

# Nonlinear Adaptive Control for Hypersonic Vehicles via Immersion and Invariance

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**Abstract:** A new nonlinear adaptive control scheme for the angle of attack of a hypersonic vehicle is presented in this paper. The new scheme is based on the immersion and invariance (I&I) methodology, aiming to tolerate various uncertainties. The control system has a modular structure consisting of the design of control law and the estimates of the unknown parameters, which can be tuned independently. Unlike the certainty-equivalent adaptive control, the parameter estimates of the I&I theory have an additional term which is a judiciously chosen nonlinear function. This term allows for shaping the dynamics of the estimation errors (off-the-manifold dynamics) to have stable zero equilibrium. The stability analysis based on a composite Lyapunov function for the closed-loop system is performed to show the asymptotical convergence of the tracking error. The proposed I&I adaptive controller is simulated in two kinds of conditions, both obtaining results of a fast response and an accurate tracking, which shows the effectiveness of the proposed approach.

**Key Words:** Nonlinear adaptive control, hypersonic vehicles, immersion and invariance, angle of attack tracking control

## 1 Introduction

In recent years the nonlinear control problem associated with hypersonic vehicles has been studied extensively for the great advantages they offer, namely increase in payload capacity and propulsion efficiency, expansion of launch windows, and reduction in operation costs [1, 2]. Due to the wide speed and altitude range, the traditional flight control design based on linear approximations of the flight model and gain scheduling becomes a complex and time-consuming work. Moreover, various uncertainties in physical and aerodynamic parameters have become troublesome problems because of the rapid change of aerodynamic characteristics and state variables during the hypersonic flight altitude. Therefore, it is necessary to design a nonlinear flight control system to overcome the variation in dynamic characteristic and these uncertainties.

Many nonlinear control methods have been used to achieve the above goals. The well-known feedback linearization theory has been widely applied to the design of flight control system [3-6]. Yet it depends on precise models which are usually impossible for hypersonic vehicles. To deal with the uncertainties in modeling, several nonlinear adaptive control methods have been developed, such as adaptive sliding mode control [4], nonlinear fuzzy adaptive control [5, 7], parameter space method [8], characteristic model based golden section adaptive control [9], neural network control [10, 11]. The adaptive Backstepping theory has caught much attention in the last years. Because the control law can be recursively derived, along with a control Lyapunov function to guarantee global stability, many efforts have been made to develop the hypersonic vehicle control systems, by the combination of Backstepping theory and other control technologies [3, 5, 10, 12-14]. However, a Lyapunov function should be designed each step for each

subsystem, and the derivative of the intermediate virtual control is needed too. These make the control law design procedure very complex, especially for high order systems.

Recently, a new method to design adaptive controllers for general nonlinear systems based on immersion and invariance (I&I) is presented [15, 16]. This approach does not require the knowledge of a Lyapunov function, and it allows for prescribed stable dynamics to be assigned to the parameter estimation error. Because of its significant nonlinear adaptive performance, I&I theory has been applied to many situations, such as [17-20]. A combination of dynamic inversion and Backstepping theory, with a parameter estimator based on I&I theory is adopted for the hypersonic vehicle control system in [21]. Similar to many high-gain observers, a dynamic scaling factor is applied in the framework of parameter estimators in that research.

This paper presents an immersion and invariance based nonlinear adaptive control system for the trajectory control of the angle of attack of the hypersonic vehicles in case of parameter uncertainties. Section 2 briefly introduces the basic idea of I&I adaptive control. The longitudinal dynamics of the hypersonic vehicles are described in Section 3 and it is assumed all the aerodynamic coefficients (except the sign of the control input gain) are unknown. The noncertainty-equivalent adaptive control law and a parameter estimator without dynamic scaling are derived in Section 4. Section 5 presents the performance of the proposed adaptive control law via numerical simulations. Finally, the conclusion can be found in Section 6.

## 2 Immersion and Invariance adaptive control

As the name suggests, I&I methodology relies upon the notions of immersion and invariance. Its basic idea is to immerse the plant into a lower-order target system which captures the desired behavior to achieve the control objective. A graphical illustration of the I&I approach is given in Fig. 1. More precisely, it consists of two steps which are first to find an invariant and attractive manifold in state-space whose internal dynamics is a copy of the desired

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closed-loop system, and then to design a control law which can render the state of the system sufficiently close to the manifold. Note that, in the I&I perspective, the Lyapunov-based method is its special case, which has one-dimensional target dynamics, as shown in Fig. 1.

