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Brief paper

Sliding mode control of MIMO Markovian jump systems[☆]

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ABSTRACT

This paper addresses the sliding mode control problem for uncertain MIMO linear Markovian jump systems. Firstly, by using the linear matrix inequality approach, sufficient conditions are proposed to guarantee the stochastically asymptotical stability of the system on the sliding surfaces. Secondly, an equivalent control based sliding mode control is proposed, such that the closed-loop system can be driven onto the desired sliding surfaces in a finite time. Finally, combining with multi-step state transition probability, the reaching and sliding probabilities are derived for situations where the control force may not be strong enough to ensure the fully asymptotical stability. Simulation results are presented to illustrate the effectiveness of the proposed design method.

1. Introduction

Nowadays, the modeling of dynamic systems subject to abrupt changes in their dynamics has been receiving a great deal of attention. These changes can be due to abrupt environmental disturbances, to actuator or sensor failure or repairs, to abrupt changes in the operation point for a nonlinear plant, etc. Such systems can be found in many fields, e.g. robotic manipulator systems, aircraft control, space stations, nuclear power plants, and wireless communication networks. The adequate operation of such devices, however, is severely compromised by the occurrence of failures, which may be intolerable in safety–critical applications, for example. Continuous Markovian jump system (MJS) is a proper model to describe these systems. A MJS is a continuous-time dynamical system with stochastic jumps, in which jumping parameter is a continuous-time, discrete-state Markov chain taking values in a finite set. A continuous-time Markov chain (CTMC) is a stochastic process that moves from state to state in accordance with a Markov chain, while its staying time in each state is exponentially distributed. Many results on MJSs have been reported in the literature (Chen, Xu, & Guan, 2003; De Farias, Geromel, Do Val, & Costa, 2000; Ji & Chizeck, 1990; Karan, Shi, & Kaya, 2006; Mahmoud & Shi, 2003; Shi, Boukas, & Agarwal, 1999; Wang, Lam, & Liu, 2004; Wu, Shi, & Gao, 2010; Xiong, Lam, Gao, & Ho, 2005; Xu, Chen, & Lam, 2003; Xu, Lam, & Mao, 2007; Zhang & Boukas, 2009), including quadratic control (Ji & Chizeck, 1990), output feedback control (De Farias et al., 2000), guarantee cost control (Chen et al., 2003), robust stabilization of MJS with uncertain switch probability (Xiong et al., 2005) and partly unknown transition probability (Zhang & Boukas, 2009), H_∞ control and filter (Xu et al., 2007), robust Kalman filter (Mahmoud & Shi, 2003), exponential filter (Wang et al., 2004), state estimation and sliding mode control of singular MJSs (Wu et al., 2010).

In the past decades, sliding mode control (SMC) has become an important method of nonlinear control, due to its inherent advantages, e.g. robustness, disturbance resistance, finite-time convergence. SMC alters the system dynamics by a discontinuous control signal that forces the system to enter and then slide along a surface, on which the system has desired properties such as stability, disturbance resistance. Most recently, significant progresses have been achieved in SMC for MJSs (Chen, Niu, & Zou, 2013a,b; Kao, Li, & Wang, 2014; Luan, Shi, & Liu, 2013; Mao, 2002; Niu & Ho, 2010; Niu, Ho, & Wang, 2007; Shi & Boukas, 1997; Shi, Xia, Liu, & Rees, 2006; Utkin & Poznyak, 2013; Wang, Liu, Yu, & Liu, 2006; Wang, Qiao, & Burnham, 2002; Wu, Shi, Su, & Chu, 2014; Yin, Shi, Liu, & Teo, 2014; Yu & Kaynak, 2009;

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Zhang, Huang, & Lam, 2003; Zhang, Wang, & Shi, 2013; Zhu, Yu, & Song, 2014,?). These include adaptive SMC for stochastic MJSs with actuator degradation (Chen et al., 2013a), SMC for stochastic MJSs with incomplete transition rate (Chen et al., 2013b), non-fragile observer-based H_∞ SMC for Ito stochastic systems with Markovian switching (Niu et al., 2007), asynchronous H_2/H_∞ filtering for discrete-time stochastic MJSs with randomly occurred sensor nonlinearities (Wu et al., 2014), observer-based H_∞ control on nonhomogeneous MJSs with nonlinear input (Yin et al., 2014), adaptive SMC with application to super-twist algorithm (Utkin & Poznyak, 2013), robust H_∞ SMC for MJSs subject to intermittent observations and partially known transition probabilities (Zhang et al., 2013), and finite-time stabilization for MJSs with Gaussian transition probabilities (Luan et al., 2013).

Aforementioned works usually assume that the strong control force is always available to overpower the stochastic uncertainties. However, in practical applications, controls are usually limited in power and sometimes insufficient. It would be beneficial to assess the risk of lowering down control force so that an economic balance between the risk and control cost can be achieved. In Zhu et al. (2014), the asymptotical stability probability was first explored for the second order MJSs under SMC where control force may not be strong enough to ensure the fully asymptotical stability. In Zhu et al. (2014), the problem on SMC of single input MJSs was studied. But it is not straightforward to extend the results for single input systems to MIMO systems as there may be strong coupling terms. Thus, the SMC scheme for MIMO MJSs (including sliding surfaces and controller design) is significantly different to the one for single input MJSs. In this paper, we explore the SMC of uncertain MIMO linear MJSs. At first, sufficient conditions are proposed in terms of LMIs to guarantee the asymptotical stability of the system on the sliding surface. Then, we derive an equivalent control based SMC paradigm, such that the closed-loop system can be driven onto the sliding surface in a finite time. Furthermore, the multi-step stochastic state transition probability function is introduced to facilitate the discussion. At last, we propose the reaching and sliding probabilities when no sufficient control is available.

The rest of this paper is organized as follows. In Section 2, the problem statement and preliminaries are presented. Next, the LMI-type sufficient conditions of asymptotical stability, the equivalent control based SMC paradigm, and the reaching probability and the sliding probability are derived for MIMO linear MJSs in Section 3. Then, the numerical simulation result is given in Section 4. A conclusion is drawn in Section 5.

