Selective Harmonic Elimination With Groebner Bases and Symmetric Polynomials

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***Abstract*—Selective harmonic elimination (SHE) technology has been widely used in many medium- and high-power converters which operates at very low switching frequency; however, it is still a challenging work to solve the switching angles from a group of nonlinear transcendental equations, especially for the multilevel converters. Based on the Groebner bases and symmetric polynomial theory, an algebraic method is proposed for SHE. The SHE equations are transformed to an equivalent canonical system which consists of a univariate high-order equations and a group of univariate linear equations, thus the solving procedure is simplified dramatically. In order to solve the final solutions from the definition of the elementary symmetric polynomials, a univariate polynomial equation is constructed according to the intermediate solutions and two criteria are given to check whether the results are true or not. Unlike the commonly used numerical and random searching methods, this method has no requirement on choosing initial values and can find all the solutions. Compared with the existing algebraic methods, such as the resultant elimination method, the calculation efficiency is improved, and the maximum solvable switching angles is nine. Experiments on three-phase two-level and 13-level inverters verify the correctness of the switching angles solved by the proposed method.**

***Index Terms*—Converter, Groebner bases, inverter, multilevel, selective harmonic elimination (SHE), symmetric polynomial.**

I. Introduction

In the medium- and high-power applications, such as the renewable generation [1], motor drive [2], harmonic compensation [3]–[5], static synchronous compensator [6], etc., the switching frequency is usually very low (300–800 Hz) due to the limitations of the switching losses and electromagnetic interferences caused by high dv/dt. With such a low switching frequency, the carrier PWM will produce lots of low-order harmonics and poor THD performance; hence, the selective harmonic eliminated pulse width modulation (SHEPWM) [7], [8], which can eliminates the low-order harmonics very precisely and improve the output equality, has been widely used in these applications. However, the biggest challenge to utilize the SHEPWM technology is to solve the switching angles from a group of nonlinear transcendental equations. The most commonly used methods to do this are the numerical methods and the random searching methods. The typical numerical methods include the Newton–Raphson algorithm [8]–[10] and the homotopy algorithm [11], [12], whereas the genetic algorithm [13], [14] and the evolutionary algorithm [15], [16] are the typical random searching methods. Although these two types of methods have shown their effectiveness in various applications, there are also some shortcomings of them.

First, the initial values must be well selected; otherwise, the iteration will probably be diverged. For the two-level and three-level converters, the initial values usually are chosen by two different approaches: Enjeti *et al.* [9] represented the first one in which some linear functions are derived to calculate the initial values for a given modulation index; these linear functions are obtained from solving the selective harmonic eliminated (SHE) equations with less switching angles and then generalize them to more switching angles. The other one is based on the assumption that "the solutions to the SHE equations vary continuously with the modulation indices," the solutions for current modulation index are directly taken as the initial values for new equations with slightly increased modulation index [17]. In [18], the initial values for the SHE equations with incremented modulation index is predicted along the tangent directions of the current point on the solution trajectory. For the multilevel converters, a method for getting initial values based on the rule of equality of area and superposition of the center of gravity of the PWM section with the sine reference signal is proposed in [19] and [20]. Although these methods for choosing initial values can work well for many cases, they may fail to handle some multilevel cases whose solutions are discontinuous with modulation indices, such as the staircase multilevel converters.

Second, due to the local convergent nature of the numerical methods and the random searching methods, usually only one solution can be found for one proper selected initial values, but actually, there exist multiple solutions for most modulation indices. In the last decade, some methods with the capability to find multiple solutions for SHE equations have been proposed. In [21] and [22], a method based on the minimization

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technique and random search has been proposed to solve the SHE equations for two-level converters. For a specific modulation index, the minimization problem to find the first solution is done by using the genetic algorithm or Nelder–Mead simplex algorithm; then, the combined random search and biased pattern for the initial values are used to find all the possible solutions, which are then used as initial values to find the solutions for new modulation index with slightly increment. Later, this minimization method is used to solve the SHE equations for three-level converters [23], and then for the multilevel converters [24]–[26] with both equal and nonequal dc sources. This minimization method does not seek the solutions that strictly eliminate the harmonics, but rather tries to find solutions that minimize the objective function. This approach could give solutions even beyond the point that other methods do not converge, or even when the solutions do not exist; moreover, it has the ability to find multiple solutions. However, due to the numerical nature of this minimization method, a formal proof of its completeness cannot be easily provided and no guarantee can be given that it would be feasible when the switching angles keep increasing. Although this method can find all the possible solutions for most modulation indices, it still has missed some solutions for some cases. For example, in the case of a two-level converter with three switching angles to eliminate 5th and 7th harmonics, this minimization method asserted that the solutions exist only when the modulation index is negative [22], but actually, there definitely exist solutions for modulation indices in $[0.92, 0.93]$. In [27], two formulas are used to compute the initial values and then the switching angles versus modulation index in full range are computed by the Newton–Raphson iteration algorithm, by varying the iteration steps, multiple solutions can be given. In their conclusions, there are two solutions for modulation indices in $[0, 0.55]$, three solutions for $[0.55, 0.59]$, and four solutions for modulation indices above 0.59, but in our investigation, there definitely exist the fifth solution for modulation indices in $[0.58, 0.7]$. If the quarter-wave symmetric of the output waveform is relaxed to half-wave symmetric, there probably exist infinite number of solutions for SHE problem [28]–[30]; however, this is beyond the subject of this paper.

Third, as the prior knowledge about the existence of the solution for the SHE equations is unknown, when these algorithms fail to provide a final result, it is not clear whether it is caused by the selection of initial values, or the parameters are unsuitable or there are indeed no solutions for the SHE equations.

In order to overcome these problems, recently, several computer algebra-based methods have been proposed. Due to their completeness in mathematics, these methods do not need to choose initial values and can find all the possible solutions. In [31] and [32], the resultant elimination theory is introduced to convert the SHE equations to an equivalent triangular form which can be solved by the way like the Gaussian elimination in solving the linear equations. The only problem of this method is the huge computation burden, it is only effective when the switching angles or the dc sources are less than five [34]. Then, the symmetric polynomial theory [33] and the power sum [34] are introduced to reduce the degrees of the polynomials and the reported maximum solvable switching angles is limited to five. In [35], the Wu method is introduced to convert the SHE equations to a characteristic triangular sets whose zeros are the same as the original polynomials; however, only a simple case with three switching angles has been studied.

