Transportation by Multiple Mobile Manipulators in Unknown Environments With Obstacles

Jile Jiao, Zhiqiang Cao, Nong Gu, Saeid Nahavandi, Senior Member, IEEE, Yuequan Yang, and Min Tan

Abstract—A novel approach to transport a large object by multiple mobile manipulators in unknown environments with obstacles is proposed in this paper. The main moving direction of the system as well as its width and the leading mobile manipulator are first assigned manually or obtained automatically according to the geometric layout of mobile manipulators. Then, three transportation modes, i.e., default mode, shrink mode, and incline mode, are defined based on constraints from the object and mobile manipulators. In default mode, multiple mobile manipulators are distributed around the object to be transported with the initial geometric layout. Mobile manipulators in shrink mode are required to lie inside the boundaries of the object’s outlined rectangle, whereas multiple mobile manipulators in incline mode endeavor to hold the object with an inclined angle. On this basis, a decision based on the multiscale passageway is proposed to facilitate the selection of best transportation mode and moving direction. The effectiveness of the approach is verified by simulations and experiments.

Index Terms—Multiple mobile manipulators, multiscale passageway, transportation, transportation modes, unknown environments.

I. INTRODUCTION

MOBILE manipulators refer to robotic systems where a manipulator is mounted on a moving platform. Due to their combined features of mobility and dexterity, mobile manipulators [1] have been found in many applications such as explosive ordnance disposal, industrial manufacturing, home care, and healthcare. Many efforts have been paid toward controlling a single mobile manipulator system. In [2], the disturbance-observer-based control of a free-floating space manipulator was investigated to deal with internal parameter uncertainty and external disturbance, and this approach was further improved by a fuzzy adaptive method [3]. Jain and Kemp [4] illustrated their progress toward the creation of an assistant mobile manipulator in a home environment, which can retrieve and deliver objects to flat surfaces. In [5], a visual servo control for grasping by wheeled mobile robots was studied, and a hybrid controller based on the image-based visual servoing and Q-learning was developed. Hamner et al. [6] reported an autonomous mobile manipulator, which is experimentally demonstrated in the insertion assembly tasks. Chitta et al. [7] showcased an approach to accomplish mobile pick-and-place tasks with a two-arm mobile manipulation system in unstructured human environments. Tang et al. [8] investigated a differential-flatness-based integrated point-to-point trajectory planning and control method for a class of nonholonomic wheeled mobile manipulators. Minca et al. described a synchronized hybrid petri net approach for a robotic manipulator mounted on a mobile platform applied in an assembly/disassembly line [9]. Jiao et al. proposed a vision-based autonomous move-to-grasp method for a compact mobile manipulator under narrow environments, and the mobile platform was controlled based on information of the object to be grasped in the image [10].

For some large-scale tasks such as manufacturing and assembly in automatic factories and space exploration, it is not practical, if not impossible, to employ a single mobile manipulator. This raises a compelling need for studying cooperation with multiple mobile manipulators [11]–[13]. Kume et al. proposed a coordinated motion control algorithm of multiple mobile manipulators handling a single object with the leader–follower type based on the caster-like dynamics [14], [15] and impedance dynamics [16], and the experimental results are demonstrated in obstacle-free environments. Liu et al. [17] conducted the research on force synchronization of multiple robot manipulators. Li et al. investigated the problem of multiple mobile manipulators grasping a common object in contact with a rigid surface [18] or a nonrigid surface [19]. Yan et al. designed a coordinated control strategy for a multioperator multi-mobile-manipulator cooperative teleoperation system [20]. Tang et al. [21] formulated a systematic framework based on screw-theoretic analysis, which is for formulation and evaluation of system-level performance of a cooperative payload transport task by a modularly composed system of multiple wheeled mobile manipulators. It shall be noted that the aforementioned researches assume that the working space of mobile manipulators is an obstacle-free zone. Unfortunately, such an assumption can be easily violated in practice. Hence, these methods cannot be applied in a more realistic context.

Considering obstacles in the working space, Tanner et al. presented a motion planning methodology applicable to multiple nonholonomic mobile manipulators handling a deformable object in a known static environment with obstacles [22], which leads to weak adaptability to unknown environments.
with obstacles. Andaluz et al. proposed a multilayer scheme for coordinated control of mobile manipulators with independent obstacle avoidance of individual one [23]. Although some efforts have been made to ensure the multiple mobile manipulators move safely, there is still a challenge to consider the collaboration in a form of system consistency, which may improve the ability to adapt to the environments. Among existing references, the global collaboration is seldom mentioned [24]. Yamashita et al. [24] conducted the research on motion planning of multiple mobile robots for transportation of a large object in a 3-D environment with obstacles whose shape and pose are known. Three core primitive operations of an object in a 3-D environment with obstacles are defined, and a decision based on the multiscale transportation modes, i.e., default mode, shrink mode, and in-}

II. PROBLEM STATEMENT

This paper is motivated by the problem of the transportation of an object by multiple mobile manipulators in unknown obstacle environments. Mobile manipulators are supposed to be distributed around the target object beforehand in a form of convex polygon. Fig. 1 demonstrates an example with four mobile manipulators handling a large object. As shown in Fig. 1, the system outlined rectangle (the orange rectangle in Fig. 1) is introduced first, which contains all mobile manipulators, and there is at least one mobile manipulator on each side of the rectangle. Among all possible system outlined rectangles, we define the one with the shortest side as the best system outlined rectangle. For the object to be transported, it also has an outlined rectangle, which covers the whole object and has the same direction as that of the whole system.

