

# Robust Control of Missiles with Reaction Jets Using Adaptive Sliding Mode and Control Allocation

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**Abstract** - A robust control scheme for high angle of attack tracking of one class of missiles with reaction jets is presented. The control scheme contains an adaptive SMC controller and a control moment allocator. In the former, an adaptive compensator based on radial basis function neural network is used to eliminate chattering by compensating the missile disturbances and uncertainties. The latter is designed to coordinate the fins and the reaction jets. The main advantage of our proposed robust control scheme is that no knowledge of the boundaries of the system uncertainties and external disturbances is required in advance; meanwhile, the chattering phenomenon that frequently appears in conventional sliding mode control systems is also eliminated without deteriorating the system fast-response characteristic and robustness. Besides, the adaptive compensator contains only one parameter updated online. With Lyapunov stability theory, the closed-loop system is proved stable and convergence properties of the system are assured. Numerical simulation results illustrate the effectiveness of the proposed robust control scheme.

**Index Terms** - *reaction jets, adaptive control, sliding mode control, RBF network, control allocation.*

## I. INTRODUCTION

During the past several decades, lots of research projects have been developed on control schemes for missiles with reaction jets (MRJ). The reason is that reaction jets could generate a huge moment in a short time, thus providing excellent maneuverability in the upper air [1]. However, the MRJ features inherently strong nonlinearity and uncertainty. Furthermore, the introduction of reaction jets brings intensive disturbance to the missile aerodynamics and draws forth control coordination problem of fins and reaction jets. Thus the design of the control scheme involves two aspects: one is to find methods to deal with the system nonlinearity and uncertainty; the other is to develop strategies to coordinate fins and reaction jets. These and other concerns have prompted researchers to look for more robust and intelligent control methods, and also coordinating strategies for the fins and reaction jets.

Considering the nonlinearity and uncertainty of the MRJ systems, some researchers designed control schemes based on dynamic inversion [2-3]. An advantage of this method is its effectiveness on solving the system nonlinearity. But it seriously depends on the model accuracy and can hardly satisfy the desired robustness. To compensate this disadvantage, some researchers combined several other methods [4-5] such as extended-mean assignment and neural

networks with dynamic inversion. However, attitude control system based on dynamic inversion consists of two loops, and each should be designed and compensated, which makes the control structure more complex. Sliding mode control (SMC) has received much attention as an efficient control technique to handle systems with large uncertainties [6-8]. The most prominent property of SMC is its insensitivity to parameter variations and external disturbances. Basic SMC needs to know information of the boundaries of disturbances in advance, and researches in [8-9] all regarded it as known in SMC design. However, the MRJ is with unknown uncertainties in practice. Chattering problem should be paid attention to in practical SMC applications, because chattering not only damages fins but also brings out continuous trigger of reaction jets. Conventional methods for eliminating the chattering include replacement of the switching term by a saturating approximation [9], integral sliding control [10], or/and boundary layer technique [11]. But these methods sacrifice the system robustness to eliminate the chattering. Researchers in [12] adopted fuzzy system to estimate the switching term. These methods can help eliminate chattering while keep robustness, but fuzzy rules are complex to design and depend on the designers' experiences. If system uncertainties are large, the sliding-mode controller would require a high switching gain, which will cause severe chattering. Then a simple idea for eliminating the chattering is to decrease the switching gain by compensating the system uncertainties. In view of the above analysis, a compensator based on radial basis function (RBF) network is designed to estimate the system uncertainties online adaptively and combine with the SMC in this paper. The idea of adaptive compensator also allows the SMC controller work well in situations without knowledge of boundaries of disturbances.

Two main strategies, command allocation [6] and control quantity allocation [13-14], have been widely used for coordination for fins and reaction jets. The angle of attack command is divided into two parts in the former strategy: one is assigned to aerodynamic system and the other is to reaction jets system. The advantage of this strategy is easy to design, but the coupling of these two systems is not considered and the entire system stability is hard to be satisfied. The latter considers the aerodynamic system design and the reaction jets system design together, and assigns the desired control quantity to these two actuators. It can assure the entire closed-loop stability. Consequently, in this paper, the control quantity allocation strategy is adopted to coordinate fins and reaction

jets. Unlike researches in [13-14], the control moment is used as the allocation quantity. Besides, the reaction jets output is treated as discrete variable which is in accordance with reality. Instead, the output of the reaction jets is regarded as continuous variable in [13], which is easy to design but cannot accurately describe the output in reality.