Except for the above three categories of methods which are all based on solving the SHE equations, in recent years, some totally different approaches of SHEPWM technology have been proposed. In [36], a modulation-based approach for harmonic elimination is proposed. In [37] and [38], the authors propose a four-equation-based harmonic elimination method which is motivated by the ideas of equal area criteria and harmonic injection in active power filter. As there are lots of works on the topic of SHE, it is difficult to give a comprehensive review about them in a short introduction, one can refer to [39] to get a detailed introduction about the formulations, solving algorithms, implementation, and applications of the SHE technologies.

This paper proposes another algebraic method to solve the SHE equations, which combines the Groebner bases and the symmetric polynomials. The main superiority of this method over the other methods is the avoidance of solving the multivariate high-order equations and all the equations need to be solved are two univariate high-order equations and a group of univariate linear equations; thus, the solving procedure is dramatically simplified, and the maximum solvable switching angles is increased to nine. Also, this method does not need to choose initial values and can find all the solutions; furthermore, it can be used for all the two-level, three-level, and multilevel inverters (not only the case of single switching per dc level but also the case of multiple switching per dc level).

II. Mathematical Model of SHE

The basic principle of the SHEPWM technology is the Fourier series of the output PWM waveform, in order to simplify the expression of the Fourier coefficients, the output waveform is usually assumed quarter-period symmetric. The equations used to compute the amplitude of harmonics for two-level and multilevel inverters are listed as follows [8], [20]:

$$b_n = \frac{4V_{\mathrm{dc}}}{n\pi}\left[1 + 2\sum_{i=1}^{N}(-1)^i \cos(n\alpha_i)\right] \tag{1}$$

$$b_n = \frac{4V_{\mathrm{dc}}}{n\pi}\sum_{i=1}^{N} \pm \cos(n\alpha_i) \tag{2}$$

where b_n is the amplitude of the nth harmonics, V_{dc} is the voltage of the dc source, N is the number of switching angles, and α_i are the switching angles in a quarter period. The "$\pm$" sign in front of $\cos(n\alpha_i)$ in (2) depends on the transition state on switching angles α_i, that is, if α_i is a rising edge, the sign will be "$+$,"; otherwise, the sign will be "$-$." If there is only one dc source, (2) degrades to the case of three-level inverter which has alternating signs of "$+$" and "$-$."

Equations (1) and (2) indicate that the amplitude of the fundamental and the harmonics are directly decided by the switching angles, if the fundamental amplitude b_1 is set to a desired value U and the amplitude of some selected harmonics are set to zero, the SHE equations will be obtained. For example, for the

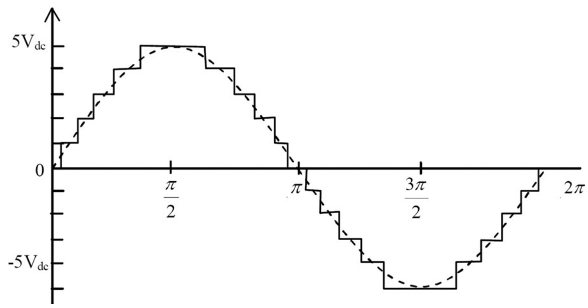

Fig. 1. Staircase waveform of multilevel inverter.

multilevel inverter with five dc sources and a staircase output waveform like Fig. 1, the SHE equations are shown as follows:

$$\begin{cases} \cos(\alpha_1)+\cos(\alpha_2)+\cos(\alpha_3)+\cos(\alpha_4)+\cos(\alpha_5)=\dfrac{\pi U}{4V_{\mathrm{dc}}} \\ \cos(5\alpha_1)+\cos(5\alpha_2)+\cos(5\alpha_3)+\cos(5\alpha_4)+\cos(5\alpha_5)=0 \\ \cos(7\alpha_1)+\cos(7\alpha_2)+\cos(7\alpha_3)+\cos(7\alpha_4)+\cos(7\alpha_5)=0 \\ \cos(11\alpha_1)+\cos(11\alpha_2)+\cos(11\alpha_3)+\cos(11\alpha_4)+\cos(11\alpha_5)=0 \\ \cos(13\alpha_1)+\cos(13\alpha_2)+\cos(13\alpha_3)+\cos(13\alpha_4)+\cos(13\alpha_5)=0 \end{cases} \tag{3}$$

with

$$0<\alpha_1<\alpha_2<\alpha_3<\alpha_4<\alpha_5<\frac{\pi}{2}$$

where U is the desired amplitude of fundamental which has a relationship with modulation index m as follows:

$$m=\frac{\pi U}{4N\cdot V_{\mathrm{dc}}}. \tag{4}$$

By using the multiple-angle formulas and substitute $\cos(\alpha_i)$ with x_i, (3) can be converted to the following polynomial equations:

$$\begin{cases} p_1(\mathbf{x}) \triangleq x_1+x_2+x_3+x_4+x_5-m=0 \\ p_2(\mathbf{x}) \triangleq \sum_{i=1}^5 16x_i^5-20x_i^3+5x_i=0 \\ p_3(\mathbf{x}) \triangleq \sum_{i=1}^5 64x_i^7-112x_i^5+56x_i^3-7x_i=0 \\ p_4(\mathbf{x}) \triangleq \sum_{i=1}^5 1024x_i^{11}-2816x_i^9+2816x_i^7 \\ \qquad -1232x_i^5+220x_i^3-11x_i=0 \\ p_5(\mathbf{x}) \triangleq \sum_{i=1}^5 4096x_i^{13}-13312x_i^{11}+16640x_i^9 \\ \qquad -9984x_i^7+2912x_i^5-364x_i^3+13x_i=0 \end{cases} \tag{5}$$

with

$$0<x_5<x_4<x_3<x_2<x_1<1.$$

III. Symmetric Polynomials and Degree Reduction of SHE Equations

A symmetric polynomial is a multivariate polynomial, such that if any of the variables are interchanged, one obtains the same polynomial. Obviously, all the polynomials in the SHE equations as (5) are symmetric polynomials. According to the fundamental theorem of symmetric polynomials [40], any symmetric polynomial has a unique representation in terms of the elementary symmetric polynomials. By using this transformation, the degrees of the SHE equations which are highly related to the computation burden of the algebraic methods can be reduced dramatically, hence, the maximum solvable switching angles can be promoted much higher.

