Proceedings of the 32nd Chinese Control Conference
July 26-28, 2013, Xi'an, China

A Nonlinear Sliding Mode Compound Autopilot for Aircrafts with Reaction Jets

CHANG Yafei[1], YUAN Ruyi[1], TAN Xiangmin[1], YI Jianqiang[1], QIAN Hanbo[2]

1. Institute of Automation, Chinese Academy of Sciences, Beijing 100190
E-mail: yafei.chang@ia.ac.cn

2. Center of Coordination & Support of SASTIND, Beijing 100081
E-mail: qianhanbo160@163.com

Abstract: A new approach to the compound autopilot design for aircrafts with blending actuators is presented. The method is based on nonlinear sliding mode control, and adopts synergistic control strategy dealing with two different actuators (dynamic fins and reaction jets). An observer is further designed to estimate and smooth the complex term whose analytical expression cannot be obtained. In addition, a chattering attenuation method is proposed to eliminate the chatter caused by SMC. The convergence domain of the system is also discussed. Then, the autopilot is tested on a aircraft model with uncertainties and disturbances, and compared with pure aerodynamic control. Simulation results show the capabilities of the synergistic control strategy and excellent performance of the compound system.

Key Words: compound control, RCS, nonlinear SMC, robustness, chattering attenuation

1 Introduction

Time response and tracking accuracy are important performance criterions to aircrafts. As the rapid development of modern aircrafts, they are desired with extraordinarily minimum response time and high tracking accuracy. Particularly, aircrafts often flight at high altitude where fin aerodynamic effectiveness is low; moreover the fin circle is with large time constant, both of which make the aircrafts with pure fin actuator hardly reach the desired performance[1, 2]. To compensate this situation, alternative control technologies have to be used. Possible options are propulsion-based control in the forms of thrust vectoring controller (TVC) and/or a reaction jet control system (RCS) [3]. The work presented here concentrates on only one of the propulsive solutions, the use of reaction jets.

The participation of RCS brings the compound system with problem of arrangement for the two actuators. Menon and Iragavarapu[4] developed a model reference adaptive control technique for the blending actuators. Ho *et al.*[5] proposed control allocation strategies with time-varying eigenvalue assignment. A new optimal control allocation algorithm was presented by Xie *et al.*[6] to adopt trust region method with global convergence to solve constrained quadratic programming problem. However, those researches all regard the output of reaction jets as continuous variable, which cannot accurately describe the output in reality.

RCS also brings disturbances to the aerodynamic control system and makes the compound system with complex features like huge nonlinearity, time-variability and close coupling. Extensive research efforts have been made over the years to design or improve the control methods for the compound system. Dynamic inversion has been widely used to deal with the system nonlinearity. However, it seriously depends on the accuracy of the model. Thus, it can hardly afford the desired robustness of the system. To compensate this situation, some researchers combine several intelligent methods[7-9] such as neural networks, fuzzy logic featuring universal approximation with dynamic inversion. Those compensating methods can enforce robustness, but they make the system structure complex. Some of researchers design the compound autopilot basing on sliding mode control (SMC)[10-12]. An advantage of this method is its robustness to parameter perturbations and bounded external disturbances. However, those researches all transform the nonlinear system to the small perturbation linear model at first, and then apply SMC on the linear model. This is unreasonable for the condition of aircrafts in big attitude angle state. In addition, the discontinuous term in the control input causes an undesirable effect called chattering, which should be seriously regarded in SMC.

In this work, a compound autopilot based on nonlinear sliding-mode control for the aircraft with blending actuators is designed to robustly improve the system time response in presence of atmospheric disturbances and dynamic uncertainty of atmospheric coefficients. The paper is organized as follows: Section 2 presents the difficulties involved in the design of the compound autopilot. The dynamic aircraft model and the compound autopilot synthesis are described in Sections 3 and 4, respectively. Section 5 describes the system performance based on extensive computer simulations. Finally, the conclusions are summarized in Section 6.

2 Problem Description

The difficulties in design of the compound autopilot involve modeling of RCS, its integration with aerodynamic fins, and approach to dealing with the compound system featuring huge nonlinearity, time-variability and close coupling.