On each mobile manipulator \( R_i \) \((i = 1, 2, \ldots, l)\), there is a 6-degree of freedom (DOF) manipulator. These six DOFs come from the translational grasp joint, the rotary joint of wrist, the pitch joint of wrist, the elbow joint, the shoulder joint, and the waist joint. The first two DOFs facilitate the manipulator to grasp the object, and they remain fixed once the object is grasped by the manipulator. Hence, only the angles of pitch joint of wrist, elbow joint, shoulder joint, and waist joint are of interest in this study for cooperative transportation, which are represented as \( \theta_1 \in [-\pi/2, \pi/2], \theta_2 \in [-\pi/2, \pi/2], \theta_3 \in [0, \pi], \) and \( \theta_4 \in [-\pi, \pi] \), respectively. \( l_1, l_2, \) and \( l_3 \) are the lengths of connecting rods for the manipulator. \( r_r \) and \( h_r \) are the radius and height of the mobile base. A coordinate system \( O_cX_cY_cZ_c \) is established with its origin and \( Y \)-axis being the center point \( O_c(x_c, y_c) \) of the best system outlined rectangle and the moving direction of the system, respectively. \( O_wX_wY_wZ_w \) is labeled as the world coordinate system. It should be noted that for each mobile manipulator, its contact point with the object remains unchanged during the transportation process.

The position \((x_{wR}^c, y_{wR}^c, z_{wR}^c)^T\) of any mobile manipulator \( O_wX_wY_wZ_w \) is calculated as follows:

\[
\begin{bmatrix}
    x_{wR}^c \\
    y_{wR}^c \\
    z_{wR}^c
\end{bmatrix} = R_w^c 
\begin{bmatrix}
    x_{wR}^c \\
    y_{wR}^c \\
    z_{wR}^c
\end{bmatrix} + \begin{bmatrix}
    x_{wR}^o \\
    y_{wR}^o \\
    z_{wR}^o
\end{bmatrix}
\tag{1}
\]

where \((x_{wR}^c, y_{wR}^c, z_{wR}^c)^T\) is the coordinates of the contact point of the manipulator in \( O_cX_cY_cZ_c \), \( R_w^c \) is the rotation matrix of coordinate system \( O_cX_cY_cZ_c \) to \( O_wX_wY_wZ_w \). \( \theta_D \) is the angle between \( O_wX_w \) and the moving direction of mobile manipulators, \( (x_{wR}^o, y_{wR}^o, z_{wR}^o)^T \) is the coordinates of point \( O_c \) in \( O_wX_wY_wZ_w \).

We denote the positions of end points of connecting rods in \( O_wX_wY_wZ_w \) as \((x_{w1}^c, y_{w1}^c, z_{w1}^c)^T, (x_{w2}^c, y_{w2}^c, z_{w2}^c)^T, \) and \((x_{w3}^c, y_{w3}^c, z_{w3}^c)^T\), respectively.

Based on the kinematic model of the manipulator, one can derive the height \( h(\theta_1, \theta_2, \theta_3) \) of the contact point of the manipulator with the object and the length \( l(\theta_1, \theta_2, \theta_3) \) of the manipulator in horizontal direction. Obviously, the installation height of the third connecting rod is \( h_r \). Thus

\[
\begin{align*}
    h(\theta_1, \theta_2, \theta_3) &= l_3 \sin \theta_3 + l_2 \sin(\theta_2 + \theta_3) \\
    &+ l_1 \sin(\theta_1 + \theta_2 + \theta_3) + h_r \\
    l(\theta_1, \theta_2, \theta_3) &= l_3 \cos \theta_3 + l_2 \cos(\theta_2 + \theta_3) \\
    &+ l_1 \cos(\theta_1 + \theta_2 + \theta_3).
\end{align*}
\tag{2}
\]
Then, we can calculate the positions of end points of the connecting rods for a mobile manipulator, i.e.,

