Guest Editorial
Special Issue on Neurodynamic Systems for Optimization and Applications

RECURRENT neural networks, as neurodynamic systems, are a class of connectionist models that capture the dynamics of sequences via cycles in artificial neurons. Since the invention of Hopfield neural network, recurrent neural networks have attracted considerable attention, which marks the beginning of the modern age of neural network studies. Thanks to their inherent nature of parallel and distributed information processing, many computationally intensive applications can be solved by recurrent neural networks in the real-time environment.

One most successful application of recurrent neural networks is to solve the optimization problem. In the 1980s, Hopfield and Tank first developed a recurrent neural network model in the continuous-time domain to solve the travelling salesmen problem and the linear programming problem. After that, a number of neurodynamic models have been proposed for solving various optimization problems ranging from discrete optimization to continuous optimization, linear programming to nonlinear optimization, convex optimization to nonconvex optimization, smooth optimization to nonsmooth optimization, numerical software to analog hardware implementations, and so on. Unlike the traditional numerical optimization solvers, recurrent neural networks can be implemented by analog circuits, which makes them extremely effective in computing the optimal solution to problems high in dimension and dense in structure. As a result, some neurodynamic models have been successfully applied to robot control, optimal control, image processing, economic prediction, and information retrival.

Another appealing feature of neurodynamic systems is that the recurrent structure can take the past information into account, which is very suitable for modeling complex dynamic processes, such as genetic networks, power systems, communication networks, and so on. Hence, neurodynamic systems are widely employed in many fields, such as dynamic systems and control, and signal/image processing. Furthermore, a neurodynamic system itself can generate many dynamical behaviors, such as stability, chaos, bifurcation, periodical solutions, and synchronization, which can further be applied to associative memories and pattern recognition. Furthermore, dynamical behaviors of neurodynamic systems are potentially useful for simulating the brain functions, which is an important topic in neuroscience. Therefore, analyzing the dynamic behaviors of neurodynamic systems becomes a popular direction in the literature as well.

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This Special Issue is dedicated to recent advances of neurodynamic systems and their engineering applications. Shortly after the announcement of the call-for-papers for this special issue, we received over one hundred submissions, and, thenceforth, started a long but rewarding review process to select 23 high-quality papers pertinent to this Special Issue. These papers have been classified into three groups.

The first group is about novel neurodynamic models, which are capable of solving various large-scale optimization problems and related applications.

1) *Linear and Quadratic Optimization Problem:* The Special Issue starts with the paper by Xia and Wang, where a novel biprojection neural network is proposed for solving a wide class of constrained quadratic optimization problems with an application to data fusion. This neural network has a successive biprojection structure, and thus, has a lower model size compared with the previous neurodynamic models. Liao *et al.* used a Taylor-type numerical differentiation approach to discretize the continuous-time recurrent neural network model proposed by Yunong Zhang for solving the dynamic/time-dependent equality-constrained quadratic programming problem. It is found that the Taylor-type discrete-time model has a better accuracy than the Newton iteration, and other models derived by the Euler-type discretization and the Lagrange-type discretization. Guo and Baruah study the real-time utility maximization scheduling problem. By the matrix vectorization technique, this NP-hard problem is further approximated by an optimization problem with the piecewise linear and concave objective function and linear inequality constraints. Then, the real-time scheduling problem can be approximately solved by the neurodynamic model, which outperforms the earliest deadline first algorithm and the fixed priority algorithm by numerical experiments.
2) *Quaternion Optimization Problem:* Xu *et al.* considered the optimization problem of quaternion variables. They propose a generalized Hamilton-real calculus to equip the quaternion analysis with the product rule and the chain rule. Then, the quaternion gradient and Hessian can be calculated analytically. With these derivative information, designing the neurodynamic model for quaternion optimization problems becomes possible.
3) *Nonlinear Optimization Problem:* Quan and Cai investigated the equality-constrained optimization problem without the regularity assumption from a feedback

control aspect. The equality constraint is transformed into a continuous-time dynamical system whose solutions always satisfy the equality constraint. Then, a controller is designed to optimize the objective function along the solution of this dynamical system to seek the local minima without convergence to the singular point. Liu *et al.* studied the l_1-norm low-rank matrix decomposition problem with missing entries. A mollifier is used to smooth the nonsmooth objective function, and the approximation error between the smoothed objective function and the original one can be properly controlled. Then, a recurrent neural network is employed to approximately solve the decomposition problem. Experimental results show that the proposed algorithm is very competitive in both approximation accuracy and computational efficiency. Yuan *et al.* studied how to minimize a sum of convex objective functions in a distributed manner. Based on an approximate projection operator, an iterative algorithm (a discrete-time recurrent neural network) is designed for each local objective function. Then, the recurrent neural networks exchange information with their neighbors via a time-varying connected network, which makes all recurrent neural networks collectively achieve an optimal solution to the sum of all individual objective functions.