Unlike most existing adaptive methods, the I&I adaptive control does not invoke certainty equivalence. The whole estimations of unknown parameters are the sum of two terms. One is obtained by an update law like the traditional adaptive methods. The other is an adequately chosen nonlinear function. The role of this additional term is to shape the manifold into which the adaptive system is immersed [16]. The manifold is usually defined as the function of parameter true values and estimations. Therefore, the additional term actually allows for shaping the dynamics of the estimation errors (off-the-manifold dynamics) to have stable zero equilibrium. When the states of the extended system are off the manifold, the trajectories will asymptotically converge to the manifold, i.e., the manifold is rendered attractive. When they are on the manifold, the trajectories will remain confined to it, i.e., it is invariant. As the trajectories asymptotically converge to the origin, the closed-loop system captures the behavior of the target system.

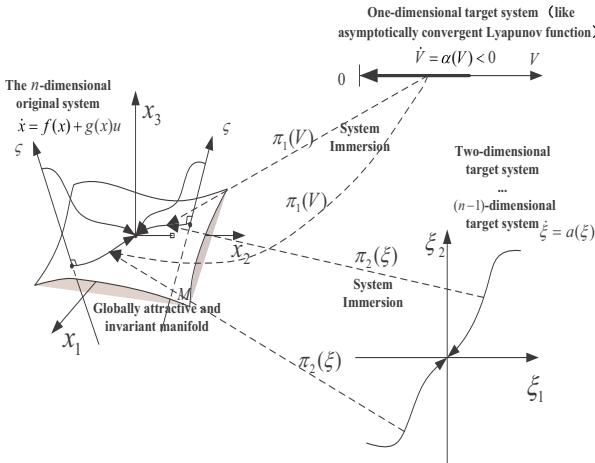


Fig. 1: Graphical illustration of the immersion and invariance approach

### 3 Hypersonic vehicle model and control problem

The model employed here is the longitudinal dynamics of an air-breathing hypersonic vehicle, derived using Lagrange's equations. Ignoring the flexibility effects of the body structure and assuming a flat Earth, the equations of the longitudinal dynamics are written as [4, 6]

$$\begin{aligned}\dot{V} &= \frac{T \cos \alpha - D}{m} - g \sin \gamma \\ \dot{h} &= V \sin \gamma \\ \dot{\gamma} &= \frac{L + T \sin \alpha}{mV} - \frac{g}{V} \cos \gamma \\ \dot{\alpha} &= -\frac{L + T \sin \alpha}{mV} + Q + \frac{g}{V} \cos \gamma \\ \dot{Q} &= \frac{M}{I_{yy}}\end{aligned}\quad (1)$$

where  $V, h, \gamma, \alpha, Q$  are the flight velocity, altitude, flight-path angle, angle of attack and pitch rate respectively. The thrust  $T$ , the aerodynamic forces  $L, D$  and the aerodynamic moment  $M$  are denoted by

$$\begin{aligned}L &= qSC_L \\ D &= qSC_D \\ T &= qSC_T\end{aligned}\quad (2)$$

$$M = qSC[C_M(\alpha) + C_M(\delta_e) + C_M(Q)]$$

where  $q$  is the dynamic pressure,  $\bar{c}$  the reference length and  $S$  the reference area. The aerodynamic force and moment coefficients are given as follows:

$$\begin{aligned}C_L &= C_{L\alpha} \alpha \\ C_D &= C_{D\alpha^2} \alpha^2 + C_{D\alpha} \alpha + C_{D0} \\ C_T &= C_{T\beta} \beta + C_{T0} \\ C_M(\alpha) &= C_{M\alpha^2} \alpha^2 + C_{M\alpha} \alpha + C_{M0} \\ C_M(\delta_e) &= c_e (\delta_e - \alpha) \\ C_M(Q) &= \bar{c} Q (C_{MQ\alpha^2} \alpha^2 + C_{MQ\alpha} \alpha + C_{MQ0}) / (2V)\end{aligned}\quad (3)$$

where elevator deflection  $\delta_e$  and throttle setting  $\beta$  are the control inputs.

