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APPROACH PROPERTY OF INTERVAL TYPE-2 FUZZY SYSTEMS USING KM AND BMM TYPE-REDUCTION METHODS

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Abstract. *Currently most of type-2 fuzzy adaptive control (FAC) approaches use the iterative Karnik-Mendel (KM) algorithm in type-reduction (TR), but other TR methods are rarely employed. This paper applies KM algorithm and Begian-Melek-Mendel (BMM) into a class of uncertain nonlinear single-input single-output (SISO) systems. And we definitely prove that the interval type-2 fuzzy logic systems (IT2 FLSs) using the KM, BMM methods are universal approximators. Numerical simulation results demonstrate that the BMM TR methods outperform the KM algorithm.*

Keywords: Interval type-2 FLSs, Type-reduction, Universal approximator

1. Introduction.

Type-1 fuzzy sets (T1 FSs) were first proposed by Zadeh in 1965 [1], and then in 1975 [2] as an extension to T1 FSs Zadeh further proposed Type-2 fuzzy sets (T2 FSs). Since the memberships in a T2 FS are T1 FSs, T2 FSs and IT2 fuzzy logic systems (FLSs) can better handle different sources of uncertainties than their T1 counterparts. There have been many different approaches for type-reduction computation which include the KM algorithm and alternative TR methods such as Begian-Melek-Mendel (BMM) Method [3]. Zhou et al. [4] have used an indirect adaptive IT2 FLC to achieve H_∞ tracking performance for a class of uncertain nonlinear SISO systems with external disturbances via YD type-reduction algorithm [5]. So we raise two key questions as follows. Firstly, except YD TR algorithm, can other TR methods be applied into estimators in adaptive fuzzy control (AFC)? Secondly, are these FLSs using the TR methods universal approximator? So far, except YD type-reduction method, most TR methods are not definitely proved whether they are of the property of universal approximator yet. In this paper, we not only formulate that the KM and BMM methods can be applied into the estimators in IT2 AFC, but also we first prove that the KM and BMM type-reducers are capable of uniformly approximating any nonlinear function over a compact set to any desired accuracy.

The paper is organized as follows. IT2 FLSs are briefly introduced in Section 2. We prove that the IT2 FLSs using the the KM and BMM TR methods are universal approximators in Section 3. A simulation example is given to compare the performance of the different TR methods in Section 4. Finally, Section 5 makes conclusions.

2. Interval Type-2 Fuzzy Logic Systems.

Assume the type-2 fuzzy rule base is composed of M IF-THEN fuzzy rules, and the rule $R^{(j_1\cdots j_n)}$ is denoted as

$$R^{(j_1\cdots j_n)}: \text{ IF } x_1 \text{ is } \widetilde{A}_1^{j_1} \text{ and } x_2 \text{ is } \widetilde{A}_2^{j_2} \text{ and } \ldots \text{and } x_n \text{ is } \widetilde{A}_n^{j_n} \text{ THEN } y \text{ is } \widetilde{B}^{(j_1\cdots j_n)} \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ is the input variable to the fuzzy system, $i = 1, 2, \ldots, n$, and y is the output variable of the fuzzy system. $\widetilde{A}_i^{j_i}$s are type-2 fuzzy sets, $j_i = 1, 2, \ldots, m_i$, there are $M = \prod_{i=1}^{n} m_i$ fuzzy rules in the base. The lower and upper membership functions of the IT2 FSs $\widetilde{A}_i^{j_i}$ are expressed as $\underline{\mu}_{\widetilde{A}_i^{j_i}}(x_i) = \underline{a}_i^{j_i} \exp\left(-\left(\frac{x_i - m_i^{j_i}}{\underline{\sigma}_i^{j_i}}\right)^2\right)$ and $\overline{\mu}_{\widetilde{A}_i^{j_i}}(x_i) = \overline{a}_i^{j_i} \exp\left(-\left(\frac{x_i - m_i^{j_i}}{\overline{\sigma}_i^{j_i}}\right)^2\right)$, respectively. Here $\underline{a}_i^{j_i}$, $\overline{a}_i^{j_i}$, $m_i^{j_i}$, $\underline{\sigma}_i^{j_i}$ and $\overline{\sigma}_i^{j_i}$ are real-valued parameters with $0 < \underline{a}_i^{j_i} \leq \overline{a}_i^{j_i} < 1$ and $\underline{\sigma}_i^{j_i} < \overline{\sigma}_i^{j_i}$. Assume the adjustable consequent parameters $\underline{y}^j$ and $\overline{y}^j$ are the points at which their membership functions achieve their maximum values, where j corresponds to an ordered grid-oriented multi-index $(j_1, j_2, \ldots, j_n)$. And we choose the membership functions of the output variable as $\underline{\mu}_{\widetilde{B}^j}(\underline{y}^j) = b$, $\overline{\mu}_{\widetilde{B}^j}(\overline{y}^j) = 1$, $0 < b \leq 1$.

