

New PEG Algorithm with Low Error Floor for Construction of Irregular LDPC Codes

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Abstract— The progressive-edge-growth (PEG) algorithm for constructing Tanner graphs with large girth by progressively establishing edges/connections between symbol nodes and check nodes in an edge-by-edge way is highly known to construct low-density parity-check (LDPC) codes at short block lengths for achieving good performance. This paper proposed an improved PEG construction for irregular codes, which heuristically selects good codes from random graphs for short block lengths. The simulation results are given to demonstrate that the proposed algorithm gains better BER performance for high signal-to-noise (SNR) ratios without performance loss for low-SNR region, i.e. low error floor.

Keywords—Irregular low density parity check (LDPC) codes, progressive-edge-growth (PEG) algorithm, girth distribution, girth average.

I. INTRODUCTION

Low density parity check (LDPC) codes, first proposed by Gallager in early 1960's [1], were rediscovered in late 1990's and proved to form a class of Shannon limit approaching codes [2]. LDPC codes have attracted significant attention because of its capacity-approaching performance and low-complexity iterative decoding methods. These codes could be decoded with iterative decoding algorithms, and it is generally known that belief-propagation (BP) or sum-product algorithm (SPA) [3] for cycle-free Tanner graphs is able to achieve optimum decoding. Therefore, it is obvious that the purpose in the iterative decoding process is trying to minimize the influence of the cycles. In particular, constructing good LDPC codes with short and intermediate block lengths is really important for practical implementation. For most currently existing LDPC codes, Tanner graph is randomly constructed, which could avoid cycles of length 4. However, randomly constructed LDPC codes have greatly relied on the sparsity of the parity-check matrix in order to avoid short cycles in the Tanner graph.

Among current methods, one of the most successful methods for the construction of finite-length LDPC codes is called progressive-edge-growth (PEG) algorithm, which was proposed by Hu et al [4]-[7]. The main principle of this method is to optimize the placement of a new edge which connects a particular symbol node to a check node on the graph, thus, the largest possible local girth could be achieved. This is simple and also flexible, and it can be applied to construct codes of arbitrary length and rate.

Besides, the PEG algorithm can be applied for both regular and irregular LDPC codes. For irregular codes, in particular, the results of [6] show that PEG construction with symbol-node degree distributions can be optimized by density evolution [8] and could result in very good performance.

Although many of LDPC codes' characteristics have been investigated, the error floor phenomenon has still remained to be an open topic. This phenomenon can be characterized as an abrupt decrease in the slope of a code's performance curve from moderate SNR waterfall region to high SNR region. Because there are plenty of systems that require extremely low error rates, for example data storage devices or optical communication systems; therefore, to improve the error floor problem has been a critical issue during the past decades.

Therefore, in this paper, a simple modification for PEG algorithm is proposed, which considerably improves the performance of irregular codes. In the PEG algorithm, it is common that one candidate has a few candidate check nodes resulting in the same local girth under the current graph structure in order to connect to a given symbol node. For the standard PEG algorithm, one chooses the one at random among such check-node candidates. However, for the proposed improved algorithm, by borrowing the main idea from [9], the average of the girth histogram is adopted as a heuristic tool to choose good codes from random graphs for short block lengths. Therefore, this paper applies the key approach of [9] in selecting good codes among PEG Tanner graphs. This method obtains further performance improvements, i.e. low error floor.

The rest of this paper is organized as follows. Firstly, a brief introduction of LDPC codes is given in Section II. The PEG algorithm of irregular LDPC codes are discussed in Section III. Section IV proposes the new construction method; Simulation results and comparisons between standard PEG schemes and the proposed new scheme are given in Section V. Section VI concludes this paper.

II. LDPC CODES

An LDPC code is defined by a parity-check matrix H having dimension $m \times n$, which is a linear code. A bipartite graph with m check nodes in one class and n symbol nodes in another class could be generated by

using matrix H as the integer-valued incidence matrix for the aforementioned two classes. This kind of graph is also called a Tanner graph, which is generally denoted as (V, E) , where V denotes the set of vertices (nodes) and E denotes the set of edges. $V = V_c \cup V_s$, where

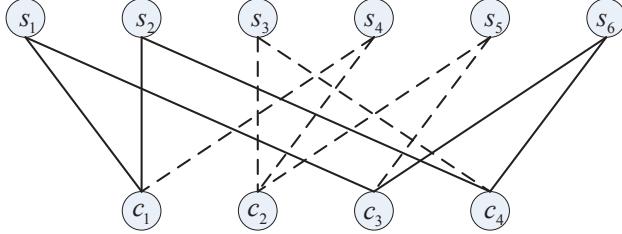


Fig. 1. An example of a symbol-node degree
 $D_s = \{2, 2, 2, 2, 2, 2\}$ Tanner graph.

