

# Direct Adaptive Type-2 Fuzzy Logic Control of a Generic Hypersonic Flight Vehicle

Fang. Yang, Ruyi. Yuan, Jianqiang. Yi, Guoliang. Fan and Xiangmin. Tan

**Abstract**—A direct adaptive type-2 fuzzy logic controller is designed in this paper. The longitudinal dynamics of a generic hypersonic flight vehicle is high-order, highly nonlinear, tight coupling and most of all includes big uncertainties. The computing of the dynamic inversion control signal is cost ineffective and an adaptive interval type-2 fuzzy logic system is used to approximate it. A  $H_\infty$  controller is implemented in order to attenuate the fuzzy approximation error and the system uncertainty. Signals' high-order derivatives are obtained by tracking differentiators and nonlinear state observers. The closed-loop stability is guaranteed by Lyapunov theory. Simulation results validate the effectiveness and robustness of the proposed controller.

## I. INTRODUCTION

HYPERSONIC flight vehicle (HFV) flies at a speed of more than 5 Mach within a very complicated environment. Due to its large thrust to weight ratio, HFV can be used as reusable orbital transport plane and intercontinental airliner. Although HFV has many application advantages, its flight control law design is highly challenging. The longitudinal dynamics of a generic hypersonic flight vehicle (GHFV) is high-order, highly nonlinear, tight coupling and most of all includes big uncertainties. Furthermore, the signals' high-order derivatives are difficult to measure. So the design of robust controller has caused extensive concern. If there is no uncertainty, the dynamic inversion (DI) control law can be a good control scheme. Many control methods based on DI control have been proposed [1], [2].

Interval type-2 fuzzy set (IT2-FS) is characterized by membership functions (MFs) which are themselves fuzzy as in Fig. 1(a), whereas MF of type-1 fuzzy set (T1-FS) is crisp as in Fig. 1(b). The domain between the upper MF (UMF) and the lower MF (LMF) is called footprint of uncertainty (FOU). Secondary MFs in FOU all equal 1. Interval type-2 fuzzy logic system (IT2-FLS) can be more capable of dealing with uncertain problems than traditional type-1 fuzzy logic system (T1-FLS). The structure of IT2-FLS is shown in Fig. 2. The type reduction of IT2-FLS just involves computing LMF and UMF of the antecedent and consequent sets. A direct adaptive interval type-2 fuzzy logic controller is proposed for a

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two-link manipulator [3]. It works well even when the system is corrupted by random noise. Adaptive IT2-FLS is more often used to approximate unknown nonlinear system online and it does not need any prior knowledge [4].

Some works have been done on type-1 fuzzy logic control of hypersonic flight vehicle. T1-FLS is used to approximate the uncertain terms of the linearized model [5]. T1-FLS is also used to approximate the nonlinear terms in backstepping control [6]. Multistage type-1 fuzzy logic controllers are used to stabilize the outer and inner loop of the flight altitude [7]. An indirect interval type-2 fuzzy logic controller for GHFV is discussed as in [8].

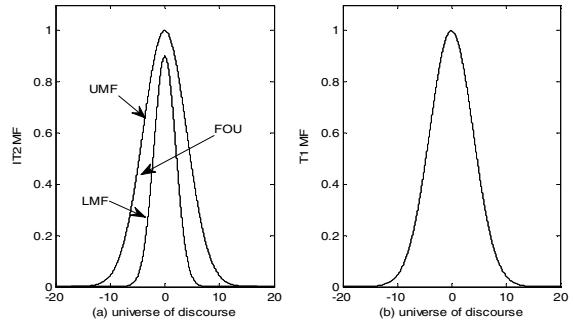


Fig. 1. (a) MF example of IT2-FS; (b) MF example of T1-FS.

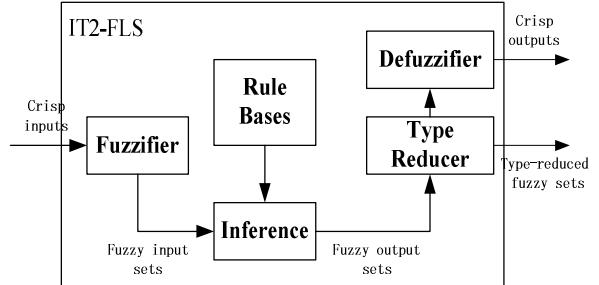


Fig. 2. The structure of IT2-FLS.

Here, a direct adaptive interval type-2 fuzzy logic controller (DAIT2-FLC) is first adopted in the tracking control of hypersonic flight vehicle. We use IT2-FLS to directly obtain the fuzzy control signals which approximate the dynamic inversion control signals.  $H_\infty$  controller is implemented to attenuate the fuzzy approximation error and the system uncertainty. This paper is organized as follows: Section 2 describes the control problem and gives some preliminary knowledge. Section 3 designs the controller in detail and also gives stability analysis. In section 4, simulations are conducted to validate the proposed controller. Conclusions are given in the final part.