Motivated by the above discussions, a novel robust control scheme using adaptive sliding mode and control moment allocation is proposed for the angle of attack tracking of the MRJ in presence of unknown uncertainties and disturbances in this paper. During the adaptive SMC design, an observer is used for estimating the complex item in SMC; an adaptive compensator based on RBF neural network is designed to compensate the system disturbances and uncertainties. The compensator contains only one parameter to be updated online, making it with fast convergence. To coordinate the fins and the reaction jets, a control moment allocator is designed. The stability proof is also given to guarantee the ultimate boundary of all signals in the closed-loop system.

The rest of the paper is organized as follows. Section II briefly describes the MRJ model introduced from [14]. Section III presents our proposed adaptive sliding mode control scheme in detail. Simulation results are demonstrated in Section IV. Also, section V provides the concluding remarks.

## II. MISSILE MODEL DESCRIPTION

In general, the dynamic model of the MRJ in pitch channel can be expressed as the following form [14]

$$\begin{cases} \dot{\alpha} = q + ((Z + F_Z) \cos \alpha - X \sin \alpha + mg \cos \mu) / mV + \Delta_\alpha \\ \dot{q} = (M_\alpha(\alpha) + M_q(q, V) + M_{\delta_e}(\alpha, \delta_e) + M_f(U)) / J_y + \Delta_q \end{cases} \quad (1)$$

where  $\alpha$  is the angle of attack,  $\mu$  is the angle of path, and  $p$  is the pitch angular rate;  $m$  is the aircraft mass,  $V$  is the aircraft velocity;  $X$  and  $Z$  are the aerodynamic forces along body axes,  $F_Z$  is the force produced by the reaction jets;  $J_y$  is the moment of inertia along the  $y$  body axis,  $M_\alpha(\alpha)$ ,  $M_q(q, V)$ ,  $M_{\delta_e}(\alpha, \delta_e)$  and  $M_f(U)$  are the static aerodynamic moment, damping moment, fin control moment and reaction jets produced moment respectively;  $\delta_e$  is the elevator deflection and  $U$  is the reaction jets fire command vector;  $\Delta_\alpha$  and  $\Delta_q$  denote total uncertainties and disturbances of the model.

The above model indicates huge nonlinearity and uncertainty of the MRJ. A controller for such missiles must be designed to handle those characteristics with strong robustness. Besides, the control moment  $M_c$  is jointly produced by the fin and the reaction jets which are totally different in control characteristics. So, the controller should be competent for control allocation for these two actuators. In this paper, a robust control scheme including an effectual allocation strategy for the fin and the reaction jets is proposed to make the angle of attack  $\alpha$  track the angle command  $\alpha_c$  accurately and rapidly in the presence of parametric uncertainties and external disturbances.

## III. ROBUST CONTROL SCHEME DESIGN

The proposed robust control scheme contains an adaptive SMC controller and a control moment allocator. The former, which consists of a traditional SMC controller, an observer and an adaptive compensator, produces the control moment  $M_c$ . The latter assigns  $M_c$  to the fins and the reaction jets. The closed-loop control system is shown in Fig.1. The detailed description of each part is given in the following subsections.

### A. Traditional SMC

For ease of notation, define  $x = [\alpha \ q]$ ,  $u = M_c$ . Then the system (1) can be rewritten as

$$\begin{cases} \dot{\alpha} = q + f(x) + \Delta_\alpha \\ \dot{q} = h(x) + cu + \Delta_q \end{cases} \quad (2)$$

where  $u$ ,  $x$  and  $\alpha$  are regarded as input, state and output, and

$$\begin{cases} f(x) = [(Z + F_Z) \cos \alpha - X \sin \alpha + mg \cos \mu] / mV \\ h(x) = [M_\alpha(\alpha) + M_q(q, V)] / J_y \\ u = M_c = M_{\delta_e}(\alpha, \delta_e) + M_f(U) \\ c = 1 / J_y \end{cases} \quad (3).$$

Let the switching surface

$$s = \dot{e} + ke, k > 0 \quad (4)$$

where  $e = \alpha - \alpha_c$ . By making  $s$  reach zero, we can get  $\dot{e} = -ke$ , which makes  $e$  converge to zero with a speed specified by the parameter  $k$ .