\[
\begin{align*}
  x^w_{j1} &= l(\theta_1, \theta_2, \theta_3) \cos \theta_R + x^w_{R} \\
  y^w_{j1} &= l(\theta_1, \theta_2, \theta_3) \sin \theta_R + y^w_{R} \\
  z^w_{j1} &= h(\theta_1, \theta_2, \theta_3) \\
  x^w_{j2} &= (l_3 \cos \theta_3 + l_2 \cos(\theta_3 + \theta_2)) \cos \theta_R + x^w_{R} \\
  y^w_{j2} &= (l_3 \cos \theta_3 + l_2 \cos(\theta_3 + \theta_2)) \sin \theta_R + y^w_{R} \\
  z^w_{j2} &= (l_3 \sin \theta_3 + l_2 \sin(\theta_3 + \theta_2)) + h_r \\
  x^w_{j3} &= l_3 \cos \theta_3 \cos \theta_R + x^w_{R} \\
  y^w_{j3} &= l_3 \cos \theta_3 \sin \theta_R + y^w_{R} \\
  z^w_{j3} &= l_3 \sin \theta_3 + h_r
\end{align*}
\]

where \( \theta_R = \arctan\left(y^w_{R} - y^w_{R} \right) \). It shall be noted that \( \theta_R \) instead of \( \theta_4 \) is used to solve the positions depicted in (3)-(5) for simplicity.

In this paper, we focus on the task where multiple mobile manipulators transport an object cooperatively to a target zone. This object has already been grasped by all mobile manipulators in advance. The environment is unknown to the system, and no preplanned path is available. The dynamic model is not considered, and we assume that there is no slippage. The transportation approach is a geometrical one, and three transportation modes, i.e., default mode, shrink mode, and incline mode, are designed to improve the adaptability to unknown environments, which will be detailed later in Section III-A. There is a leading mobile manipulator who is in charge of environmental detection with a range sensor. A decision of the system is made by this leader in each control cycle. The decision results include the optimal transportation mode for current environment and the optimal moving direction, which are sent to other robots in a broadcasting way. On this basis, each individual robot calculates its expected position and postures according to selected optimal transportation mode as well as the kinematics of mobile manipulators. With this optimal decision, a coordinated collision-free movement can be achieved by the proposed transportation system.

The objective of this paper is stated as follows. Given a system of mobile manipulators, find a transportation control scheme under constraints from the system itself and the object, which steers mobile manipulators whose positions and postures are expected to be adjusted simultaneously with the relationship between \( O_x Y_x Z_x \) and \( O_w X_w Y_w Z_w \), such that the whole system has the capability to adapt to unknown environments with obstacles in a form of system consistency.

Next, we extract the initial system information based on the distribution of the mobile manipulators. The initial system information includes the main moving direction of the system, the leading mobile manipulator, and the width of the system. Certainly, it can be manually specified. A more desirable way is to obtain this initial system information automatically based on the geometric relationship of mobile manipulators.

**Lemma 2.1:** For a multiple-mobile-manipulator system in a form of an arbitrary convex polygon, the best system outlined rectangle should satisfy that there are at least two mobile manipulators on one side of the rectangle.

**Proof:** First, we assume that there is only one mobile manipulator on each side of the best system outlined rectangle \( O_r \), as shown in Fig. 2, where \( l_i (i=1,\ldots,4) \) are four sides of \( O_r \), respectively. Assume that \( \theta_{ks} = \min(\theta_{ks} (k=1,\ldots,4)) \), and we obtain lines \( l'_{2} \) and \( l'_{3} \) by rotating \( l_{3} \) around \( P_3 \) and \( l_{4} \) around \( P_4 \) for the angle \( \theta_{s} \), respectively. It is obvious that the width \( l_{o} \) of the new system outlined rectangle \( O'_r \) is shorter than the width \( l_{o} \), which is the width of \( O_r \). Hence, the assumption is invalid, which means that there are at least two mobile manipulators on one side of the best system outlined rectangle.

According to Lemma 2.1, the best system outlined rectangle can be generated by traversing those rectangles whose one side is formed by two adjacent mobile manipulators. For each candidate rectangle with two neighboring mobile manipulators \( R_l \) and \( R_r \), its length and the corresponding system information are determined based on the projection of the positions of all mobile manipulators in the line connecting \( R_l \) and \( R_r \). The rectangle with the shortest side is considered as the best system outlined rectangle, and then, initial system information is extracted. More details are shown in Algorithm 1.

**Algorithm 1. Initial Information Extraction**

**Input:** the positions \( P_i (i=1,2,\ldots,I) \) of all mobile manipulators \( R_i (i=1,2,\ldots,I) \), where \( I \) is the number of mobile manipulators.