Notations: $P(\cdot)$ denotes the probability of an event. $P(A|B)$ denotes the conditional probability of event A given event B. $\|\cdot\|$ denotes the Euclidean norm of a vector or the Frobenius norm of a matrix. $M > 0 (< 0)$ denotes that matrix M is a positive(negative) definite matrix. Bold $\mathbf{0}$ denotes a zero vector with compatible dimensions.

2. Problem statement and preliminaries

The MIMO linear MJS under investigation is

$$\begin{cases} \dot{X}_1(t) = (A_{11}(\eta_t) + \Delta_{11}(\eta_t))X_1(t) \\ \quad + (A_{12}(\eta_t) + \Delta_{12}(\eta_t))X_2(t), \\ \dot{X}_2(t) = (A_{21}(\eta_t) + \Delta_{21}(\eta_t))X_1(t) \\ \quad + (A_{22}(\eta_t) + \Delta_{22}(\eta_t))X_2(t) + B_2(\eta_t)U(t), \\ Y(t) = X_1(t), \\ \eta_0 = s_0, \quad t \geq 0, \end{cases} \tag{1}$$

where $X(t) = [X_1^T(t), X_2^T(t)]^T$ is the system state, $X_1 \in R^{(n-m)}$, $X_2 \in R^m$; $U(t) \in R^m$ is the control input; $Y(t) \in R^{(n-m)}$ is the system output; $A_{ij}(\eta_t), i, j = 1, 2$, $B_2(\eta_t)$ are stochastic coefficient matrices with compatible dimensions, $\Delta_{ij}(\eta_t), i, j = 1, 2$ are stochastic uncertain matrices and $\{\eta_t, t \in [0, T]\}$ is a finite-state Markovian process having a state-space $S = \{1, 2, \ldots, v\}$, $\det(B_2(j)) \neq 0$, $j \in S$, generator (q_{ij}) with transition probability from mode i at time t to mode j at time $t + \delta$, $i, j \in S$,

$$\begin{aligned} P_{ij}(\delta) &= P(\eta_{t+\delta} = j | \eta_t = i) \\ &= \begin{cases} q_{ij}\delta + o(\delta), & \text{if } i \neq j, \\ 1 + q_{ii}\delta + o(\delta), & \text{if } i = j, \end{cases} \end{aligned} \tag{2}$$

where

$$q_{ii} = -\sum_{m=1, m\neq i}^{v} q_{im}, \; q_{ij} \geq 0, \quad \forall i, j \in S, \; i \neq j, \tag{3}$$

$\delta > 0$ and $\lim_{\delta \to 0} o(\delta)/\delta = 0$.

Assumption 1. The uncertain matrices satisfy

$$\|\Delta_{ij}(k)\| \leq \delta_{ij}(k), \quad i, j = 1, \ldots, 2, \; k = 1, \ldots, v, \tag{4}$$

where $\delta_{ij}(k)$ are known constants.

Definition 1 (*Zhu et al., 2014*)**.** The system (1) is called mean-square stable, if for each $\varepsilon > 0$, there exists $\delta > 0$, such that

$$\sup_{t_0 \leq t < \infty} \mathbf{E}\|X(t)\|^2 < \varepsilon, \quad \text{for all } \|X(t_0)\| < \delta. \tag{5}$$

In addition, the system (1) is called asymptotically mean-square stable, if it is mean-square stable and

$$\lim_{t \to \infty} \mathbf{E}\|X(t)\|^2 = 0, \quad \text{for all } \|X(t_0)\| < \delta. \tag{6}$$

Furthermore, if Eq. (6) holds for arbitrary positive constant δ, then the system (1) is called globally asymptotically mean-square stable.

The sliding surfaces are defined as

$$S(X, \eta_t) = C_1(\eta_t)X_1 + X_2 = \mathbf{0}. \tag{7}$$

Denotes $\mathscr{L}$ as the weak infinitesimal operator of the process $\{X_1(t), \eta_t, t \geq 0\}$ at the point $\{t, X_1, j\}$. Substituting (7) and $\eta_t = j$ into system (1) yields

$$\begin{cases} \dot{X}_1 = \bar{A}_{11}(j)X_1 + \bar{A}_{12}(j)S, \\ \mathscr{L}S = \bar{A}_{21}(j)X_1 + \bar{A}_{22}(j)S + B_2(j)U(t), \\ \eta_0 = s_0, \quad t \geq 0, \end{cases} \tag{8}$$

where $\eta_t = j, j = 1, \ldots, v$, and

$$\begin{aligned} \bar{A}_{11}(j) &= A_{11}(j) + \Delta_{11}(j) - (A_{12}(j) + \Delta_{12}(j))C_1(j), \\ \bar{A}_{12}(j) &= A_{12}(j) + \Delta_{12}(j), \\ \bar{A}_{21}(j) &= C_1(j)(A_{11}(j) + \Delta_{11}(j)) \\ &\quad - C_1(j)(A_{12}(j) + \Delta_{12}(j))C_1(j) + (A_{21}(j) + \Delta_{21}(j)) \\ &\quad - (A_{22}(j) + \Delta_{22}(j))C_1(j) + \sum_{i=1}^{v} \alpha_{ji} C_1(i), \\ \bar{A}_{22}(j) &= C_1(j)(A_{12}(j) + \Delta_{12}(j)) + (A_{22}(j) + \Delta_{22}(j)). \end{aligned} \tag{9}$$

An important step for analyzing the probability problems of MJSs under SMC is to derive multi-step state transition probability. Some events are defined as follows.

$$B: \text{ The initial condition is } \eta_0 = s_0. \tag{10}$$

$$A_j: \; \eta_t = j, 0 \leq t \leq t_r. \tag{11}$$

$$\begin{aligned} &A^0(t): \text{ The stochastic process parameter does not jump,} \\ &\quad \eta_\tau = s_0, \; 0 \leq \tau \leq t, \end{aligned} \tag{12}$$