To compare the IT2 FLSs with T1 FLSs, we consider the T1 fuzzy rule which is identical to the above IT2 fuzzy rule $R^{(j_1 \cdots j_n)}$ except that the fuzzy membership functions $\mu_{A_i^{j_i}}(x_i)$ and $\mu_{B^{(j_1 \cdots j_n)}}(z)$ of the input variable and the output variable are the upper membership functions of the IT2 FSs $\widetilde{A}_i^{j_i}$ and $\widetilde{B}^{(j_1 \cdots j_n)}$, respectively.

3. An IT2 FLS Using KM or BMM TR Methods.

In this section, we will give the FBFs and FBF expansions of the FLSs using the KM and BMM methods, and prove that the FLSs using the two TR methods are universal approximators.

3.1. The Karnik-Mendel algorithm.

In this subsection, we will give the theorem that the FLSs using the KM algorithm are universal approximators.

Theorem 3.1. *Consider IT2 FLSs (1). For any given real continuous function $g(\boldsymbol{x})$ on the compact set $U \subset \mathbb{R}$ and arbitrary $\epsilon > 0$, there exists $y(\boldsymbol{x}) \in \widetilde{Y}$ such that $\sup_{x \in U} |g(\boldsymbol{x}) - y(\boldsymbol{x})| < \epsilon$ where $y : U \subset \mathbb{R}^n \to \mathbb{R}$, $\boldsymbol{x} \in U$; $\widetilde{Y}$ is the set of all the fuzzy basis function expansions (2).*

$$y(\boldsymbol{x}) = \begin{pmatrix} \boldsymbol{\xi}_l^T Q_l^{-1} & \boldsymbol{\xi}_r^T Q_r^{-1} \end{pmatrix} \begin{pmatrix} \underline{\boldsymbol{y}} \\ \overline{\boldsymbol{y}} \end{pmatrix} \tag{2}$$

where both Q_l and Q_r are the matrices of row-switching transformations which make the elements in vectors $\underline{\boldsymbol{y}}$ and $\overline{\boldsymbol{y}}$ arranged in ascending order, and $Q_l = Q_l^{-1}$, $Q_r = Q_r^{-1}$, $\boldsymbol{\xi}_l = \begin{pmatrix} \xi_l^1 & \ldots & \xi_l^L & \xi_l^{L+1} & \ldots & \xi_l^M \end{pmatrix}^T$, $\boldsymbol{\xi}_r = \begin{pmatrix} \xi_r^1 & \ldots & \xi_r^R & \xi_r^{R+1} & \ldots & \xi_r^M \end{pmatrix}^T$, Here

$$\xi_l^s = \frac{f_l^s}{2\left(\sum_{i=1}^{L} \overline{f}^i + \sum_{i=L+1}^{M} \underline{f}^i\right)} \tag{3}$$

$$\xi_r^t = \frac{f_r^t}{2\left(\sum_{i=1}^{R} \underline{f}^i + \sum_{i=R+1}^{M} \overline{f}^i\right)} \tag{4}$$

in which L and R can be computed by the iterative Karnik-Mendel algorithm. If $1 \leq s \leq L$, then $f_l^s = \overline{f}^i$; if $L+1 \leq s \leq M$, then $f_l^s = \underline{f}^i$. If $1 \leq t \leq R$, then $f_r^t = \underline{f}^i$; If $R+1 \leq t \leq M$, then $f_r^t = \overline{f}^i$. ξ_l^s and ξ_r^t $(s, t = 1, \ldots, M)$ are fuzzy basis functions.