In general, there are various uncertainties in the hypersonic vehicle dynamics. Here all aerodynamic coefficients are treated as unknown constant parameters. The control objective is to design a control law to make the states of the system track desired trajectories in case of such uncertainties. According to Eq. (1), the hypersonic vehicle longitudinal dynamics can be decomposed into three parts, namely, the velocity subsystem, the altitude and flight-path subsystem and the angle of attack and pitch rate subsystem [6, 13]. Each subsystem is controlled separately using the available inputs or the intermediate virtual control. Only a nonlinear adaptive controller for the angle of attack and pitch rate subsystem is presented in this paper due to the page limitation. Note, however, that the proposed approach can also be applied to the other two subsystems and used to deal with other uncertainties.

### 4 Nonlinear adaptive control system design

Substituting Eq. (2) and (3) into the last equation of Eq. (1), the dynamics of the angle of attack and pitch rate subsystem can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 - \dot{\gamma} \\ \dot{x}_2 = \sigma \theta_2 u + \sigma \varphi(x)^T \theta_1 \end{cases}\quad (4)$$

where

$$[x_1, x_2]^T = [\alpha, Q]^T, \sigma = qS\bar{c}/I_{yy},$$

$$\varphi(x) = [x_1^2, x_1, 1, \frac{\bar{c}x_2}{2V}x_1^2, \frac{\bar{c}x_2}{2V}x_1, \frac{\bar{c}x_2}{2V}]^T,$$

$$\theta_1 = [C_{M\alpha^2}, C_{M\alpha} - c_e, C_{M0}, C_{MQ\alpha^2}, C_{MQ\alpha}, C_{MQ0}]^T,$$

$$\theta_2 = c_e, u = \delta_e, \theta_2 > 0,$$

$\theta_1$  and  $\theta_2$  are unknown constant parameters.

The I&I approach provides a modular scheme which is easier to regulate than the one obtained from Lyapunov-based methods. The modular structure of the

control system consists of two parts. One is the control law, and the other is the estimates of the unknown parameters. These two parts can be done separately. The design procedure can be described as follows. First, find a control law by using unknown parameters as known ones [22], such that the closed-loop system has an asymptotically stable equilibrium, and treat the closed-loop system as the target system. Second, design the update law. Then add an additional term to the parameter estimates, which is judiciously chosen to render the manifold attractive and invariant. Finally, the augmented system will have an asymptotically stable equilibrium. The configuration of the whole control system is shown in Fig. 2.

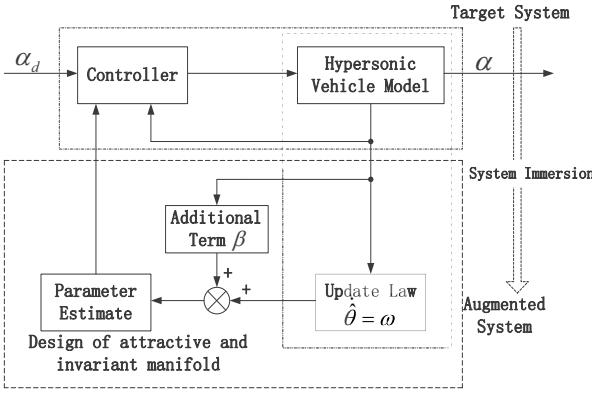


Fig. 2: The configuration of the whole control scheme

#### 4.1 Control law design

Assuming the demanded angle of attack is  $\alpha_d$ , the tracking error is  $\tilde{\alpha} = \alpha - \alpha_d$ . If there exists  $s$  that satisfies

$$s = \dot{\tilde{\alpha}} + \lambda \tilde{\alpha} = 0, \quad \lambda > 0 \quad (5)$$

the tracking error  $\tilde{\alpha}$  will converge to zero.

Select the update law

$$\begin{cases} \dot{\hat{\theta}}_1 = \omega_1 \\ \dot{\hat{\theta}}_2 = \omega_2 \end{cases} \quad (6)$$

where  $\hat{\theta}_1$  and  $\hat{\theta}_2$  are only partial estimates of the unknown parameters  $\theta_1$  and  $\theta_2^{-1}$ . The full estimates of  $\theta_1$  and  $\theta_2^{-1}$  are defined as  $\hat{\theta}_1 + \beta_1(x)$  and  $\hat{\theta}_2 + \beta_2(x, \hat{\theta}_1)$  respectively. This is very different from the estimators based on the certainty equivalence because of the additional terms  $\beta_1$  and  $\beta_2$ . So the estimate errors are

$$\begin{cases} z_1 = \hat{\theta}_1 + \beta_1(x) - \theta_1 \\ z_2 = \hat{\theta}_2 + \beta_2(x, \hat{\theta}_1) - \theta_2^{-1} \end{cases} \quad (7)$$