A. Elementary Symmetric Polynomials

Denote $f(x)=x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n$ as a univariate polynomial defined on the real number field, and $x_1,x_2,\ldots,x_n$ are n roots for equation $f(x)=0$. Then, $f(x)$ can be rewritten as follows:

$$f(x)=(x-x_1)(x-x_2)\ldots(x-x_n). \tag{6}$$

If (6) is expanded and compared with the original form of $f(x)$, it can be seen that there exist the following relationship between the coefficients a_i and the roots x_i, $i=1,2,\ldots,n$:

$$\begin{cases} a_n=(-1)^n\prod_{i=1}^n x_i \\ a_{n-1}=(-1)^{n-1}\sum_{i=1}^n\prod_{j\neq i}x_j \\ \quad\vdots \\ a_2=(-1)^2\sum_{1<i<j<n}x_ix_j \\ a_1=(-1)^1\sum_{i=1}^n x_i. \end{cases} \tag{7}$$

If the signs of a_i are omitted, (7) turns into the definition of elementary symmetric polynomials. For example, the elementary symmetric polynomials for $n=5$ are listed as follows:

$$\begin{cases} e_1=x_1+x_2+x_3+x_4+x_5 \\ e_2=x_1x_2+x_1x_3+x_1x_4+x_1x_5+x_2x_3 \\ \qquad +x_2x_4+x_2x_5+x_3x_4+x_3x_5+x_4x_5 \\ e_3=x_1x_2x_3+x_1x_2x_4+x_1x_2x_5+x_1x_3x_4 \\ \qquad +x_1x_3x_5+x_1x_4x_5+x_2x_3x_4+x_2x_3x_5 \\ \qquad +x_2x_4x_5+x_3x_4x_5 \\ e_4=x_1x_2x_3x_4+x_1x_2x_3x_5+x_1x_2x_4x_5 \\ \qquad +x_1x_3x_4x_5+x_2x_3x_4x_5 \\ e_5=x_1x_2x_3x_4x_5. \end{cases} \tag{8}$$

B. Degree Reduction of SHE Equations

As mentioned above, (5) can be represented in terms of elementary symmetric polynomials $e_i, i=1,2,\ldots,5$, and this conversion can be done by calling the $SymmetricReduction$ command in the symbolic computing software *Mathematica*.

TABLE I
COMPARISON OF THE DEGREES OF $p_5(\mathbf{x})$ AND $p_5(\mathbf{e})$

	x_1/e_1	x_2/e_2	x_3/e_3	x_4/e_4	x_5/e_5
$p_5(\mathbf{x})$	13	13	13	13	13
$p_5(\mathbf{e})$	0	6	4	3	2

The conversion results are listed as follows:

$$\begin{cases} p_1(\mathbf{e}) \triangleq e_1 - m = 0 \\ p_2(\mathbf{e}) \triangleq 5e_1 - 20e_1^3 + 16e_1^5 + 60e_1e_2 - 80e_1^3e_2 \\ \qquad + 80e_1e_2^2 - 60e_3 + 80e_1^2e_3 - 80e_2e_3 \\ \qquad - 80e_1e_4 + 80e_5 = 0 \\ p_3(\mathbf{e}) \triangleq -7e_1 + 56e_1^3 + \cdots - 448e_2e_5 = 0 \\ p_4(\mathbf{e}) \triangleq -11e_1 + 220e_1^3 + \cdots + 11264e_1e_5^2 = 0 \\ p_5(\mathbf{e}) \triangleq 13e_1 - 364e_1^3 + \cdots + 53248e_3e_5^2 = 0. \end{cases} \tag{9}$$

As the expressions of p_3, p_4, and p_5 are too long, their intermediate items are omitted here.

In (9), as the modulation index m is a constant, when $e_1 = m$ is substituted to the other four equations, one variable is eliminated and the degrees of (9) are reduced dramatically. For example, Table I is the comparison of the degrees of p_5 before and after the conversion.

As the computation complexity of the algebraic methods are highly related to the number and the degrees of the variables, Table I implies that the computation burden to solve the SHE equations would be reduced tremendously after this conversion.

For the two-level, three-level, and multilevel converters whose output waveforms are not staircase as shown in Fig. 1, the original SHE equations are not symmetric; therefore, the elementary symmetric polynomials cannot be directly used to reduce the degrees. However, if the cosine items in the SHE equations which are leaded by "$-$" are substituted with $x_i = \cos(\pi - \alpha_i)$, the SHE equations turn into symmetric, and just the constraints for (5) need to be slightly modified by adding a "$-$" in front of the corresponding x_i.

IV. Groebner Bases and Solving the SHE Equations

A. Groebner Bases

The Groebner bases theory [40] was proposed by an Austria mathematician Bruno Buchberger in his doctoral dissertation in 1965, and this provided an efficient algebraic method to solve the nonlinear algebraic system which is different from the traditional numerical iterative methods. About half a century later, the Groebner bases theory has been widely used not only in the applied mathematics field but also in many engineering fields. Like the resultant elimination method, the Groebner bases method also has no requirement on choosing initial values and can find all the possible solutions for the SHE equations; furthermore, it has some nice properties that the resultant elimination method does not have. As the Groebner bases theory involves lots of concepts, definitions, and theorems in commutative algebra, it is difficult to give a detailed introduction about it here; therefore, just some important conclusions and the tools used to compute the Groebner bases will be introduced from a viewpoint of engineering application. For more details about solving the SHE equations with Groebner bases, one can refer to [41].