Proof: Assume the elements in the vectors $\underline{\boldsymbol{y}}$ and $\overline{\boldsymbol{y}}$ are arranged in ascending order as $\underline{y}_a^1 \leq \underline{y}_a^2 \leq \ldots \leq \underline{y}_a^M$; $\overline{y}_a^1 \leq \overline{y}_a^2 \leq \ldots \leq \overline{y}_a^M$, and let $\underline{\boldsymbol{y}}_a = [\underline{y}_a^1, \underline{y}_a^2, \ldots, \underline{y}_a^M]^T$ and $\overline{\boldsymbol{y}}_a = [\overline{y}_a^1, \overline{y}_a^2, \ldots, \overline{y}_a^M]^T$. Then, it follows that

$$Q_l \underline{\boldsymbol{y}} = \underline{\boldsymbol{y}}_a \tag{5}$$

$$Q_r \overline{\boldsymbol{y}} = \overline{\boldsymbol{y}}_a \tag{6}$$

According to the KM TR algorithm [6], we can obtain that

$$y(\boldsymbol{x}) = \begin{pmatrix} \boldsymbol{\xi}_l^T & \boldsymbol{\xi}_r^T \end{pmatrix} \begin{pmatrix} \underline{\boldsymbol{y}}_a \\ \overline{\boldsymbol{y}}_a \end{pmatrix} \tag{7}$$

Substituting (5) and (6) into (7), we deduce (2).

Since both the upper and the lower membership functions are Gaussian membership functions, without loss of generality, we can rewrite the membership functions used in (3) as $\mu^l_{\tilde{A}_i^{j_i^l}}(x_i)$ where $j_i^l \cong j_i$ in (3). Similarly, the membership functions used in (4) are rewritten as $\mu^r_{\tilde{A}_i^{j_i^r}}(x_i)$ where $j_i^r \cong j_i$ in (4). So Equation (7) is expressed as

$$y(\boldsymbol{x}) = \frac{\sum_{j_1^l=1}^{m_1} \cdots \sum_{j_n^l=1}^{m_n} \underline{y}^{j_1\cdots j_n} \Pi_{i=1}^n \mu^l_{\tilde{A}_i^{j_i^l}}(x_i)}{2\sum_{j_1^l=1}^{m_1} \cdots \sum_{j_n^l=1}^{m_n} \Pi_{i=1}^n \mu^l_{\tilde{A}_i^{j_i^l}}(x_i)} + \frac{\sum_{j_1^r=1}^{m_1} \cdots \sum_{j_n^r=1}^{m_n} \overline{y}^{j_1\cdots j_n} \Pi_{i=1}^n \mu^r_{\tilde{A}_i^{j_i^r}}(x_i)}{2\sum_{j_1^r=1}^{m_1} \cdots \sum_{j_n^r=1}^{m_n} \Pi_{i=1}^n \mu^r_{\tilde{A}_i^{j_i^r}}(x_i)} \tag{8}$$

By reduction of fractions in (8) to a common denominator, we obtain (9).

$$y(\boldsymbol{x}) = \frac{\sum_{j_1^l=1}^{m_1} \cdots \sum_{j_n^l=1}^{m_n} \sum_{j_1^r=1}^{m_1} \cdots \sum_{j_n^r=1}^{m_n} \left(\frac{\underline{y}^{j_1^l\cdots j_n^l} + \overline{y}^{j_1^r\cdots j_n^r}}{2} \right) \Pi_{i=1}^n \mu^l_{\tilde{A}_i^{j_i^l}}(x_i) \mu^r_{\tilde{A}_i^{j_i^r}}(x_i)}{\sum_{j_1^l=1}^{m_1} \cdots \sum_{j_n^l=1}^{m_n} \sum_{j_1^r=1}^{m_1} \cdots \sum_{j_n^r=1}^{m_n} \Pi_{i=1}^n \mu^l_{\tilde{A}_i^{j_i^l}}(x_i) \mu^r_{\tilde{A}_i^{j_i^r}}(x_i)} \tag{9}$$