The derivative of  $s$  is

$$\dot{s} = \sigma(\theta_2 u + \varphi^T \theta_1) - \ddot{\alpha}_d - \ddot{\gamma} + \lambda(\dot{x}_1 - \dot{\alpha}_d) \quad (8)$$

Select control law as follows

$$u = (\hat{\theta}_2 + \beta_2(x, \hat{\theta}_1))v \quad (9)$$

Substituting Eq. (9) and (7) into Eq. (8) yields,

$$\dot{s} = \theta_2 z_2 v + v + \sigma \varphi^T (\hat{\theta}_1 + \beta_1(x) - z_1) - \ddot{\alpha}_d - \ddot{\gamma} + \lambda(\dot{x}_1 - \dot{\alpha}_d) \quad (10)$$

Now  $v$  is a new input to be determined. According to Eq. (10),  $v$  should be chosen to render  $s$  to converge to zero. One choice is

$$v = -[\sigma \varphi^T (\hat{\theta}_1 + \beta_1(x)) - \ddot{\alpha}_d - \ddot{\gamma} + \lambda(\dot{x}_1 - \dot{\alpha}_d) + cs] \quad (11)$$

where  $c > 0$ . The derivation of stability analysis will be given in Section 4.3.

#### 4.2 Parameter Estimates based on non-certainty equivalence

Parameter Estimates presented here are the sum of a partial estimate generated by an update law and a judiciously chosen nonlinear function, as shown in Eq. (7). Differentiating the Eq. (7) gives

$$\begin{cases} \dot{z}_1 = \dot{\hat{\theta}}_1 + \frac{\partial \beta_1}{\partial x_1}(x_2 - \dot{\gamma}) + \frac{\partial \beta_1}{\partial x_2}(\sigma \theta_2 u + \sigma \varphi(x)^T \theta_1) \\ \dot{z}_2 = \dot{\hat{\theta}}_2 + \frac{\partial \beta_2}{\partial x_1}(x_2 - \dot{\gamma}) + \frac{\partial \beta_2}{\partial x_2}(\sigma \theta_2 u + \sigma \varphi(x)^T \theta_1) + \frac{\partial \beta_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 \end{cases} \quad (12)$$

Substituting Eq. (6) and (9) into Eq. (12) yields

$$\begin{cases} \dot{z}_1 = \omega_1 + \frac{\partial \beta_1}{\partial x_1}(x_2 - \dot{\gamma}) + \frac{\partial \beta_1}{\partial x_2}(\sigma \varphi^T \theta_1 + \theta_2 z_2 v + v) \\ \dot{z}_2 = \omega_2 + \frac{\partial \beta_2}{\partial x_1}(x_2 - \dot{\gamma}) + \frac{\partial \beta_2}{\partial x_2}(\sigma \varphi^T \theta_1 + \theta_2 z_2 v + v) + \frac{\partial \beta_2}{\partial \hat{\theta}_1} \omega_1 \end{cases} \quad (13)$$

The update law can be chosen as

$$\begin{cases} \dot{\hat{\theta}}_1 = -\frac{\partial \beta_1}{\partial x_1}(x_2 - \dot{\gamma}) - \frac{\partial \beta_1}{\partial x_2}[\sigma \varphi^T (\hat{\theta}_1 + \beta_1) + v] \\ \dot{\hat{\theta}}_2 = -\frac{\partial \beta_2}{\partial x_1}(x_2 - \dot{\gamma}) - \frac{\partial \beta_2}{\partial x_2}[\sigma \varphi^T (\hat{\theta}_1 + \beta_1) + v] - \frac{\partial \beta_2}{\partial \hat{\theta}_1} \omega_1 \end{cases} \quad (14)$$

Substituting Eq. (14) into Eq. (13) yields

$$\dot{z} = -\frac{\partial \beta}{\partial x_2} \Phi(x, \hat{\theta}_1)^T z \quad (15)$$

where

$$z = [z_1^T, z_2]^T, \quad \beta = [\beta_1^T, \beta_2]^T, \quad \Phi(x, \hat{\theta}_1) = [\sigma \varphi^T, -\theta_2 v]^T.$$

