Desired output y_d and actual output y. (c) Control signal u_1 and u_2. (d) Error of output e_1 and e_2.

where $\theta \in \mathbb{R}^{n_\theta}$ is the undetermined coefficient and n_θ is the degree of freedom. For convenience, the controller is simplified as

$$u_\theta(k) = c_\theta(e(k), IC_c(k)). \tag{7}$$

The candidate controller set is denoted by $\{c_\theta(\cdot); \theta \in \mathbb{R}^{n_\theta}\}$. According to (2) and (7),

$$\begin{aligned} y(k) &= p(u_\theta(k-1), IC_p(k-1)) \\ &= p(c_\theta(e(k-1), IC_c(k-1)), IC_p(k-1)). \end{aligned} \tag{8}$$

The reference model is a mapping from r to y_d as follows:

$$y_d(k) = m(r(k-1), \ldots, r(k-n_{m_r}), y_d(k-1), \ldots, y_d(k-n_{m_y})), \tag{9}$$

where $y_d(k) \in \mathbb{R}^n$ is the desired output signal and $r(k) \in \mathbb{R}^n$ is the reference signal. n_{m_r} and n_{m_y} are the orders of reference signal r and the output y in the reference model, respectively. Let

$$\begin{aligned} IC_m(k-1) = \{&r(k-2), \ldots, r(k-n_{m_r}), \\ &y_d(k-1), \ldots, y_d(k-n_{m_y})\}, \end{aligned}$$

then (9) can be rewritten as

$$y_d(k) = m(r(k-1), IC_m(k-1)). \tag{10}$$

We assume that the function $m(\cdot)$ is continuous with all variables and the initial condition is known which is denoted by

$$IC_m(0) = \{r(-1), \ldots, r(1-n_{m_y}), y_d(0), \ldots, y_d(1-n_{m_r})\}$$

For the implementation of VRFT, we assume that the function $m(\cdot)$ is invertible with respect to $r(k-1)$.

The reference model is the desired performance of the system under ideal conditions and depends on the actual demand. The reference model has an important effect on the performance of the controller designed by VRFT. However, there are no effective methods to acquire an ideal reference model, which is designed just by experience. The reference model can be linear or nonlinear, and it is hard to obtain such a nonlinear model to achieve the system's demand. In most cases, the reference model is set to be a linear system given as follows:

$$\begin{aligned} y_d(k) = &A_1 r(k-1) + \cdots + A_{n_{m_r}} r(k-n_{m_r}) \\ &-B_1 y_d(k-1) - \cdots - B_{n_{m_y}} y_d(k-n_{m_y}). \end{aligned} \tag{11}$$

It is obvious that this linear system is invertible with respect to $r(k-1)$ when A_1 is nonsingular.

In view of the observations above, the model reference control problem shown in Fig. 1 is to find a nonlinear functional from $\mathcal{Y}$ to $\mathcal{U}$ satisfying the following optimization problem:

$$\begin{aligned} \min_{c(\cdot)} \quad & J_{MR}(c) = \sum_{k=1}^{N} \| y(k) - m(r(k-1), IC_m(k-1)) \|^2 \\ \text{s.t.} \quad & y(k) = p(c(e(k-1), IC_c(k-1)), IC_p(k-1)) \\ & e(k-1) = r(k-1) - y(k-1) \\ & k = 1, 2, \ldots, N \\ & IC(0) = IC_p(0) \cup IC_c(0) \cup IC_m(0). \end{aligned} \tag{12}$$

The solution of (12) is a nonlinear function, which is difficult to obtain. We usually choose a candidate controller set $\{c_\theta(\cdot); \theta \in \mathbb{R}^{n_\theta}\}$ at first, then the problem (12) can be converted into

$$\begin{aligned} \min_{\theta} \quad & J_{MR}(\theta) = \sum_{k=1}^{N} \| y_\theta(k) - m(r(k-1), IC_m(k-1)) \|^2 \\ \text{s.t.} \quad & y_\theta(k) = p(c_\theta(e(k-1), IC_c(k-1)), IC_p(k-1)) \\ & e(k-1) = r(k-1) - y_\theta(k-1) \\ & k = 1, 2, \ldots, N \\ & IC(0) = IC_p(0) \cup IC_c(0) \cup IC_m(0). \end{aligned} \tag{13}$$

Remark 2. Although the optimization problem (13) is defined by a certain trajectory, this trajectory is required to make $y(k)$ and $u(k)$ fully explore the domains of definitions $\mathcal{Y}$ and $\mathcal{U}$. To meet this requirement, the trajectory must be long enough and the input must be sufficiently excited. If the plant p is known, we can directly solve the nonlinear optimization problem by gradient descent algorithm to obtain the optimal solution θ^*. However, we cannot acquire the precise mathematical model of the plant. Thus we will introduce the data-driven method to solve the optimization problem (13) in the next section.

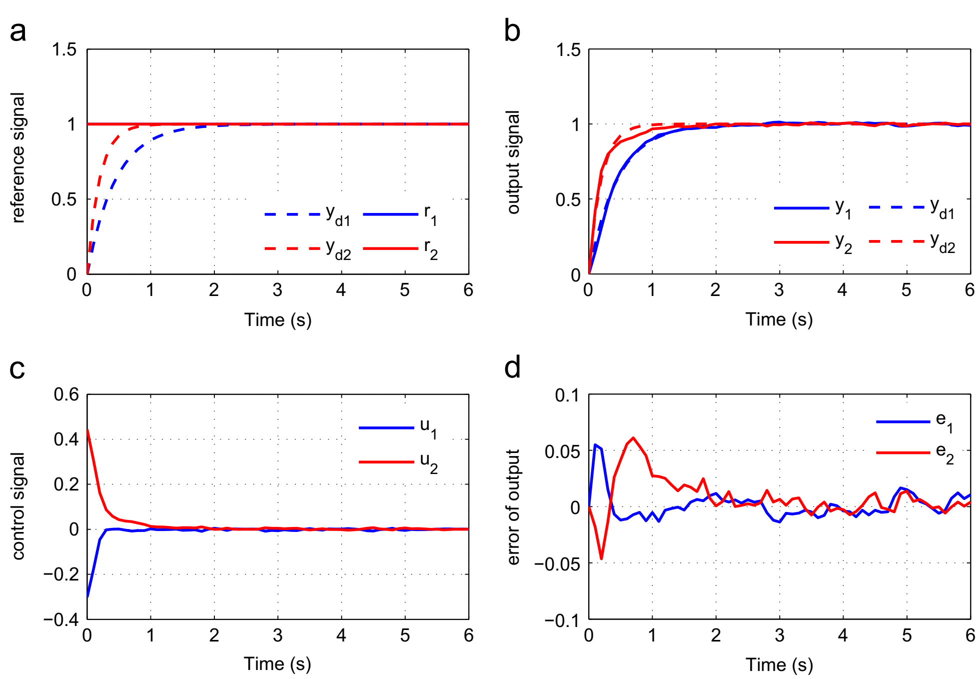

Fig. 4. Responses of the system with reference to unit step signal in noisy environment. (a) Reference input r and desired output y_d. (b) Desired output y_d and actual output y. (c) Control signal u_1 and u_2. (d) Error of output e_1 and e_2.