Since both $\mu^l_{\tilde{A}_i^{j_i^l}}(x_i)$ and $\mu^r_{\tilde{A}_i^{j_i^l}}(x_i)$ are Gaussian in form, their product $\mu^l_{\tilde{A}_i^{j_i^l}}(x_i)\mu^r_{\tilde{A}_i^{j_i^r}}(x_i)$ is also Gaussian in form. Hence, (9) is in the form of (4) in [7]. According to the theorem in [7], we can deduce that the type-2 fuzzy systems are universal approximators.

3.2. **The Begian-Melek-Mendel (BMM) method.** Begian et al. [3] proposed a closed-form type-reduction and defuzzification method for TSK interval type-2 fuzzy logic controllers. The BMM method requires $\underline{y}^n = \overline{y}^n = y^n$. Li et al. [8] extended it to the case that $\underline{y}^n \neq \overline{y}^n$, i.e.,

$$y(\boldsymbol{x}) = \alpha \frac{\sum_{i=1}^M \underline{f}^i(\boldsymbol{x}) \underline{y}^i}{\sum_{i=1}^M \underline{f}^i(\boldsymbol{x})} + (1-\alpha) \frac{\sum_{i=1}^M \overline{f}^i(\boldsymbol{x}) \overline{y}^i}{\sum_{i=1}^M \overline{f}^i(\boldsymbol{x})} \tag{10}$$

Here, we apply the BMM method into the IT2 FLSs described by (1), and assume the adjustable parameter vector is $\boldsymbol{\theta}_{t2} = \begin{pmatrix} \underline{\boldsymbol{y}}^T & \overline{\boldsymbol{y}}^T \end{pmatrix}^T$ where $\underline{\boldsymbol{y}} = [\underline{y}^1, \underline{y}^2, \ldots, \underline{y}^M]^T$, $\overline{\boldsymbol{y}} = [\overline{y}^1, \overline{y}^2, \ldots, \overline{y}^M]^T$. Then we propose the following theorem:

Theorem 3.2. *Consider the IT2 FLSs (1). For any given real continuous function $g(\boldsymbol{x})$ on the compact set $U \subset \mathbb{R}$ and arbitrary $\epsilon > 0$, there exists $y(\boldsymbol{x}) \in \check{Y}$ such that $\sup_{x\in U} |g(\boldsymbol{x}) - y(\boldsymbol{x})| < \epsilon$ where $y : U \subset \mathbb{R}^n \to \mathbb{R}$, $\boldsymbol{x} \in U$; $\check{Y}$ is the set of all the fuzzy basis function expansions (11).*

$$y(\boldsymbol{x}) = \begin{pmatrix} \alpha \check{\boldsymbol{\xi}}_l^T & (1-\alpha)\check{\boldsymbol{\xi}}_r^T \end{pmatrix} \begin{pmatrix} \underline{\boldsymbol{y}} \\ \overline{\boldsymbol{y}} \end{pmatrix} \tag{11}$$

where $\check{\boldsymbol{\xi}}_l = \begin{pmatrix} \check{\xi}_l^1 & \ldots & \check{\xi}_l^M \end{pmatrix}$, $\check{\boldsymbol{\xi}}_r = \begin{pmatrix} \check{\xi}_r^1 & \ldots & \check{\xi}_r^M \end{pmatrix}$, (12) are fuzzy basis functions.

$$\check{\xi}_l^i = \frac{\underline{f}^i}{\sum_{i=1}^M \underline{f}^i}, \qquad \check{\xi}_r^i = \frac{\overline{f}^i}{\sum_{i=1}^M \overline{f}^i} \tag{12}$$