Denote $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ as a group of polynomials in which p_i are polynomials in variables $x_1, x_2, \ldots, x_n$. According to Hilbert's basis theorem, there exist Groebner bases $\mathbf{g} = (g_1, g_2, \ldots, g_n)$ whose zeros are the same as that of $\mathbf{p}$. Generally speaking, the Groebner bases is not unique unless the monomial order is given, and the reduced Groebner bases of $\mathbf{p}$ under the pure lexicographical monomial order is always in a triangular form as follows:

$$\mathbf{g} = \begin{cases} g_1(x_n) \\ g_2(x_{n-1}, x_n) \\ \vdots \\ g_n(x_1, x_2 \ldots, x_n). \end{cases} \tag{10}$$

This nice property of Groebner bases is guaranteed by the *elimination theorem*. Once the SHE equations have been converted to their equivalent triangular form like (10), they can be solved in the following iterative way: First, solve the first univariate polynomial equation and substitute the solutions of x_n to the second equation to solve x_{n-1}; then, repeat this procedure until the last equation is solved. The stringency of this procedure is guaranteed by the *extension theorem*. The *elimination theorem* and the *extension theorem* are two important theorems in Groebner bases theory and their detailed explanation can be found in [40].

Now, the core problem is how to compute the Groebner bases for a given polynomials set? This can be done by using the famous Buchberger algorithm and some improved versions of it, due to the constructive nature of these algorithms, the Groebner bases of a certain group of polynomials can always be constructed and there are no conditions to compute them; however, the detailed description of these algorithms is beyond the subject of this paper. As the implementation of these algorithms requires solid knowledge about the polynomials and professional programming skills, it is unrealistic to develop the algorithm by engineers. Actually, some commercial symbolic computing software such as the *Maple* or the *Mathematica* provide some functions to compute the Groebner bases, and it is user friendly for the engineers. For example, the command to compute Groebner bases in *Maple* is

$$\text{Basis}([p_1, p_2, \ldots, p_n], plex(x_1, x_2, \ldots, x_n))$$

where $[p_1, p_2, \ldots, p_n]$ are the polynomials and "$plex$" designates the monomial order is pure lexicographical order which ensures that the output polynomials are triangular.

The approach described above is very similar to the method used in [31] –[34], where the SHE equations are triangularized by the resultant elimination method. The main difference between these two methods is that the degrees of the triangular

TABLE II
COMPARISON OF THE TRIANGULAR POLYNOMIAL EQUATIONS (NO DEGREE REDUCTION AND $N = 4$)

	x_1	x_2	x_3	x_4	operands	
g_1	1	1	1	1	5	GB
	1	1	1	1	5	RE
g_2	23	2	2	0	31	GB
	4	4	4	0	53	RE
g_3	23	3	0	0	51	GB
	22	22	0	0	451	RE
g_4	24	0	0	0	25	GB
	884	0	0	0	885	RE

TABLE III
COMPARISON OF THE TRIANGULAR POLYNOMIAL EQUATIONS (WITH DEGREE REDUCTION AND $N = 5$)

	e_2	e_3	e_4	e_5	operands	
g_1	0	0	0	9	10	GB
	0	0	0	262	263	RE
g_2	0	0	1	8	10	GB
	0	0	15	13	129	RE
g_3	0	1	0	8	10	GB
	0	3	3	2	17	RE
g_4	1	0	0	8	10	GB
	2	1	1	1	7	RE

equations given by Groebner bases method are much lower than that of the resultant elimination method. Take the SHE equations with four switching angles, for example, the degree reduction by symmetric polynomials is not applied, Table II gives a detailed comparison of the degrees in variables x_1, x_2, x_3, x_4 and the number of operands of each equation for these two methods, the last column of this table identifies which method is used, that is, "GB" represents the Groebner bases method, whereas "RE" represents the resultant elimination method. It can be seen that except for g_1, both the degrees and the number of operands produced by the Groebner bases method are much less than that produced by the resultant elimination method.

If the degree reduction is applied before computing the Groebner bases, the output triangular equations will be further simplified to one univariate high-order equation and a group of univariate linear equations. This amazing property is very useful and can dramatically simplify the subsequent solving procedure. For example, by computing the Groebner bases under the pure lexicographical monomial order of $e_2 \succ e_3 \succ e_4 \succ e_5$, (9) can be converted to the following equivalent equations:

$$\begin{cases} g_1(e_5) \triangleq a_9 e_5^9 + a_8 e_5^8 + \cdots + a_1 e_5 + a_0 = 0 \\ g_2(e_4, e_5) \triangleq b_1 e_4 + f_1(e_5) = 0 \\ g_3(e_3, e_5) \triangleq b_2 e_3 + f_2(e_5) = 0 \\ g_4(e_2, e_5) \triangleq b_3 e_2 + f_3(e_5) = 0 \end{cases} \tag{11}$$

where $a_0, a_1, \ldots, a_8, a_9$ and b_1, b_2, b_3 are all big integers, and f_1, f_2, and f_3 are all eight-order univariate polynomials in e_5. It can be seen from (11) that the first equation is a univariate polynomial in e_5, as the topic of how to solve a univariate high order polynomial equation has been well developed in the algebra, it is easy to find all the solutions of e_5. Once the solutions of e_5 are obtained, the other three equations in (11) are all turned into univariate linear equations, and e_2, e_3, e_4 can be solved very easily. This property is the main advantage of Groebner bases method, whereas the resultant elimination method does not have. Table III is the comparison of (11) with the triangular form of (9) computed by the resultant elimination method. The elimination order is the same as the monomial order of the Groebner bases method. It can be seen that the degrees and the number of operands produced by the Groebner bases method are much lower than that produced by the resultant elimination method, this property will dramatically simplify the subsequent solving procedure. As there is only one high-order equation in (11), the maximum number of intermediate solutions produced by the Groebner bases method is nine, whereas this number produced by the resultant elimination method is $262 \times 15 \times 3 \times 2 = 23\,580$. This means that the Groebner bases method will save lots of time to remove the error solutions. Furthermore, the triangularization procedure of the Groebner bases method is much faster than the resultant elimination method, in this example, the execution time is 0.063 versus 2.247 s on a 2.2-GHz quad-core i7-2720QM CPU with 8 GB RAM and the symbolic computing software is *Maple18*.