The aircraft with blending actuators discussed in this paper is based on PAC-3 configuration with a group of fixed fins in the middle and another group of all-moving fins in the tail corresponding to a standard cruciform

[*]This work is supported by National Natural Science Foundation (NNSF) of China under Grant 61273149, 61203003, the Special Project for Innovation Methods of MOST under Grant 2012IM010200 and B1320133020.

axial-symmetric shape shown in Fig.1[13]. 180 reaction jets of 10 circles are located in radical directions evenly at the distance of 1 meter before the center of mass. It features as follows:

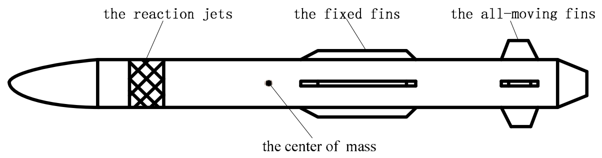

Fig. 1:Aircraft configuration

a) the force and moment each jet produces are unalterable and discrete;

b) it is with huge aerodynamic uncertainties such as the disturbance of atmospheric environment change and/or reaction jets affect on aerodynamic coefficients;

c) it is close-coupled in three channels (pitch, yaw and roll), as the aircraft is often in state of high angle of attack.

The characteristics a) and b) indicate that as the consumption of jets and non-adjustability of each jet output, it can hardly always generate the desired control power, i.e. there always exists a deviation between real reaction jets output and the RCS command. Moreover, the aerodynamic control is continuous while RCS is discontinuous. In addition, those two actuators (fins and reaction jets) have different time constant. Those all bring difficulties to design the strategy arranging their contribution to control reasonably and making the compound system work steadily and successfully. The characteristics c) and d) require the control system possessing strong robust performance in presence of disturbance.

The purpose is to establish a compound control structure which can coordinate the blending actuators opportunely and to design a robust control method, to achieve the goal of rapid time response and high accuracy for the aircraft control system.

3 Model Description

3.1 Aerodynamic Data

For the purpose of this study, the aerodynamic coefficients are expressed as functions of the angle of attack α, the angle of sideslip β, angle of fin deflections δ_a, δ_e, δ_r and angular rate of pitch, yaw, roll p, q, r as

$$\begin{cases} C_{xt} = C_x(\alpha,\beta),\ C_{yt} = C_y(\alpha,\beta,\delta_r),\ C_{zt} = C_z(\alpha,\beta,\delta_e) \\ C_{lt} = C_l(\alpha,\beta,\delta_a) + C_{lp}(\alpha)\dfrac{pL_{ref}}{2V} + C_{lr}(\alpha)\dfrac{rL_{ref}}{2V} \\ C_{mt} = C_m(\alpha,\beta,\delta_e) + C_{mq}(\alpha)\dfrac{qL_{ref}}{2V} \\ C_{nt} = C_n(\alpha,\beta,\delta_r) + C_{np}(\alpha)\dfrac{pL_{ref}}{2V} + C_{nr}(\alpha)\dfrac{rL_{ref}}{2V} \end{cases} \tag{1}$$

where C_{xt}, C_{yt}, C_{zt} and C_{lt}, C_{mt}, C_{nt} are the aerodynamic force coefficients and moment coefficients along the three axes of the body in American coordinate system, L_{ref} is the reference length and V the flight velocity of the aircraft.

3.2 RCS Modeling

The model of RCS is composed with the model of reaction jets and fire logic as shown in Fig.2. The fire logic gives fire command basing on the desired moment and the temporal fire state. The model of the reaction jets produces the force and moment and upgrades the fire state in response to the fire command.

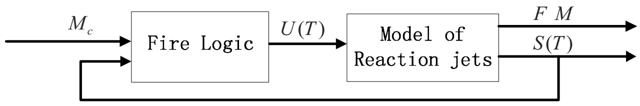

Fig. 2: RCS Model

Firstly, we consider the model of the reaction jets. We define two 10×18 matrixes, the state matrix $S(T)$ to describe the fire conditions of the reaction jets and the fire matrix $U(T)$ to describe the fire command in the modeling. The element $S_{ij}(T)$ of the matrix $S(T)$ can be 0 or 1, where $S_{ij}(T)=0$ indicates the jet in row i, column j has been fired while $S_{ij}(T)=1$ means it hasn't been used and can be fired at the moment T. Similarly, the element $U_{ij}(T)$ of the matrix U can be 0 or 1, where $U_{ij}(T)=0$ indicates the RCS commands the jet in the row i, column j to be fired while $U_{ij}(T)=1$ means the RCS commands it to keep the state, i.e. it won't be fired at the moment T.