## II. PROBLEM STATEMENT AND PRELIMINARIES

### A. GHFV Model and Feedback Linearization

The longitudinal dynamics of the GHFV and its engine dynamics are given by (1)-(6) [1], [2], [9]

$$\dot{V} = \frac{T \cos \alpha - D}{m} - \frac{\mu \sin \gamma}{r^2} \quad (1)$$

$$\dot{\gamma} = \frac{L + T \sin \alpha}{mV} - \frac{(\mu - V^2 r) \cos \gamma}{V r^2} \quad (2)$$

$$\dot{h} = V \sin \gamma \quad (3)$$

$$\dot{\alpha} = q - \dot{\gamma} \quad (4)$$

$$\dot{q} = M_y / I_y \quad (5)$$

$$\beta = -2\xi_t \omega \beta - \omega^2 \beta + \omega^2 \delta_t \quad (6)$$

where the forces and the pitch moment are  $L = \frac{1}{2} \rho V^2 s C_L$ ,

$D = \frac{1}{2} \rho V^2 s C_D$ ,  $T = \frac{1}{2} \rho V^2 C_T$ ,  $M_y = \frac{1}{2} \rho V^2 s \bar{c} C_M$  and coefficient  $C_M = c \delta_e + C_M^0$ .

We use Lie derivatives to differentiate  $V$  three times and  $h$  four times separately and finally achieve the complete input/output linearized model [10]

$$\begin{cases} \ddot{V} = f_1 + g_{11} \delta_e + g_{12} \delta_t \\ h^{(4)} = f_2 + g_{21} \delta_e + g_{22} \delta_t \end{cases} \quad (7)$$

where  $f_i$  are unknown but bounded nonlinearities and  $g_{ij}$  are approximately known but bounded nonlinearities. The relative degrees of  $V$  and  $h$  are  $n_1=3$  and  $n_2=4$  respectively.

Denote the system input  $u \triangleq [u_1 \ u_2]^T = [\delta_e \ \delta_t]^T$ , output  $y \triangleq [y_1 \ y_2]^T = [V \ h]^T$ , reference command  $y_r \triangleq [y_{r1} \ y_{r2}]^T = [V_r \ h_r]^T$ , system uncertainty  $d \triangleq [d_1 \ d_2]^T$ , and control matrix

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \quad (8)$$

Hence system (1)-(6) can be rewritten as

$$\begin{bmatrix} y_1^{(3)} \\ y_2^{(4)} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + G \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \quad (9)$$

Assume that  $G$  is nonsingular and  $d_i$  is bounded. If the system is free of uncertainty, by dynamic inversion control law, the control signal can be

$$\begin{bmatrix} u_1^* \\ u_2^* \end{bmatrix} = G^{-1} \left( - \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} - \begin{bmatrix} k_1^T e_1 \\ k_2^T e_2 \end{bmatrix} + \begin{bmatrix} y_{r1}^{(3)} \\ y_{r2}^{(4)} \end{bmatrix} \right) \quad (10)$$

where  $e_1 = y_1 - y_{r1}$ ,  $e_2 = y_2 - y_{r2}$ ,  $\dot{e}_1 = [e_1 \ \dot{e}_1 \ \ddot{e}_1]^T$ ,  $\ddot{e}_2 = [e_2 \ \dot{e}_2 \ \ddot{e}_2 \ \dddot{e}_2]^T$ .  $k_i = [k_{i0}, \dots, k_{in_i-1}]^T$  ( $i=1, 2$ ) are coefficients such that the polynomials  $k_i^T e_i$  are Hurwitz.

### B. Interval Type-2 Fuzzy Logic System

The fuzzy rule bases consist of a collection of IF-THEN rules in the following form:

$R^i$ : If  $x_1$  is  $\widetilde{F}_1^i$  and  $x_2$  is  $\widetilde{F}_2^i$ , then  $\widehat{u}_1$  is  $F_3^i$  and  $\widehat{u}_2$  is  $F_4^i$   $i=1, 2, \dots, M$ , where  $M$  is the number of rules,  $x_1 = \Delta V$ ,  $x_2 = \Delta h$ .  $\widetilde{F}_1^i$  and  $\widetilde{F}_2^i$  are antecedent interval type-2 fuzzy sets (IT2-FS).  $F_3^i$  and  $F_4^i$  are consequent type-1 fuzzy sets (T1-FS). Denote  $x = [x_1 \ x_2]^T$  and the firing set  $F^i(x)$  and degree of firing  $f^i$  associated with the  $i^{\text{th}}$  rule are

$$f^i \in F^i(x) = \prod_{j=1}^2 \mu_{\widetilde{F}_j^i}(x_j) = [\underline{f}^i, \bar{f}^i] \quad (11)$$

$$\underline{f}^i = \prod_{j=1}^2 \underline{\mu}_{\widetilde{F}_j^i}(x_j), \bar{f}^i = \prod_{j=1}^2 \bar{\mu}_{\widetilde{F}_j^i}(x_j) \quad (12)$$