$V_c = \{c_0, c_1, \dots, c_{m-1}\}$ is the set of check nodes and $V_s = \{s_0, s_1, \dots, s_{n-1}\}$ is the set of symbol nodes. $E \subseteq V_c \times V_s$, where edge $(c_i, s_j) \in E$ if and only if $h_{i,j} \neq 0$, and $h_{i,j}$ is the entry of parity-check matrix H at the i th row and j th column and $0 \leq i \leq m-1$, $0 \leq j \leq n-1$. A Tanner graph is named *regular* if every symbol node involves d_s check nodes and every check node participates in d_c symbol nodes; otherwise, it is named *irregular*. The symbol degree sequence is denoted by

$$D_s = \{d_{s_0}, d_{s_1}, \dots, d_{s_{n-1}}\} \quad (1)$$

where d_{s_j} denotes the degree of symbol node s_j in a nondecreasing order $d_{s_0} \leq d_{s_1} \leq \dots \leq d_{s_{n-1}}$, and the parity-check degree sequence is given by

$$D_c = \{d_{c_0}, d_{c_1}, \dots, d_{c_{m-1}}\} \quad (2)$$

where d_{c_i} is the degree of parity-check node c_i in a nondecreasing order $d_{c_0} \leq d_{c_1} \leq \dots \leq d_{c_{m-1}}$.

Fig. 1 shows one example of a $D_s = \{2, 2, 2, 2, 2, 2\}$ regular Tanner graph, where the check degree sequence has uniform degree 5, i.e. $D_c = \{3, 3, 3, 3\}$.

The parity-check equation is given as

$$\begin{cases} s_1 + s_2 + s_4 = 0 \\ s_3 + s_4 + s_5 = 0 \\ s_5 + s_6 + s_1 = 0 \\ s_2 + s_3 + s_6 = 0 \end{cases} \quad (3)$$

and the parity-check matrix is given by

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

A graph is called *simple* if it has three characteristics: 1) it does not include a self-loop that means an edge joining a vertex to itself; 2) there is at most one edge between a pair of vertices; 3) all edges in the graph are non-directed. For a simple graph, vertices a and b are *adjacent* if (a, b) is an edge. The set including all vertices that are adjacent to a is called a 's *neighbors*. A subgraph of a graph $G = (V, E)$ is a graph whose vertex and edge set are subsets of those in graph G . A sequence of distinct vertices, starting from a and ending with b is called a *path* between a and b if any two consecutive vertices in this sequence are joined by one edge. If there is at least one path between these two vertices, they are called *connected*. If two vertices a and b in a graph are *connected*, the distance $d(a, b)$ is then defined to be the length of the shortest path between them, i.e., number of edges. A closed path with edges starting from a and ending at a is called a *cycle* of a . *Girth* g is defined to be the length of the shortest cycle in a graph. For a symbol node s_j , a local girth g_{s_j} is defined as the length of the shortest cycle passing through the symbol node. The girth of a graph is defined to be $g = \min_j \{g_{s_j}\}$, for example, the girth of the graph in Fig. 1 is 6.

III. PEG ALGORITHM

As demonstrated in Section II, same definitions and notions are used to describe the conventional PEG algorithm for constructing a Tanner graph with n variable nodes and m check nodes as shown in the following Table I.

Assuming the graph parameters is given, i.e. the number of symbol nodes n , the umber of check nodes m , and the symbol-node-degree sequence D_s , an edge selection procedure could be executed, thus the placement of a new edge on the graph would have as small impact on the girth as possible. The underlying graph increases in an edge-by-edge manner to optimize each local girth. Consequently, the resulting Tanner graph is thought to be PEG Tanner graph. The basic idea of PEG algorithm is to find the most distant check node and then to replace it by a new edge connecting the symbol node and that most distant check node.