3. Data-driven control design of MIMO nonlinear systems via VRFT

Notations: In this section, $\tilde{\bullet}$ denotes measured values of the variables, e.g., $\tilde{u}$ and $\tilde{y}$ are the measured values of the input and output of the plant, respectively. $\hat{\bullet}$ denotes estimated values of the variables, e.g., $\hat{u}$, $\hat{y}$ and $\hat{\theta}$ are the estimated values of the input, output and parameters of the controller, respectively. $\|\bullet\|$ denotes Euclidean norm.

We assume that there is a sequence of input/output data generated by the plant as

$$\tilde{u}(k),\ k=0,1,\ldots,N-1;\quad \tilde{y}(k),\ k=1,2,\ldots,N. \tag{14}$$

If the plant is noiseless, $\{\tilde{u}(k),\tilde{y}(k)\}$ is equivalent to $\{u(k),y(k)\}$. In this section, we have the following assumption.

Assumption 1.

(a) The plant is noiseless.
(b) The initial condition is known.
(c) The sampling time N is long enough and the control sequence is sufficiently excited.
(d) $\tilde{u}(k)$ and $\tilde{y}(k)$ are bounded.

The reference model $\mathcal{M}$ is given in advance. Due to the assumption of invertibility, we can get $\tilde{r}(k), k=0,1,\ldots,N-1$ from $\tilde{r}(k-1)=m^{-1}(\tilde{y}(k),\tilde{IC}_m(k-1))$ with the initial condition $\tilde{IC}_m(0)$ which is usually zero. $\tilde{r}(k)$ is called virtual reference signal, because it is computed by the inverse of reference model and is not the desired reference trajectory that is used as the reference of the system. However, $\tilde{r}(k)$ coincides with the reference model $m(\cdot)$ so as to design the controller to adjust to $m(\cdot)$. If $\{c_\theta(\cdot);\theta\in\mathbb{R}^{n_\theta}\}$ is sufficiently rich, $\tilde{r}(k)$ is constructed by $\tilde{y}(k)$ and $m^{-1}(\cdot)$ is sufficiently excited, the optimal controller $c_{\theta^*}(\cdot)$ designed by $\tilde{r}(k)$ is also suitable for other desired reference trajectories.

Theorem 1. *If $p(\cdot)$ and $m(\cdot)$ satisfy Assumption 1, for arbitrary initial conditions $IC(0)$ and reference trajectory $r(k)$ in the domain of definition, there is an optimal controller $c^*(\cdot)$ such that the closed-loop system is equivalent to the selected reference model $m(\cdot)$, i.e., $J_{\mathrm{MR}}(c^*)=0$.*

Proof. For arbitrary initial conditions $IC(0)$ and reference trajectory $r(k)$, according to the reference model $m(\cdot)$, we can obtain

$$y_d(k)=m(r(k-1),IC_m(k-1)). \tag{15}$$

As $p(\cdot)$ is invertible, its inverse is

$$u(k-1)=p^{-1}(y(k),IC_p(k-1)). \tag{16}$$

According to (16) and the desired output trajectory $y_d(k)$,

$$u^*(k-1)=p^{-1}(y_d(k),IC_p(k-1)). \tag{17}$$

Then we let $e(k)=r(k)-y_d(k)$ and construct the mapping $c^*(\cdot)$ from $\mathbb{R}^n$ to $\mathbb{R}^m$, such that

$$c^*(e(k),IC_c(k))=u^*(k),\quad k=0,1,\ldots,N-1.$$

It is easy to verify $J_{\mathrm{MR}}(c^*)=0$. □

Theorem 1 shows that the invertibility of $p(\cdot)$ ensures the existence of the optimal controller. The invertibility of $m(\cdot)$ is not used in this theorem, as it is not the necessary condition of the existence of optimal controller. However, we make the reference model invertible, as it is indispensable in the implementation of VRFT.

In view of SISO nonlinear systems [38], the optimization problem (13) can be simplified as

$$\begin{aligned}\min\quad & J(\theta)=\|y_\theta-M[r]\|^2\\ \text{s.t.}\quad & y_\theta=P[C_\theta[r-Dy_\theta]],\end{aligned} \tag{18}$$

where $M:r\to y$ is the reference model and D is the delay matrix defined as

$$D=\begin{bmatrix}0&0&\cdots&0&0\\1&0&\cdots&0&0\\0&1&\cdots&0&0\\\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&1&0\end{bmatrix}. \tag{19}$$

$C[\cdot]$ and $P[\cdot]$ are derived by $c(\cdot)$ and $p(\cdot)$, respectively. The exact definitions of $C[\cdot]$ and $P[\cdot]$ can refer to [38]. The objective function

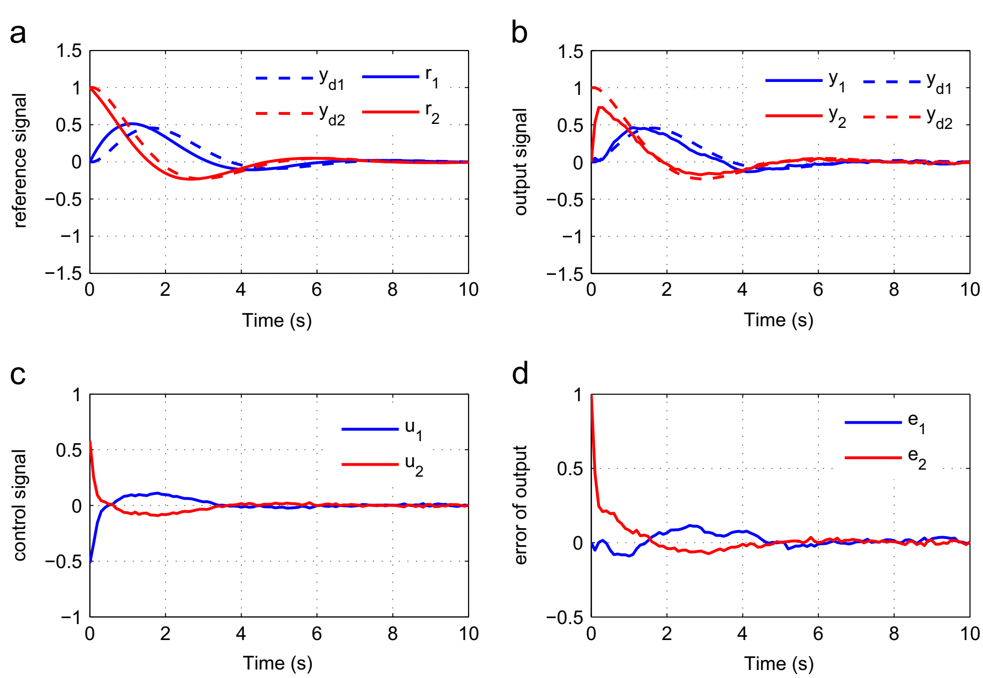

Fig. 5. Responses of the system with reference to damping sine and cosine signals in noisy environment. (a) Reference input r and desired output y_d. (b) Desired output y_d and actual output y. (c) Control signal u_1 and u_2. (d) Error of output e_1 and e_2.