where  $\underline{\mu}_{\widetilde{F}_j^i}(x_j)$  and  $\bar{\mu}_{\widetilde{F}_j^i}(x_j)$  are the lower and upper MFs of  $\mu_{\widetilde{F}_j^i}(x_j)$  respectively. Assume  $y_1^i$  and  $y_2^i$  are the centroid of  $F_3^i$  and  $F_4^i$  which have the maximum membership value 1. By using the singleton fuzzification, product inference, centre-average defuzzification and the center-of-sets (COS) type reducer, the type-reduced T1-FS is given by [11]

$$\begin{aligned} \widehat{u}_{1 \text{ cos}} &= \int_{y_1^i} \dots \int_{y_1^M} \int_{f^1} \dots \int_{f^M} \prod_{i=1}^M \mu_{F_3^i}(y_1^i) \prod_{i=1}^M \mu_{F_4^i}(f^i) \Big/ \frac{\sum_{i=1}^M f^i y_1^i}{\sum_{i=1}^M f^i} \\ \widehat{u}_{2 \text{ cos}} &= \int_{y_2^i} \dots \int_{y_2^M} \int_{f^1} \dots \int_{f^M} \prod_{i=1}^M \mu_{F_3^i}(y_2^i) \prod_{i=1}^M \mu_{F_4^i}(f^i) \Big/ \frac{\sum_{i=1}^M f^i y_2^i}{\sum_{i=1}^M f^i} \end{aligned} \quad (13)$$

For IT2-FLS,  $F^i$  is T1-FS,  $\mu_{F^i}(f^i) = 1$ . Considering the definition of  $y_1^i$  and  $y_2^i$ ,  $\mu_{F_3^i}(y_1^i) = \mu_{F_4^i}(y_2^i) = 1$ , so

$$\widehat{u}_{1 \text{ cos}} = \int_{y_1^i} \dots \int_{y_1^M} \int_{f^1} \dots \int_{f^M} 1 \Big/ \frac{\sum_{i=1}^M f^i y_1^i}{\sum_{i=1}^M f^i} = [\widehat{u}_{1l}, \widehat{u}_{1r}] \quad (14)$$

$$\widehat{u}_{2 \text{ cos}} = \int_{y_2^i} \dots \int_{y_2^M} \int_{f^1} \dots \int_{f^M} 1 \Big/ \frac{\sum_{i=1}^M f^i y_2^i}{\sum_{i=1}^M f^i} = [\widehat{u}_{2l}, \widehat{u}_{2r}] \quad (15)$$

In the adaptive controller,  $y_1^i, y_2^i$  ( $i=1, \dots, M$ ) are free adaptive parameters and we use new symbols to denote

$$\theta_1 = (\theta_1^1, \theta_1^2, \dots, \theta_1^M)^T, \theta_2 = (\theta_2^1, \theta_2^2, \dots, \theta_2^M)^T \quad (16)$$

Then choose the fuzzy basis function with

$$\begin{aligned} \xi_{1l}^i &= \frac{f_{1l}^i}{\sum_{i=1}^M f_{1l}^i}, \xi_{1r}^i = \frac{f_{1r}^i}{\sum_{i=1}^M f_{1r}^i} \\ \xi_{2l}^i &= \frac{f_{2l}^i}{\sum_{i=1}^M f_{2l}^i}, \xi_{2r}^i = \frac{f_{2r}^i}{\sum_{i=1}^M f_{2r}^i} \end{aligned} \quad (17)$$

$$\xi_{1l} = (\xi_{1l}^1, \xi_{1l}^2, \dots, \xi_{1l}^M)^T, \xi_{1r} = (\xi_{1r}^1, \xi_{1r}^2, \dots, \xi_{1r}^M)^T$$

$$\xi_{2l} = (\xi_{2l}^1, \xi_{2l}^2, \dots, \xi_{2l}^M)^T, \xi_{2r} = (\xi_{2r}^1, \xi_{2r}^2, \dots, \xi_{2r}^M)^T \quad (18)$$

$$\xi_1 = (\xi_{1l} + \xi_{1r})/2, \xi_2 = (\xi_{2l} + \xi_{2r})/2 \quad (19)$$

where  $f_{1l}^i, f_{1r}^i, f_{2l}^i, f_{2r}^i$  denote the firing values used to compute the bounds  $\Delta_{1l}, \Delta_{1r}, \Delta_{2l}, \Delta_{2r}$  respectively which can

be obtained using the Karnik-Mendel iterative method [11]. By IT2-FLS theory, the uncertain term  $\hat{u}_1$  and  $\hat{u}_2$  can be achieved by

$$\hat{u}_1 = (\hat{u}_{1l} + \hat{u}_{1r})/2 = \theta_1^T \xi_1, \quad \hat{u}_2 = (\hat{u}_{2l} + \hat{u}_{2r})/2 = \theta_2^T \xi_2 \quad (20)$$

### III. CONTROL DESIGN

#### A. Overall Control Scheme

The overall control scheme is shown in Fig.3. High-order derivatives of  $y_r$  and  $y$  are obtained by tracking differentiator (TD) and nonlinear state observer (NSO) respectively. The direct adaptive IT2-FLC is used to get the fuzzy control signal  $\hat{u}$  which approximates the dynamic inversion control signal  $u^*$ .  $H_\infty$  control signal  $u_s$  is used to attenuate the fuzzy approximation error and the system uncertainty. The total control signal  $u$  is the subtraction of  $\hat{u}$  and  $G^{-1}u_s$ .