Basing on the traditional SMC [9], let  $V = s^2 / 2$  as a Lyapunov function candidate, then

$$\dot{V} = ss = s[h(x) + cu + \Delta_q + \dot{f}(x) + \dot{\Delta}_\alpha - \ddot{\alpha}_c + kq + kf(x) + k\Delta_\alpha - k\dot{\alpha}_c] \quad (5)$$

Taking

$$u = -[h(x) + \dot{f}(x) - \ddot{\alpha}_c + kq + kf(x) - k\dot{\alpha}_c + k_w \operatorname{sgn}(s)] / c \quad (6)$$

where  $k_w > \max(|\dot{\Delta}_\alpha + \Delta_q + k\Delta_\alpha|) + k_{w0}$ ,  $k_{w0} > 0$ , and

$$\operatorname{sgn}(s) = \begin{cases} 1, s > 0 \\ 0, s = 0 \\ -1, s < 0 \end{cases} \quad (7)$$

yields

$$\begin{aligned} \dot{V} &= ss \\ &= s[\Delta_q + \dot{\Delta}_\alpha + k\Delta_\alpha - k_w \operatorname{sgn}(s)] \\ &\leq |\Delta_q + \dot{\Delta}_\alpha + k\Delta_\alpha| |s| - k_w |s| \\ &< -k_{w0} |s| \end{aligned} \quad (8)$$

Therefore, the system trajectory reaches the manifold  $s = 0$  in finite time.

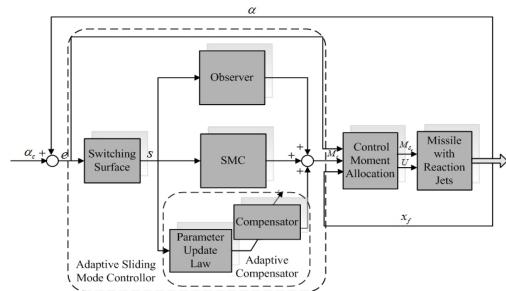


Fig. 1. The structure of the closed-loop control system.

It is worth noting that, according to the expression of  $f(x)$  in (3),  $X$  and  $Z$  are time-varying, thus the term  $\dot{f}(x)$  is very complex and its analytical expression is hardly obtained. To solve this problem, an observer is utilized to estimate it based on the dynamic of the switching surface. As for the switching surface (4), taking the derivative of  $S$  yields

$$\dot{s} = \dot{q} - \ddot{\alpha}_c + \dot{\alpha} - \dot{\alpha}_c + \dot{f}(x) + \dot{\Delta}_\alpha \quad (9)$$

Rewrite (9) as  $\dot{s} = \hat{f}(x) + \hat{u}$ , where  $\hat{f}(x) = \dot{f}(x) + \dot{\Delta}_\alpha$  and  $\hat{u} = \dot{q} - \ddot{\alpha}_c + \dot{\alpha} - \dot{\alpha}_c$ . Consider the following observer [15]:

$$\begin{cases} \dot{z}_0 = v_0 + \hat{u}, & v_0 = -2L^{1/3}|z_0 - s|^{2/3} \operatorname{sign}(z_0 - s) + z_1 \\ \dot{z}_1 = v_1, & v_1 = -1.5L^{1/2}|z_0 - v_0|^{1/2} \operatorname{sign}(z_1 - v_0) + z_2 \\ \dot{z}_2 = -1.1L \operatorname{sign}(z_2 - v_1) \end{cases} \quad (10)$$

where  $z_0$ ,  $z_1$ , and  $z_2$  are states of the observer;  $L > 0$  is a constant. If  $s$  and  $\hat{u}$  are measured without noise, then  $z_1$  will converge to  $\hat{f}(x)$  in a finite time  $t^*$ , i.e.,  $\lim_{t \rightarrow t^*} |\hat{f}(x) - z_1| = 0$ .