**Output:** the 3-D information \( C^*_R = (w^*, R^*, D^*) \) to represent the best system outlined rectangle, which indicates its width, the leading mobile manipulator of the system, and the main moving direction.

```
for i = 1, 2, ... , I do
  l_i = line(P_i, P_i) (t = i+1 when i < I; t = 1 when i = I);
  //generate a line \( l_i \) passing through \( P_i \) and \( P_i \)
  p_i = dis(P_r, l_i), Q_r = projection(P_r, l_i); (r = 1, 2, ... , I)
  //obtain the distance \( p_r \) from \( P_r \) to \( l_i \) and corresponding foot \( Q_r \)
  p_i = max(p_i), D_i = direction(p_i), R_{i,l} = R_{l, \text{arg max}(p_r)};
  //\( D_i \) is the direction of the system corresponding to \( p_i \)
  //\( R_{i,l} \) is the leader candidate corresponding to \( p_i \)
  w_i = max(w_i(r), s) = dis(Q_r, Q_s); (r = 1, 2, ... , I; s = r + 1, r + 2, ... , I)
  w_i = max(w_i(r), s) // the index of the minimum \( w_i \)
  i_p = arg min(p_i) // the index of the minimum \( p_i \)
  if w_{i_p} ≤ p_{i_p} then
    C^*_R = C_R(i_p); // take \( i_w \) as the best one
  else
    l_{i_p} = line(P_{i_p}, l_{i_p});
    //generate the line with \( P_{i_p} \) and \( l_{i_p} \)
    C^*_R = (p_{i_p}, R^*, D^*);
    //\( D^* \) is the direction perpendicular to \( l_{i_p} \) and \( R^* \) is the mobile manipulator with a maximum distance to \( l_{i_p} \)
end
```
It is noted that the leading mobile manipulator and the main moving direction derived by Algorithm 1 may be shifted if the width and the length of the best system outlined rectangle is very close. In other words, a small disturbance in the position of one manipulator may lead to an obvious change in the leadership and main moving direction. However, even if this shift happens, the system width shall not change dramatically since the best system outlined rectangle can almost be represented by a square in this case. This shift shall only affect the initial motion condition of the system. In this case, the initial selection of leadership and main moving direction are trivial for the following optimal decisions due to the symmetrical nature of the square.

III. TRANSPORTATION APPROACH

The proposed approach involves three transportation modes of mobile manipulators, including default mode, shrink mode, and incline mode. In each control cycle, the leader first determines the optimal transportation mode and moving direction with an optimal decision-making algorithm. On this basis, a new position of system center \((x_0^w, y_0, z_0^w)^T\) is derived by the leader based on the optimal moving direction and a fixed step length \(d\). Each mobile manipulator calculates the expected position and postures based on the optimal transportation mode and updated position of system center. This process will be repeated until the task is completed.

A. Transportation Modes

In this paper, we define three transportation modes, namely, default mode, shrink mode, and incline mode, with the target object locating inside the best system outlined rectangle. In default mode, mobile manipulators conform to the initial geometric layout related to the main moving direction, and all manipulators maintain their given postures. The mobile manipulators in shrink mode are required to lie inside the boundaries of the object’s outlined rectangle. Moreover, the minimum change of joint angles is employed for calculating the postures of manipulators. As for incline mode, all mobile manipulators endeavor to hold the object with an inclined angle \(\varphi\).

Fig. 3(a)–(c) illustrates these three modes, respectively. Without loss of generality, four mobile manipulators are considered in this example. In Fig. 3, one can easily see that the default mode results in a larger system width, and the incline mode achieves a smaller system width. This indicates that the incline mode enables mobile manipulators to pass a narrow space more easily.

1) Default Mode: In this mode, the angles of joints are denoted as \(\theta_{1i}^{m1}, \theta_{2i}^{m1}, \theta_{3i}^{m1}\), and \(\theta_{4i}^{m1}\), where \(\theta_{1i}^{m1}, \theta_{2i}^{m1}\), and \(\theta_{3i}^{m1}\) are predefined. The angle \(\theta_{4i}^{m1}\) is solved with all manipulators pointing at \(O_c\), and we have \(\theta_{4i}^{m1} = f(\theta_R, \theta_D) = \{\pi/2 - \theta_R + \theta_D\mid \theta_R \in [-\pi/2, \pi]\}

\[-3\pi/2 - \theta_R + \theta_D\mid \theta_R \in (-\pi, -\pi/2)\}

The positions of end points of connecting rods \((x_{i1}^w, y_{i1}^w, z_{i1}^w)^T\), \((x_{i2}^w, y_{i2}^w, z_{i2}^w)^T\), and \((x_{i3}^w, y_{i3}^w, z_{i3}^w)^T\) in default mode are obtained by (2)–(5) with \(\theta_{1i}^{m1}, \theta_{2i}^{m1}, \theta_{3i}^{m1}\).

\[\text{Algorithm 2. The System Width of Default Mode}\]

**Input:** the positions \((x_{i1}^w, y_{i1}^w, z_{i1}^w), (x_{i2}^w, y_{i2}^w, z_{i2}^w), (x_{i3}^w, y_{i3}^w, z_{i3}^w)\) of end points of the connecting rods for mobile manipulator \(R_i (i = 1, 2, \ldots, I)\), \(O_c(x_o^w, y_o, z_o^w)\).