of the VRFT is

$$\min J_{\text{VRFT}} = \| F[C_\theta[\tilde{e}]] - F[\tilde{u}] \|^2, \tag{20}$$

where $\tilde{e} = \tilde{r} - D\tilde{y}$. F is a linear time-varying filter

$$F = (I - MD)\frac{\partial P}{\partial u}\Big|_{\tilde{u}}. \tag{21}$$

In [38], the authors have proven that when the filter F is defined by (21), the optimization problem (20) is a second order approximation to the optimization problem (18). However, the method of designing a linear filter is difficult to apply to MIMO nonlinear systems as the estimation of the derivative of plant is hard to obtain. Rough estimation cannot ensure the second order approximation. More importantly, the estimation of the derivative is based on system identification, which makes the method not a pure data-driven method. Hence, we remove the filter F in MIMO nonlinear systems and redefine the objective function of the VRFT as

$$\min J_{\text{VRFT}}(\theta) = \sum_{k=0}^{N-1} \| c_\theta(\tilde{e}(k)) - \tilde{u}(k) \|^2. \tag{22}$$

From (22), we can find that the core idea of VRFT method is to design a controller which produces $\tilde{u}$ when fed by $\tilde{e}$. The performance function of VRFT stands for the error between the designed controller and the desired controller, while the performance function of model reference control is the error between the actual output and the desired output. In what follows, we will demonstrate that when $c^* \in \{c_\theta\}$, the solution of problem (22) is equivalent to that of problem (13). Moreover, when $c^* \notin \{c_\theta; \theta \in \mathbb{R}^{n_\theta}\}$, the solution of problem (22) can be an effective estimation of the solution of problem (13) which can make the objective value of problem (13) sufficiently small.

Theorem 2. *If there exists $\theta^* \in \mathbb{R}^{n_\theta}$ such that*

$$J_{\text{MR}}(\theta^*) = \sum_{k=1}^{N} \| y_{\theta^*}(k) - m(\tilde{r}(k-1), I\tilde{C}_m(k-1)) \|^2 = 0, \tag{23}$$

then we have

$$J_{\text{VRFT}}(\theta^*) = \sum_{k=0}^{N-1} \| c_\theta(\tilde{e}\,|\,(k), I\tilde{C}_c(k)) - \tilde{u}(k) \|^2 = 0, \tag{24}$$

and vice versa.

Proof. (i) By the assumption that

$$\sum_{k=1}^{N} \| y_{\theta^*}(k) - m(\tilde{r}(k-1), I\tilde{C}_m(k-1)) \|^2 = 0,$$

we can acquire, for $k = 1, 2, \ldots, N$,

$$\| y_{\theta^*}(k) - m(\tilde{r}(k-1), I\tilde{C}_m(k-1)) \| = 0.$$

Noticing that $\tilde{y}(k) = m(\tilde{r}(k-1), I\tilde{C}_m(k-1))$, we have

$$y_{\theta^*}(k) = \tilde{y}(k), \quad k = 1, 2, \ldots, N. \tag{25}$$

By (8), we can derive, for $k = 1, 2, \ldots, N$,

$$y_{\theta^*}(k) = p(c_{\theta^*}(\tilde{r}(k-1) - y_{\theta^*}(k-1), I\tilde{C}_c(k-1)), I\tilde{C}_p(k-1)),$$

and

$$\tilde{y}(k) = p(\tilde{u}(k-1), I\tilde{C}_p(k-1)).$$

By the invertibility of $p(\cdot)$, we can obtain, for $k = 1, 2, \ldots, N$,

$$c_{\theta^*}(\tilde{r}(k-1) - y_{\theta^*}(k-1), I\tilde{C}_c(k-1)) = \tilde{u}(k-1). \tag{26}$$

In addition, with (25), we have

$$\tilde{r}(k-1) - y_{\theta^*}(k-1) = \tilde{r}(k-1) - \tilde{y}(k-1) = \tilde{e}(k-1). \tag{27}$$

Substituting (27) into (26), we can get, for $k = 1, 2, \ldots, N$,

$$c_{\theta^*}(\tilde{e}(k-1), I\tilde{C}_c(k-1)) = \tilde{u}(k-1).$$

Hence, we can conclude

$$J_{\text{VRFT}}(\theta^*) = \sum_{k=0}^{N-1} \| c_{\theta^*}(\tilde{e}(k), I\tilde{C}_c(k)) - \tilde{u}(k) \|^2 = 0.$$

(ii) If there exists θ^* such that $J_{\text{VRFT}}(\theta^*) = 0$, i.e.,

$$\sum_{k=0}^{N-1} \| c_{\theta^*}(\tilde{e}(k), I\tilde{C}_c(k)) - \tilde{u}(k) \|^2 = 0,$$

we have

$$c_{\theta^*}(\tilde{e}(k), I\tilde{C}_c(k)) = \tilde{u}(k).$$

When the initial condition $I\tilde{C}_p(0)$ is fixed, we can derive, for $k = 0, 1, \ldots, N-1$,

$$p(c_{\theta^*}(\tilde{e}(k), I\tilde{C}_c(k)), I\tilde{C}_p(k)) = p(\tilde{u}(k), I\tilde{C}_p(k)). \tag{28}$$

On the right hand side of (28), it implies

$$\begin{aligned} p(\tilde{u}(k), I\tilde{C}_p(k)) &= \tilde{y}(k+1) \\ &= m(\tilde{r}(k), I\tilde{C}_m(k)). \end{aligned} \tag{29}$$

On the left hand side of (28), for $k=0$,

$$\begin{aligned} &p(c_{\theta^*}(\tilde{e}(0), I\tilde{C}_c(0)), I\tilde{C}_p(0)) \\ &\quad = p(c_{\theta^*}(\tilde{r}(0) - y(0), I\tilde{C}_c(0)), I\tilde{C}_p(0)) \\ &\quad = y_{\theta^*}(1), \end{aligned}$$

and for $k = 1, 2, \ldots, N-1$,

$$\begin{aligned} &p(c_{\theta^*}(\tilde{e}(k), I\tilde{C}_c(k)), I\tilde{C}_p(k)) \\ &\quad = p(c_{\theta^*}(\tilde{r}(k) - y(k), I\tilde{C}_c(k)), I\tilde{C}_p(k)) \\ &\quad = y_{\theta^*}(k+1). \end{aligned}$$

Then, we can derive

$$y_{\theta^*}(k+1) - m(\tilde{r}(k), I\tilde{C}_m(k)) = 0.$$

Therefore, we can conclude

$$\sum_{k=1}^{N} \| y_{\theta^*}(k) - m(\tilde{r}(k-1), I\tilde{C}_m(k-1)) \|^2 = 0.$$

The equivalence of (23) and (24) is shown.□

Theorem 2 shows that when the set of candidate controllers is sufficiently rich, the optimization problem (13) is equivalent to (22).