Once e_1, e_2, e_3, e_4, e_5 are known, the last step is to solve x_1, x_2, x_3, x_4, x_5 from (8). In [33], the resultant elimination method is used again to triangularize (8), although this method can work well, it is much more complicated than the method used here. According to the description of elementary symmetric polynomials in Section III-A, the roots of a univariate polynomial equation have a relationship with the coefficients as (7). Since the values of e_1, e_2, e_3, e_4, e_5 are known, the following univariate polynomial can be constructed:

$$f(x) = x^5 - e_1 x^4 + e_2 x^3 - e_3 x^2 + e_4 x - e_5. \tag{12}$$

Then, substitute each e_1, e_2, e_3, e_4, e_5 into (12) and solve the resulted univariate equations $f(x) = 0$. As the solutions for (5) must satisfy $0 < x_5 < x_4 < x_3 < x_2 < x_1 < 1$, if $f(x) = 0$ has complex roots or multiple roots or real roots outside $(0, 1)$, the corresponding five roots are not the correct solutions for (5). The following two criteria can be used to check whether the five roots are correct or not:

1) all the roots of $f(x) = 0$ are real roots and they are not equal to each other;
2) all the roots are located in the range $(0, 1)$.

If the solutions satisfy the above two criteria simultaneously, they are the true solutions for the SHE equations. Then, sort the solutions in descending order and compute their anticosine values, the switching angles are obtained.

For the cases with more switching angles, a similar polynomial as (12) can be constructed by the same way. Also, this method can be easily extended to the two-level, three-level, and the general multilevel cases, just the above second criterion and the order of the solutions need to be slightly modified.

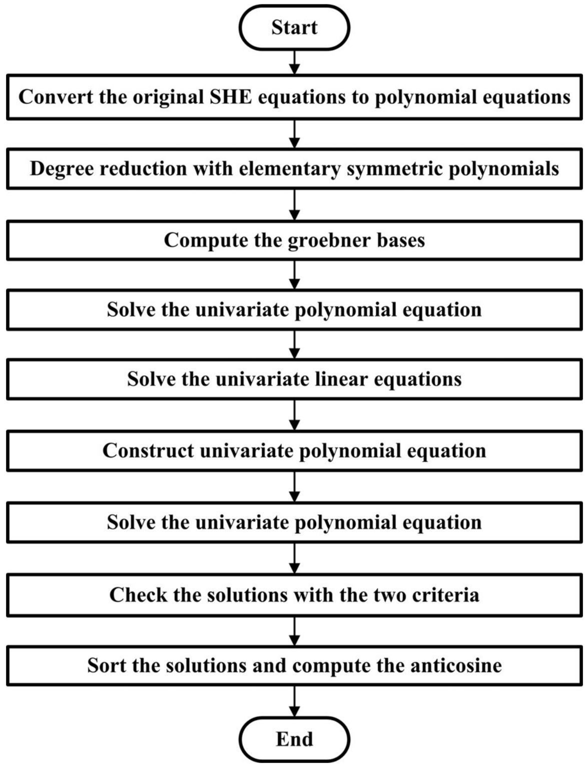

Fig. 2. Flowchart of the proposed algorithm.

B. Implementation of the Proposed Algorithm

The proposed algorithm is implemented under the symbolic computing software *Maple*, and the maximum solvable switching angles is nine on the mainstream personal computers or workstations. It can be used to solve the switching angles for the two-level, three-level, and multilevel converters. The main program flow of the proposed algorithm is shown in Fig. 2.

V. Computation Results

The computing hardware is a workstation computer with XEON E3-1230 CPU and 16 GB RAM and the symbolic computing software is *Maple18*. The range of m is set to $[0, 1]$ for three-level and multilevel converters, for the two-level converters, as there exist solutions for the negative modulation indices, the range of m is set to $[-1, 1]$. The increment steps are set to $\Delta m = 0.01$ for two-level and three-level converters and $\Delta m = \frac{0.01}{N}$ for multilevel converters. The computation results for eight switching angles are shown in Figs. 3–5, respectively, in which the blue represents α_8, the black represents α_7, the yellow represents α_6, the mauve represents α_5, the cyan represents α_4, the red represents α_3, the green represents α_2, and the brown represents α_1.

Fig. 3 shows the four groups of solutions for three-phase two-level converter with eight switching angles. It can be seen that there exist solutions for the SHE equations when $m \in [-0.81, 0.91]$, and almost all the modulation indices have four solutions except $m = -0.81$ which has only two solutions.

Fig. 4 shows the switching angles versus modulation indices in full range for the three-phase three-level converter with eight switching angles, and Table IV is the classification of the solution number versus modulation index. It can be seen that there are at least six solutions for $m \leq 0.5$, and for $m > 0.5$, as there are some isolated solution trajectories, the solution number is much more complicated than $m \leq 0.5$.

Fig. 5 shows the switching angles versus modulation index in full range for the three-phase multilevel staircase converter with eight switching angles. It can be seen that the trajectories are much more disorganized than the two-level and

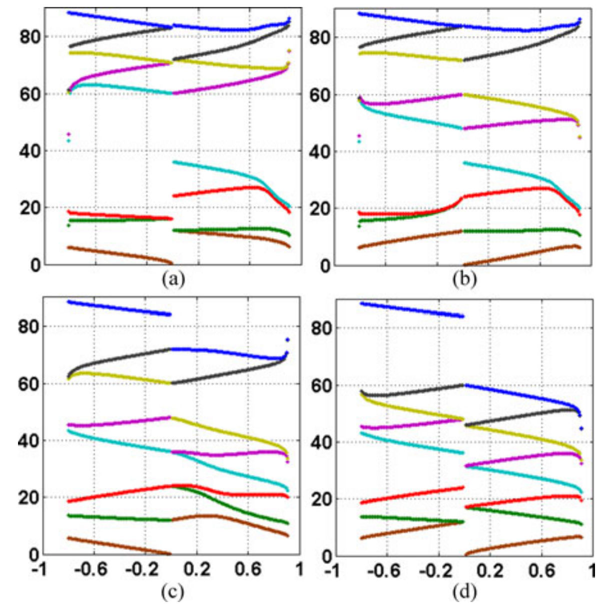

Fig. 3. Four groups of solutions for three-phase two-level converter with eight switching angles, and each of them are shown separately as (a), (b), (c), and (d).