In addition, the output of the above observer  $z_1$  is smooth [16].

Replace  $\dot{f}(x)$  in (6) with the observer output  $z_1$  described above. Then the new control can be obtained

$$u = -[h(x) + z_1 - \ddot{\alpha}_c + kq + kf(x) - k\dot{\alpha}_c + k_w \operatorname{sgn}(s)]/c \quad (11)$$

Here, the switching gain  $k_w$  satisfies

$$k_w > \max(|\varepsilon_o + \Delta_q + k\Delta_\alpha|) + k_{w0} \quad (12)$$

where  $\varepsilon_o$  is the observer error.

In view of (12), knowledge of the system uncertainties and disturbances is required in the traditional SMC described above. Additionally, the switching gain  $k_w$  is directly related to the switching surface parameter  $k$ . The increase of  $k$  will improve the system time response. Correspondingly, the switching gain  $k_w$  must be increased. However, this will bring out severe chattering. On the other hand, if the switching gain  $k_w$  is not increased, the system robustness will be deteriorated. To cope with this situation, an adaptive compensator added to the SMC controller is designed to compensate the system uncertainties, disturbances, and the observer error, which are lumped as  $\hat{d} = \varepsilon_o + \Delta_q + k\Delta_\alpha$ . Then the control becomes

$$u = -[h(x) + z_1 - \ddot{\alpha}_c + kq + kf(x) - k\dot{\alpha}_c + u_{comp}(x) + k_w \operatorname{sgn}(s)]/c \quad (13)$$

where  $u_{comp}(x)$  is the adaptive compensator output which is described next subsection. Under the assumption that the compensation error is  $\varepsilon$ , the switching gain  $k_w$  only need to satisfy  $k_w > \max(|\varepsilon|) + k_{w0}$  to make the system trajectory reach the manifold  $s = 0$ . Obviously,  $k_w$  only relates to the compensation error and no knowledge of system uncertainties and disturbances is required. It can be decreased by improving the accuracy and convergence of the compensator. In addition, the term  $u_{comp}(x)$  can help improve the system response.

### B. Adaptive Compensator

Here, the adaptive compensator based on RBF neural

network is designed. Firstly, a RBF neural network with strong ability to uniformly approximate smooth functions is built to approximate  $\hat{d}$  over a compact region of the state space. The general form of a RBF network is shown as

$$\begin{cases} h_j = \exp(-\|\mathbf{x}_f - \mathbf{c}_j\|^2 / 2b_j^2), j = 1, \dots, m \\ F = \Theta^T h(\mathbf{x}_f) \end{cases} \quad (14)$$

where  $\Theta \in R^m$  is a vector of adjustable weights and  $h(\mathbf{x}_f) = [h_1, h_2, \dots, h_m]$  is a vector of Gaussian basis functions;  $\mathbf{x}_f$  is the input vector;  $\mathbf{c}_j$  and  $b_j$  are the mean and standard deviation of the Gaussian functions;  $j$  is the node number of the hidden layer.

Suppose  $\hat{d}$  is smooth enough, then there exists a Gaussian basis function vector  $h$  and a weight vector  $\Theta^*$  so that

$$\hat{d} = \Theta^{*T} h(x_f) + \varepsilon \quad (15)$$

where  $|\varepsilon| \leq \varepsilon_N$ ,  $\varepsilon_N$  is a constant [17].

If the RBF neutral network described above is utilized to compensate  $\hat{d}$ ,  $m$  parameters need to be updated, making the adaptive law complex and with slow convergence. In order to accelerate the adaptive law, parameter  $\phi = \|\Theta\|^2$  is chosen as the single parameter which needs to be updated online [18, 19]. Then, the adaptive compensator is designed as follow:

$$u_{comp} = s\phi h^T h / 2 + \mu s \quad (16)$$

where  $\mu > 0$ .

Integrating the compensator above, the compensated control  $u$  can be expressed as

$$u = -[h(x) + z_1 - \ddot{\alpha}_c + kq + kf(x) - k\dot{\alpha}_c + s\phi h^T h / 2 + \mu s + k_w \operatorname{sgn}(s)]/c \quad (17)$$

Next the update law of the parameter  $\phi$  is derived to make it approach its best value  $\phi^*$  ( $\phi^* = \|\Theta^*\|^2$ ), and to make the system trajectory reach the manifold  $s = 0$  under the Lyapunov theory.