Proof: Substituting (12) into (10), we can obtain (11). To differentiate j_i in the different terms, we rewrite the j_i in the lower membership function as j_i^l, and the j_i in the upper membership function as j_i^r. Equation (10) can be rewritten as

$$y(\boldsymbol{x}) = \alpha \frac{\sum_{j_1^l=1}^{m_1} \cdots \sum_{j_n^l=1}^{m_n} \underline{y}^{j_1^l \cdots j_n^l} \Pi_{i=1}^n \underline{\mu}_{\tilde{A}_i^{j_i^l}}(x_i)}{\sum_{j_1^l=1}^{m_1} \cdots \sum_{j_n^l=1}^{m_n} \Pi_{i=1}^n \underline{\mu}_{\tilde{A}_i^{j_i^l}}(x_i)} + (1-\alpha) \frac{\sum_{j_1^r=1}^{m_1} \cdots \sum_{j_n^r=1}^{m_n} \overline{y}^{j_1^r \cdots j_n^r} \Pi_{i=1}^n \overline{\mu}_{\tilde{A}_i^{j_i^r}}(x_i)}{\sum_{j_1^r=1}^{m_1} \cdots \sum_{j_n^r=1}^{m_n} \Pi_{i=1}^n \overline{\mu}_{\tilde{A}_i^{j_i^r}}(x_i)} \tag{13}$$

By reduction of fractions in (13) to a common denominator, we can deduce (14), and (14) is in the form of (4) in [7]. According to the theorem in [7], we can deduce that the type-2 fuzzy systems are universal approximators.

$$y(\boldsymbol{x}) = \frac{\sum_{j_1^l=1}^{m_1} \cdots \sum_{j_n^l=1}^{m_n} \sum_{j_1^r=1}^{m_1} \cdots \sum_{j_n^r=1}^{m_n} \left(\alpha \underline{y}^{j_1^l \cdots j_n^l} + (1-\alpha)\overline{y}^{j_1^r \cdots j_n^r}\right) \Pi_{i=1}^n \underline{\mu}_{\tilde{A}_i^{j_i^l}}(x_i) \overline{\mu}_{\tilde{A}_i^{j_i^r}}(x_i)}{\sum_{j_1^l=1}^{m_1} \cdots \sum_{j_n^l=1}^{m_n} \sum_{j_1^r=1}^{m_1} \cdots \sum_{j_n^r=1}^{m_n} \Pi_{i=1}^n \underline{\mu}_{\tilde{A}_i^{j_i^l}}(x_i) \overline{\mu}_{\tilde{A}_i^{j_i^r}}(x_i)} \tag{14}$$

4. **Simulation.** In this section, we consider the nonlinear system (15) in [9]. The control objective is to force y to follow a given bounded reference signal $y_m = 0.2\sin(t)$, and the indirect H_∞ adaptive control scheme in [10] is used. The IT2 FLSs using different TR methods are employed to approach the unknown functions. The simulation demonstrates the performance and the computational cost of the TR methods. The platform is a desktop computer with Intel Cerleron CPU E3300 @2.5GHz and 2 GB memory, running Windows XP and Matlab 6.5.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1-e^{-x_1}}{1+e^{-x_1}}(x_2^2 + 2x_1)\sin x_2 + \left(1+e^{-x_1}\right)u + d \\ y &= x_1 \end{aligned} \tag{15}$$

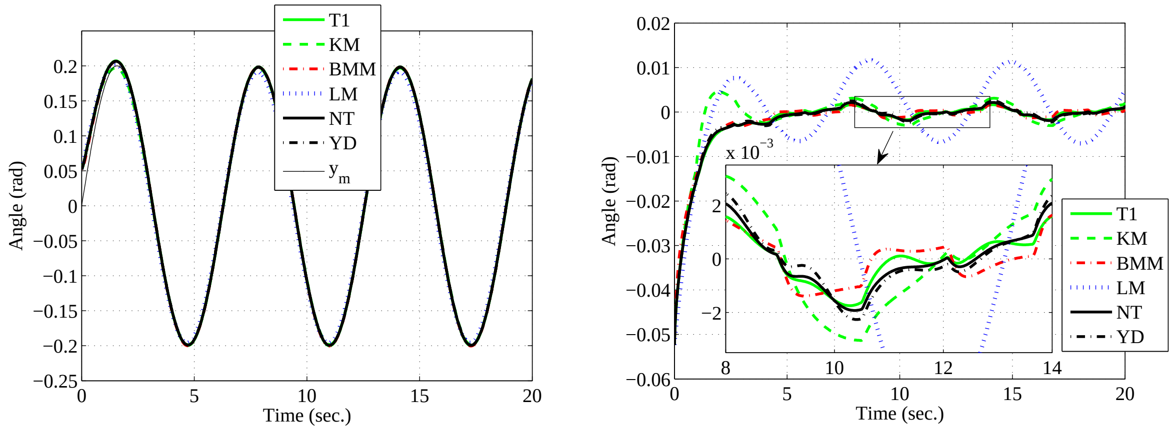
where d is external disturbance signal which includes $d_1 = 0.1 * \mathrm{sign}\big(\sin(\omega * t + \phi)\big)$, $d_2 = 0.1 * \mathrm{rand}()$, $d_3 = g_\tau$, where d_1 is a square wave, and period is $\frac{2\pi}{\omega}$. d_2 is white noise signal on interval [0 1], and g_τ is a gate function.