Table I: Progressive Edge-Growth Algorithm

```

1  for  $j = 0$  to  $n-1$  do
2  begin
3  for  $k = 0$  to  $d_{s_j} - 1$  do

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4 begin
5 if  $k = 0$ 
6  $E_{s_j}^0 \leftarrow$  edge  $E_{s_j}^0$ , where  $E_{s_j}^0$  is the first edge incident
  to  $s_j$  and  $c_j$  is a check node, thus it has the lowest
  check-node degree under the current graph setting
   $E_{s_0} \cup E_{s_1} \cup \dots \cup E_{s_{j-1}}$ ;
7 else
8 expand a subgraph from symbol node  $s_j$  up to depth  $l$ 
  under the current graph setting, thus the cardinality of
   $N_{s_j}^l$  stops increasing, but it is less than  $m$ , or  $\bar{N}_{s_j}^l \neq \phi$ 
  but  $\bar{N}_{s_j}^{l+1} = \phi$ , then  $E_{s_j}^k \leftarrow$  edge  $E_{s_j}^0$ , where  $E_{s_j}^k$  is the
   $k$  th edge incident to  $s_j$  and  $c_j$  is a check node which
  is picked from the set  $N_{s_j}^l$  which has the lowest check-
  node degree.
9 end
10 end

```

In this algorithm, both symbol nodes and check nodes are ordered based on their degrees in non-decreasing order, where d_{s_j} is the degree of symbol-node j , $N_{s_j}^l$ and $\bar{N}_{s_j}^l$ denote the set of all check nodes reached by a tree spreading from symbol-node s_j within depth l , and its complement set, respectively [6].

In conventional PEG algorithm, if a new edge is connected to a symbol node, it may have multiple available candidate check nodes. By using PEG algorithm, one randomly chooses the one among such check-node candidates. PEG construction with symbol-node degree distributions could result in a very good performance; however, this good performance of highly optimized irregular codes in the low-SNR waterfall region is usually counterbalanced by a relatively poor performance in the high-SNR error-floor region.

IV. IMPROVED PEG ALGORITHM

In this paper, a modification to the conventional PEG algorithm is proposed, which effectively improves the performance of irregular codes especially for high SNR regime.

In PEG algorithm, it is common that one has a few candidate check nodes to connect to a specified symbol node. Among those check node candidates, either the one with the smallest index or just one at random would be selected.

The proposed algorithm focuses on finding the code with “best” girth distribution from a given ensemble. The girth of a symbol node refers to the length of the shortest cycle that passes through this symbol node. Girth distribution plays an

very important role in the performance of belief propagation (BP) iterative decoding methods, because it is same to the smallest number of iterations for a message sent by that node so as to propagate back to the node by itself.

A. Improved PEG Algorithm

The proposed PEG algorithm exploits the girth histogram to choose good codes from random graphs. In fact, the girth histogram is more important than the girth for the performance of iterative decoding [6]. By using the improved PEG algorithm, one can obtain an ensemble of LDPC codes among such check-node candidates, and all of them are with the same block length and the same Tanner graph degree sequences. Here, the proposed method selects the code by girth distribution. Based on the selection criterion for girth distribution, it chooses the average and selects the code with the highest girth average. As shown in the simulation results of section V, the increase in the girth average could improve the performance of the codes.

Table II: Improved Progressive Edge-Growth Algorithm

```

11 for  $j = 0$  to  $n - 1$  do
12 begin
13 for  $k = 0$  to  $d_{s_j} - 1$  do
14 begin
15 if  $k = 0$ 
16  $E_{s_j}^0 \leftarrow$  edge  $E_{s_j}^0$ , where  $E_{s_j}^0$  is the first edge incident
  to  $s_j$  and  $c_j$  is a check node such that it has the highest
  girth average under the current graph setting
   $E_{s_0} \cup E_{s_1} \cup \dots \cup E_{s_{j-1}}$ ;
17 else
18 expand a subgraph from symbol node  $s_j$  up to depth  $l$ 
  (as shown in Fig. 2) under the current graph setting so
  that the cardinality of  $N_{s_j}^l$  stops increasing, but it is less
  than  $m$ , or  $\bar{N}_{s_j}^l \neq \phi$  but  $\bar{N}_{s_j}^{l+1} = \phi$ , then  $E_{s_j}^k \leftarrow$  edge
   $E_{s_j}^0$ , where  $E_{s_j}^k$  is the  $k$  th edge incident to  $s_j$  and  $c_j$ 
  is a check node picked from the set  $N_{s_j}^l$ , which has the
  highest girth average.
19 end
20 end

```

B. Belief Propagation Iterative Decoding

The belief propagation (BP) algorithm [10] is an efficient method to solve the aforementioned problems when the factor graph is a tree; however, it only approximate when the factor graph has cycles. The details of BP algorithm is referred to [11].

V. SIMULATION RESULTS

In this section, BPSK modulation is applied over AWGN channel with BP iterative decoding algorithm. Similar to [6,12], the maximum number of iterations for the BP decoder is chosen to be 80. Furthermore, at least 100 block errors per simulation point are collected. To obtain a reasonable comparison between the conventional and the improved PEG algorithms, the same noise vectors are chosen for the codes with the same block lengths.