Let the Lyapunov-like function candidate be

$$\bar{V} = s^2 / 2 + \tilde{\phi}^2 / 2\gamma \quad (18)$$

where  $\gamma > 0$ ,  $\tilde{\phi} = \phi - \phi^*$ . Taking the derivative of  $\bar{V}$  yields

$$\begin{aligned} \dot{\bar{V}} &= ss + \tilde{\phi}\dot{\phi}/\gamma \\ &= s[h(x) + cu + \Delta_q + \dot{f}(x) + \Delta_\alpha - \ddot{\alpha}_c + kq + kf(x) + k\Delta_\alpha - k\dot{\alpha}_c] + \tilde{\phi}\dot{\phi}/\gamma \\ &= s[h(x) + cu + z_1 - \ddot{\alpha}_c + kq + kf(x) - k\dot{\alpha}_c + \hat{d}] + \tilde{\phi}\dot{\phi}/\gamma \end{aligned} \quad (19)$$

Substitute the control (17) into (19), so that

$$\dot{\bar{V}} = s[\hat{d} - s\phi h^T h / 2 - \mu s - k_w \operatorname{sgn}(s)] + \tilde{\phi}\dot{\phi}/\gamma \quad (20)$$

Substituting (15) into (20) yields

$$\begin{aligned} \dot{\bar{V}} &= s[\Theta^{*T} h + \varepsilon - s\phi h^T h / 2 - \mu s - k_w \operatorname{sgn}(s)] + \tilde{\phi}\dot{\phi}/\gamma \\ &= s\Theta^{*T} h - s^2\phi h^T h / 2 - \mu s^2 - k_w |s| + \tilde{\phi}\dot{\phi}/\gamma \\ &\leq (s^2 \|\Theta^*\|^2 + 1)/2 - s^2\phi h^T h / 2 - \mu s^2 - k_w |s| + \tilde{\phi}\dot{\phi}/\gamma \\ &= -s^2\tilde{\phi}h^T h / 2 + 1/2 - k_w |s| - \mu s^2 + \tilde{\phi}\dot{\phi}/\gamma \\ &= \tilde{\phi}(-s^2h^T h / 2 + \dot{\phi}/\gamma) + 1/2 - k_w |s| - \mu s^2 \end{aligned} \quad (21)$$

Then the update law of the parameter  $\phi$  is chosen as

$$\dot{\phi} = \gamma s^2 h^T h / 2 - 2\mu\phi \quad (22)$$

The Lyapunov stability of the closed-loop control system under the control (17) and the parameter update law (22) will be analyzed in the next subsection.

### C. Lyapunov Stability Analysis

Here, the system stability theorem is given below:

**Theorem:** Consider the dynamic nonlinear system represented by (2) with the control law (17) where the adaptive compensator is designed as (16) and the parameter  $\phi$  is updated by the learning rules (22). If there exists  $k_w > 0$  satisfying the relational expression:

$$k_w > \max(|\varepsilon|) + k_{w0} \quad (23)$$

Then, the parameter  $\tilde{\phi}$  will remain bounded, and the error of  $\alpha$  will approach in a scope decided by  $\mu$  and  $\gamma$ .

#### Proof:

In order to analyze the stability of the closed-loop system, the Lyapunov candidate function  $\bar{V}$  is chosen as same as (18). Substituting the parameter update law (22) into (21), yields

$$\dot{\bar{V}} = -2\mu\phi\tilde{\phi} / \gamma + 1/2 - k_w |s| - \mu s^2 \quad (24)$$