$A_j^k(t)$: The stochastic process parameter jumps k times

$$\eta_\tau = \begin{cases} s_0 & 0 \le \tau < t_1, \\ s_1 & t_1 \le \tau < t_2, \\ \vdots & \\ s_k & t_k \le \tau \le t \end{cases}$$

where the route $r_j^k = (s_0, \ldots, s_k) \in S^k$ (13)

$$S^k = \{(s_0, s_1, \ldots, s_k) | s_i \in S, s_{i-1} \neq s_i, 1 \le i \le k\}. \quad (14)$$

$A^k(t)$: The stochastic process parameter jumps k times

$$A^k(t) = \bigcup_{r_j^k \in S^k} A_j^k(t), \quad k = 1, 2, \ldots, n. \quad (15)$$

Lemma 1 (Lin, 2001; Zhu et al., 2014)**.** *For the continuous Markovian stochastic process, multi-step state transition conditional probability is*

$$P(A^0(t)|B) = e^{-q_{s_0} t}, \quad (16)$$

$$P(A_j^n(t)|B) = \int_0^t P(A_j^{n-1}(t-s)|B) q_{s_0 s_1} e^{-q_{s_0} s} ds, \quad (17)$$

$$\begin{aligned} P(A_j|B) &= P(\eta_t = j | \eta_0) \\ &= \sum_{n=0, s_n = j}^{n=\infty} \sum_{r_i^n \in S^n} P(A_i^n(t)|B). \end{aligned} \quad (18)$$

3. SMC of MIMO linear MJSs

There are two key steps in designing a SMC scheme. The first step is to design sliding surfaces which have desired system dynamics performance (e.g. asymptotical stability). The second step is to design a discontinuous controller which can drive the system to reach the sliding surface in a finite time.

At the beginning, we design sliding surfaces (7) and give sufficient conditions in terms of LMIs which can guarantee that the sliding motion is asymptotically stable. The result is given in the following theorem.

Theorem 1. *The system (1) is asymptotically stable on the sliding surface (7), if there exist symmetric positive-definite matrices $P(j)$, general matrices $Q(j)$ and positive scalars $b_1(j)$, $b_2(j)$ such that the following inequalities hold for all $j \in S$.*

$$\begin{bmatrix} N_{11}(j) & N_{12}(j) \\ N_{12}^T(j) & N_{22}(j) \end{bmatrix} < 0, \quad (19)$$

where

$$\begin{aligned} N_{11}(j) &= A_{11}(j)P(j) + P(j)A_{11}^T(j) - A_{12}(j)Q(j) \\ &\quad - Q^T(j)A_{12}^T(j) + \alpha_{jj}P(j) + b_1(j)\delta_{11}^2(j)I + b_2(j)\delta_{12}^2(j)I, \end{aligned} \quad (20)$$

$$\begin{aligned} N_{12}(j) &= [\alpha_{j1}^{1/2}P(j), \ldots, \alpha_{jj-1}^{1/2}P(j), \ \alpha_{jj+1}^{1/2}P(j), \ldots, \\ &\quad \alpha_{jv}^{1/2}P(j), \ P(j), \ Q^T(j)], \end{aligned} \quad (21)$$

$$\begin{aligned} N_{22}(j) &= \mathrm{diag}(-P(1), \ldots, -P(j-1), \ -P(j+1), \\ &\quad -P(v), \ -b_1(j)I, \ -b_2(j)I), \end{aligned} \quad (22)$$

$$Q(j) = C_1(j)P(j). \quad (23)$$

Proof. By Schur complement lemma, we get that (19) is equivalent to

$$\begin{aligned} &A_{11}(j)P(j) + P(j)A_{11}^T(j) - A_{12}(j)Q(j) - Q^T(j)A_{12}^T(j) \\ &+ \alpha_{jj}P(j) + b_1\delta_{11}^2 I + b_2\delta_{12}^2 I + \sum_{i=1, i\neq j}^{v} \alpha_{ji}P(j)P^{-1}(i)P(j) \\ &+ b_1^{-1}P^2(j) + b_2^{-1}Q^T(j)Q(j) < 0. \end{aligned} \quad (24)$$

It leads to

$$\begin{aligned} &A_{11}(j)P(j) + P(j)A_{11}^T(j) - A_{12}(j)Q(j) - Q^T(j)A_{12}^T(j) \\ &+ \alpha_{jj}P(j) + b_1\Delta_{11}\Delta_{11}^T + b_2\Delta_{12}\Delta_{12}^T \\ &+ \sum_{i=1, i\neq j}^{v} \alpha_{ji}P(j)P^{-1}(i)P(j) + b_1^{-1}P^2(j) \\ &+ b_2^{-1}Q^T(j)Q(j) < 0. \end{aligned} \quad (25)$$

Let $M(j) = P^{-1}(j)$. Pre- and post multiplied by $M(j)$, it gives

$$\begin{aligned} &M(j)A_{11}(j) + A_{11}^T(j)M(j) - M(j)A_{12}(j)C_1(j) \\ &- C_1^T(j)A_{12}^T(j)M(j) + \sum_{i=1}^{v} \alpha_{ji}M(i) \\ &+ b_1 M_j\Delta_{11}\Delta_{11}^T M_j + b_2 M_j \Delta_{12}\Delta_{12}^T M_j \\ &+ b_1^{-1}I + b_2^{-1}C_1^T(j)C_1(j) < 0. \end{aligned} \quad (26)$$

The Lyapunov function candidate is given as

$$V_1 = X_1^T M(j) X_1. \quad (27)$$

The weak infinitesimal operator $\mathscr{L}$ of the process $\{X_1(t), \eta_t, t \ge 0\}$ at the point $\{t, X_1, j\}$ is given by

$$\begin{aligned} \mathscr{L}V_1 &= 2X_1^T M(j)\dot{X}_1 + \sum_{i=1}^{v} X_1^T \alpha_{ji} M(i) X_1 \\ &= 2X_1^T M(j)(A_{11}(j) + \Delta_{11}(j) - A_{12}(j)C_1(j) \\ &\quad - \Delta_{12}(j)C_1(j))X_1 + \sum_{i=1}^{v} X_1^T \alpha_{ji} M(i) X_1. \end{aligned} \quad (28)$$