**Output:** the system width \(w_{m1}\).

1 \(d_i (i = 1, 2, \ldots, I) = 0\);
2 \(w_{t1} = w_{t2} = 0\);
3 for \(i = 1, 2, \ldots, I\) do
4 \(d_i = \max_{i=J2, J3}\{\text{projection}(\text{segment}((x_{i1}^w, y_{i1}^w, z_{i1}^w), (x_{i2}^w, y_{i2}^w), O_c, X_c))\}\);
5 if \(d_i > 0\) then
6 \(w_{t1} = \max(d_i, w_{t1})\);
7 else \(w_{t2} = \max(d_i, w_{t2})\);
8 end
9 end
10 \(w_{m1} = \max(w^o + 2r, w_{t1} + w_{t2})\).

2) Shrink Mode: In this mode, mobile manipulators are required to lie inside the boundaries of the object’s outlined rectangle, and the system width \(w_{m2}\) is changed to the width \(w_o\) of the object.
The expected coordinates \((x^e, y^e, z^e)^T\) of the mobile manipulator in \(O_cX_cY_cZ_c\) with its current position \((x^w, y^w, z^w)^T\) is given as follows:

\[
[x^e, y^e, z^e]^T = \begin{cases} 
-(w_o/2 - r_r) & -(w_o/2 - r_r) \\
((w_o/2 - r_r), (w_o/2 - r_r)) & y^e/R, z^e/R
\end{cases} \begin{bmatrix} x^R \end{bmatrix} \begin{bmatrix} x^R \end{bmatrix} < 0
\begin{bmatrix} x^R \end{bmatrix} \geq 0
\]

(6)

where \((x^R, y^R, z^R)^T\) is the position of \((x^w, y^w, z^w)^T\) in \(O_cX_cY_cZ_c\).

Therefore, its expected coordinates \((x^w, y^w, z^w)^T\) in \(O_wX_wY_wZ_w\) shall be obtained by (1), which provides the basis of the motion for mobile manipulators.

To ensure that all points of manipulators are within the boundaries of the object, mobile manipulators should not only change their positions but also adjust their postures. The left/right boundary of the object is denoted as \(l^w_{1/2}\) and \(r^w_{1/2}\), and we have \(l^w_{1/2} : y - k^w_{1/2} x - b^w_{1/2} = 0, l^w_{1/2} x - b^w_{1/2} = 0\), where \(k^w_{1/2}, b^w_{1/2}\) and \(k^w_{1/2}, b^w_{1/2}\) are the slope and intercept of \(l^w_{1/2}\) and \(l^w_{1/2}\), respectively. On this basis, the optimal posture \((\theta^w_{1/2}, \theta^w_{3/4})\) of the manipulator is solved by (7) with the evaluation of minimum change of joint angles, where the angle \(\theta^w_{1/2}\) of waist joint is the same as that of default mode and \(\theta^w_{3/4} = \theta^w_{m3}\). It should be pointed out that the end point of the manipulator remains unchangeable relative to its fixed position, i.e.,

\[
\theta^w_{1/2}, \theta^w_{2/3}, \theta^w_{3/4} = \operatorname{arg} \min_{\theta^w_{1/2}, \theta^w_{2/3}, \theta^w_{3/4}} \sum_{q=1}^{3} \{\theta^w_{q1} - \theta^w_{q2}\}
\]

s.t. \(x^w_{j1} = x^w_{j2} = l \left( \theta^w_{1/2}, \theta^w_{2/3}, \theta^w_{3/4} \right) \cos \theta_R\)

\(y^w_{j1} - y^w_{j2} = l \left( \theta^w_{1/2}, \theta^w_{2/3}, \theta^w_{3/4} \right) \sin \theta_R\)

\(z^w_{j1} = h \left( \theta^w_{1/2}, \theta^w_{2/3}, \theta^w_{3/4} \right)\)

(7)

3) Incline Mode: A feasible solution for mobile manipulators moving in a narrower environment is to decrease the width of the system by inclining the object for an angle. The incline angle is labeled as \(\varphi\), and the width \(w_{m3}\) of the system then becomes \(w_o \cos \varphi\).

First, the maximum and minimum heights of the end point of the manipulator shall be solved according to the mechanical parameters of the manipulator. The maximum height \(h_{max}\) shall be reached when the mobile manipulator is tangent to the boundary of the object and the length of the manipulator in the horizontal direction is equal to the radius of the mobile manipulator. The value of the minimum height \(h_{min}\) shall be under the constraints that the distance between the mobile manipulator and its contact point with the object is large and all points of the manipulator shall be lower than the height of the contact point. Therefore, \(\varphi\) is solved as follows:

\[
\varphi = \alpha \sin \left( \min \{h_{max} - h_{init}, h_{init} - h_{min} \}/w_o \right)
\]

\[
h_{max} = \max_{\theta_{i_{1/2}} \in \left[0, \pi/2 \right]} \{h(\theta_{i_{1/2}}, \theta_{i_{2/3}}, \theta_{i_{3/4}})\}
\]

s.t. \(l(\theta_{i_{1/2}}, \theta_{i_{2/3}}, \theta_{i_{3/4}}) = r_r\)

\[
h_{min} = \min_{\theta_{i_{1/2}} \in \left[0, \pi/2 \right]} \{h(\theta_{i_{1/2}}, \theta_{i_{2/3}}, \theta_{i_{3/4}})\}
\]

s.t. \(l(\theta_{i_{1/2}}, \theta_{i_{2/3}}, \theta_{i_{3/4}}) = w_o/2 - r_r\)

(8)

Therefore, its expected coordinates \((x^w_{j}, y^w_{j}, z^w_{j})^T\) in \(O_wX_wY_wZ_w\) shall be obtained by (1).