According to the definition of the state matrix $S(T)$ and the fire matrix $U(T)$ above, the total forces and moments of the reaction jet model are as

$$\begin{cases} F_X = 0 \\ F_Y = \sum\limits_{\substack{i=1,2..10 \\ j=1,2...18}} F_{Yij}\bullet(1-U_{ij}) \\ F_Z = \sum\limits_{\substack{i=1,2..10 \\ j=1,2...18}} F_{Zij}\bullet(1-U_{ij}) \end{cases} \quad \begin{cases} M_X = 0 \\ M_Y = \sum\limits_{\substack{i=1,2..1 \\ j=1,2...18}} M_{Yij}\bullet(1-U_{ij}) \\ M_Z = \sum\limits_{\substack{i=1,2..10 \\ j=1,2...18}} M_{Zij}\bullet(1-U_{ij}) \end{cases} \tag{2}$$

where F_{Xij}, F_{Yij}, F_{Zij} and M_{Xij}, M_{Yij}, M_{Zij} are respectively the force and moment of single reaction jet located in row i, column j in three directions under the coordinate system of the body, and the updated state is as

$$S_{ij}(T+1) = S_{ij}(T)\bullet U_{ij}(T) \tag{3}$$

Next the fire logic is discussed in detail. We integrate moments in pitch and yaw channels into logic designation and adopt symmetrical ignition method[14], using optimal searching algorithm to obtain the desired fire vector. For the simplicity of analysis we merge the jets in adjacent circles into one circle. Consequently there are 5 circles with 36 jets in one circle.

The fire logic is designed as:

1) according to the giving moment $M_c = [M_{yc}, M_{zc}]$, calculate the magnitude M and direction θ of the moment M_c;

2) search the jet which is the closest to the direction θ from 1 to 36 in one circle , and mark its position as column j_0;

3) search the optimal fire vector $[U_{1,j_0}\ U_{2,j_0}\ U_{3,j_0}\ U_{4,j_0}\ U_{5,j_0}]$ to make M_0 closest to M, where $M_0 = \sum_{i=1}^{5}(1\text{-}U_{i,j_0})\bullet M_{i,j_0}$. If $M_0 \geq M$, go to step 6) else go to step 4);

4) search the optimal fire vector $[U_{1,j_{01}}\ U_{2,j_{01}}\ U_{3,j_{01}}\ U_{4,j_{01}}\ U_{5,j_{01}} U_{1,j_{02}}\ U_{2,j_{02}}\ U_{3,j_{02}}\ U_{4,j_{02}}\ U_{5,j_{02}}]$ to make M_1 closest to M, where the jets of column j_{01} and j_{02} is adjacent to the jets of column j_0. M_1 is expressed as function of M_0 as $M_1 = \sum_{i=1}^{5}(U_{i,j_{01}} M_{i,j_{01}} + U_{i,j_{02}} M_{i,02}) + M_0$. If $M_1 \geq M$, go to step 6) else go to step 5);

5) search the optimal fire vector $[U_{1,j_{c1}}\ U_{2,j_{c1}}\ U_{3,j_{c1}}\ U_{4,j_{c1}}\ U_{5,j_{c1}} U_{1,j_{c2}}\ U_{2,j_{c2}}\ U_{3,j_{c2}}\ U_{4,j_{c2}}\ U_{5,j_{c2}}]$ to make M_2 closest to M, where the jet of column j_{c1} is the other jet adjacent to the jet of column j_{01} and the jet of column j_{c2} is the other jet adjacent to the jet of column j_{02}. The new moment M_2 is expressed as function of M_1 as $M_2 = \sum_{i=1}^{5}(U_{i,j_{11}} M_{i,j_{11}} + U_{i,j_{12}} M_{i,12}) + M_1$. If $M_2 < M$ and the number of jets between the jets of column j_{c1} (or j_{c2}) and the jets of column j_0 is less than 8, upgrade $j_{01} = j_{c1}$, $M_1 = M_2$ and go to step 5), else go to step 6);

6) output the fire matrix $U(T)$.

The fire logic designed here gives the fire command, making the RCS consume fewer numbers of reaction jets than the method of independent design of pitch and yaw channels[14] and produce the moments with small errors. The validation of this method is not discussed further.

3.3 Aircraft Model

As acceleration command from guidance law can be converted to attitude command, the main concern of this paper is the designation of attitude compound controller. The aircraft model is as follow:

$$\begin{cases} \dot{\alpha} = q - \tan\beta(p\cos\alpha + r\sin\alpha) + \dfrac{1}{mV\cos\beta}(Z\cos\alpha \\ \quad +F_z\cos\alpha - X\sin\alpha + mg\cos\mu\cos\gamma) \\ \dot{\beta} = p\sin\alpha - r\cos\alpha + \dfrac{1}{mV} mg\cos\mu\sin\mu + \dfrac{1}{mV} \\ \quad (\mathrm{Y}\cos\beta + \mathrm{F}_y\cos\beta - X\cos\alpha\sin\beta - \mathrm{F}_z\sin\alpha\sin\beta \\ \quad -\mathrm{Z}\sin\alpha\sin\beta)\cos\beta \\ \dot{p} = c_1 L \\ \dot{q} = c_2 pr + c_3(M + M_f) \\ \dot{r} = c_4 pq + c_5(N + N_f) \end{cases} \quad (4)$$

where μ is angle path; X, Y, Z and L, M, N are the aerodynamic forces and moments along the three axes of the body relatively. c_1, c_2, c_3, c_4, c_5 are coefficients related with the moment of inertia.