By (26), we get $\mathscr{L}V_1 < 0$, if $X_1 \neq \mathbf{0}$. Referring (7), it can be drawn that the system state $[X_1^T, X_2^T]^T$ is stochastically asymptotically stable. This completes the proof. □

There are two phases in sliding mode dynamics, the reaching phase and sliding phase. If the control force is large enough, it can suppress any matched uncertainties to realize the sliding motion. However, if it is not sufficiently large, it will affect the reaching and sliding abilities of the SMC, that is, the reaching ability and sliding ability would be in a probabilistic sense. In this section, we will derive the conditions to guarantee the globally asymptotical stability of the MJSs under SMC. We will also derive the reaching probability function and the sliding probability function. For convenience, we introduce the *reaching probability* (Zhu et al., 2014), denoted by $P(E_r|B_r)$, which represents the condition probability of the system reaching the sliding mode in a finite time t_r, and the *sliding probability* (Zhu et al., 2014), denoted by $P(E_s|B_s)$, which represents the condition probability of the system maintaining on the sliding mode.

Some events are defined as

$$\begin{aligned} B_r &: \text{The condition is } \eta_0 = s_0, X(0) = X_0, \\ &\quad U(t) = F_u(t), \ 0 \le t \le t_r. \end{aligned} \quad (29)$$

$$\begin{aligned} B_s &: \text{The condition is } \eta_{t_r} = \eta_{t_r}, X(t_r) = X_{t_r}, \\ &\quad S(t_r) = \mathbf{0}, \ U(t) = F_u(t), \ t > t_r. \end{aligned} \quad (30)$$

$$\begin{aligned} E_r &: \text{The system first reaches the sliding mode} \\ &\quad \text{at time } t_r, \ S(t_r) = \mathbf{0}. \end{aligned} \quad (31)$$

$$\begin{aligned} E_s &: \text{The system maintains on the sliding mode}, \\ &\quad S(t) = \mathbf{0}, \ t > t_r \end{aligned} \quad (32)$$

where t_r is a given constant, $F_u(t)$ represents the designed control input.

The following SMC is proposed for the MIMO MJS as

$$U(t) = \begin{cases} U_{eq}(t) - B_2^{-1}(j) \\ \quad \times \left(k_1\|X_1\| \dfrac{S}{\|S\|} + k_2 S + k_r \dfrac{S}{\|S\|} \right) & S \neq \mathbf{0}, \\ \mathbf{0} \quad S = \mathbf{0}, \end{cases} \tag{33}$$

where variable S is defined by (7), k_1, k_2 are control parameters, and $U_{eq}(t)$ is the equivalent control defined as

$$U_{eq}(t) = -B_2^{-1}(j) \Bigg[(C_1 A_{11}(j) - C_1 A_{12}(j) C_1 + A_{21}(j) - A_{22}(j) C_1 + \sum_{i=1}^{\upsilon} \alpha_{ji} C_1(i)) X_1 + (C_1 A_{12}(j) + A_{22}(j)) S \Bigg]. \tag{34}$$

The following theorem is given.

Theorem 2. *Assume the condition in Theorem 1 holds, and SMC is designed as (33), the close-loop system is asymptotically stable, if the control parameters in (33) are given as follows.*

$$k_1 = \max_{j\in S}\{k_{1j}\}, \qquad k_2 = \max_{j\in S}\{k_{2j}\}, \tag{35}$$

$$k_{1j} = \|C_1(j)\|\delta_{11}(j) + \|C_1(j)\|^2\delta_{12}(j) + \delta_{21}(j) + \|C_1(j)\|\delta_{22}(j), \tag{36}$$

$$k_{2j} = \|C_1(j)\|\delta_{12}(j) + \delta_{22}(j), \quad j = 1, \ldots, \nu. \tag{37}$$

Proof. The Lyapunov function candidate is given as

$$V(S) = \frac{1}{2} S^T(X) S(X). \tag{38}$$

Substituting SMC (33) and (35) into it, the weak infinitesimal operator $\mathscr{L}$ of the process $\{X_1(t), \eta_t, t \geq 0\}$ at the point $\{t, X_1, j\}$ is given by

$$\begin{aligned} \mathscr{L}V(S) &= S^T[\bar{A}_{21}(j)X_1 + \bar{A}_{22}(j)S + B_2(j)U(t)] \\ &= S^T[(C_1\Delta_{11}(j) - C_1\Delta_{12}(j)C_1 + \Delta_{21}(j) \\ &\quad - \Delta_{22}(j)C_1)X_1 + (C_1\Delta_{12}(j) + \Delta_{22}(j))S] \\ &\quad - S^T\left(k_1\|X_1\|\frac{S}{\|S\|} + k_2 S + k_r \frac{S}{\|S\|}\right) \\ &\leq -k_r\|S\| \leq 0. \end{aligned} \tag{39}$$

The equality holds if and only if $S = \mathbf{0}$. It means under SMC (33), the system can reach the switching surface in a finite time $t_r = \frac{\|S(0)\|}{k_r}$ and then maintain its state on the switching surface afterwards.

Here arises a question. If the control is insufficient, what will the reaching probability and sliding probability be? This question is answered in the following.

In the reaching phase, the control objective is to drive the system state to reach the sliding mode in finite time t_r. With the following definitions of auxiliary parameters

$$k_{1j}^n = \max_{i\in r_j^n}\{k_{1i}\}, \qquad k_{2j}^n = \max_{i\in r_j^n}\{k_{2i}\}, \tag{40}$$

where k_{1i}, k_{2i} are defined by (36) and (37), and $r_j^n = (s_{j0}, \ldots, s_{jn}) \in S^{n+1}$ represents the Markovian process state transition route, we derive the reaching probability formulae.