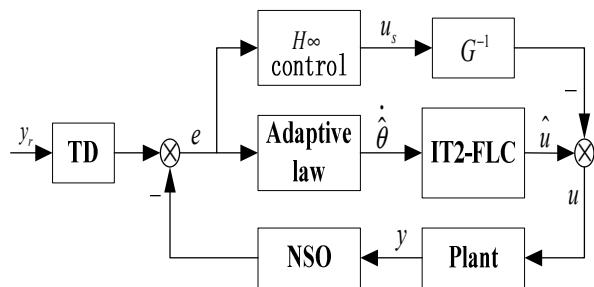


Fig. 3. The overall control scheme of DAIT2-FLS.

The control objective is to design elevator deflection  $\delta_e$  and throttle setting  $\delta_t$  so that the  $H_\infty$  tracking performance (21) can be achieved for a prescribed attenuation level  $\rho_h$ .

$$\int_0^{T_s} e^T Q e dt \leq e(0)^T P e(0) + \frac{1}{\eta} \tilde{\theta}^T(0) \tilde{\theta}(0) + \rho_h^2 \int_0^{T_s} w^T w dt \quad (21)$$

where  $Q=Q^T > 0$ ,  $P=P^T > 0$ ,  $\eta > 0$ . Details will be seen later.

#### B. Adaptive Law and Control Law

First of all, we try to get the closed-loop system. From (10), we get

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = -Gu^* - \begin{bmatrix} k_1^T e_1 \\ k_2^T e_2 \end{bmatrix} + \begin{bmatrix} y_{r1}^{(3)} \\ y_{r2}^{(4)} \end{bmatrix} \quad (22)$$

The total control signal is

$$u = \hat{u} - G^{-1}u_s = \begin{bmatrix} \hat{u}_1(x|\theta_1) \\ \hat{u}_2(x|\theta_2) \end{bmatrix} - G^{-1} \begin{bmatrix} u_{s1} \\ u_{s2} \end{bmatrix} \quad (23)$$

Substituting (22)-(23) into (9) and after some manipulation, we get

$$\begin{bmatrix} e_1^{(3)} + k_{12} \ddot{e}_1 + k_{11} \dot{e}_1 + k_{10} e_1 \\ e_2^{(4)} + k_{23} \ddot{e}_2 + k_{22} \dot{e}_2 + k_{21} e_2 + k_{20} e_2 \end{bmatrix} = G \left( \begin{bmatrix} \hat{u}_1(x|\theta_1) \\ \hat{u}_2(x|\theta_2) \end{bmatrix} - \begin{bmatrix} u_1^*(x) \\ u_2^*(x) \end{bmatrix} \right) + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + \begin{bmatrix} u_{s1} \\ u_{s2} \end{bmatrix} \quad (24)$$

From (24), it is obvious that  $H_\infty$  control signal compensates the fuzzy approximation error and system uncertainty.

**Definition** The optimal approximation of  $u^*$  is  $\hat{u}^*$ , where the optimal parameter is

$$\theta_i^* = \arg \min_{\theta_i \in \Omega_{\theta_i}} [\sup_{x \in \Omega_{\theta_i}} |\hat{u}_i(x|\theta_i) - u_i^*|] \quad (25)$$

By the universal approximation theorem, the optimal approximation error

$$w_{ci} = \sum_{j=1}^2 g_{ij} (\hat{u}_j(x|\theta_j^*) - u_j^*) \quad (26)$$

can be arbitrarily small. Denote  $\tilde{\theta}_i = (\hat{\theta}_i - \theta_i^*)$ ,  $w_i = w_{ci} + d_i$ ,  $\tilde{\theta} = [\tilde{\theta}_1 \ \tilde{\theta}_2]^T$ ,  $B_1 = [0 \ 0 \ 1]^T$ ,  $B_2 = [0 \ 0 \ 0 \ 1]^T$ ,

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_{10} & -k_{11} & -k_{12} \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_{20} & -k_{21} & -k_{22} & -k_{23} \end{bmatrix},$$

$$A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \quad B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}, \quad \xi = \begin{bmatrix} \xi_1 & 0 \\ 0 & \xi_2 \end{bmatrix}, \quad \tilde{\theta} = \begin{bmatrix} \tilde{\theta}_1 \\ \tilde{\theta}_2 \end{bmatrix}, \quad e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}.$$