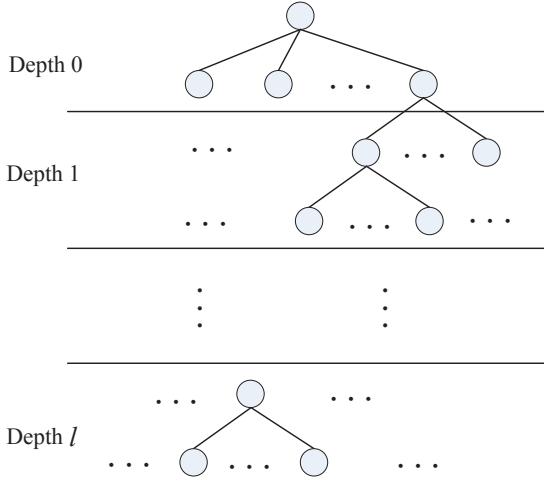


Fig. 2 A subgraph spreading from symbol node s_j up to depth l .

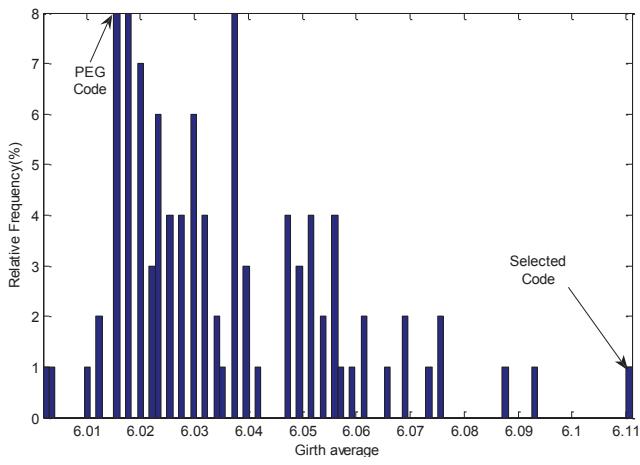


Fig. 3. Histogram of girth averages for the ensembles of (1008, 504) codes

In the simulations, an ensemble of irregular LDPC codes with the same rate and symbol-node degree distribution are constructed, similar as those in [6]. All the codes have code rate 0.5, and are constructed with maximum symbol-node

degree 15 [8]. All the codes parameters are set to be (1008, 504). In this paper, the ensemble consists of 100 codes (Therefore, one may find better code if the ensemble consists of more codes).

The histogram of the girth averages for the ensemble is given in Fig. 3. For the ensemble, the highest girth average is selected. For the sake of comparison, this paper also indicates the girth average of standard PEG code in Fig. 3.

Fig. 4 demonstrates the bit error rate (BER) and the message error rate (MER) curves for the standard PEG code and selected code. It can be seen that, for high SNRs, the codes constructed by the proposed PEG algorithm outperforms that of the conventional PEG algorithm.

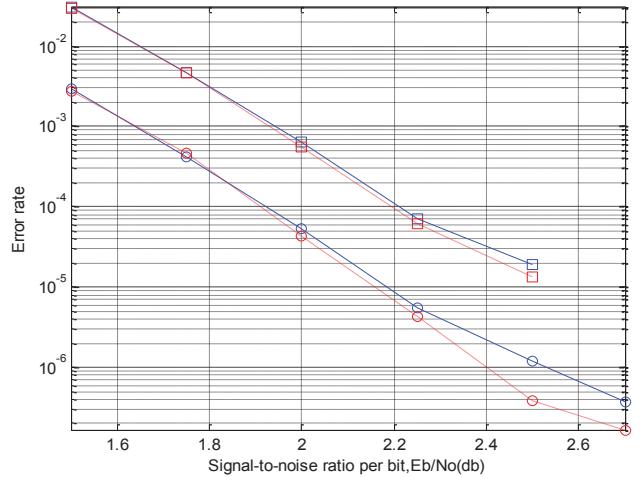


Fig. 4. BER (○) and MER (□) curves for the PEG (—) code and selected (---) code.

VI. CONCLUSIONS

In this paper, an improved PEG algorithm was proposed, which enhanced the performance at high SNR region by finding the code with “best” girth distribution from a given ensemble. Girth distribution plays an important role in the performance of BP iterative decoding. The main idea of this improved algorithm is based on maximizing the girth average by using the average of the girth histogram to select good codes. Finally, the simulation results and discussions further approve the performance improvements of the proposed algorithm. The improved PEG algorithm achieves more than 0.1 dB performance gain than the conventional PEG algorithm.

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