Theorem 3. *For the MIMO linear MJS (1) under SMC (33), the reaching probability is given as follows*

$$P(E_r|B_r) = \sum_{n=0}^{m} P(E_r A^n|B_r), \tag{41}$$

where

$$\begin{aligned} m &= \arg\min_n \Bigg\{ \sum_{i=0}^{n} P(E_r A^i|B_r) > 1-\varepsilon,\ 0 < \varepsilon \ll 1, \\ &\qquad \textit{if } k_1 = \max_{j\in S}\{k_{1j}\}, k_2 = \max_{j\in S}\{k_{2j}\} \Bigg\}, \end{aligned} \tag{42}$$

$$P(E_r A^n|B) = \sum_{r_j^n \subset S_n} P(E_r|A_j^n B) P(A_j^n|B), \tag{43}$$

$$P(E_r|A_j^n B) = \begin{cases} 1 & k_1 \geq k_{1j}^n \text{ and } k_2 \geq k_{2j}^n, \\ 0 & k_1 < k_{1j}^n \text{ or } k_2 < k_{2j}^n. \end{cases} \tag{44}$$

Proof. We consider the situations where the control parameters k_1 and k_2 are not sufficient enough to guarantee the asymptotical stability of the MJS under SMC. Therefore, there exists a probability that the sliding mode cannot be reached.

Firstly, consider that the stochastic parameter η_t does not jump. $\eta_t = s_0$. The reaching probability for no jump is

$$\begin{aligned} P(E_r A^0|B_r) &= P(E_r|A^0 B_r) P(A^0|B) \\ &= \begin{cases} e^{-q_{s_0} t} & k_1 \geq k_{1s_0} \text{ and } k_2 \geq k_{2s_0}, \\ 0 & k_1 < k_{1s_0} \text{ or } k_2 < k_{2s_0}, \end{cases} \end{aligned} \tag{45, 46}$$

where $P(A^0|B)$ is defined by (16) and

$$P(E_r|A^0 B_r) = \begin{cases} 1 & k_1 \geq k_{1s_0} \text{ and } k_2 \geq k_{2s_0}, \\ 0 & k_1 < k_{1s_0} \text{ or } k_2 < k_{2s_0}. \end{cases} \tag{47}$$

Secondly, consider the stochastic parameter η_t jumps n times ($n \in [1, \infty)$). Denote k_{1j}^n, k_{2j}^n as the sufficient control parameters which enable the system to reach the control objective while the stochastic parameter η_t jumps along the transition route r_j^n (13). It holds that

$$k_{1j}^n = \max_{i\in r_j^n}\{k_{1i}\}, \qquad k_{2j}^n = \max_{i\in r_j^n}\{k_{2i}\}. \tag{48}$$

The reaching probability for n jumps is

$$P(E_r A^n|B_r) = \sum_{r_j^n \subset S_n} P(E_r|A_j^n B_r) P(A_j^n|B) \tag{49}$$

where $P(A_j^n|B)$ is defined by (17),

$$P(E_r|A_j^n B_r) = \begin{cases} 1 & k_1 \geq k_{1j}^n \text{ and } k_2 \geq k_{2j}^n, \\ 0 & k_1 < k_{1j}^n \text{ or } k_2 < k_{2j}^n. \end{cases} \tag{50}$$

The total reaching probability is

$$P(E_r|B_r) = \sum_{i=0}^{\infty} P(E_r A^i|B_r) \approx \sum_{j=0}^{m} P(E_r A^j|B_r) \tag{51}$$

where m is chosen as (42). The residual items $P(E_r A^j|B_r), j > m$ can be omitted. This completes the proof. □

In the sliding phase, the control objective is to keep the system state sliding on the switching surface. Insufficient control force may result in violation of the existence condition of the sliding mode $S^T\dot{S} < 0$ in the neighborhood of $S = \mathbf{0}$.

The following theorem is given for the sliding probability.

Theorem 4. *For the MIMO linear MJS (1) under SMC (33), the sliding probability is given as follows*

$$P(E_s|B_s) = \sum_{j\in S} P(E_s|A_j B_s) P(A_j|B_s) \tag{52}$$

where

$$P(E_s|A_jB_s) = \begin{cases} 1 & k_1 \geq k_{1j} \text{ and } k_2 \geq k_{2j}, \\ 0 & k_1 < k_{1j} \text{ or } k_2 < k_{2j}. \end{cases} \tag{53}$$

Proof. In the sliding phase, since the system state is already on the switching surface, the control objective is to keep the system state to stay on it at anytime.

For each time t, $\eta_t = j$, $j \in S$, as seen from (39), the MJS under SMC (33) can ensure that the system state stays in the switching surface. However, if the control parameters k_1 and k_2 are somehow insufficient, we get

$$P(E_s|A_jB_s) = \begin{cases} 1 & k_1 \geq k_{1j} \text{ and } k_2 \geq k_{2j}, \\ 0 & k_1 < k_{1j} \text{ or } k_2 < k_{2j}. \end{cases} \tag{54}$$

Synthesizing all possibilities, then we have the sliding probability calculated as

$$P(E_s|B_s) = \sum_{j\in S} P(E_s|A_jB_s)P(A_j|B_s). \tag{55}$$

This completes the proof. □

Remark 1. The reaching probability (sliding probability) is a piecewise non-decreasing function with a parameter value domain $\{[0, +\infty), [0, +\infty)\}$ and a function value domain $[0, 1.0]$. It indicates that if the control parameters k_1, k_2 are sufficiently large, then the function value equals one, which means the control objective is to be definitely reached. If the control parameters are sufficiently small, then the function value equals zero, which means the control objective is definitely unreachable. If the control parameters take middle values, then the function takes a value between zero and one, which means the control objective is to be reached in a probability sense.

Remark 2. It is well-known that there exist controller chattering phenomena in classical SMC schemes. For single-input SMC, a simple method to handle the chattering is to use a saturate function sat(s) instead of the sign function sgn(s). In this paper, if the norms of control gain matrices $B_2(j)$ are large enough, by (33), (34), the chattering will be slight. Furthermore, we can use the saturate function $S \cdot \text{sat}(\frac{1}{\|S\|})$ instead of $\frac{S}{\|S\|}$ in (33) to handle the chattering. The chattering-free SMC is given as

$$U(t) = \begin{cases} U_{eq}(t) - B_2^{-1}(j)\left(k_1\|X_1\|S \cdot \text{sat}\left(\frac{1}{\|S\|}\right)\right. \\ \qquad \left. + k_2 S + k_r S \cdot \text{sat}\left(\frac{1}{\|S\|}\right)\right) \quad S \neq \mathbf{0}, \\ \mathbf{0} \quad S = \mathbf{0}. \end{cases}$$

The result is shown in Fig. 7.