4 Compound Autopilot Design

This section describes the design of the compound autopilot involving the combining strategy, the aerodynamic control technique for three channels (pitch, yaw, and roll) and the control method for the RCS. The structure of the compound control system is shown as Fig.3.

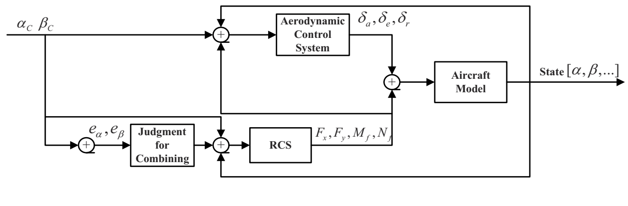

Fig. 3: Compound Control System Structure

4.1 Combining Strategy

In this paper, the strategy of synergistic control is suggested for several reasons. First of all, as mentioned in the section 2, the force and moment the reaction jets produce in a specific direction are limited and fixed, thus there always exists a deviation between the practical reaction jet output and the RCS command. Thus, the strategy of allocation of attitude command and/or dynamic allocation will cause errors to the compound system. Second, if the RCS and aerodynamic control systems work independently just under one kind of distribution, the compound system is likely to be unstable once RCS is paused. Third, the coming airflow interferes with RCS, appending excess forces and moments on its output. The strategy of synergistic control adopted here can address all these issues by arranging the RCS and the aerodynamic control system to work together cooperatively. Moreover, it makes the aerodynamic control system work on the foundation of RCS, consequently compensating the differences between the practical jet output and the RCS command and helping relieve the interference of the coming airflow. The RCS and the aerodynamic control systems work synergistically in the sense that the aerodynamic control system gives the control quantities of δ_a, δ_e and δ_r on the basis of the output of RCS, which can be seen in the design of the aerodynamic control system.

The combining process of the two different actuators will be discussed in details here. The module of "judgment for combining" in Fig.3 decides when RCS operates according to the follow condition:

$$\begin{cases} |e| > \dfrac{|A_c|}{k} \\ M_{fc} > M \end{cases} \quad (5)$$

where e is the present error of controlled state, A_c is the desired state, M_{fc} is the control quantities of RCS, M is the threshold to M_{fc} which is assumed above the moment each jet produces, and k is a coefficient which can be changed according to the simulation result. It implies that only when the error and the output of the RCS are large enough, the RCS begins to work. The condition proposed here can improve the time response of the system at the same time avoiding the waste of the reaction jets.

4.2 Aerodynamic Control System Design

The design of the aerodynamic control system involves three channels (pitch, yaw and roll). Two sliding mode controllers for the pitch channel and the yaw channel respectively are designed to track the expected angles, and decouple the two channels at the same time. The roll channel is controlled through a PID controller in joint with the feedback of the roll rate p. The roll angle γ is controlled to zero, aiming at mitigating the couple of the pitch and the yaw channels.

We adopt the nonlinear sliding mode control (NSMC) approach for several reasons. First of all, the aircraft characteristics present nonlinear dynamic with large parameter variations. Second, the aircraft is close-coupled in three channels (pitch, yaw and roll), as it is often in state of high angle of attack. Third, the use of the reaction jets brings huge interference to the aerodynamic control system. NSMC, with due care, is one of the methodologies capable of addressing all these issues.

In this paper, we will only discuss the design for the pitch channel in details as the design of the yaw channel is similar to the pitch channel. The state variable x_α and control variable u_α are denoted as $x_\alpha = [x_{\alpha 1}, x_{\alpha 2}]$, $u_\alpha = -\delta_e$ where $\begin{cases} x_{\alpha 1} = \alpha - \alpha_c \\ x_{\alpha 2} = q \end{cases}$, and the pitching moment coefficient $C_m(\alpha,\beta,\delta_e)$ is linearly fitted as

$$C_m(\alpha,\beta,\delta_e) \approx C_{m0}(\alpha,\beta) + C_{m\delta_e}(\alpha,\beta)\delta_e \tag{6}$$