Consider

$$\begin{aligned} (\tilde{\phi} + \phi^*)^2 &= \tilde{\phi}^2 + 2\tilde{\phi}\phi^* + \phi^{*2} \\ &= \tilde{\phi}^2 + 2\tilde{\phi}(\phi - \tilde{\phi}) + \phi^{*2} \\ &\geq 0 \end{aligned} \quad (25)$$

which yields

$$2\tilde{\phi}\phi \geq \tilde{\phi}^2 - \phi^{*2} \quad (26)$$

Substitute (26) into (24), so that

$$\begin{aligned} \dot{\bar{V}} &\leq -\mu(\tilde{\phi}^2 - \phi^{*2}) / \gamma + 1/2 - k_w |s| - \mu s^2 \\ &= -2\mu(s^2 / 2 + \tilde{\phi}^2 / 2\gamma) + 1/2 + \mu\phi^{*2} / \gamma - (k_w - |\varepsilon|)|s| \\ &= -2\mu\bar{V} + 1/2 + \mu\phi^{*2} / \gamma - (k_w - |\varepsilon|)|s| \end{aligned} \quad (27)$$

Taking the switching gain  $k_w > \max(|\varepsilon|) + k_{w0}$ , yields

$$\dot{\bar{V}} \leq -2\mu\bar{V} + 1/2 + \mu\phi^{*2} / \gamma \quad (28)$$

For ease of notation, let  $W = 1/2 + \mu\phi^{*2} / \gamma$ . Considering (28),  $\bar{V}$  is bounded and

$$\bar{V} \leq [\bar{V}(0) - W / 2\mu] \exp(-2\mu t) + W / 2\mu \quad (29)$$

which implies that

$$\lim_{t \rightarrow \infty} \bar{V} = W / 2\mu = 1/4\mu + \phi^{*2} / 2\gamma \quad (30)$$

Therefore,  $e$  and  $\tilde{\phi}$  are bounded, and the stability of the closed-loop system is guaranteed based on the above results and the Lyapunov stability theorem. Besides, the parameter  $\mu$  and  $\gamma$  can be chosen sufficiently big to obtain the desired accuracy.

The adaptive compensator described above is derived from the RBF network. It adopts the knowledge of the RBF network Gaussian basis functions and regards the sum of square of the RBF neural network weights as the single adjustable parameter. Differently from traditional RBF network compensators, the adaptive compensator also adopts the knowledge of the

switching surface.

### D. Control Moment Allocator

Considering the reaction jets could provide a large moment in a short time, we command the reaction jets work dominantly when the error of the angle of attack is large while the fins work primarily when the error is small. Thus, the moment assigned to the reaction jets  $\hat{M}_f$  can be expressed as  $\hat{M}_f = M_c \text{sat}(e_\alpha / e_{\alpha \max})$ , where  $e_{\alpha \max}$  is the possible maximum of  $e_\alpha$ .

Because of the discrete characteristics of the reaction jets and their limited amount, they can hardly provide the desired moment  $\hat{M}_f$ . To compensate this situation, we calculate the best fire command vector  $U^*$ , which provides the moment  $M_f$  closest to  $\hat{M}_f$  basing on the present reaction jets states and the possible moment they can provide. Then we assign the remaining moment to the fins. It is worth mentioning that in order to avoid waste of the reaction jets, we only make the reaction jets work under the condition  $M_f(U^*) \geq \tilde{M}_f$ , where  $\tilde{M}_f$  is the moment single reaction jet can provide. Then, the allocation function can be written as follow:

$$\begin{cases} U^* = \arg \min_U |M_f(U) - \hat{M}_f|, U \subset A_U(x_f) \\ M_f = \begin{cases} M_f(U^*), & \text{if } M_f(U^*) \geq \tilde{M}_f \\ 0, & \text{if } M_f(U^*) < \tilde{M}_f \end{cases} \\ M_{\delta_e} = M_c - M_f \end{cases} \quad (31)$$

where  $U$  is the possible reaction jets fire vector,  $A_U(x_f)$  is the set decided by the present state of the reaction jets, containing all the possible fire vector presently. The allocator here can not only help the fins and reaction jets work sufficiently, but also guarantee the desired control moment with no deviation.

## IV. SIMULATION RESULTS

In this section, the performance of the proposed scheme is evaluated for the MRJ as shown in [14]. The control objective is to make the angle of attack  $\alpha$  track the angle command  $\alpha_c$  rapidly and accurately in the presence of disturbances and uncertainties.