It is noted that the coordinates of the contact points with the object in \(O_wX_wY_wZ_w\) will change after mobile manipulators switch to the incline mode. The expected coordinates \((x^e_{j1}, y^e_{j1}, z^e_{j1})^T\) of the contact point with the object in \(O_wX_wY_wZ_w\) is determined by (1) based on the expected coordinates \((x^e_{j1}, y^e_{j1}, z^e_{j1})^T\) of the contact point with the object in \(O_cX_cY_cZ_c\):

\[
[x^e_{j1}, y^e_{j1}, z^e_{j1}]^T = \begin{bmatrix} x^e_{j1} \cos \varphi, y^e_{j1} + |z^e_{j1}| \sin \varphi \end{bmatrix}^T \begin{bmatrix} x^e_{j1} \sin \varphi, y^e_{j1} - |z^e_{j1}| \sin \varphi \end{bmatrix}^T \begin{bmatrix} x^e_{j1} \geq 0 \end{bmatrix}
\]

(10)

where \(\varphi_{j1} = [R^w_{j1}]^{-1} [x^w_{j1}, y^w_{j1}, z^w_{j1}]^T\). To ensure that all points of the manipulator are in the horizontal projection of the outlined rectangle of the object, the manipulators shall also adjust their postures. The left and right boundaries of the horizontal projection of the inclined object are represented as \(l^w_{3/4} : y - k^w_{3/4} x - b^w_{3/4} = 0\) and \(l^w_{3/4} : y - k^w_{3/4} x - b^w_{3/4} = 0\), where \(k^w_{3/4}, b^w_{3/4}, k^w_{3/4}, b^w_{3/4}\) are the slope and intercept of \(l^w_{3/4}\) and \(l^w_{3/4}\), respectively. The optimal posture
(θ^m1, θ^m2, θ^m3) of the manipulator is solved by (11),
where θ^m1 = f(atan2(y^w_o - y^w_R, x^w_o - x^w_R), θ_D). Thus

\[ \theta^m_{1}, \theta^m_{2}, \theta^m_{3} = \arg \min_{\theta^m_{1}, \theta^m_{2}, \theta^m_{3} \in [0, \pi]} \sum_{q=1}^{3} |\theta^m_{q} - \theta^m_{3}| \]

s.t.  
\[ x^w_{e,J1} - x^w_R = l(\theta^m_{1}, \theta^m_{2}, \theta^m_{3}) \cos \theta_R \]
\[ y^w_{e,J1} - y^w_R = l(\theta^m_{1}, \theta^m_{2}, \theta^m_{3}) \sin \theta_R \]
\[ z^w_{e,J1} = h(\theta^m_{1}, \theta^m_{2}, \theta^m_{3}) \]
\[ k^{ol} \left( y^w_{J3} - k^{ol} x^w_{J3} - b^{ol}_3 \right) < 0 \land k^{or} \left( y^w_{J3} - k^{or} x^w_{J3} - b^{or}_3 \right) > 0 \]
\[ k^{ol} \left( y^w_{J2} - k^{ol} x^w_{J2} - b^{ol}_3 \right) < 0 \land k^{or} \left( y^w_{J2} - k^{or} x^w_{J2} - b^{or}_3 \right) > 0. \]

(11)

### B. Decision-Making

To select the proper mode, a decision shall be designed based on environmental information obtained by a range sensor installed on the leading mobile manipulator. Moreover, the optimal moving direction to reach the target point TP shall be also derived.

Herein, the multiscale passageway is introduced where the passageway is a rectangle with a specific direction [25]. This means that the passageways in a direction are dependent on transportation modes and constrained by environmental information (see Fig. 4).

Within the angle range of ±π/2 of the current direction, the best passageway P^\ast_{v,m} is determined by evaluating the lengths of passageways, transportation modes, the angles between the directions of passageways and TP, and the effect of distances among obstacles, where m^\ast represents the optimal transportation mode suitable for current environment, and v\ast refers to the optimal moving direction. The evaluation is shown as

\[ (v^\ast, m^\ast) = \arg \max_{v \in [1, 180], m \in [1, 3]} \left( k_1 L_{P^\ast_{v,m}} + C_m \right) \]
\[ + \frac{k_2}{\sqrt{2\pi}\sigma} e^{-\frac{v^2}{2\sigma^2}} + C_{obs_{v,m}} \]

s.t.  
\[ L_{P^\ast_{v,m}} = \min \left( S_{\text{laser}}, \arg \max_l (R^l_{v,m}) \right) \forall \ O_p \notin R^l_{v,m} \]
\[ L_{P^\ast_{v,m}} \geq L_T \]

(12)

where \( L_{P^\ast_{v,m}} \) represents the length of the passageway \( P^\ast_{v,m} , C_m \) is the evaluation of the transportation mode \( m , \theta^l \) is the angle between the direction of the passageway and \( T_P , C_{obs} \) reflects the effect of distances among obstacles, \( S_{\text{laser}} \) is the maximum
sensing range of the range sensor, $R_{l,m}^l$ is a rectangle corresponding to $P_{v,m}$ with the length of $l$, and $\Omega$ is the point set of obstacle distribution.