Then we obtain the pitch plane dynamic model as

$$\begin{cases} \dot{x}_{\alpha 1} = x_{\alpha 2} + f_\alpha(x) \\ \dot{x}_{\alpha 2} = h_\alpha(x) + g_\alpha(x)u_\alpha + \Delta E(x) \end{cases} \tag{7}$$

with

$$\begin{cases} f_\alpha(x) = -\tan\beta(p\cos\alpha + r\sin\alpha) + \dfrac{1}{mV\cos\beta}(Z\cos\alpha \\ \qquad + F_z\cos\alpha - X\sin\alpha + mg\cos\mu\cos\gamma - T\sin\alpha) \\ h_\alpha(x) = c_2 pr + c_3 QSL_{ref}(C_{m0}(\alpha,\beta) + C_{mq}(\alpha)\dfrac{qL_{ref}}{2V}) \\ \qquad + c_3 M_f \\ g_\alpha(x) = -c_3 QSL_{ref}C_{m\delta_e}(\alpha,\beta), \quad g_\alpha(x) > g_{\alpha 0} > 0 \end{cases} \tag{8}$$

$\Delta E(x)$ is the error caused by the linear fitting which is not known accurately.

Let the sliding variable S_α be of the form $S_\alpha = a_\alpha x_{\alpha 1} + x_{\alpha 2} + f_\alpha(x), a_\alpha > 0$. By stabilizing S_α to zero, we can get the dynamic of x_α as $\dot{x}_{\alpha 1} = -a_\alpha x_{\alpha 1}$, which makes $x_{\alpha 1}$ converge to zero with a speed specified by the parameter a_α. Then, the main concern here is to decide the dynamic of S_α with a desired performance of finite-time convergence by controlling the aerodynamic fin deflection.

From the expression of S_α, we can obtain its rate of change

$$\begin{aligned} \dot{S}_\alpha &= a_\alpha \dot{x}_{\alpha 1} + \dot{x}_{\alpha 2} + \dot{f}_\alpha(x) \\ &= a_\alpha x_{\alpha 2} + a_\alpha f(x) + h(x) + \dot{f}_\alpha(x) + g_\alpha(x)u + \Delta E(x) \end{aligned} \tag{9}$$

For the system above, the following control

$$u_\alpha = -\left(\left|\frac{a_\alpha x_{\alpha 2} + a_\alpha f(x) + h_\alpha(x) + \dot{f}_\alpha(x)}{g_\alpha(x)}\right| + \lambda_{\alpha 0}\right)\mathrm{sgn}(S_\alpha) \tag{10}$$

where $\lambda_{\alpha 0} > 0$ provides for the finite-time convergence of S_α to zero robustly to any disturbance including $\Delta E(x)$ with a known limit for its derivative .

During operation in the sliding mode, the discontinuous control u_α chatters about the switching surface at high frequency. High frequency switching may be destructive for control devices or may cause system resonance. There are several ways to eliminate the effects of chattering, with little loss in performance. These include the definition of a boundary layer near the sliding surface as introduced in [15], and/or the introduction of the high-order sliding mode techniques to design a continuous sliding mode control [16]. Here we will combine two ideas for reducing or eliminating chattering.

The first idea is to divide the control into continuous and switching sections so as to reduce the amplitude of the switching section. We choose $\left(\left|\frac{\dot{f}_\alpha(x)}{g_\alpha(x)}\right| + \lambda_{\alpha 0}\right)$ as the switching section for its amplitude is most likely smaller than an upper bound on the whole function, which has been validated through simulations. However, $\dot{f}_\alpha(x)$ in reality is hardly measured. To settle this problem, we build an observer. Rewrite (9) as $\dot{S}_\alpha = \dot{f}_\alpha(x) + \bar{u}$ where

$$\begin{aligned} \bar{u} &= a_\alpha \dot{\alpha} + \dot{q} \\ &= a_\alpha(q - \tan\beta(p\cos\alpha + r\sin\alpha) + \frac{1}{mV\cos\beta} \\ &\quad (Z\cos\alpha + F_z\cos\alpha - X\sin\alpha + mg\cos\mu\cos\gamma \\ &\quad - T\sin\alpha)) + c_2 pr + c_3(M + M_f) \end{aligned} \tag{11}$$

Consider the following observer [16]:

$$\begin{cases} \dot{z}_0 = v_0 + \bar{u}, \quad v_0 = -2L^{\frac{1}{3}}|z_0 - S_\alpha|^{\frac{2}{3}} sign(z_0 - S_\alpha) + z_1 \\ \dot{z}_1 = v_1, \quad v_1 = -1.5L^{\frac{1}{2}}|z_0 - v_0|^{\frac{1}{2}} sign(z_1 - v_0) + z_2 \\ \dot{z}_2 = -1.1L sign(z_2 - v_1) \end{cases} \tag{12}$$

where $L > 0$. Then z_1 converges to $\dot{f}_\alpha(x)$ in a finite time, if the sliding variable S_α and control $\bar{u}$ are measured without noise[17]. In addition, the output of the above observer $z_1(t)$ is smooth.