Then the closed-loop system can be rewritten as

$$\dot{e} = Ae + BG\xi^T \tilde{\theta} + B(w - u_s) \quad (27)$$

**Theorem** For system (7), if the  $H_\infty$  control law  $u_s$ , the fuzzy adaptive law  $\dot{\tilde{\theta}}$  and the total control input are chosen as

$$u_s = \frac{1}{\lambda} B^T Pe \quad (28)$$

$$\dot{\tilde{\theta}} = \text{Proj}\{-\eta \xi^T G^T B^T Pe\} \quad (29)$$

$$u = \xi^T \tilde{\theta} - G^{-1}u_s \quad (30)$$

then the closed-loop system (27) will be stable and the  $H_\infty$  tracking performance (21) will be achieved for a prescribed disturbance attenuation level  $\rho_h$ , where  $\text{Proj}(\bullet)$  is a projection operator,  $\lambda > 0$ ,  $\eta > 0$  is the learning rate,  $Q=Q^T > 0$  is a prescribed weighting matrix, and  $P=P^T > 0$  is the positive definite solution of the Riccati-like equation

$$A^T P + PA + Q - PB \left( \frac{2}{\lambda} - \frac{1}{\rho_h^2} \right) B^T P = 0 \quad (31)$$

#### C. Stability Analysis

Consider the following Lyapunov function candidate

$$V_L = \frac{1}{2} e^T P e + \frac{1}{2\eta} \tilde{\theta}^T \tilde{\theta} \quad (32)$$

The time derivative of  $V_L$  along the system trajectory is

$$\dot{V}_L = \frac{1}{2} \dot{e}^T P e + \frac{1}{2} e^T P \dot{e} + \frac{1}{2\eta} \dot{\tilde{\theta}}^T \tilde{\theta} \quad (33)$$

Substituting (27), (28), (31) to (33), we get

$$\begin{aligned}\dot{V}_L &= -\frac{1}{2}e^T Q e - \frac{1}{2} \left( \frac{1}{\rho_h} B^T P e \right)^T \left( \frac{1}{\rho_h} B^T P e \right) \\ &\quad + \frac{1}{2} \rho_h^2 w^T w + \tilde{\theta}^T \left( \frac{1}{\eta} \dot{\hat{\theta}} + \xi G^T B^T P e \right)\end{aligned}\quad (34)$$

So the adaptive law can be chosen as

$$\dot{\hat{\theta}} = -\eta \xi G^T B^T P e \quad (35)$$

Since  $\frac{1}{2} \left( \frac{1}{\rho_h} B^T P e \right)^T \left( \frac{1}{\rho_h} B^T P e \right) \geq 0$ , we get

$$\dot{V}_L \leq -\frac{1}{2}e^T Q e + \frac{1}{2} \rho_h^2 w^T w \quad (36)$$

Integrating (36) from  $t=0$  to  $t=T_s$  and considering  $V_L(T_s) \geq 0$ , we get the  $H_\infty$  tracking performance (21).

Moreover, we use projection operator [12] to guarantee  $\hat{\theta}$  is in the allowed compact set.

#### D. Tracking Differentiator and Nonlinear State Observer

High-order derivatives of  $V$ ,  $V_r$  and  $h$ ,  $h_r$  are used when computing error  $e$ . Here we use tracking differentiator (TD) to get high-order derivatives of  $V_r$  and  $h_r$ , use nonlinear state observer (NSO) to get high-order derivatives of  $V$  and  $h$  [13]. TD is also used to form arranged transition process (ATP) and filter high frequency noise. The discrete algorithms implemented in TD are as follows:

Velocity channel:

$$\begin{cases} f_{s_1}(k) = -r(r(r(v_{11}(k)-V_c)+3v_{12}(k))+3v_{13}(k)) \\ v_{11}(k+1) = v_{11}(k) + \tau * v_{12}(k) \\ v_{12}(k+1) = v_{12}(k) + \tau * v_{13}(k) \\ v_{13}(k+1) = v_{13}(k) + \tau * f_{s_1}(k) \end{cases} \quad (37)$$

Altitude channel:

$$\begin{cases} f_{s_2}(k) = -r(r(r(r(v_{21}(k)-h_c)+4v_{22}(k))+6v_{23}(k))+4v_{24}(k)) \\ v_{21}(k+1) = v_{21}(k) + \tau * v_{22}(k) \\ v_{22}(k+1) = v_{22}(k) + \tau * v_{23}(k) \\ v_{23}(k+1) = v_{23}(k) + \tau * v_{24}(k) \\ v_{24}(k+1) = v_{24}(k) + \tau * f_{s_2}(k) \end{cases} \quad (38)$$

where “velocity factor”  $r$  and time step  $\tau$  are taken as 1 and 0.01 separately. In hypersonic flight environment, it is very difficult to measure or compute the exact flight states and their high derivatives. NSO can estimate them online. The discrete algorithms implemented in NSO are as follows:

Velocity channel:

$$\begin{cases} e = z_{11} - V \\ z_{11}(k+1) = z_{11}(k) + \tau * (z_{12}(k) - \beta_{11} * e) \\ z_{12}(k+1) = z_{12}(k) + \tau * (z_{13}(k) - \beta_{12} * fal(e, 0.5, \tau)) \\ z_{13}(k+1) = z_{13}(k) + \tau * (-\beta_{13} * fal(e, 0.25, \tau) + U1) \end{cases} \quad (39)$$

Altitude channel:

$$\begin{cases} e = z_{21} - h \\ z_{21}(k+1) = z_{21}(k) + \tau * (z_{22}(k) - \beta_{21} * e) \\ z_{22}(k+1) = z_{22}(k) + \tau * (z_{23}(k) - \beta_{22} * fal(e, 0.5, \tau)) \\ z_{23}(k+1) = z_{23}(k) + \tau * (z_{24}(k) - \beta_{23} * fal(e, 0.25, \tau)) \\ z_{24}(k+1) = z_{24}(k) + \tau * (-\beta_{24} * fal(e, 0.125, \tau) + U2) \end{cases} \quad (40)$$

where  $fal(x, a, \delta) = xf_{sg}(x, \delta) / \delta^{1-a} + |f_{db}(x, \delta)| |x|^a sign(x)$ ,  $f_{sg}(x, \delta) = (sign(x+\delta) - sign(x-\delta)) / 2$  and  $\beta_{11} = 100$ ,  $\beta_{12} = 300$ ,  $\beta_{13} = 2000$ ,  $\beta_{21} = 100$ ,  $\beta_{22} = 300$ ,  $\beta_{23} = 1000$ ,  $\beta_{24} = 1800$ .

## IV. SIMULATIONS

At the trimmed condition  $V_0 = 4590.3m/s$ ,  $h_0 = 33528m$ ,  $\gamma_0 = 0rad$ ,  $q_0 = 0rad/s$ ,  $\alpha_0 = 0.04799rad$ ,  $\delta_{t0} = 0.2124$ ,  $\delta_{e0} = -0.5507^\circ$ , the reference command signals are chosen as  $\Delta V = 0$  and  $\Delta h = 600 \sin(0.04\pi t)$  whose period is 50 seconds. Learning rate  $\eta = 0.01$  and  $\lambda = 0.001$ . Disturbance attenuation level is chosen as  $\rho_h = 0.05$ . Weighting matrix  $Q = 10I_7$  where  $I_7$  is 7-dimension identity matrix.  $k_{10} = 1$ ,  $k_{11} = 3$ ,  $k_{12} = 3$ ,  $k_{20} = 1$ ,  $k_{21} = 4$ ,  $k_{22} = 6$ ,  $k_{23} = 4$ . The antecedent sets  $\widetilde{F}_1$  and  $\widetilde{F}_2$  each has 5 Gaussian fuzzy sets with centers -5, -2, 0, 2, 5, and the rule number  $M = 5 \times 5 = 25$ . For IT2-FS, the variances of the Gaussian UMF are  $\sigma_{v1} = 1$ ,  $\sigma_{h1} = 1$  whereas the variances of the Gaussian LMF are  $\sigma_{v2} = 0.2$ ,  $\sigma_{h2} = 0.2$ ; For T1-FS, the variances of the Gaussian MF are  $\sigma_v = 1$ ,  $\sigma_h = 1$ . The initial values of the adaptive consequent sets parameter vector  $\theta$  are randomly chosen. The parameter uncertainties are chosen as

$$\begin{cases} m = m_0(1+U_f+U_g * GWN) \\ I_y = I_{y0}(1+U_f+U_g * GWN) \\ \rho = \rho_0(1+U_f+U_g * GWN) \\ s = s_0(1+U_f+U_g * GWN) \\ c = c_0(1+U_f+U_g * GWN) \end{cases} \quad (41)$$

where  $m_0, I_{y0}, \rho_0, s_0, c_0$  are in canonical parameter values,  $U_f$  is the fixed parameter uncertainty and  $U_g$  is the strength of the Gaussian white noise (GWN) whose power is 0.002. Simulations are conducted for IT2-FLC and T1-FLC in two circumstances: A) without uncertainty and B) with uncertainty.

### A. Simulation Without Uncertainty

Set  $U_f = U_g = 0$ , then all parameters are in canonical values and there exists no uncertainty. Conduct the simulation for 100 seconds in two cases: IT2-FLC and T1-FLC. The simulation results are shown in Fig. 4-Fig. 7.