The effect $C_{v,m}^{obs}$ of distances among obstacles is obtained based on environmental data $\rho_u(u = 1, 2, \ldots, u_{\text{max}})$ given by the range sensor, and two filters are designed as shown in Fig. 5.

- Peak filter, to filter all peak data except the head and tail data, that is $\rho_u = 0(|\rho_u = S_{\text{laser}} \land u \neq 1, u_{\text{max}})$.
- Difference filter, to extract the pairs of feature points reflecting the distances among obstacles corresponding to current view. The head/tail data keeps unchanged if it is equal to $S_{\text{laser}}$, except that, for $\rho_u$ with $\rho_u \neq 0$, it keeps unchanged if there is an abrupt change between it and $\rho_{u-1}/\rho_{u+1}$ ($\rho_{u_{\text{min}}} = 0$ and $\rho_{u_{\text{max}}+1} = \rho_{u_{\text{max}}}$), or else, $\rho_u = 0$. Thus, a series of pairs of feature points with nonzero $\rho_u$ is obtained, and the distances among obstacles are consequently acquired.

Finally, the effect $C_{v,m}^{obs}$ of distances among obstacles is solved as follows. When $v$-related $u$ is outside of any pair of feature points, $C_{v,m}^{obs} = 0$; otherwise, the effect shall be $|\min(2, d_G/2w_m - 1)|$, where $d_G$ is the distance caused by obstacles corresponding to $u$, and $C_{v,m}^{obs}$ is positive when $d_G \geq w_m$.

IV. SIMULATIONS AND EXPERIMENTS

The transportation of multiple mobile manipulators is verified by simulations and experiments.

A. Simulations

The parameters of mobile manipulators are as follows. $l_1 = 20$ cm, $l_2 = 16$ cm, $l_3 = 16$ cm, $h_r = 10.9$ cm, $r_r = 12$ cm, $S_{\text{laser}} = 3$ m, and $w_o = 73.4$ cm. The parameters in the proposed approach are as follows: $k_1 = 1, k_2 = 4.5, C_1 = 1, C_2 = 0.7, C_3 = 0.4, d = 30$ cm.

Fig. 6. Moving trajectories of mobile manipulators in simulation 1.

Fig. 7. Changes of joint angles of manipulators in simulation 1. (a) MM1. (b) MM2. (c) MM3.
In simulation 1, the object is transported by three mobile manipulators $\text{MM}_1 - \text{MM}_3$. The initial information of the system is manually assigned, and $\text{MM}_2$ is the leading mobile manipulator. Figs. 6 and 7 depict the moving trajectories and changes of joint angles of manipulators, respectively. The variation of transportation modes is shown in Fig. 8, where $m_{\text{mode}} = 1, 2, 3$ correspond to the default mode, shrink mode, and incline mode, respectively. One can see that these three mobile manipulators coordinately move from their starting positions $S_1 - S_3$, adjust their poses according to the selected transportation mode with environment adaptability, and, finally, smoothly arrive at the positions $G_1 - G_3$.

Simulation 2 considers the case with four mobile manipulators $\text{MM}_1 - \text{MM}_4$, and the initial information of the system is automatically generated according to their geometric distribution. From the simulation results shown in Figs. 9–11, it can be easily found that four mobile manipulators traverse through the environment without any collisions with obstacles, which demonstrates the adaptability of the proposed approach.

Simulation 3 is designed to validate the effect factor of distances among obstacles. Fig. 12(a) and (b) gives the trajectories of mobile manipulators $\text{MM}_1 - \text{MM}_4$ with and without consideration of $C_{v,m}^{\text{obs}}$ under the same initial condition, respectively. The corresponding variations of transportation modes are shown in Fig. 13(a) and (b), respectively. It is seen that the one with the consideration of $C_{v,m}^{\text{obs}}$ leads to a positive result.

B. Experiments

The following experiments consider the case with two mobile manipulators with the parameters shown in Fig. 14. Moreover, considering the complexity of environments, the effect of distances among obstacles is not considered. $k_1 = 0.7$, $k_2 = 2$, $C_1 = 2$, $C_2 = 1.2$, $C_3 = 1$.