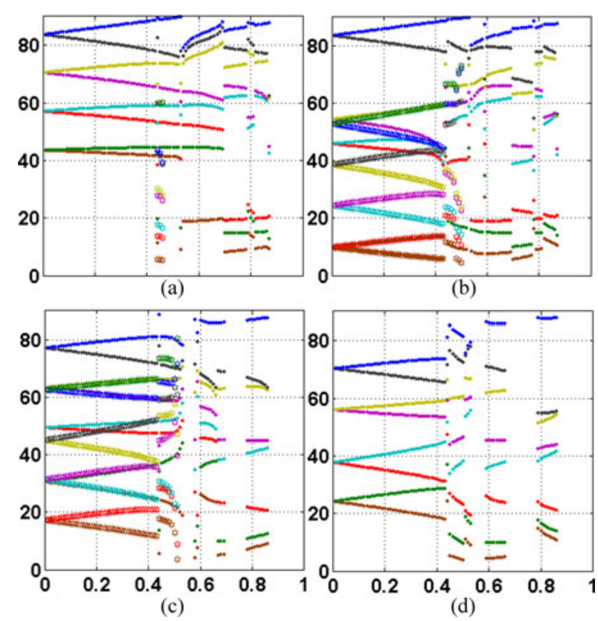

Fig. 4. Switching angles versus modulation indices for three-phase three-level converter with eight switching angles. There are two groups of solutions in (a), (b), and (c), and one group of solutions in (d). In (a), (b), and (c), the second group of solutions is represented by "☆."

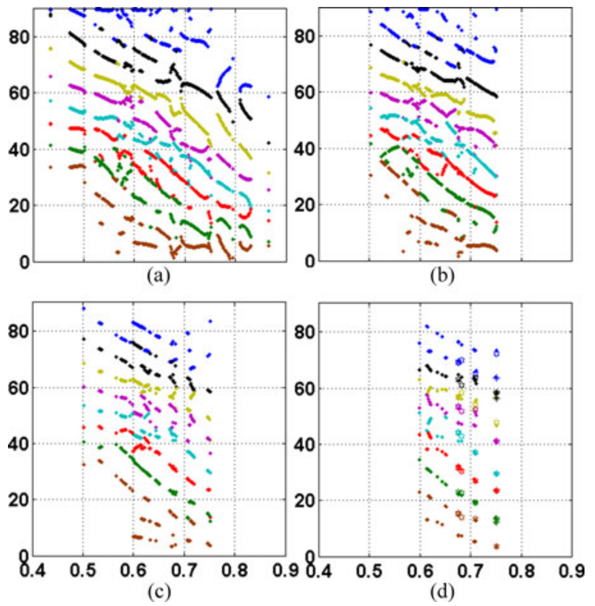

Fig. 5. Switching angles versus modulation indices for three-phase multilevel converter with eight switching angles. There are one group of solutions in (a), (b), and (c), and three groups of solutions in (d). In (d), the second group of solutions are represented by "☆" and the third group of solutions are represented by "∗."

TABLE IV
THREE-PHASE THREE-LEVEL CONVERTER $N = 8$

No.	Range of m
2	[0.54, 0.57], [0.7, 0.77], 0.87
3	0.58, [0.67, 0.69], 0.78
4	[0.52, 0.53], [0.59, 0.66], [0.79, 0.86]
5	0.51
6	[0.01, 0.43], [0.47, 0.5]
7	[0.44, 0.46]
0	[0.88, 1]

three-level cases, and there are lots of intervals where the SHE equations have no solutions. The modulation index range where the SHE equations have solutions is $m \in [\frac{347}{800}, \frac{348}{800}] \cup [\frac{378}{800}, \frac{402}{800}] \cup [\frac{418}{800}, \frac{601}{800}] \cup [\frac{612}{800}, \frac{631}{800}] \cup \frac{636}{800} \cup [\frac{647}{800}, \frac{664}{800}] \cup \frac{692}{800}$.

For most numerical methods [9], [10], usually, some empirical formulas are used to compute the initial values for $m = 0$, and then, the current solutions are directly taken as the initial values for the new SHE equations with slightly increased m. This approach is based on the following two assumptions: 1) There exist solutions for $m = 0$; 2) the solution trajectories vary continuously with the modulation index. For the two-level and three-level converters, as shown in Figs. 3 and 4, these two assumptions are both true, so the numerical methods can work well in these cases although they would probably miss some true solutions. But for the multilevel converter, as shown in Fig. 5, there are no solutions for $m = 0$, so, it is almost impossible to establish the empirical formulas; furthermore, the solution trajectories are discontinuous on many modulation indices. The above two assumptions are no longer tenable, thus, this approach would probably fail for multilevel converters. But with the proposed method, as its completeness in mathematics and no requirement on choosing initial values, all the solutions for any given modulation index can be found.

The computation results indicate that there are no solutions in some ranges of modulation index, especially for the multilevel cases. In order to extend the range of modulation index, some strategies has been proposed. By eliminating some constraints in the SHE equations, a modified SHE technique has been proposed in [42], which employs reduced number of transcendental equations to find the N desired switching angles, the range of modulation index can be extended with the same switching frequency, however, the number of eliminated harmonics is reduced. The Groebner bases and symmetric polynomials-based method proposed in this paper can also handle this modified SHE strategy very well. For example, for the case of three-level converters with four switching angles, the traditional SHE strategy has no solutions for modulation indices in $(0.87, 1)$, if the equation used to eliminate 11th harmonic is removed, this method can definitely tell us that there exist an infinite number of solutions for $m = 0.9$, so an optimal solution can be further identified to improve the THD performance. Furthermore, if the modulation index is assigned values started from 0.87 to 1, this method can further figure out how much of the modulation index can be extended by this modified SHE strategy.

For the multilevel converter, if the output is not limited to the staircase waveform, then more switching patterns can be used to control the converter and this increases the possibility to find valid solutions for specific modulation indices, especially for low modulation indices [43]. As mentioned in Section IV-A, the Groebner bases and symmetric polynomials-based method proposed in this paper can be easily extended to the general multilevel cases; hence, the optimal SHE strategy can be realized in the full range of modulation indices, and the number of eliminated harmonics does not need to be reduced.