Now the nonlinear function  $\beta$  should be chosen to guarantee the  $z$  dynamics have stable behavior. One choice is [19]

$$\begin{cases} \frac{\partial \beta_1}{\partial x_2} = r_1 \sigma \varphi \\ \frac{\partial \beta_2}{\partial x_2} = -r_2 v \end{cases} \quad (16)$$

where  $r_1 > 0, r_2 > 0$ . So

$$\begin{cases} \beta_1 = r_1 \sigma \Omega \\ \beta_2 = r_2 \sigma \Omega^T \hat{\theta}_1 + r_1 r_2 \sigma^2 \Psi + \Lambda \end{cases} \quad (17)$$

where

$$\Omega = [x_1^2 x_2, x_1 x_2, x_2, \frac{\bar{c}}{4V} x_1^2 x_2^2, \frac{\bar{c}}{4V} x_1 x_2^2, \frac{\bar{c}}{4V} x_2^2]^T,$$

$$\Lambda = r_2 (\chi_1 + \chi_2),$$

$$\begin{aligned}
\chi_1 &= (c + \lambda)x_2^2/2, \\
\chi_2 &= (\ddot{\alpha}_d + \dot{\gamma}) + (c + \lambda)(\dot{\alpha}_d + \dot{\gamma}) - c\lambda(x_1 - \alpha_d), \\
\Psi &= \psi_1 + \psi_2, \\
\psi_1 &= x_1^4 x_2 + x_1^2 x_2 + x_2, \\
\psi_2 &= \left(\frac{\bar{c}}{2V}\right)^2 \frac{x_2^3}{2} x_1^4 + \left(\frac{\bar{c}}{2V}\right)^2 \frac{x_2^3}{2} x_1^2 + \left(\frac{\bar{c}}{2V}\right)^2 \frac{x_2^3}{2}.
\end{aligned}$$

Substituting  $\beta_1$  into the first equation of Eq. (14) gives  $\omega_1$ . Then according to Eq. (11),  $v$  can be obtained. Finally,  $\beta_2$  and  $\omega_2$  can be derived from Eq. (17) and (14), respectively.

### 4.3 Stability analysis

Substituting Eq. (11) into Eq. (10), and Eq. (16) into Eq. (15), the dynamics of  $s$  and  $z$  are written as

$$\begin{cases} \dot{s} = -cs - \Phi^T(x, \hat{\theta}_1)z \\ \dot{z} = -\Gamma\Phi(x, \hat{\theta}_1)\Phi^T(x, \hat{\theta}_1)z \end{cases} \quad (18)$$

where  $\Gamma = \text{diag}(r_1 I_{6 \times 6}, r_2 \theta_2^{-1})$ .  $\Gamma$  is a positive definite matrix due to  $r_1 > 0$ ,  $r_2 > 0$  and  $\theta_2 > 0$ .

To examine the stability of (18), consider a Lyapunov function

$$V(s, z) = \frac{1}{2}s^2 + c^{-1}z^T \Gamma^{-1}z \quad (19)$$

Differentiating  $V(s, z)$  gives

$$\dot{V} = -cs^2 - s\Phi(x, \hat{\theta}_1)z - 2c^{-1}[\Phi(x, \hat{\theta}_1)z]^2 \quad (20)$$

By Young's inequality,

$$-s\Phi(x, \hat{\theta}_1)z \leq \frac{1}{2}cs^2 + \frac{1}{2}c^{-1}[\Phi(x, \hat{\theta}_1)z]^2 \quad (21)$$

Substituting Eq. (20) into Eq. (19) yields,

$$\begin{aligned}
\dot{V} &\leq -cs^2 + \frac{1}{2}cs^2 + \frac{1}{2}c^{-1}[\Phi(x, \hat{\theta}_1)z]^2 \\
&\quad - 2c^{-1}[\Phi(x, \hat{\theta}_1)z]^2 \\
&= -\frac{1}{2}cs^2 - \frac{3}{2}c^{-1}[\Phi(x, \hat{\theta}_1)z]^2
\end{aligned} \quad (22)$$

Therefore, if  $c$  is selected as  $c > 0$ ,  $s$  and  $z$  both asymptotically converge to zero. As analyzed above, the tracking error  $\tilde{\alpha}$  will asymptotically converge to zero.