Next, we will prove that when the result of optimization problem (22) is not zero but bounded, the problem (13) is also bounded. First, we will prove that it is true when the order of the system is one. The system (1) is reduced to

$$y(k) = p(u(k-1), y(k-1)), \tag{30}$$

The controller is simplified as

$$u_\theta(k) = c_\theta(e(k), u(k-1)), \tag{31}$$

and the reference model is

$$y_d(k) = m(r(k-1), y(k-1)). \tag{32}$$

Then, we have the following theorem.

Theorem 3. *Assume that for a given reference model $m(\cdot)$ and arbitrary initial condition $y(0)$, there exist $\hat{\theta} \in \mathbb{R}^{n_\theta}$ and a positive number $\epsilon > 0$, such that*

$$J_{\text{VRFT}}(\hat{\theta}) = \sum_{k=1}^{N} \| c_{\hat{\theta}}(\tilde{e}(k), \tilde{u}(k-1)) - \tilde{u}(k) \|^2 < \epsilon^2 \tag{33}$$

and $c_{\hat{\theta}}(\cdot)$ is continuously differentiable with respect to all variables in the domain of definition. Assume that the derivative of $p(\cdot)$ satisfies

$$\left\|\frac{\partial p}{\partial u}(u(k),y(k))\right\| < M_{p_u}, \quad \left\|\frac{\partial p}{\partial y}(u(k),y(k))\right\| < M_{p_y}, \tag{34}$$

and the derivative of $c_{\hat{\theta}}(\cdot)$ satisfies

$$\begin{aligned}&\left\|\frac{\partial c_{\hat{\theta}}}{\partial u}(e(k),u(k-1))\right\| < M_{c_u},\\ &\left\|\frac{\partial c_{\hat{\theta}}}{\partial e}(e(k),u(k-1))\right\| < M_{c_e}.\end{aligned} \tag{35}$$

Then, there exists a positive number M, such that

$$J_{\mathrm{MR}}(\hat{\theta}) = \sum_{k=1}^{N} \| y_{\hat{\theta}}(k) - m(\tilde{r}(k-1),\tilde{y}(k-1))\|^2 < M\epsilon^2. \tag{36}$$

Proof. From (33), we can derive

$$\| c_{\hat{\theta}}(\tilde{e}(k),\tilde{u}(k-1)) - \tilde{u}(k)\| < \epsilon. \tag{37}$$

By the inverse function of $m(\cdot)$ and the measured trajectory $\tilde{u}(k)$ and $\tilde{y}(k)$, the virtual reference signal is

$$\tilde{r}(k) = m^{-1}(\tilde{y}(k+1),\tilde{y}(k)). \tag{38}$$

For each k, the error of output is

$$\begin{aligned}\Delta y(k) &= y_{\hat{\theta}}(k) - m(\tilde{r}(k-1),\tilde{y}(k-1))\\ &= p(c_{\hat{\theta}}(\hat{e}(k-1),\hat{u}(k-2)),\hat{y}(k-1)) - \tilde{y}(k)\\ &= p(c_{\hat{\theta}}(\tilde{r}(k-1)-y_{\hat{\theta}}(k-1),\hat{u}(k-2)),\hat{y}(k-1))\\ &\quad - p(\tilde{u}(k-1),\tilde{y}(k-1)).\end{aligned} \tag{39}$$

Let $\hat{u}(k-1) = c_{\hat{\theta}}(\tilde{r}(k-1)-y_{\hat{\theta}}(k-1),\hat{u}(k-2))$. By the mean value theorem, there is a real number γ, $0<\gamma<1$, such that

$$\begin{aligned}\Delta y(k) &= \frac{\partial p}{\partial u}(\hat{u}_\gamma(k-1),\hat{y}_\gamma(k-1))(\hat{u}(k-1)-\tilde{u}(k-1))\\ &\quad + \frac{\partial p}{\partial y}(\hat{u}_\gamma(k-1),\hat{y}_\gamma(k-1))(\hat{y}(k-1)-\tilde{y}(k-1)),\end{aligned} \tag{40}$$

where $\hat{u}_\gamma(k) = \gamma\hat{u}(k)+(1-\gamma)\tilde{u}(k)$ and $\hat{y}_\gamma(k) = \gamma\hat{y}(k)+(1-\gamma)\tilde{y}(k)$.

Let $\Delta u(k-1) = \hat{u}(k-1)-\tilde{u}(k-1)$ and take the norm on both sides of (40). Then, we can obtain

$$\begin{aligned}\|\Delta y(k)\| &\le \left\|\frac{\partial p}{\partial u}(\hat{u}_\gamma(k-1),\hat{y}_\gamma(k-1))\right\| \|\Delta u(k-1)\|\\ &\quad + \left\|\frac{\partial p}{\partial y}(\hat{u}_\gamma(k-1),\hat{y}_\gamma(k-1))\right\| \|\Delta y(k-1)\|\\ &\le M_{p_u}\|\Delta u(k-1)\| + M_{p_y}\|\Delta y(k-1)\|.\end{aligned} \tag{41}$$

For each k, the error of control input is

$$\begin{aligned}\Delta u(k) &= \hat{u}(k) - \tilde{u}(k)\\ &= c_{\hat{\theta}}(\hat{e}(k),\hat{u}(k-1)) - \tilde{u}(k)\\ &= c_{\hat{\theta}}(\hat{e}(k),\hat{u}(k-1)) - c_{\hat{\theta}}(\tilde{e}(k),\tilde{u}(k-1))\\ &\quad + c_{\hat{\theta}}(\tilde{e}(k),\tilde{u}(k-1)) - \tilde{u}(k).\end{aligned} \tag{42}$$

By the mean value theorem, there is a real number α, $0<\alpha<1$, such that

$$\begin{aligned}\Delta u(k) &= \frac{\partial c_{\hat{\theta}}}{\partial e}(\hat{e}_\alpha(k),\hat{u}_\alpha(k-1))(-\Delta y(k))\\ &\quad + \frac{\partial c_{\hat{\theta}}}{\partial u}(\hat{e}_\alpha(k),\hat{u}_\alpha(k-1))\Delta u(k-1)\\ &\quad + c_{\hat{\theta}}(\tilde{e}(k),\tilde{u}(k-1)) - \tilde{u}(k),\end{aligned} \tag{43}$$

where $\hat{e}_\alpha(k) = \alpha\hat{e}(k)+(1-\alpha)\tilde{e}(k)$, $\hat{u}_\alpha(k-1) = \alpha\hat{u}(k-1)+(1-\alpha)\tilde{u}(k-1)$ and $\Delta y(k) = \hat{y}(k)-\tilde{y}(k) = \tilde{e}(k)-\hat{e}(k)$. Taking norm on both sides of (43), we obtain

$$\begin{aligned}\|\Delta u(k)\| &\le \left\|\frac{\partial c_{\hat{\theta}}}{\partial e}(\hat{e}_\alpha(k),\hat{u}_\alpha(k-1))\right\| \|\Delta y(k)\|\\ &\quad + \left\|\frac{\partial c_{\hat{\theta}}}{\partial u}(\hat{e}_\alpha(k),\hat{u}_\alpha(k-1))\right\| \|\Delta u(k-1)\|\\ &\quad + \| c_{\hat{\theta}}(\tilde{e}(k),\tilde{u}(k-1)) - \tilde{u}(k)\|\\ &\le M_{c_e}\|\Delta y(k)\| + M_{c_u}\|\Delta u(k-1)\| + \epsilon.\end{aligned} \tag{44}$$