VI. EXPERIMENTS

Experiments on both three-phase two-level inverter and 13-level cascade H-bridge inverter have been carried out to verify the correctness of the switching angles computed by the proposed method. The power stage for the two-level inverter is a 600-V/18-A intelligent power modular STGIPS20K60 and the power stage for the 13-level inverter uses FDD8424H (40 V/20 A) as the switching device and ADuM3220 as the gate driver and isolator. An ARM Cortex-m3-based microcontroller STM32F103R is used to generate the PWM gating signal and control the power stage.

A. Three-Phase Two-Level Inverter

The modulation index is set to $m = 0.7$ and the number of switching angles is $N = 9$. There exist four groups of switching angles which are listed in Table V, and the last row are the theoretical THDs calculated up to 49th harmonics. Figs. 6–8 are the phase voltages, line voltages, and their FFT results for the first solutions, respectively, and Figs. 9–11 are for the second solutions. It can be seen that the aimed 5th, 7th, 11th, 13th, 17th, 19th, 23rd, and 25th harmonics are eliminated very well and the

TABLE V
FOUR GROUPS OF SWITCHING ANGLES FOR TWO-LEVEL INVERTER ($m = 0.7$)

	I	II	III	IV
α_1	$7.1736°$	$4.4553°$	$7.0120°$	$4.3595°$
α_2	$10.195°$	$10.360°$	$9.919°$	$10.194°$
α_3	$18.653°$	$18.736°$	$16.012°$	$16.076°$
α_4	$20.156°$	$20.217°$	$20.840°$	$20.863°$
α_5	$64.668°$	$52.510°$	$40.243°$	$40.237°$
α_6	$67.487°$	$55.330°$	$43.429°$	$43.424°$
α_7	$76.547°$	$76.553°$	$64.677°$	$52.496°$
α_8	$79.730°$	$79.736°$	$67.500°$	$55.321°$
α_9	$88.053°$	$88.055°$	$88.065°$	$88.068°$
THD	67.86%	65.68%	65.34%	62.11%

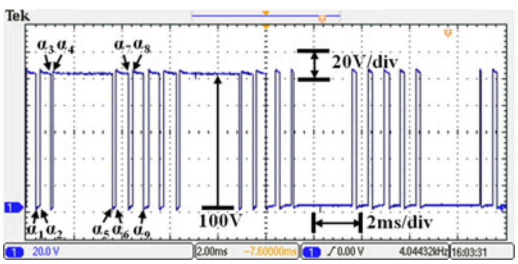

Fig. 6. Phase voltage generated by switching angles I in Table V.

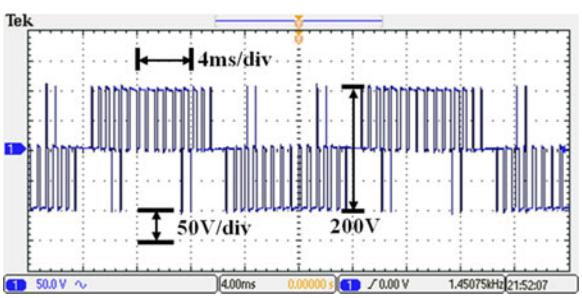

Fig. 7. Line voltage generated by switching angles I in Table V.

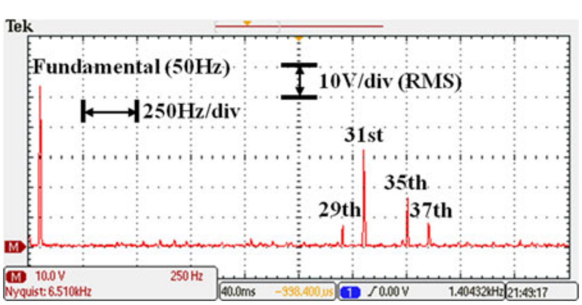

Fig. 8. FFT results of the line voltage shown in Fig. 7.

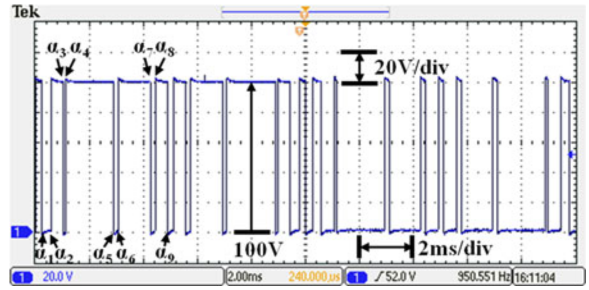

Fig. 9. Phase voltage generated by switching angles II in Table V.

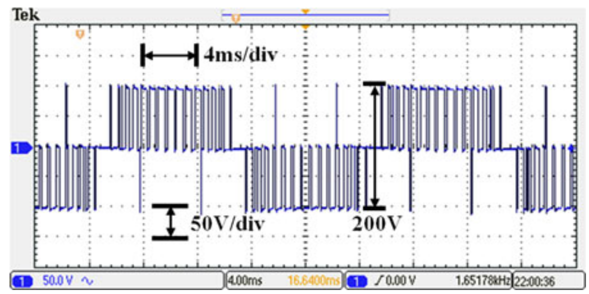

Fig. 10. Line voltage generated by switching angles II in Table V.

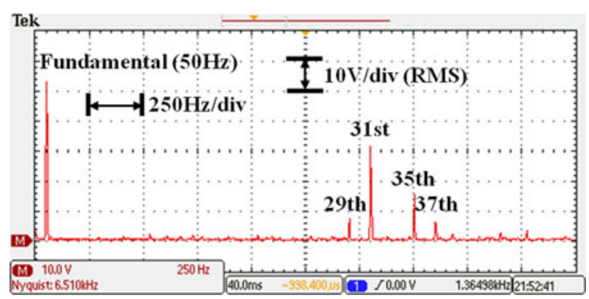

Fig. 11. FFT results of the line voltage shown in Fig. 10.