The second idea to eliminate chattering is to replace the signum function by a high-slope saturation function. Then we combine the two ideas above and obtain the following compensated control

$$u_\alpha^* = -\frac{a_\alpha x_{\alpha 2} + a_\alpha f_\alpha(x) + h_\alpha(x)}{g_\alpha(x)} - \left(\left|\frac{z_1}{g_\alpha(x)}\right| + \lambda_\alpha\right)sat\left(\frac{S_\alpha}{\varepsilon}\right) \tag{13}$$

where ε is a positive constant, and $sat(\bullet)$ is the saturation function defined by

$$sat(y) = \begin{cases} y, & if\,|y| \le 1 \\ \mathrm{sgn}(y), & if\,|y| > 1 \end{cases} \tag{14}$$

Then, the accuracy of finite-time convergence for S_α and $x_{\alpha 1}$ under control u_α^* will be discussed.

At first, we consider the condition of $|S_\alpha| > \varepsilon$. With $V = \frac{1}{2}S_\alpha{}^2$ as a Lyapunov function candidate for $\dot{S}_\alpha = a_\alpha x_{\alpha 2} + a_\alpha f(x) + h_\alpha(x) + \dot{f}_\alpha(x) + g_\alpha(x)u + \Delta E(x)$, we have

$$\begin{aligned}\dot{V} &= S_\alpha \dot{S}_\alpha = S_\alpha(\dot{f}_\alpha(x) + \Delta E(x) - g_\alpha(x)(\frac{|\dot{f}_\alpha(x)|}{g_\alpha(x)} + \lambda_{\alpha 0})\mathrm{sgn}(S_\alpha)) \\ &= S_\alpha(\dot{f}_\alpha(x) + \Delta E(x)) - |\dot{f}_\alpha(x)||S_\alpha| - g_\alpha(x)\lambda_{\alpha 0}|S_\alpha| \\ &\le |S_\alpha||\dot{f}_\alpha(x)| + |S_\alpha||\Delta E(x)| - |\dot{f}_\alpha(x)||S_\alpha| - g_\alpha(x)\lambda_{\alpha 0}|S_\alpha| \\ &\le -(g_{\alpha 0}\lambda_{\alpha 0} - |\Delta E(x)|)|S_\alpha|\end{aligned} \tag{15}$$

under the control of u_α^*.

Let $y_0 = (g_{\alpha 0}\lambda_{\alpha 0} - |\Delta E(x)|)$, we choose $\lambda_{\alpha 0}$ which satisfies the inequality $\lambda_{\alpha 0} > \frac{|\Delta E(x)|}{g_{\alpha 0}}$ to make $y_0 > 0$. Thus, $W = \sqrt{2V} = |S_\alpha|$ satisfies the differential inequality $\dot{W} = \frac{\dot{V}}{|S_\alpha|} \le -y_0$ and we can obtain that $|S_\alpha(t)| \le |S_\alpha(0)| - y_0 t$.

Therefore, the trajectory reaches the manifold $|S_\alpha| \le \varepsilon$ in finite time. Once on the manifold, it cannot leave it, as seen from the inequality $\dot{W} \le -y_0$.

Next, we consider the condition of $|S_\alpha| \le \varepsilon$. We get from the analysis above that whenever $|S_\alpha(0)| > \varepsilon$, $|S_\alpha(t)|$ will be strictly decreasing, until it reaches the set $\{|S_\alpha| \le \varepsilon\}$ in finite time and remains inside thereafter. Inside the boundary layer, we have $\dot{x}_{\alpha 1} = -a_\alpha x_{\alpha 1} + S_\alpha$, where $|S_\alpha| \le \varepsilon$. The derivative of $V_1 = \frac{1}{2}x_{\alpha 1}{}^2$ satisfies

$$\dot{V}_1 = -a_\alpha x_{\alpha 1}{}^2 + x_{\alpha 1}S_\alpha \le -a_\alpha x_{\alpha 1}{}^2 + |x_{\alpha 1}|\varepsilon = -(a_\alpha x_{\alpha 1} - \varepsilon)|x_{\alpha 1}| \tag{16}$$

When $x_{\alpha 1} > \frac{\varepsilon}{a_\alpha}$, we get $\dot{V}_1 < 0$. Thus, the trajectory reaches the set $\{|x_{1\alpha}| \le \frac{\varepsilon}{a_\alpha}, |S_\alpha| \le \varepsilon\}$ in finite time. We achieve an ultimate boundary that can be reduced by decreasing ε.

4.3 RCS Design

The RCS design involves the pitch channel and yaw channel, and we design them separately. The control law of each channel adopts PID control technology in joint with the feedback of angular rate. The structure of the controller is shown as Fig.4. The parameters of the controller are determined through simulations.