According to (41) and (44), we have

$$\begin{aligned}\|\Delta y(k+1)\| &\le M_{p_u}\|\Delta u(k)\| + M_{p_y}\|\Delta y(k)\|\\ &\le M_{p_u}(M_{c_e}\|\Delta y(k)\| + M_{c_u}\|\Delta u(k-1)\| + \epsilon)\\ &\quad + M_{p_y}\|\Delta y(k)\|\\ &\le (M_{p_u}M_{c_e} + M_{p_y})\|\Delta y(k)\|\\ &\quad + M_{p_u}M_{c_u}\|\Delta u(k-1)\| + M_{p_u}\epsilon.\end{aligned} \tag{45}$$

Substituting (44) and (45), we can obtain

$$\begin{aligned}&\|\Delta y(k+1)\| + \|\Delta u(k)\|\\ &\quad \le (M_{p_u}M_{c_e} + M_{p_y} + M_{c_e})\|\Delta y(k)\|\\ &\qquad + (M_{p_u}M_{c_u} + M_{c_u})\|\Delta u(k-1)\|\\ &\qquad + (M_{p_u}+1)\epsilon.\end{aligned} \tag{46}$$

Adding ϵ to both sides of inequality (46), we have

$$\begin{aligned}&\|\Delta y(k+1)\| + \|\Delta u(k)\| + \epsilon\\ &\quad \le (M_{p_u}M_{c_e} + M_{p_y} + M_{c_e})\|\Delta y(k)\|\\ &\qquad + (M_{p_u}M_{c_u} + M_{c_u})\|\Delta u(k-1)\| + (M_{p_u}+2)\epsilon\\ &\quad \le M_0(\|\Delta y(k)\| + \|\Delta u(k-1)\| + \epsilon),\end{aligned} \tag{47}$$

where

$$M_0 \triangleq \max\{M_{p_u}M_{c_e} + M_{p_y} + M_{c_e}, M_{p_u}M_{c_u} + M_{c_u}, M_{p_u}+2\}. \tag{48}$$

Therefore, we have

$$\|\Delta y(k+1)\| + \|\Delta u(k)\| + \epsilon \le M_0^k(\|\Delta y(1)\| + \|\Delta u(0)\| + \epsilon). \tag{49}$$

As the initial values are the same, i.e., $\hat{y}(0) = \tilde{y}(0)$ and $\hat{u}(-1) = \tilde{u}(-1)$, we have $\Delta y(0) = 0$ and $\Delta u(-1) = 0$. Considering (44) and (45), we obtain

$$\|\Delta u(0)\| \le \epsilon, \tag{50}$$

and

$$\|\Delta y(1)\| \le M_{p_u}\epsilon. \tag{51}$$

Hence, we can find

$$\|\Delta y(1)\| + \|\Delta u(0)\| + \epsilon \le M_1\epsilon, \tag{52}$$

where $M_1 = M_{p_u}+2$.

Combining (52) with (49), we can acquire

$$\begin{aligned}\|\Delta y(k+1)\| + \|\Delta u(k)\| &\le M_0^k(\|\Delta y(1)\| + \|\Delta u(0)\| + \epsilon) - \epsilon\\ &\le (M_0^k M_1 - 1)\epsilon.\end{aligned} \tag{53}$$

Then, $\|\Delta y(k)\| \le (M_0^{k-1}M_1 - 1)\epsilon$. Hence, we can derive

$$\begin{aligned}J_{\mathrm{MR}}(\hat{\theta}) &= \sum_{k=1}^{N} \| y_{\hat{\theta}}(k) - m(\tilde{r}(k-1),y(k-1))\|^2\\ &= \sum_{k=1}^{N} \|\Delta y(k)\|^2\\ &\le \sum_{k=1}^{N} (M_0^{k-1}M_1 - 1)^2\epsilon^2.\end{aligned} \tag{54}$$

Let $M = \sum_{k=1}^{N}(M_0^{k-1}M_1 - 1)^2$. Thus we can conclude

$$\begin{aligned}J_{\mathrm{MR}}(\hat{\theta}) &= \sum_{k=1}^{N} \| y_{\hat{\theta}}(k) - m(\tilde{r}(k-1),y(k-1))\|^2\\ &< M\epsilon^2,\end{aligned} \tag{55}$$

which completes the proof of the theorem.□

Remark 3. Theorem 3 shows that when J_{VRFT} is bounded, J_{MR} is also bounded. M is a constant which is determined by the plant

$p(\cdot)$ and the candidate controller set $\{c_\theta(\cdot); \theta \in \mathbb{R}^{n_\theta}\}$. In practice, as the plant is unknown, J_{MR} is unavailable. However, we can obtain J_{VRFT} by input and output data. Theorem 3 ensures that when J_{VRFT} converges to zero, J_{MR} converges to zero. Moreover, $M\epsilon^2$ is just an upper bound which may be far greater than J_{MR}. However, in practice, J_{MR} will be close to J_{VRFT} in most situations.

Theorem 4. *Assume that the plant, the controller and the reference model are defined by (2), (7) and (10), respectively. For arbitrary initial condition IC(0), there exist $\hat{\theta} \in \mathbb{R}^{n_\theta}$ and a positive number $\epsilon > 0$, such that*

$$\sum_{k=0}^{N-1} \| c_{\hat{\theta}}(\tilde{e}(k), I\tilde{C}_c(k)) - \tilde{u}(k) \|^2 < \epsilon^2. \tag{56}$$