TABLE VI
FOUR GROUPS OF SWITCHING ANGLES FOR 13-LEVEL INVERTER ($m = 0.7$)

	α_1	α_2	α_3	α_4	α_5	α_6	THD
I	$6.714°$	$14.62°$	$24.00°$	$37.33°$	$58.15°$	$89.84°$	4.28%
II	$6.648°$	$14.73°$	$35.65°$	$37.71°$	$58.15°$	$83.79°$	4.04%
III	$6.614°$	$23.71°$	$37.12°$	$45.30°$	$58.14°$	$74.79°$	4.37%
IV	$14.80°$	$23.69°$	$37.16°$	$53.53°$	$58.02°$	$66.64°$	4.46%

lowest order of the surplus harmonics in the line voltage is 29th. The experiment THDs calculated up to 49th harmonic for these four groups of switching angles are 69.72%, 67.50%, 67.11%, and 64.11%, respectively, which show a good consistency with the theoretical THDs that both the solution IV has the lowest THD.

B. Three-Phase 13-Level Inverter

The modulation index is set to $m = 0.7$ and there exist four groups of solutions which are listed in Table VI, and the last column are the theoretical THDs which are calculated up to 49th harmonics. Figs. 12 and 13 are the phase voltage and its FFT results for the second solutions, respectively. Figs. 14 and 15 are the phase voltage and its FFT results for the third solutions, respectively. It can be seen that the aimed 5th, 7th, 11th, 13th, and 17th harmonics are eliminated very well from the phase voltage and the lowest order of the surplus nontriplen

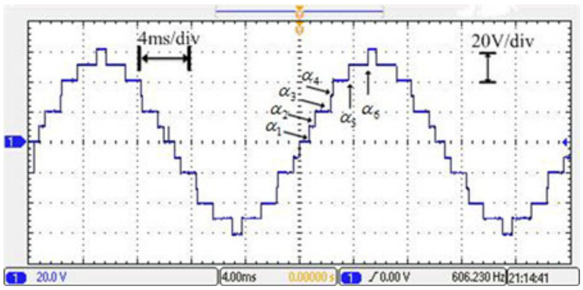

Fig. 12. Phase voltage generated by switching angles II in Table VI.

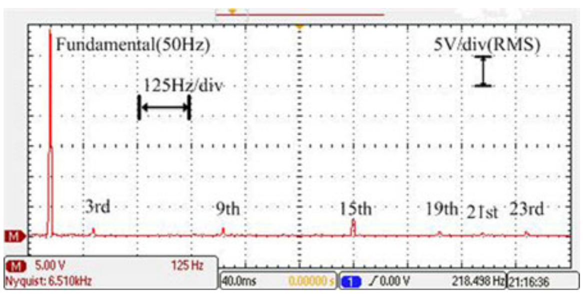

Fig. 13. FFT results of the phase voltage shown in Fig. 12.

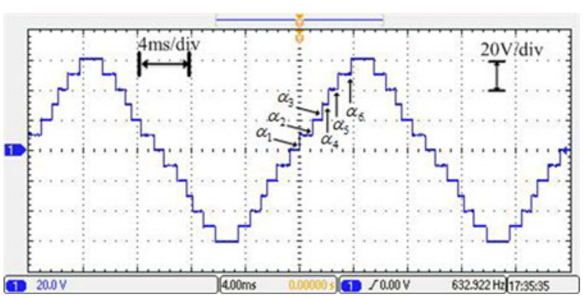

Fig. 14. Phase voltage generated by switching angles III in Table VI.

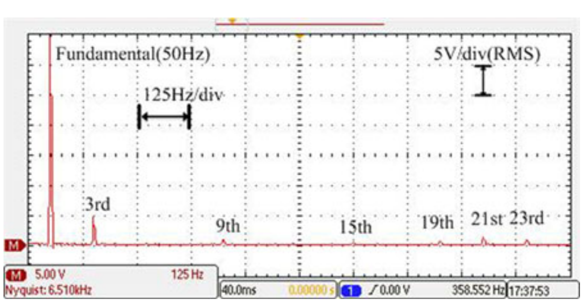

Fig. 15. FFT results of the phase voltage shown in Fig. 14.

harmonics is 19th. The experiment THDs calculated up to 49th harmonic for these four groups of switching angles are 5.41%, 5.26%, 5.51%, and 5.60%, respectively, which show a good consistency with the theoretical THDs that both the solution II has the lowest THD.

VII. Conclusion

In this paper, an algebraic method which is based on the Groebner bases and symmetric polynomial theory is proposed to solve the SHE problem. With this method, the multivariate high-order SHE equations are converted to two univariate high-order equations and a set of univariate linear equations, thus the solving procedure is simplified dramatically. This method can be used for two-level, three-level, and multilevel converters (not only the cases of single switching per dc level but also the cases of multiple switching per dc level). Compared with the numerical and random searching methods, the main advantages of this method are that it has no requirement on choosing initial values and can find all the solutions; compared with the existing algebraic methods, the efficiency is improved and the maximum solvable switching angles is increased to nine. The experiments on three-phase two-level and 13-level inverters show that all the solutions solved by this method can eliminate the aimed harmonics very well.

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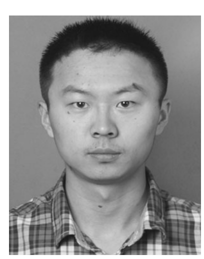

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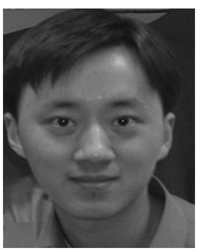

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Dr. Wang received multiple teaching and research awards including the IEEE Power Electronics Society Richard M. Bass Young Engineer Award and the National Science Foundation's CAREER Award, both in 2011; Ralph L. Boyer Award for Excellence in Undergraduate Teaching Innovation from the College of Engineering at The Ohio State University in 2012, and the Lumley Research Award of the College of Engineering at The Ohio State University in 2013. Dr. Wang has over 100 peer-reviewed journal and conference publications and three patents. Dr. Wang had been an Associate Editor for IEEE TRANSACTIONS ON INDUSTRY APPLICATIONS from 2008 to 2014. He initiated and served as the General Chair for the 1st IEEE Workshop on Wide Bandgap Power Devices and Applications in 2013. Currently, Dr. Wang serves as an Associate Editor for the IEEE TRANSACTIONS ON POWER ELECTRONICS and the IEEE JOURNAL OF EMERGING AND SELECTED TOPICS IN POWER ELECTRONICS.