## 5 Simulation results

Simulations have been performed to verify the proposed nonlinear adaptive control system. The hypersonic vehicle parameters of [4] are used for computation. The initial flying state of the hypersonic vehicle is chosen at an altitude  $H = 33528m$  and a speed  $V = 4590m/s$ , with a trimmed angle of attack  $\alpha_0 = 1.87^\circ$ , flight-path angle  $\gamma_0 = 0^\circ$  and pitch rate  $Q_0 = 0^\circ/s$ . The initial control inputs are chosen as  $[\delta_{e0}, \beta_0] = [0, 0.21]$ . Since the angle of attack and pitch rate subsystem is controlled through the pitch moment by means of the elevator deflection  $\delta_e$ , we assume the throttle setting  $\beta$  remains constant during the simulation (This is appropriate because the angle of attack

and pitch rate subsystem is the fast variable system). The elevator is assumed as a first order dynamics with a time constant of 0.05, deflection limits of  $\pm 20$  deg [13] and rate limits of  $\pm 50$  deg/s. To prove the effectiveness of the proposed I&I adaptive controller, two kinds of reference trajectories of the angle of attack are defined. In both cases, control parameters are chosen as  $\lambda = 25$ ,  $c = 1$ ,  $r_1 = 0.001$ ,  $r_2 = 5$ . The simulation results are shown in Fig. 3~8.

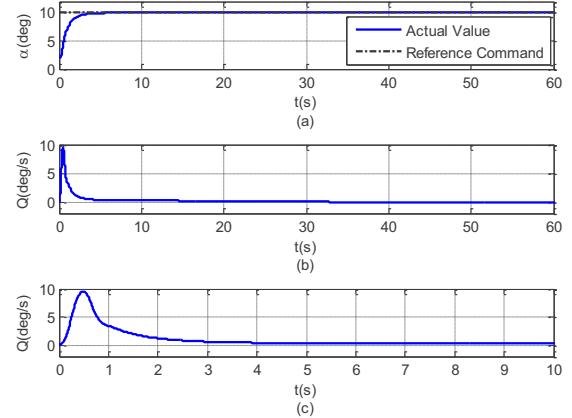


Fig. 3: Angle of attack and pitch rate trajectory ( $\alpha_d = 10^\circ$ )

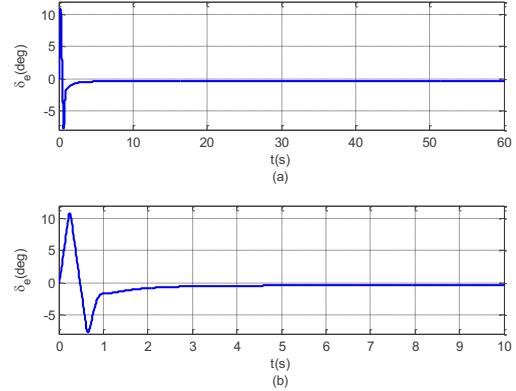


Fig. 4: Elevator deflection

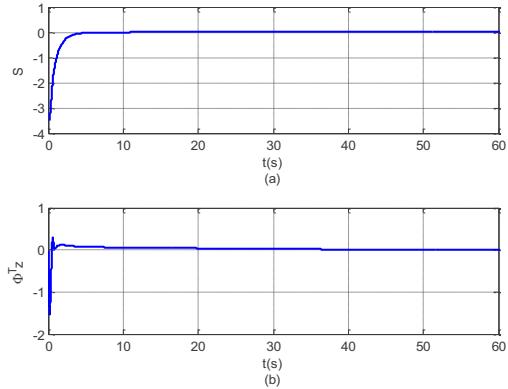


Fig. 5:  $s$  and  $\Phi^T(x, \hat{\theta}_1)z$

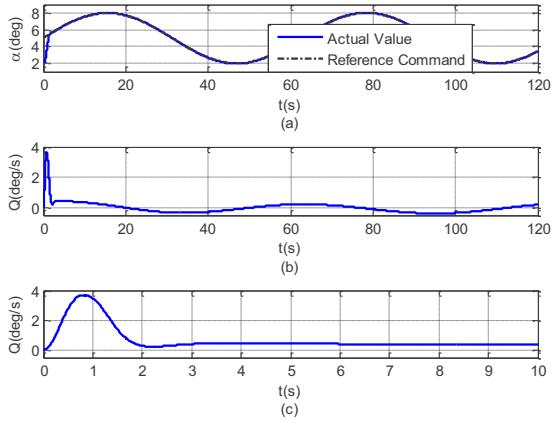


Fig. 6: Angle of attack and pitch rate trajectory  
( $\alpha_d = 3\sin(0.1t) + 5$ )