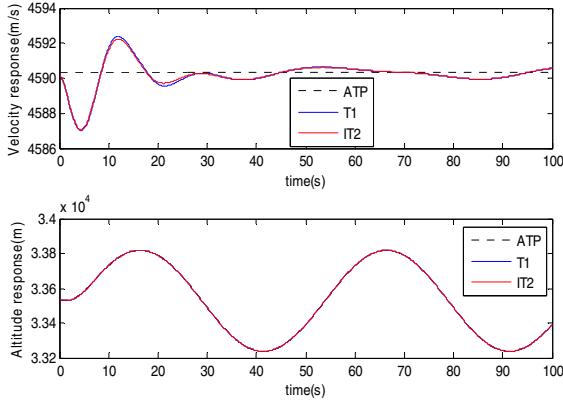


Fig. 4. Command response without uncertainty.

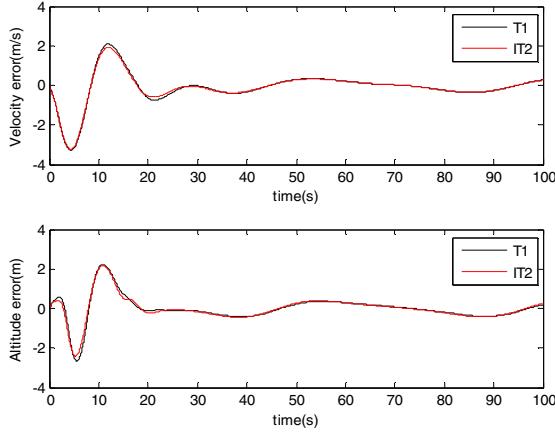


Fig. 5. Tracking error without uncertainty.

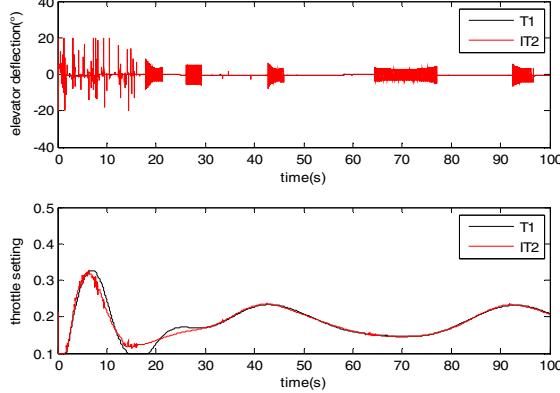


Fig. 6. Control signals without uncertainty.

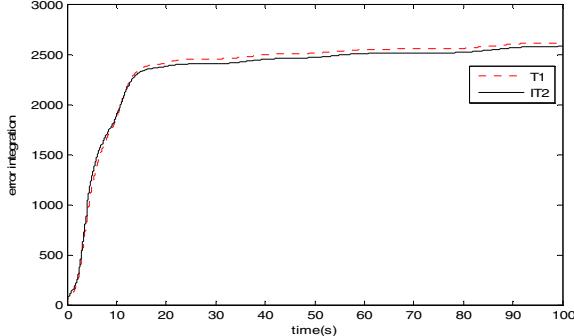


Fig. 7. Integration of tracking error without uncertainty

Fig. 4 and Fig. 5 show that IT2-FLC and T1-FLC have similar good tracking performance. Fig. 6 shows that the control signals are within control authority in both cases. Fig. 7 shows the integration of the tracking error  $\int_0^{100} e^T Q e dt$  which again indicates IT2-FLC and T1-FLC have very little difference in tracking when there is no uncertainty.

### B. Simulation With Uncertainty

Set  $U_f = 0.2$  and  $U_g = 0.8$ , then all parameters are corrupted by fixed uncertainty and noise. Conduct the simulation as chapter 4.1 does and the results are shown in Fig. 8-Fig. 11.

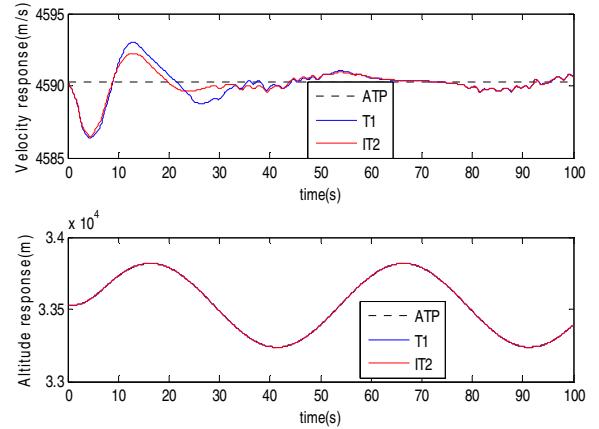


Fig. 8. Command response with uncertainty.

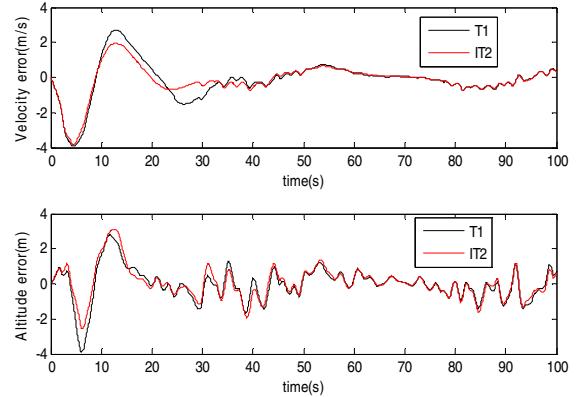


Fig. 9. Tracking error with uncertainty.