4. Numerical simulations

In this section, we present numerical simulations to show the effectiveness of the theoretical results.

A linear MIMO MJS (1) is considered, where $\eta_t \in S$ is a continuous Markovian process and

$S = \{1, 2, 3\}$,

$$A_{11}(1) = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \Delta_{11}(1) = \begin{bmatrix} 0.0848 & 0 \\ 0 & 0.0848 \end{bmatrix},$$

$$A_{12}(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \Delta_{12}(1) = \begin{bmatrix} 0.1664 & 0 \\ 0 & 0.1664 \end{bmatrix},$$

$$A_{21}(1) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \qquad \Delta_{21}(1) = \begin{bmatrix} 0.1664 & 0 \\ 0 & 0.1664 \end{bmatrix},$$

$$A_{22}(1) = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}, \qquad \Delta_{22}(1) = \begin{bmatrix} 0.3526 & 0 \\ 0 & 0.3526 \end{bmatrix},$$

$$A_{11}(2) = \begin{bmatrix} -5 & 0 \\ 0 & 2 \end{bmatrix}, \qquad \Delta_{11}(2) = \begin{bmatrix} 0.2932 & 0 \\ 0 & 0.2932 \end{bmatrix},$$

$$A_{12}(2) = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}, \qquad \Delta_{12}(2) = \begin{bmatrix} 0.3526 & 0 \\ 0 & 0.3526 \end{bmatrix},$$

$$A_{21}(2) = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, \qquad \Delta_{21}(2) = \begin{bmatrix} 0.2319 & 0 \\ 0 & 0.2319 \end{bmatrix},$$

$$A_{22}(2) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, \qquad \Delta_{22}(2) = \begin{bmatrix} 0.2932 & 0 \\ 0 & 0.2932 \end{bmatrix},$$

$$A_{11}(3) = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}, \qquad \Delta_{11}(3) = \begin{bmatrix} 0.1664 & 0 \\ 0 & 0.1664 \end{bmatrix},$$

$$A_{12}(3) = \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}, \qquad \Delta_{12}(3) = \begin{bmatrix} 0.2932 & 0 \\ 0 & 0.2932 \end{bmatrix},$$

$$A_{21}(3) = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}, \qquad \Delta_{21}(3) = \begin{bmatrix} 0.2932 & 0 \\ 0 & 0.2932 \end{bmatrix},$$

$$A_{22}(3) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \Delta_{22}(3) = \begin{bmatrix} 0.1664 & 0 \\ 0 & 0.1664 \end{bmatrix},$$

$$B_2(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad B_2(2) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_2(3) = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix},$$

generator (q_{ij}) with transition probability from mode i at time t to mode j at time $t + \delta$, $i, j \in S$ as

$$(q_{ij}) = \begin{bmatrix} -5 & 3 & 2 \\ 1 & -1 & 0 \\ 2 & 1 & -3 \end{bmatrix}.$$

By Theorem 1, we get

$$C_1(1) = \begin{bmatrix} -0.3896 & 0.0371 \\ 0.0813 & 2.2635 \end{bmatrix},$$

$$C_1(2) = \begin{bmatrix} -1.1129 & 0.6470 \\ 0.0491 & 1.2559 \end{bmatrix},$$

$$C_1(3) = \begin{bmatrix} 2.5956 & 1.7389 \\ -0.0561 & 3.2910 \end{bmatrix}.$$

The sliding variable is defined as

$$S = C_1(\eta_t)X_1 + X_2.$$

The initial parameters are set that $X(0) = [0.1 \; -0.1 \, 0.1 \, 0]^T$, $\eta_0 = 1$, and $t_r = 0.1$.

Using $\hat{P}_j = P(\eta_{\tau_1} = j|\eta_0 = i)$ to estimate $P(\eta_t = j|\eta_0 = i)$, we get

$$\hat{P}_1 = 0.5, \qquad \hat{P}_2 = 0.3, \qquad \hat{P}_3 = 0.2.$$

The SMC law is given as (33). According to Theorem 2, we have

$$k_1(1) = 3.6512, \qquad k_1(2) = 3.3683, \qquad k_1(3) = 10.8426,$$
$$k_2(1) = 1.2795, \qquad k_2(2) = 1.4251, \qquad k_2(3) = 2.2938.$$

If control parameters satisfy $k_1 \geq 10.8426$, $k_2 \geq 2.2938$, then the system is asymptotically stable. The results are shown in Figs. 4–5.

Next, we study the sliding and reaching probabilities when the control is insufficient.

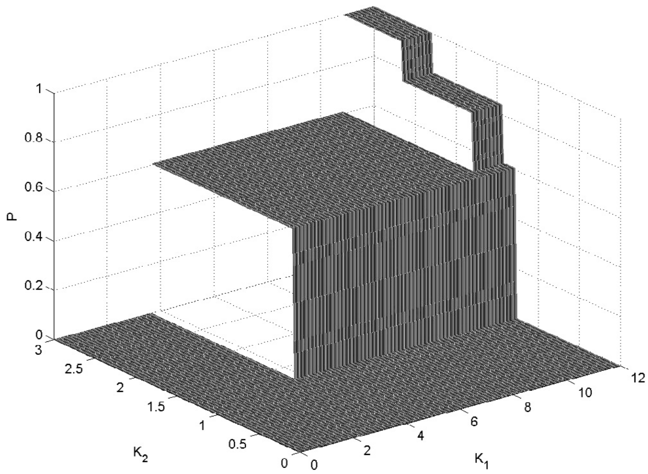

Fig. 1. Reaching probability.

For the reaching phase ($0 \le t \le t_r$), first, consider that the stochastic parameter does not jump. From Theorem 2, we get

$$P(EA^0|B) = P(E|A^0B)P(A^0|B) = \begin{cases} 0.6065 & k_1 \ge 3.6512 \text{ and } k_2 \ge 1.2795, \\ 0 & k_1 < 3.6512 \text{ or } k_2 < 1.2795 \end{cases}$$

where

$$P(E|A^0B) = \begin{cases} 1 & k_1 \ge 3.6512 \text{ and } k_2 \ge 1.2795, \\ 0 & k_1 < 3.6512 \text{ or } k_2 < 1.2795. \end{cases}$$

and $P(A^0|B)$ is defined by (16).