5 Simulation Results

The compound autopilot described in the previous section, is tested in the Matlab-Simulink environment. The participation of the reaction jets can improve the system time response. However, it depends on the suitability of the

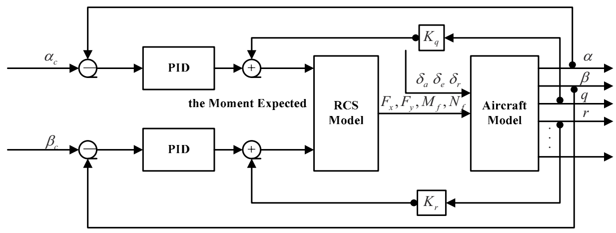

Fig4: RCS Control Structure

combining strategy. The main objectives of the simulation are to test the suitability of the combined strategy, to validate the compound autopilot with improved time response performance by comparing with pure aerodynamic control, and to verify the robustness of the method with respect to parameter variations such as aerodynamic coefficients, moment of inertia, and mass of the aircraft. In addition, the effectiveness of the approach to chattering attenuation is demonstrated.

The simulation begins from the trimmed equilibrium state $\delta_a = 0^\circ$, $\delta_e = -0.70^\circ$, $\delta_r = 0^\circ$, $\alpha = 1.15^\circ$, $\beta = 0$, $\gamma = 0$.

5.1 The Time Response and the Robustness Synthesis

At first, we consider the time response and robustness of the compound system. We set angle commands as $\begin{cases}\alpha_c = 20^\circ \\ \beta_c = 20^\circ\end{cases}$ at 1 sec after the simulation begins, then we add to disturbances of aerodynamic coefficients, moment of inertia, and mass of aircraft. The simulation results are presented in Fig.5-6.

Fig.5 shows the response time of α and β in the compound system is absolutely less than that in the system with fins only. Thus we can confirm that the combined strategy is impactful and the reaction jets can improve the time response remarkably. Fig.5 also shows the output of the reaction jets, from which we are informed the reaction jets work at the beginning of the response and stop while the angle achieves the desired. It indicates the combined strategy can help the reaction jets work effectively and avoid waste at the same time. From Fig.6 we can obtain that the system can preserve the control accuracy which means it possesses excellent robustness to disturbances.

5.2 Validation of Approach to Chattering Attenuation

Next, we test the effectiveness of the approach to chattering attenuation. Fig.7 shows the response of α and β under the control u_α and u_α^*. We can see the angular response in the control system without chattering attenuation has bad performance with huge oscillations while the response with chattering attenuation possess good performance with smooth transition process and small overshoot. It also shows the rudder response has huge chatter in the control system without chattering attenuation while the response in the control system with chattering attenuation is relatively smooth and without chatter in steady. Thus, the approach to chattering attenuation proposed in this paper is effective.