$p(\cdot)$ and $c_{\hat{\theta}}(\cdot)$ are continuously differentiable with respect to all variables in the domain of definition. Assume that the derivative of $p(\cdot)$ satisfies

$$\left\| \frac{\partial p}{\partial u(k-1)} \right\| < M_1^{p_u}, \ldots, \left\| \frac{\partial p}{\partial u(k-n_{p_u})} \right\| < M_{n_{p_u}}^{p_u},$$
$$\left\| \frac{\partial p}{\partial y(k-1)} \right\| < M_1^{p_y}, \ldots, \left\| \frac{\partial p}{\partial y(k-n_{p_y})} \right\| < M_{n_{p_y}}^{p_y}, \tag{57}$$

and the derivative of $c_{\hat{\theta}}(\cdot)$ satisfies

$$\left\| \frac{\partial c_{\hat{\theta}}}{\partial u(k-1)} \right\| < M_1^{c_u}, \ldots, \left\| \frac{\partial c_{\hat{\theta}}}{\partial u(k-n_{c_u})} \right\| < M_{n_{c_u}}^{c_u},$$
$$\left\| \frac{\partial c_{\hat{\theta}}}{\partial e(k)} \right\| < M_0^{c_e}, \ldots, \left\| \frac{\partial c_{\hat{\theta}}}{\partial e(k-n_{c_e})} \right\| < M_{n_{c_e}}^{c_e}. \tag{58}$$

Then there exists a positive number $\overline{M}$, such that

$$J_{\mathrm{MR}}(\hat{\theta}) = \sum_{k=1}^{N} \| y_{\hat{\theta}}(k) - \tilde{y}(k) \|^2 < \overline{M}\epsilon^2. \tag{59}$$

Proof. From (56), we can obtain

$$\| c_{\hat{\theta}}(\tilde{e}(k), I\tilde{C}_c(k)) - \tilde{u}(k) \| < \epsilon. \tag{60}$$

By the invertibility of $m(\cdot)$ defined by (10) and the measured trajectory $\tilde{u}(k)$ and $\tilde{y}(k)$, the virtual reference signal is

$$\tilde{r}(k-1) = m^{-1}(\tilde{y}(k), I\tilde{C}_m(k-1)). \tag{61}$$

For each k, the error of output is

$$\begin{aligned}\Delta y(k) &= p(c_{\hat{\theta}}(\hat{e}(k-1), \widehat{IC}_c(k-1)), \widehat{IC}_p(k-1)) - \tilde{y}(k) \\ &= p(c_{\hat{\theta}}(\hat{e}(k-1), \widehat{IC}_c(k-1)), \widehat{IC}_p(k-1)) \\ &\quad - p(\tilde{u}(k-1), I\tilde{C}_p(k-1)).\end{aligned} \tag{62}$$

Let $\hat{u}(k) = c_{\hat{\theta}}(\tilde{r}(k) - y_{\hat{\theta}}(k), \widehat{IC}_c(k-1))$. By the mean value theorem, there is a real number γ, $0 < \gamma < 1$, which makes the following equality holds:

$$\begin{aligned}\Delta y(k) &= \frac{\partial p}{\partial u(k-1)}(\hat{u}_\gamma(k-1), \widehat{IC}_c^\gamma(k-1))\Delta u(k-1) \\ &+ \frac{\partial p}{\partial u(k-n_{c_u})}(\hat{u}_\gamma(k-1), \widehat{IC}_c^\gamma(k-1))\Delta u(k-n_{c_u}) \\ &+ \frac{\partial p}{\partial y(k-1)}(\hat{u}_\gamma(k-1), \widehat{IC}_c^\gamma(k-1))\Delta y(k-1) \\ &+ \frac{\partial p}{\partial y(k-n_{p_y})}(\hat{u}_\gamma(k-1), \widehat{IC}_c^\gamma(k-1))\Delta y(k-n_{p_y}),\end{aligned} \tag{63}$$

where $\hat{u}_\gamma(k) = \gamma\hat{u}(k) + (1-\gamma)\tilde{u}(k)$, $\hat{y}_\gamma(k) = \gamma\hat{y}(k) + (1-\gamma)\tilde{y}(k)$ and $\Delta u(k-1) = \hat{u}(k-1) - \tilde{u}(k-1)$. Taking the norm on both sides of (63), we obtain

$$\begin{aligned}\| \Delta y(k) \| &\le M_1^{p_u} \| \Delta u(k-1) \| + \cdots + M_{n_{p_u}}^{p_u} \| \Delta u(k-n_{p_u}) \| \\ &+ M_1^{p_y} \| \Delta y(k-1) \| + \cdots + M_{n_{p_y}}^{p_y} \| \Delta y(k-n_{p_y}) \| \\ &\le M^{p_u}(\| \Delta u(k-1) \| + \cdots + \| \Delta u(k-n_{p_u}) \|) \\ &+ M^{p_y}(\| \Delta y(k-1) \| + \cdots + \| \Delta y(k-n_{p_y}) \|),\end{aligned} \tag{64}$$

where $M^{p_u} = \max\{M_1^{p_u}, \ldots, M_{n_{p_u}}^{p_u}\}$ and $M^{p_y} = \max\{M_1^{p_y}, \ldots, M_{n_{p_y}}^{p_y}\}$. Let $\delta_y(k) = \max\{\| \Delta y(k) \|, \ldots, \| \Delta y(0) \|\}$ and $\delta_u(k) = \max\{\| \Delta u(k) \|, \ldots, \| \Delta u(0) \|\}$. We can derive

$$\delta_y(k) \le n_{p_u} M^{p_u} \delta_u(k) + n_{p_y} M^{p_y} \delta_y(k). \tag{65}$$

For each k, the error of control input is

$$\begin{aligned}\Delta u(k) &= \hat{u}(k) - \tilde{u}(k) \\ &= c_{\hat{\theta}}(\hat{e}(k), \widehat{IC}_c(k-1)) - \tilde{u}(k) \\ &= (c_{\hat{\theta}}(\hat{e}(k), \widehat{IC}_c(k-1)) - c_{\hat{\theta}}(\tilde{e}(k), I\tilde{C}_c(k-1))) \\ &+ (c_{\hat{\theta}}(\tilde{e}(k), I\tilde{C}_c(k-1)) - \tilde{u}(k)).\end{aligned} \tag{66}$$

By the mean value theorem, there is a real number α, $0 < \alpha < 1$, which makes the following equality holds:

$$\begin{aligned}\Delta u(k) &= \frac{\partial c_{\hat{\theta}}}{\partial e(k)}(\hat{e}_\alpha(k), \widehat{IC_c^\alpha}(k-1))(-\Delta y(k)) + \cdots \\ &+ \frac{\partial c_{\hat{\theta}}}{\partial e(k-n_{c_e})}(\hat{e}_\alpha(k), \widehat{IC_c^\alpha}(k-1))(-\Delta y(k-n_{c_e})) \\ &+ \frac{\partial c_{\hat{\theta}}}{\partial u(k-1)}(\hat{e}_\alpha(k), \widehat{IC_c^\alpha}(k-1))\Delta u(k-1) + \cdots \\ &+ \frac{\partial c_{\hat{\theta}}}{\partial u(k-n_{c_u})}(\hat{e}_\alpha(k), \widehat{IC_c^\alpha}(k-1))\Delta u(k-n_{c_u}) \\ &+ (c_{\hat{\theta}}(\tilde{e}(k), I\tilde{C}_c(k-1)) - \tilde{u}(k)).\end{aligned} \tag{67}$$