### A. Scenario

The angle command is set as  $\alpha_c = 20^\circ$  at 0.1 sec after the simulation starts, and the simulation begins from the trimmed equilibrium state  $\delta_e = -0.70^\circ$ ,  $\alpha = 1.15^\circ$ , and  $V_t = 1000 m/s$ . In this robust control scheme, the RBF network is designed with the structure of two inputs ( $\alpha$  and  $q$ ), five hidden nodes with the Gaussian function mean  $c = [-0.35 \ 0.17 \ 0.17 \ 0.35 \ -1.7 \ -0.85 \ 0.085 \ 1.7]$  and standard deviation  $b = [1 \ 1 \ 1 \ 1 \ 1]$ . In the following subsections, the effectiveness of the proposed adaptive SMC method and the coordination method using the control moment allocator are verified.

### B. Effectiveness of the Adaptive SMC Method

In this subsection, to show the effectiveness of the proposed adaptive SMC method, the performance of the proposed control scheme is compared with the SMC scheme without the adaptive compensator. The control moment allocator proposed in this paper is used to coordinate the fins and the reaction jets. Four simulation cases are done in condition of periodic aerodynamic moment disturbances  $M_{ad}(\alpha) = 0.1\cos(0.1t) + 0.1\sin(0.1t)$ ,  $M_{qd}(q, V_t) = 0.1\cos(0.1t) + 0.1\sin(0.1t)$  and an uncertainty  $\Delta M_\alpha(\alpha) = 0.5M_\alpha(\alpha)$ ,  $\Delta M_q(q, V_t) = 0.5M_q(q, V_t)$ .

Robustness, fast-response characteristic, and chattering phenomenon will be evaluated. The parameters of four cases and the corresponding performances are shown in Table 1, where Y implies chattering phenomenon happens while N implies not. Table 1 shows that the adaptive compensator helps reduce the chattering and preserve the robustness and fast-response characteristic at the same time.

The time histories of the angle of attack  $\alpha$ , control output  $M_c$ , fin deflection  $\delta_e$ , and reaction jets moment output  $M_f$  in each case are demonstrated in Fig. 3 to Fig. 6, respectively. Fig. 3 shows that, under the control in Case 1, the angle of attack arrives the required angle rapidly and accurately; the output of the adaptive sliding mode controller is smooth and without chattering. Accordingly, the fin response is with no chattering and the continuous trigger does not happen to the reaction jets. Fig. 4 shows that under the control in Case 2, the closed-system is unstable. The reason is the switching gain  $k_w$  is too small to conquer the disturbances to the control system. Fig. 5 shows that by increasing the switching gain  $k_w$ , the angle of attack can track the angle command accurately; but the ultimate state is with oscillation and chattering happens to the control output, which brings out the chattering of the fins and the continuous trigger of the reaction jets. Fig. 6 shows the performance under the control in Case 4. It can be observed that  $\alpha$  reaches the desired state with a slow response speed.

The comparison of Case 1 and Case 2 shows that the adaptive compensator strengthens the system robustness. The comparison of Case 1 and Case 3 indicates it can allow the sliding mode control with small switching gain, thus eliminating the chattering phenomenon. Besides, the comparison of Case 1 and Case 4 shows the adaptive compensator allows the system adopting big switching surface parameter  $k$ , thus increasing the system dynamic response speed. In conclusion, the adaptive compensator is effective; it can guarantee the system robustness and fast-response characteristic, and eliminate the chattering meanwhile.

### C. Effectiveness of the Coordination Method Evaluation

The reaction jets can accelerate the system time response when the coordination method is designed effectively. Bad coordination method can not only hardly improve the system dynamic response, but also damage the system stability. Here, the time response of the missile with reaction jets under the control scheme proposed in this paper is compared to the

missile with fins only. The adaptive SMC method proposed in this paper is used in simulation. The result shown in Fig. 6 indicates that the participation of the reaction jets improves the system time response significantly. So the coordination strategy in this paper is effective.