In experiment 1, two mobile manipulators transport an object in the environment with a narrow obstacle zone. Initially, the system information is manually given, and two mobile manipulators take the default mode. The video snapshots of the experiment and trajectories of two mobile manipulators are shown in Figs. 15 and 16, respectively. The mobile
manipulator system first turns to shrink mode due to the narrow obstacle zone. After the system successfully passes the obstacle zone, it switches to default mode for continuing marching.

Experiment 2 considers the case where an obstacle is suddenly pushed in, which will bring the disturbance to the movement of the system. Fig. 17 gives the video snapshots. The variation of transportation modes is given in Fig. 18. Clearly, the mobile manipulator system starts to transport an object with default mode. When it perceives the intrusive obstacle, the shrink mode is preferable to deal with the emergency. After the influence of the obstacle disappears, the system recovers to default mode.

It should be noted that there is no obvious decrease in system width with incline mode compared with shrink mode due to the limitation of the small mobile manipulator used here, which will greatly prevent the incline mode.

Fig. 12. Trajectories of mobile manipulators in simulation 3. (a) With consideration of $C_{\text{obs}}, (b)$ Without consideration of $C_{\text{obs}}$.

Fig. 13. Variation of transportation modes in simulation 3. (a) With consideration of $C_{\text{obs}}$, (b) Without consideration of $C_{\text{obs}}$.

<table>
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<th>Items</th>
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<td>Lengths of Connecting Rods</td>
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<td>Load</td>
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<tr>
<td>Velocity</td>
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<td>Sensing Range</td>
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<tr>
<td>Running time</td>
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<tr>
<td>Communication Mode</td>
<td>Wireless Communication</td>
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<td>Radius of mobile manipulator</td>
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Fig. 14. Parameters of the experimental mobile manipulator.

Fig. 15. Video snapshots of experiment 1.
V. Conclusion

This paper has presented an approach to object transportation by multiple mobile manipulators. Three transportation modes, i.e., the default mode, the shrink mode, and the incline mode, are given. The shrink and incline modes enable mobile manipulators to pass narrow obstacle zones. A decision based on the multiscale passageway facilitates the selection of the best transportation mode and moving direction of mobile manipulators. Simulations and experiments indicate that multiple mobile manipulators achieve a flexible and adaptable transportation.

One major assumption in this paper is that there is no slippage. In our future work, we shall incorporate a dedicated controller for the mobile base to mitigate the slipping factor. Moreover, the dynamic model shall be considered.

REFERENCES


Jile Jiao received the B.S. degree from Central South University, Changsha, China, in 2009 and the Ph.D. degree in control theory and control engineering from the Institute of Automation, Chinese Academy of Sciences, Beijing, China, in 2014. She is currently an Assistant Professor with the Research Center of Precision Sensing and Control, Institute of Automation, Chinese Academy of Sciences. Her research interests include multirobot systems and mobile manipulator control.

Zhiquiang Cao received the B.S. and M.S. degrees from Shandong University of Technology, Jinan, China, in 1996 and 1999, respectively, and the Ph.D. degree in control theory and control engineering from the Institute of Automation, Chinese Academy of Sciences, Beijing, China, in 2002. He is currently an Associate Professor with the State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences. His research interests include visual scene cognition, multirobot systems, and networked robotic systems.

Nong Gu received the B.S. degree from the University of Electronic Science and Technology of China, Chengdu, China, in 1997; the M.S. degree from Beijing Institute of Technology, Beijing, China, in 2000; and the Ph.D. degree in control theory and control engineering from the Institute of Automation, Chinese Academy of Sciences, Beijing, in 2003. He is currently a Research Fellow with the Centre for Intelligent Systems Research, Deakin University, Waurn Ponds, Australia. His research interests include multirobot systems and signal processing.

Saeid Nahavandi (SM’07) received the Ph.D. degree from Durham University, Durham, U.K. He is an Alfred Deakin Professor, Chair of Engineering, and the Director of the Centre for Intelligent Systems Research at Deakin University, Waurn Ponds, Australia. He has published over 550 papers in various international journals and conferences. His research interests include modeling of complex systems, robotics, and haptics. Dr. Nahavandi is the Co-Editor-in-Chief of the IEEE/ASME TRANSACTIONS ON MECHATRONICS. He is a Fellow of the Institute of Engineers Australia and the Institution of Engineering and Technology.

Yuequan Yang received the B.S. degree in mathematics and the M.S. degree in computer application techniques from Yangzhou University, Yangzhou, China, in 1994 and 2002, respectively, and the Ph.D. degree in control theory and control engineering from Chinese Academy of Sciences, Beijing, China, in 2005. He is currently a Professor with Yangzhou University. His research interests include multirobot systems, intelligent control, and complex networks.

Min Tan received the B.S. degree in control theory and control engineering from Tsinghua University, Beijing, China, in 1986 and the Ph.D. degree in control theory and control engineering from the Institute of Automation, Chinese Academy of Sciences, Beijing, in 1990. He is currently a Professor with the State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences. His research interests include robotics and intelligent control systems.