We use the same IT2 fuzzy membership functions as those of simulation example in [4]. When the elements in the vectors $\boldsymbol{\theta}_f(0)$ and $\boldsymbol{\theta}_g(0)$ are chosen as a group of random values on interval [0 1], and the initial system states are $x_1(0) = \frac{\pi}{20}$ and $x_2(0) = 0$ with external disturbance. The performance indices are listed in Table 1, in which $e(t) = y_m - y$, ISE $= \int_{t=0}^{\infty} |e(t)|^2 dt$, IAE $= \int_{t=0}^{\infty} |e(t)| dt$, and ITAE $= \int_{t=0}^{\infty} t|e(t)| dt$. The corresponding curves of the output y_1 and the error e_1 are plotted in Figure 1. Under the initial conditions, the responses of the system (15) indicate that the BMM method is the best among the six methods, and according to the performance of the methods the descending order of the overall performance is listed as follows: BMM, type-1 fuzzy system (T1), Nie-Tan method (NT), YD, KM, Liang-Mendel method (LM). The computation time of the methods varies from 0.375 to 4.218 seconds, and the BMM method is the fastest.

When the elements in the vectors $\boldsymbol{\theta}_f(0)$ and $\boldsymbol{\theta}_g(0)$ are chosen as a group of random values on interval [0 1], and the initial system states are $x_1(0) = \frac{\pi}{20}$ and $x_2(0) = 0$ with external disturbance $d_2 + d_3$, the performance indices are listed in Table 2, Also the

Table 1. Comparison of the different TR or defuzzification methods with $x_1(0) = \frac{\pi}{20}$ and $x_2(0) = 0$ when disturbance is d_1

Methods	ISE	IAE	ITAE	Time
T1	0.0991	5.3347	16.6938	0.6100
KM	0.0916	5.8743	30.9470	1.7180
BMM	0.0839	4.8807	15.5829	0.3750
LM	0.1872	13.6426	110.8375	3.3440
NT	0.0993	5.5347	19.1161	0.5160
YD	0.0985	5.6012	20.7340	4.2180