However, if the stochastic parameter jumps n times, we have the following scenarios.

For case $n = 1$, there are two parameter transition routes which are

$$r_1^1 = (1, 2), \qquad r_2^1 = (1, 3).$$

From Theorem 3, we have

$$k_{11}^1 = \max\{k_1(1), k_1(2)\} = 3.6512,$$
$$k_{21}^1 = \max\{k_2(1), k_2(2)\} = 1.4251,$$
$$k_{12}^1 = \max\{k_1(1), k_1(3)\} = 10.8426,$$
$$k_{22}^1 = \max\{k_2(1), k_2(3)\} = 2.2938.$$

From Lemma 1, we have the solution of $P(A_j^1|B)$, which leads to

j	r_j^1	k_{1j}^1	k_{2j}^1	$P(A_j^1\|B)$
1	(1,2)	3.6512	1.4251	0.2237
2	(1,3)	10.8426	2.2938	0.1343

Thus it gives that

$$P(EA^1|B) = \sum_{r_j^1 \in S_1} P(E|A_j^1B)P(A_j^1|B) = \begin{cases} 0 & k_1 < 3.6512 \text{ or } k_2 < 1.4251, \\ 0.2237 & \begin{array}{l} 3.6512 \le k_1 < 10.8426 \\ \quad \text{and } k_2 \ge 1.4251 \\ \text{or } 1.4251 \le k_2 < 2.2938 \\ \quad \text{and } k_1 \ge 3.6512, \end{array} \\ 0.3580 & k_1 \ge 10.8426 \text{ and } k_2 \ge 2.2938. \end{cases}$$

For case $n = 2$, similar to the above, we get

j	r_j^2	k_{1j}^2	k_{2j}^2	$P(A_j^2\|B)$
1	(1,2,1)	3.6512	1.4251	0.0105
2	(1,2,3)	10.8426	2.2938	0
3	(1,3,1)	10.8426	2.2938	0.0130
4	(1,3,2)	10.8426	2.2938	0.0074

$$P(EA^2|B) = \sum_{r_j^2 \in S_2} P(E|A_j^2B)P(A_j^2|B) = \begin{cases} 0 & k_1 < 3.6512 \text{ or } k_2 < 1.4251, \\ 0.0105 & \begin{array}{l} 3.6512 \le k_1 < 10.8426 \\ \quad \text{and } k_2 \ge 1.4251 \\ \text{or } 1.4251 \le k_2 < 2.2938 \\ \quad \text{and } k_1 \ge 3.6512, \end{array} \\ 0.0309 & k_1 \ge 10.8426 \text{ and } k_2 \ge 2.2938. \end{cases}$$

Finally, for case $n = 3$, the same derivations can be done and we have the total reaching probability approximated as

$$P(E_r|B_r) = \sum_{i=0}^{3} P(EA^i|B) = \begin{cases} 0 & k_1 < 3.3683 \text{ or } k_2 < 1.2795 \\ 0.6065 & \begin{array}{l} 3.6512 \le k_1 < 10.8426 \\ \quad \text{and } k_2 \ge 1.2795 \\ \text{or } 1.2795 \le k_2 < 1.4251 \\ \quad \text{and } k_1 \ge 3.6512, \end{array} \\ 0.8407 & \begin{array}{l} 3.6512 \le k_1 < 10.8426 \\ \quad \text{and } k_2 \ge 1.4251 \\ \text{or } 1.4251 \le k_2 < 2.2938 \\ \quad \text{and } k_1 \ge 3.6512, \end{array} \\ 1.0 & k_1 \ge 10.8426 \text{ and } k_2 \ge 2.2938. \end{cases}$$

For the sliding phase ($t > t_r$), let $t_s = 0.2$. By Lemma 1 and $\hat{P}_j = P_{ij}(t_s)$, we get

$$\hat{P}_1 = 0.4324, \qquad \hat{P}_2 = 0.3751, \qquad \hat{P}_3 = 0.1925.$$

Referring to Theorem 4, we get the sliding probability

$$P(E_s|B_s) = \sum_{j \in S} P(E_s|A_jB_s)P(A_j|B_s) = \begin{cases} 0 & k_1 < 3.3683 \text{ or } k_2 < 1.2795, \\ 0.4324 & k_1 \ge 3.6512 \text{ and } 1.2795 \le k_2 < 1.4251, \\ 0.3751 & \begin{array}{l} 3.3683 \le k_1 < 3.6512 \text{ and} \\ 1.4251 \le k_2 < 2.2938, \end{array} \\ 0.8075 & \begin{array}{l} 3.6512 \le k_1 < 10.8426 \\ \quad \text{and } k_2 \ge 1.4251 \\ \text{or } 1.4251 \le k_2 < 2.2938 \\ \quad \text{and } k_1 \ge 3.6512, \end{array} \\ 1.0 & k_1 \ge 10.8426 \text{ and } k_2 \ge 2.2938. \end{cases}$$

The results are shown in Figs. 1–7.

5. Conclusion

In this paper, an equivalent control based SMC for MIMO uncertain linear MJSs has been proposed which guarantees the system's stochastically asymptotical stability. Sufficient conditions in terms of LMIs are presented, which guarantee that the sliding motion is asymptotically stable. Furthermore, the reaching and sliding probabilities have been studied for situations where the control force is not strong enough to ensure fully asymptotical stability. Simulation results have been presented to illustrate the effectiveness of the proposed design method.