where $\hat{e}_\alpha(k) = \alpha\hat{e}(k) + (1-\alpha)\tilde{e}(k)$, $\hat{u}_\alpha(k-1) = \alpha\hat{u}(k-1) + (1-\alpha)\tilde{u}(k-1)$ and $\Delta y(k) = \hat{y}(k) - \tilde{y}(k) = \tilde{e}(k) - \hat{e}(k)$. Taking norm on both sides of (67), we obtain

$$\begin{aligned}\| \Delta u(k) \| &\le M_0^{c_e} \| \Delta y(k) \| + \cdots + M_{n_{c_e}}^{c_e} \| \Delta y(k-n_{c_e}) \| \\ &+ M_1^{c_u} \| \Delta u(k-1) \| + M_{n_{c_u}}^{c_u} \| \Delta u(k-n_{c_u}) \| + \epsilon \\ &\le M^{c_e}(\| \Delta y(k) \| + \cdots + \| \Delta y(k-n_{c_e}) \|) \\ &+ M^{c_u}(\| \Delta u(k-1) \| + \cdots + \| \Delta u(k-n_{c_e}) \|) + \epsilon,\end{aligned} \tag{68}$$

where $M^{c_u} = \max\{M_1^{c_u}, \ldots, M_{n_{c_u}}^{c_u}\}$ and $M^{c_e} = \max\{M_0^{c_e}, \ldots, M_{n_{c_e}}^{c_e}\}$. Then, we can derive

$$\delta_u(k) \le n_{c_u} M^{c_u} \delta_u(k-1) + n_{c_e} M^{c_e} \delta_y(k) + \epsilon. \tag{69}$$

By (65) and (69), we have

$$\begin{aligned}\delta_y(k+1) &\le n_{p_u} M^{p_u} \delta_u(k) + n_{p_y} M^{p_y} \delta_y(k) \\ &\le n_{p_u} M^{p_u}(n_{c_e} M^{c_e} \delta_y(k) + n_{c_u} M^{c_u} \delta_u(k-1) + \epsilon) \\ &+ n_{p_y} M^{p_y} \delta_y(k) \\ &\le (n_{p_u} n_{c_e} M^{p_u} M^{c_e} + n_{p_y} M^{p_y})\delta_y(k) \\ &+ n_{p_u} M^{p_u} n_{c_u} M^{c_u} \delta_u(k-1) + n_{p_u} M^{p_u} \epsilon.\end{aligned} \tag{70}$$

By (69) and (70), we obtain

$$\begin{aligned}&\delta_y(k+1) + \delta_u(k) + \epsilon \\ &\le (n_{p_u} n_{c_e} M^{p_u} M^{c_e} + n_{p_u} M^{p_y} + n_{c_u} M^{c_u})\delta_y(k) \\ &\quad + (n_{p_u} M^{p_u} M^{c_u} + n_{c_u} M^{c_u})\delta_u(k-1) \\ &\quad + (n_{p_u} M^{p_u} + 2)\epsilon \\ &\le \overline{M}_0(\delta_y(k) + \delta_u(k-1) + \epsilon),\end{aligned} \tag{71}$$

where $\overline{M}_0 = \max\{n_{p_u} n_{c_e} M^{p_u} M^{c_e} + n_{p_u} M^{p_y} + n_{c_u} M^{c_u}, n_{p_u} M^{p_u} M^{c_u} + n_{c_u} M^{c_u}, n_{p_u} M^{p_u} + 2\}$. From (71), we have

$$\delta_y(k+1) + \delta_u(k) + \epsilon \le \overline{M}_0^k(\delta_y(1) + \delta_u(0) + \epsilon). \tag{72}$$

As the initial values are the same, i.e., $\delta_y(0) = 0$ and $\delta_u(-1) = 0$, considering (69) and (70), we obtain

$$\delta_u(0) \le \epsilon, \tag{73}$$

and

$$\delta_y(1) \le n_{p_u} M^{p_u} \epsilon. \tag{74}$$

Thus, we can find

$$\delta_y(1)+\delta_u(0)+\epsilon \le \overline{M}_1\epsilon, \tag{75}$$

where $\overline{M}_1 = n_{p_u}M^{p_u}+2$. Combining with (72), we can acquire

$$\delta_y(k+1)+\delta_u(k) \le \overline{M}_0^k(\delta_y(1)+\delta_u(0)+\epsilon)-\epsilon$$
$$\le (\overline{M}_0^k\overline{M}_1-1)\epsilon. \tag{76}$$

Then, $\delta_y(k) \le (\overline{M}_0^{k-1}\overline{M}_1-1)\epsilon$. Hence, we can derive

$$J_{\text{MR}}(\hat{\theta}) = \sum_{k=1}^{N} \| y_{\hat{\theta}}(k) - m(\tilde{r}(k-1), I\tilde{C}_m(k-1)) \|^2$$
$$= \sum_{k=1}^{N} \| \Delta y(k) \|^2$$
$$\le \sum_{k=1}^{N} \delta_y(k)^2$$
$$\le \sum_{k=1}^{N} (\overline{M}_0^{k-1}\overline{M}_1-1)^2\epsilon^2. \tag{77}$$

Let $\overline{M} = \sum_{k=1}^{N}(\overline{M}_0^{k-1}\overline{M}_1-1)^2$. Therefore, we can conclude

$$J_{\text{MR}}(\hat{\theta}) = \sum_{k=1}^{N} \| y_{\hat{\theta}}(k) - m(\tilde{r}(k-1), I\tilde{C}_m(k-1)) \|^2$$
$$< \overline{M}\epsilon^2, \tag{78}$$

which completes the proof.□

Similar to Theorem 3, Theorem 4 shows the relationship between the bounds of the optimization problems (13) and (22) for general nonlinear systems. When $c^* \notin \{c_\theta(\cdot); \theta \in \mathbb{R}^{n_\theta}\}$, we can still obtain $\hat{\theta}$ by solving the problem (22) to make the objective value of problem (13) as small as possible. Furthermore, when the problem (22) converges to zero, problem (13) also converges to zero, which demonstrates the validity of Theorem 2.

Remark 4. From problem (13), we can see that the problem of VRFT for MIMO nonlinear systems is very different from linear systems [33,36] and much more complex than SISO nonlinear systems [38]. Hence, Theorems 3 and 4 provide a totally new idea which is different from the previous work on VRFT. Our work demonstrates theoretically the validity of VRFT in nonlinear MIMO case for the first time.