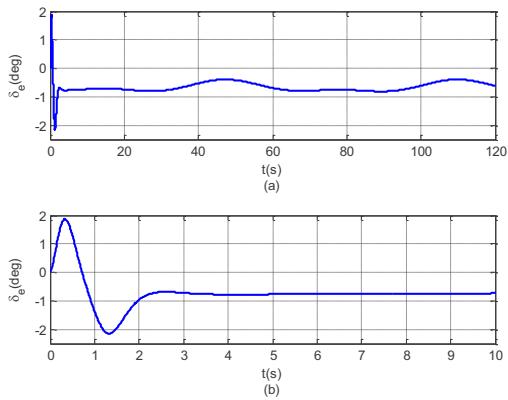


Fig. 7: Elevator deflection

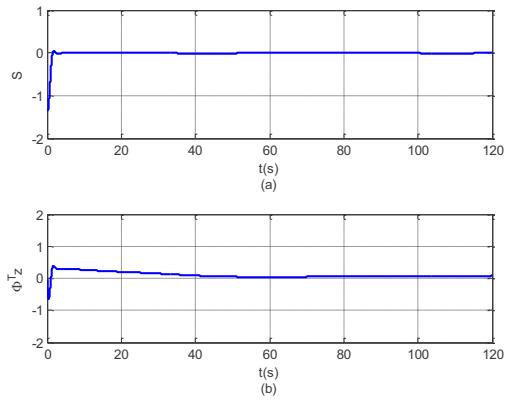


Fig. 8:  $s$  and  $\Phi^T(x, \hat{\theta}_1)z$

In the first simulation study, a step signal with  $\alpha_d = 10^\circ$  is defined as the reference trajectory. Fig. 3 shows the angle of attack and pitch rate trajectories. Fig. 3 (a) indicates the proposed I&I adaptive controller has a fast response characteristic and a great tracking performance. The angle of attack increases to the command without steady state error in a limited time, which is a very important characteristic for the inner loop of attitude control. Fig. 3 (b) and (c) are the pitch rate response and the partial enlarged view for the first

ten seconds of the pitch rate response, respectively. They show the pitch rate first increases rapidly, and then decreases to zero when the angle of attack reaches the desired value. The deflections of the elevator are shown in Fig. 4. Fig. 4 (a) and (b) are the elevator deflection trajectory and the partial enlarged view for the first ten seconds, respectively. This figure demonstrates the elevator conforms to the deflection limits of  $\pm 20^\circ$  and rate limits of  $\pm 50^\circ/\text{s}$ , although the transient process behaves a slightly worse. Fig. 5 shows both  $s$  and  $\Phi^T(x, \hat{\theta}_1)z$  convergent to zero. According to the above analyzed, this implies the tracking error satisfies that  $\lim_{t \rightarrow \infty} \tilde{\alpha} = 0$ .

A more aggressive maneuver is considered in the second case. The reference trajectory of the angle of attack is set as  $\alpha_d = 3\sin(0.1t) + 5$ . Fig. 6 shows the angle of attack and pitch rate trajectories. It again shows the angle of attack closely follows the reference trajectory, which confirms the proposed I&I adaptive controller responses fast and has an accurate tracking. Fig. 7 is the time history of the elevator deflections. Due to the big difference at the initial time, the elevator deflects severely at first. But when the angle of attack closes to the desired value, the deflection becomes smooth. Fig. 8 shows both  $s$  and  $\Phi^T(x, \hat{\theta}_1)z$  convergent to zero, too.

## 6 Conclusion

It is well-known how to overcome various uncertainties is a very important problem for the design of hypersonic flight control system. This paper aims to design a nonlinear adaptive flight controller based on immersion and invariance to deal with this problem. A brief theoretical introduction of the I&I theory is addressed. This approach can naturally lead a control law and a Lyapunov function when the stability of the system is analyzed. An adaptive controller based on the I&I approach for the angle of attack and pitch rate subsystem is proposed, under the condition where all aerodynamic coefficients are treated as unknown parameters. Two kinds of cases are simulated in the generic hypersonic vehicle longitudinal model. Simulation results show the effectiveness of this methodology. Note that the proposed approach can also be applied on the other hypersonic subsystems and used to deal with other uncertainties.

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