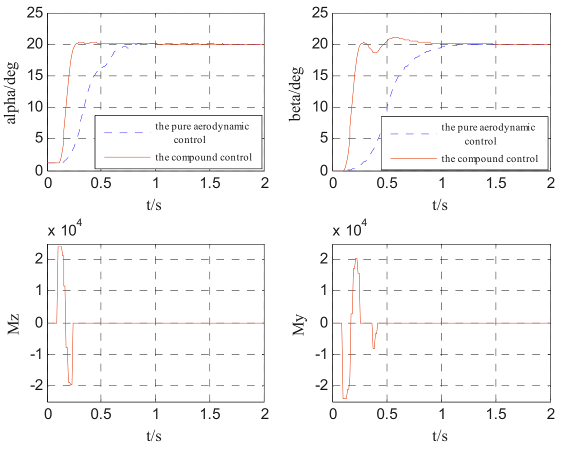

Fig5: α, β and M_z, M_y

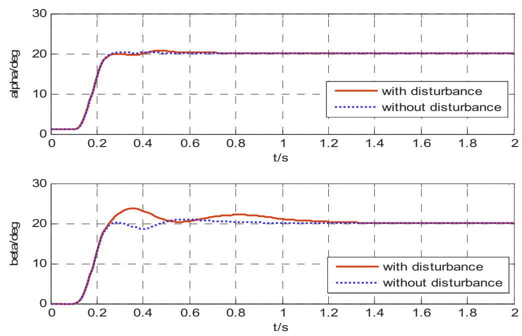

Fig 6: α and β with and without 30% disturbance

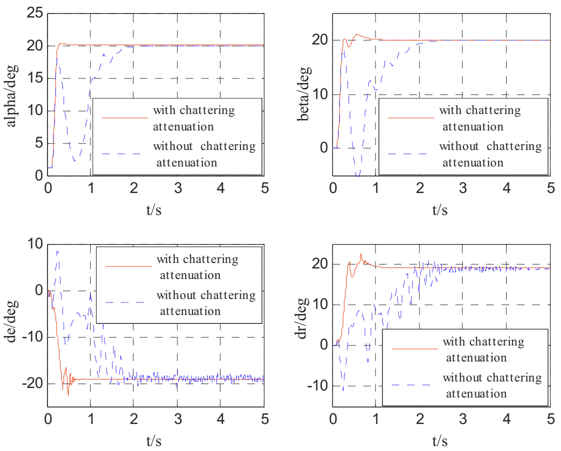

Fig 7: α , β and δ_e , δ_r with and without chattering attenuation

6 Conclusion

A new compound autopilot has been designed, which uses a combination of aerodynamic control and RCS, in order to improve the system time response. We propose synergistic control strategy dealing with the two different actuators on the basis of nonlinear sliding mode control. The performances of the compound system with the designed autopilot are validated via simulation, showing the significant time response improvements and robustness with respect to aerodynamic uncertainties. Future work is directed toward the research of new effectual strategy dealing with two different actuators, and to the selection of the new dynamic of sliding mode.

References

[1] R. S. Barnes, K. N. Brown, Patriot Advanced Capability-Three(PAC-3)-The First Deployable Hit-to-Kill Technology, in *AIAA-19881127-015*, Monterey, 1998.

[2] L. K. Zhu, G. J. Tan, and M. L. Yu, A Survey of Combined Control Methods with Lateral Thrust and Aerodynamic Force for Antiaircraft Missiles, *Journal of Projectiles, Rockets, Missiles and Guidance*, 29(1): 11-14, 2009.

[3] Z. H. Zhang, Russia S-400 air defence system, *Modern Defence Technology*, 30(1): 27-31, 2002.

[4] P. K. Menon, V. R. Iragarapu, Adaptive Techniques for Multiple Actuator Blending, *AIAA Guidance, Navigation and Control Conference*, Boston, MA, 1998.

[5] H. C. Lee, J. W. Choi, T. L. Song, and C. H. Song, Agile Missile Autopilot Design via Time-Varying Eigenvalue Assignment, in *the 8th International Conference on Control, Automation, Robotics and Vision*, Kunming, China, 2004.

[6] Z. H. Xie, Z. C. Liu, Y. W. Fang, and A. Q. Huang, Study on Optimal Control Allocation of Comound Control Missiles, *Journal of North University of China(Natural Science editon)*, 32(4): 465-469, 2011.

[7] W. Chen, X. H. Jia, Y. N. Li, and Z. liu, Improving Maneuverability and Guidance Performance of Air-to-air Missile with Compound Control System, *Journal of Northwestern Polytechnical University*, 24(4): 423-426, 2006.

[8] X. T. Luo, X. G. Liang, J. Yang, and J. Jia, Decoupling Design for High Angle of Attack Missile Based on Neural Network and Dynamic Inversion, *Computer Measurement & Control*, 19(11): 2733-2735, 2011.

[9] H. P. Xuan, J. W. Ma, X. H. Jia, and Y. Yang, Adaptive Robust Flight Control Based on Online Neural Network, *Electronics Optics & Control,* 18(1), 2011.

[10] H. Z. Zhao, J. Zhou, Terminal Sliding Mode Control for Attitude Control Antiaircraft Missile," *Computer Simulation,* 26(2): 73-76, 2009.

[11] Y. H. Fan, X. Li, J. Yang, and Y. Z. Zhang, "Design of Autopilot for Aerodynamic/Reaction-jet Multiple Control Missile Using Variable Structure Control," in *Proceeding of the 27th Chinese Control Conference*, Kunming, Yunnan, China, 2008: 642-645.

[12] R. D. Weil, K. A. Wise, Blending Aero & Reaction Jet Missile Autopilot Design Using VSS techniques, in *the 30th Conference on Decision and Control*, Brighton, England, 1991: 2828-2829.

[13] T. Wang, J. Zhou, Modeling and Simulation Research of PAC-3 Interceptor, *Journal of System Simulation*, 19(20): 4642-4645, 2007.

[14] Y. T. Bi, Research on Guidance and Control for Missile with Lateral Jets and Aerodynamic Surfaces, Thesis of Harbin Institute of Technology, Harbin, 2010

[15] V. I. Utkin, *Sliding Mode in Control Optimization,* New York: Springer-Verlag, 1992.

[16] L. A. Shkolnikov, Y. B. Shtessel, A second-order smooth sliding mode control, in *the 40th IEEE Conference on Decision and Control*, Orlando, Florida USA, 2001.

[17] A. Levant, High-order sliding modes, differentiation and output-feedback control, *International Journal of Control,* 924-941, 2003.