4. Neural network implementation of VRFT

In this section, a three-layer neural network is used to approximate the controller $c(\cdot)$. The number of hidden layer neurons is denoted by l, the weight matrix between the input layer and the hidden layer is denoted by $V \in \mathbb{R}^{l\times\overline{n}}$ and the weight matrix between the hidden layer and the output layer is denoted by $W \in \mathbb{R}^{m\times l}$. Then the output of three-layer neural network is represented as

$$\hat{u}(k) = \hat{c}(x_k; V, W) = W\sigma(Vx_k) \tag{79}$$

where $x_k = [e^{\mathsf{T}}(k), \ldots, e^{\mathsf{T}}(k-n_{c_e}), u^{\mathsf{T}}(k-1), \ldots, u^{\mathsf{T}}(k-n_{c_u})]^{\mathsf{T}}$ is the input of the neural network, $\overline{n} = n(n_{c_e}+1)+mn_{c_u}$ is the dimension of x_k, $\sigma(Vx_k) \in \mathbb{R}^l$ and $[\sigma(z)]_i = (e^{z_i}-e^{-z_i})/(e^{z_i}+e^{-z_i})$, $i = 1, 2, \ldots, l$, are the activation functions. Let $X = [x_1, x_2, \ldots, x_N]$ and $U = [u(1), u(2), \ldots, u(N)]$. For convenience of computing, only the hidden-output weight W is undetermined, while the input-hidden weight V is initialized randomly and fixed. Then, we can use the least square method to train neural network.

In the following, the neural network expression is simplified by $\hat{c}(x_k; W) = W\sigma(Vx_k) = W\sigma_V(x_k)$. The objective function can be rewritten as

$$\hat{J}_{\text{VRFT}}(W) = \sum_{k=1}^{N} \| \hat{c}(\tilde{x}_k; W) - \tilde{u}(k) \|^2. \tag{80}$$

Our goal is to select W to make the performance function minimized, i.e.,

$$W^* = \arg\min_W \left\{ \sum_{k=1}^{N} \| \hat{c}(\tilde{x}_k; W) - \tilde{u}(k) \|^2 \right\}. \tag{81}$$

Let $\tilde{X} = [\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_N]$ and $\tilde{U} = [\tilde{u}(1), \tilde{u}(2), \ldots, \tilde{u}(N)]$. Y is defined as follows:

$$Y = \sigma_V(\tilde{X}) = \sigma(V\tilde{X}). \tag{82}$$

So W^* can be calculated by

$$W^* = \tilde{U}Y^{\mathsf{T}}(YY^{\mathsf{T}})^{-1}. \tag{83}$$

The above equation is true only when the data matrix Y is full row rank. Fortunately, this condition is satisfied in most cases as the number of data N is sufficiently large. Even if Y is singular, the generalized inverse can be introduced to calculate

$$W^* = \tilde{U}Y^{\mathsf{T}}(YY^{\mathsf{T}})^{+} \tag{84}$$

where $(YY^{\mathsf{T}})^{+}$ stands for the generalized inverse of YY^{T}.

5. Simulation

In this section, we will verify the effectiveness of the developed method for MIMO nonlinear systems with different reference signals under noiseless and noisy environment, respectively.

5.1. Noiseless environment

Consider the given discrete-time MIMO nonlinear system:

$$y(k+1) = 10\tanh(0.1Ay(k)) + \tanh(Bu(k)), \tag{85}$$

where $y(k) \in \mathbb{R}^2$, $u(k) \in \mathbb{R}^2$, and

$$A = \begin{bmatrix} 0.88 & 0.123 \\ 0.123 & 0.88 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}. \tag{86}$$

This system extends the SISO nonlinear system introduced by [38] into MIMO case. The reference model is a linear transfer function as follows:

$$y(k) = \begin{bmatrix} \frac{0.2}{s-0.8} & 0 \\ 0 & \frac{0.4}{s-0.6} \end{bmatrix} r(k), \tag{87}$$

which can be represented by

$$y(k+1) = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.6 \end{bmatrix} y(k) + \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix} r(k). \tag{88}$$

The controller neural network is chosen as a three-layer neural network with the structure of 6–20–2. The input $x = [e(k)^{\mathsf{T}}, e(k-1)^{\mathsf{T}}, u(k-1)^{\mathsf{T}}]^{\mathsf{T}}$ and the output $u = u(k)$. The weight matrix from input layer to hidden layer is assigned randomly in $[-1, 1]$ and the weight matrix from hidden layer to output layer is undetermined. We choose $N = 10\,000$ groups of input–output data $\{u(k), y(k)\}$, where $u(k)$ is assigned from -1 to 1. Then we can determine the weight matrix from hidden layer to output layer by VRFT algorithm to obtain the controller. Finally, we can test the performance of the designed controller by different reference input signals.

When the reference signal is chosen as a unit step response and the initial output of the system is $[0, 0]^{\mathsf{T}}$, the performance of the designed controller is illustrated in Fig. 2. When the reference signal is chosen as damping sine and cosine curve and the initial output of the system is $[0,0]^{\mathrm{T}}$, the performance of the designed controller is illustrated in Fig. 3.

From Figs. 2 and 3, we can see that the designed controller has a good performance with the reference signal of step response and damping sine–cosine signal. Actually, it also works well with the reference signal as ramp signal, sine–cosine signal and other familiar signals. This result verifies the effectiveness of VRFT and that the controller designed by the virtual reference signal is also suitable for other desired reference signals.

5.2. Noisy environment

Consider the following discrete-time MIMO nonlinear system:

$$y(k+1) = 10\tanh(0.1Ay(k)) + \tanh(Bu(k)) + \nu \tag{89}$$

where $y(k) \in \mathbb{R}^2$, $u(k) \in \mathbb{R}^2$, A and B are defined by (86) and ν is the white noise with expectation 0 and variance 0.01. The reference model is also defined by (88).

The structure of the neural network is the same as the one in the noiseless environment. We choose $N = 10\,000$ groups of input–output data $\{u(k), y(k)\}$ generated by system (89). Then we determine the weight matrix from hidden layer to output layer by VRFT algorithm to obtain the controller and test the performance of the designed controller by different reference input signals.

When the reference signal is chosen as a unit step signal and the initial output of the system is $[0,0]^{\mathrm{T}}$, the performance of the designed controller is illustrated in Fig. 4. When the reference signal is chosen as damping sine and cosine curve and the initial output of the system is $[0,0]^{\mathrm{T}}$, the performance of the designed controller is illustrated in Fig. 5.

From Figs. 4 and 5, we can see that when the system contains white noise, the developed method can also design a controller which has the similar performance with the situation of noiseless. This result shows that our method can deal with noise and obtain the optimal controller in noisy environment.

6. Conclusion

In this paper, we developed a data-driven controller design method for MIMO nonlinear systems by VRFT. We presented the optimization problems of model reference control and VRFT in MIMO nonlinear systems and proved the equivalence of them under ideal conditions. For the first time, we provided the relationship between the bounds of optimization problems of model reference control and VRFT. We introduced neural network as a parameterized controller trained by the least square method in the implementation of VRFT and the simulation results demonstrate the validity of the developed method. As shown in Theorems 3 and 4, the derivative of the plant has an influence on the bound of optimization problem of model reference control. In the future, we will try to introduce a linear or nonlinear filter to reduce the influence of the derivative and improve the control performance.

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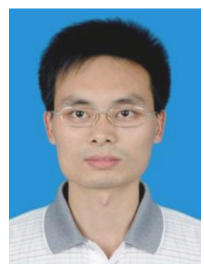

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