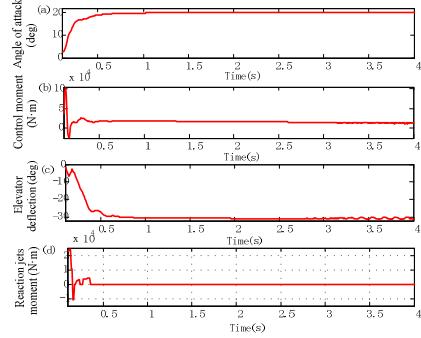


Fig.2.  $\alpha$ ,  $M_c$ ,  $\delta_e$  and  $M_f$  in Case 1.

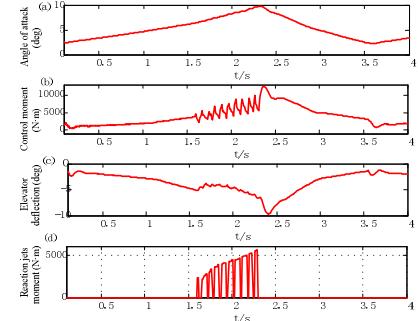


Fig.3.  $\alpha$ ,  $M_c$ ,  $\delta_e$  and  $M_f$  in Case 2.

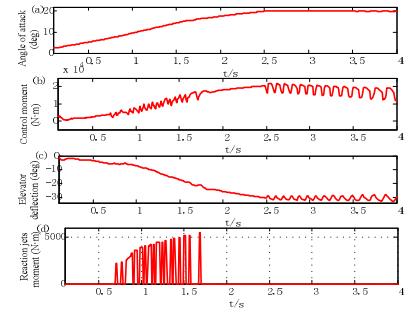


Fig.4.  $\alpha$ ,  $M_c$ ,  $\delta_e$  and  $M_f$  in Case 3.

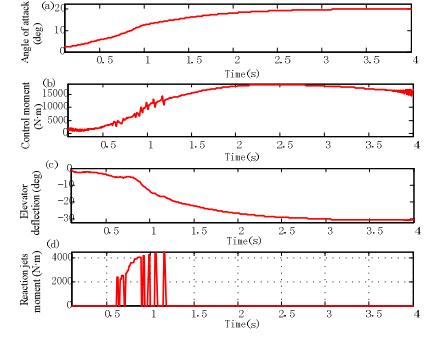


Fig.5.  $\alpha$ ,  $M_c$ ,  $\delta_e$  and  $M_f$  in Case 4.

TABLE I. COMPARISON OF FOUR CASES

Control Case	Parameters				Performances		
	$k$	$k_w$	$\mu$	$\gamma$	Robustness	Reaching-time	Chattering
Case1: Control with adaptive compensating	40	1.4	5	1	Stable without error	1s	N
Case2: Control without adaptive compensating	40	1.4	-	-	Unstable	-	-
Case3: Control without adaptive compensating	40	5	-	-	Stable with Oscillation	2.5s	Y
Case4: Control without adaptive compensating	10	1.4	-	-	Stable without error	3.5s	N

## V. CONCLUSION

In this paper, the robust control scheme is investigated to control the missiles with reaction jets to reach the desired state accurately and rapidly in the presence of system uncertainties and external disturbances. The control scheme contains two aspects, one is the control law of control moment, and the other is the allocation method which assigns the control moment to the fins and the reaction jets. To design the law of the control moment, the adaptive sliding mode control method compensated by the adaptive compensator is designed. The main contribution of this method is that no knowledge of the boundaries of the system uncertainties and external disturbances is required in advance; meanwhile, the chattering phenomenon is also eliminated without deteriorating the system fast-response characteristic and robustness. To allocate the control moment to the fins and the reaction jets, the moment allocator is designed, which can make the system respond fast and avoid the waste of reaction jets. The stability and the convergence of the overall system are proved by the Lyapunov theory. Simulation results show the effectiveness of the proposed control scheme.

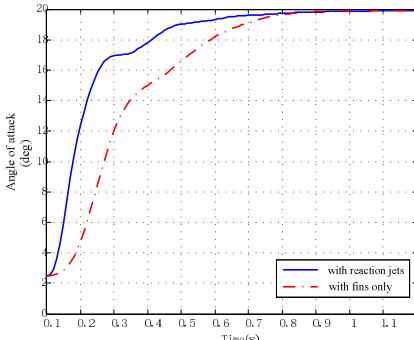


Fig.6.  $\alpha$  with reaction jets or with fins only.

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