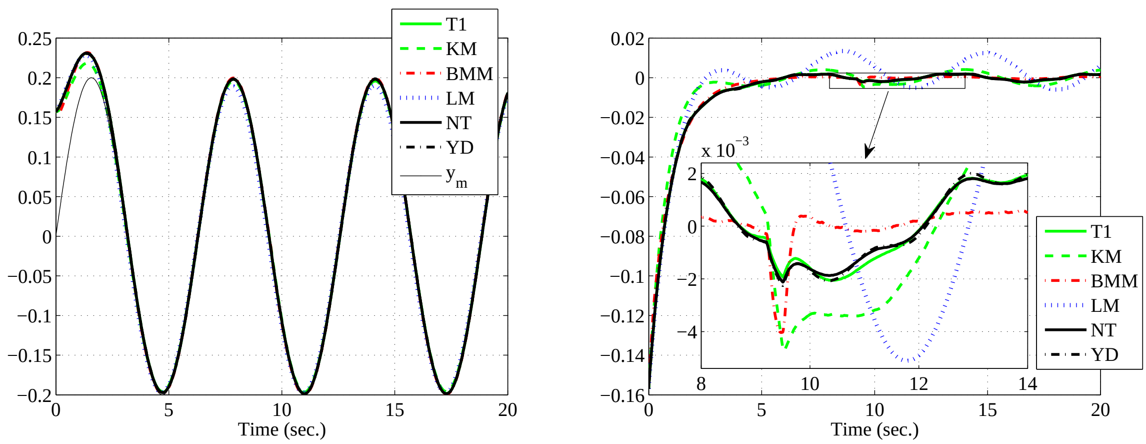


(a) The curves of the output y_1 (b) The curves of the error e_1

FIGURE 1. The curves of the output y_1 and the error e_1 with $x_1(0) = \frac{\pi}{20}$ and $x_2(0) = 0$ when disturbance is d_1

TABLE 2. Comparison of the different TR or defuzzification methods with $x_1(0) = \frac{\pi}{20}$ and $x_2(0) = 0$ when disturbance is $d_2 + d_3$

Methods	ISE	IAE	ITAE	Time
T1	1.1799	17.2338	39.5512	4.7350
KM	0.9592	15.8688	58.8138	20.1090
BMM	1.0674	15.0078	21.0257	4.9840
LM	1.2357	22.3868	118.0996	8.2500
NT	1.1825	17.1542	38.2995	5.2030
YD	1.1733	17.1825	39.5865	52.5630



(a) The curves of the output y_1 (b) The curves of the error e_1

FIGURE 2. The curves of the output y_1 and the error e_1 with $x_1(0) = \frac{\pi}{20}$ and $x_2(0) = 0$ when disturbance is $d_2 + d_3$

corresponding curves of the output y_1 and the error e_1 are plotted in Figure 2. Under this initial conditions, the responses of the system (15) show that the BMM method is still the best among the six methods, and according to the performance of the methods the descending order of the overall performances is listed as follows: BMM, NT, T1, YD, KM, LM. Except NT, the order is identical to the one in the first case. The computation time of the methods varies from 4.735 to 52.563 seconds, and the BMM method is still the fastest.

In order to avoid particularity, we choose 100 groups of random values on interval [0 1] as the elements in the vectors $\boldsymbol{\theta}_f(0)$ and $\boldsymbol{\theta}_g(0)$, and the initial system states are $x_1(0) = \frac{\pi}{20}$ and $x_2(0) = 0$ with external disturbance $d_2 + d_3$, the statistical performance indices which are the arithmetic mean of all the corresponding performance indices are listed in Table 3. Under the initial conditions, from Table 3, the descending order of the overall performances is the same as the second case in the example. Thus, we can make a conclusion that in different initial conditions the performance of most cases follows the above-mentioned order, and the average computational time is similar to that consumed under the second case. For the example, the BMM algorithm is the best on the whole.

Table 3. Comparison of the different TR or defuzzification methods with $x_1(0) = \frac{\pi}{20}$ and $x_2(0) = 0$ under different initial conditions of $\boldsymbol{\theta}_f(0)$ and $\boldsymbol{\theta}_g(0)$ when disturbance is $d_2 + d_3$

Methods	ISE	IAE	ITAE	Time
T1	1.1409	16.5649	34.7975	4.5953
KM	1.0952	17.2513	51.6925	18.8382
BMM	1.1457	15.6271	21.9439	4.6531
LM	1.2494	22.5702	114.4127	31.0953
NT	1.1427	16.5606	34.6101	5.0752
YD	1.1365	16.6261	36.2116	55.1987

5. **Conclusions.** In this paper, we have proved that the IT2 FLSs using the KM and BMM methods are universal approximators, and have shown the FBFs of the different IT2 FLSs using the two type-reduction methods. The numerical simulations demonstrate that although for each simulation the different IT2 FLSs have different performance depending on the different plants or the system states initial values, generally speaking, the BMM TR method outperforms the KM algorithm according to the statistical performance and computational time. Except the KM and BMM TR methods, there exist other non-iterative TR methods for IT2 FLS. Approach property of IT2 FLSs using other non-iterative TR is our next research direction.

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