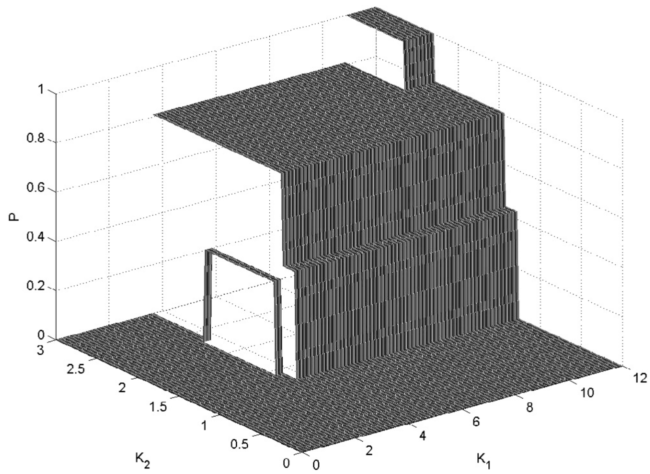

Fig. 2. Sliding probability.

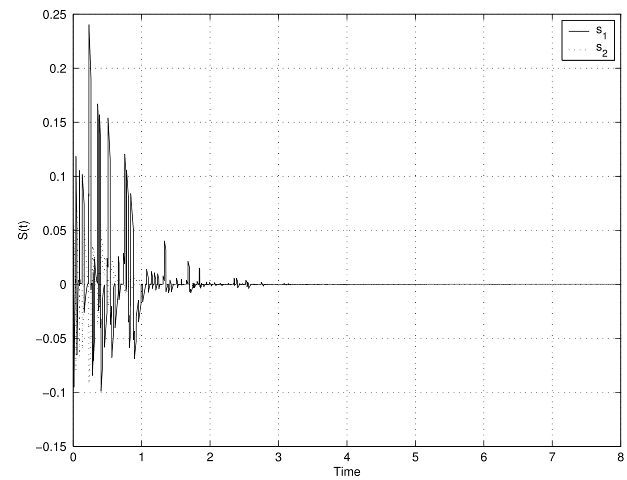

Fig. 5. Sliding surface.

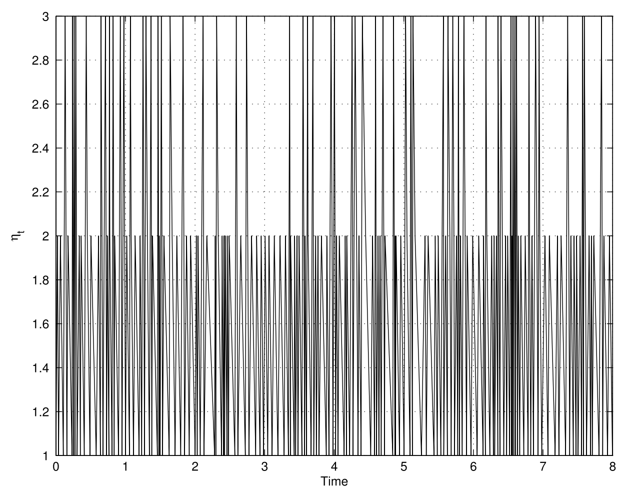

Fig. 3. Continuous Markovian process.

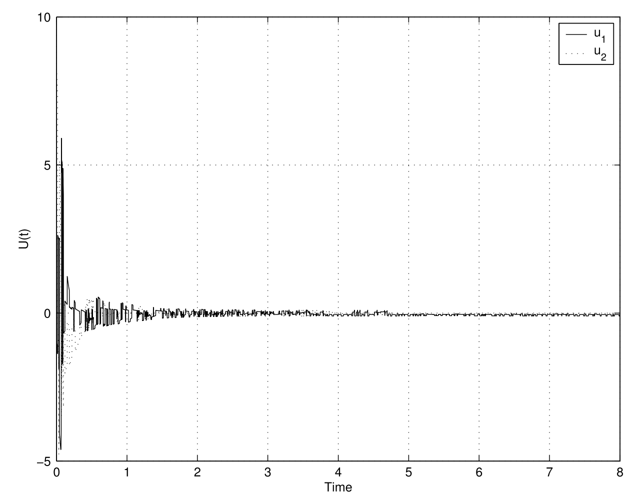

Fig. 6. Control input.

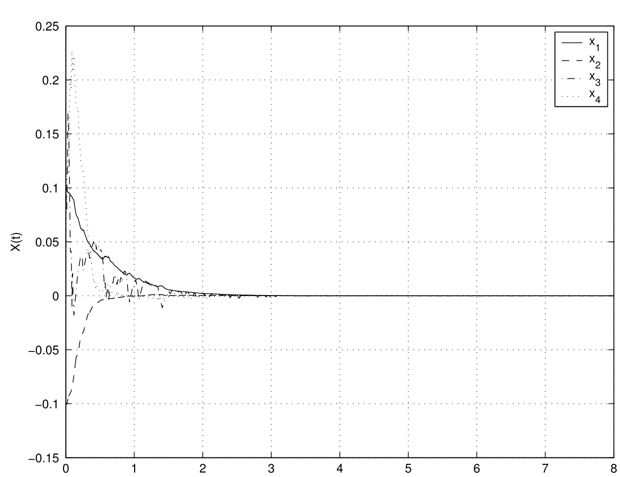

Fig. 4. System state.

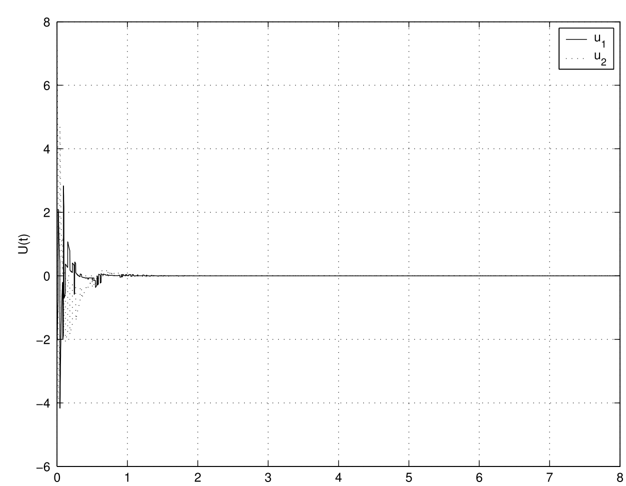

Fig. 7. Chattering-free control input.

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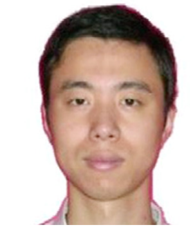

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