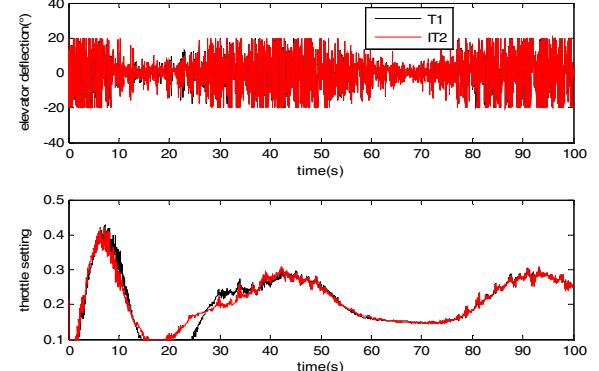


Fig. 10. Control signals with uncertainty.

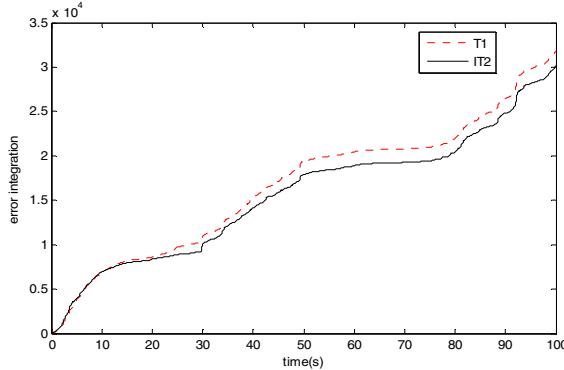


Fig. 11. Integration of tracking error with uncertainty.

Because of the strong white noise, there exists strong chattering in control signals as in Fig. 10, which results in tracking chattering as in Fig. 8 and Fig. 9. IT2-FLC has smaller tracking error and better tracking performance than T1-FLC as in Fig. 8, Fig. 9 and Fig. 11.

If we enlarge the fixed parameter uncertainty  $U_f$  to 0.3, IT2-FLC can still track well whereas T1-FLC will diverge after a few seconds (28s) as in Fig. 12 and Fig. 13. This validates IT2-FLS can be more capable of handling uncertain problems than T1-FLS.

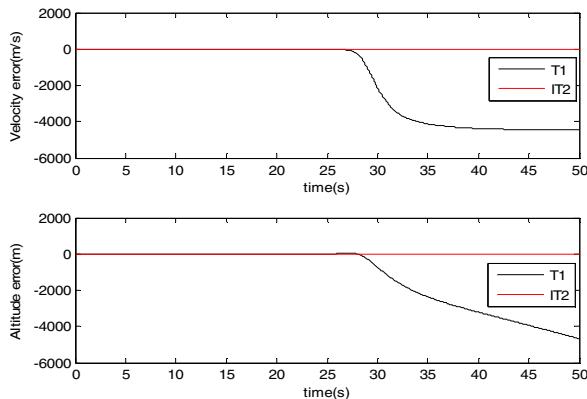


Fig. 12. Tracking error with big uncertainty.

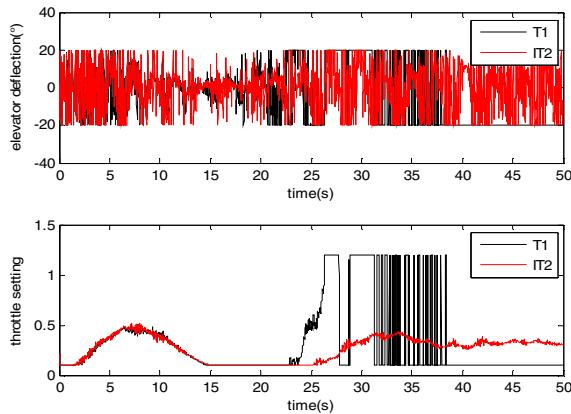


Fig. 13. Control signals with big uncertainty.

## V. CONCLUSION

In this paper, a direct adaptive type-2 fuzzy logic controller for a generic hypersonic flight vehicle was discussed.

IT2-FLS was used to directly obtain the control signals.  $H_\infty$  controller was implemented to attenuate the fuzzy approximation error and the system uncertainty. TD and NSO were used to obtain signals' high-order derivatives. When there existed no uncertainty, IT2-FLC and T1-FLC had similar control effects. When there existed uncertainties, IT2-FLC had better control effects. Especially when the uncertainties became much bigger, IT2-FLC could still be effective whereas T1-FLC failed. These results showed IT2-FLS is more capable of handling uncertain problems than T1-FLS.

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