4) *Nonsmooth Optimization Problem:* Di Marco *et al.* presented a nonsmooth neural network model, which is able to find a feasible solution within the time-varying admissible set formed by the dynamic/time-dependent nonsmooth inequality constraints. Based on the exact penalty method, the trajectory of this neural network displays two phases of motions: the first one is to reach the time-varying admissible set in finite time if the initial state of this neural network is not in the initial admissible range, and the second one is to track the time-varying admissible set right after the first phase. Li *et al.* generalized the Hopfield neural network for solving nonsmooth convex optimization problems. By the Lie derivative, the trajectory of this neural network is proved to be convergent to an equilibrium point; the optimality of this equilibrium point is guaranteed by the enhanced Fritz conditions. Wang *et al.* employed a nonsmooth recurrent neural network for the optimal formation of multirobot systems by the shape theory. The total distances traveled by all robots can be minimized. In addition, they consider the case where the robots' positions are exchangeable. This leads to a combinational optimization problem, which can be approximately solved by the proposed recurrent neural network.

The second group of the selected papers focuses on neurodynamic systems for signal processing and control.

1) *Signal Processing and Pattern Recognition:* Li and Príncipe proposed a novel kernelized neurodynamic system (so-called kernel adaptive autoregressive-moving-average algorithm), which bridges the adaptive signal processing and the recurrent neural network. By doing so, the kernel adaptive filtering can take the feedback into account. Numerical results show that this algorithm has the potential of effectively solving the identification and synthesis of deterministic finite automata. Liu *et al.* presented a recurrent self-evolving fuzzy neural network as a predictor of brain dynamics to identify the driving fatigue. Since the human's cognition process is a continuous and cumulative process, using the recurrent structure allows to memorize past events and, therefore, improve the identification performance, which has been verified by the cross-subject approach in the driving simulator. Chien and Ku employed the recurrent neural network to capture the dynamics of a human's language model and apply it for the continuous speech recognition. A Bayesian approach is proposed to regularize the training of the recurrent neural network to avoid the ill-posed problem. Experiments on different corpora demonstrate the robustness of this approach. Boguslawski *et al.* studied how to improve the performance of the sparse associative memory with nonuniformly stored messages. A twin-neuron model is proposed, which can increase the system performance close to the optimal level. The proposed algorithm is further applied to optimize the power consumption of electronic circuits.

2) *Systems and Control:* Hwang and Jan proposed a recurrent neural network with the residue compensation approach for the adaptive control of a class of multivariable nonlinear dynamic systems with time-varying delays. The semiglobally ultimately bounded tracking is guaranteed by the Lyapunov stability theory. Han *et al.* developed a new nonlinear model predictive controller based on a self-organizing recurrent radial basis function neural network. The structure and parameters of this recurrent neural network can be updated online by a spiking-based growing and pruning algorithm, and the control performance is analyzed by the Lyapunov stability theory. Wang *et al.* studied a class of multirate networked nonlinear systems with a dual-layer structure. The adaptive neural network approach is employed to guarantee the tracking performance of the local control plant in the device layer, while the overall performance is optimized by the nonlinear model predictive control approach in the operation layer. Han *et al.* considered the networked control problem of a class of discrete-time neural networks with distributed time delay. An optimal H_∞ quantized controller is designed, which subjects to the signal logarithmic quantization and the random packet dropout with Bernoulli distribution. Yu *et al.* investigated the transmission scheduling of smart grid users in a cognitive radio-based smart grid communication network satisfying the potential requirement of differential quality of service. The entire transmission scheduling problem is formulated as a semi-Markov decision process, and then, is solved by the adaptive dynamic programming method. Wei *et al.* also used the adaptive dynamic programming approach to solve the zero-sum optimal control problem of a class of continuous-time unknown nonlinear systems

whose dynamic behavior is approximately modeled by the recurrent neural network.

The third group includes the papers concerning the dynamical behaviors of neurodynamic systems.

1) *Synchronization:* Zhang *et al.* considered a class of discrete-time hierarchical recurrent neural networks with time-varying delays. The proposed neural network model consists of a higher level nondeterministic switching and a lower level stochastic switching. Despite of the mode-dependent time-varying delays, data missing and quantization, the exponential H_∞ synchronization, and state estimation can still be ensured. Liu *et al.* studied how to design an optimal pinning controller for the finite-time synchronization of a class of nonlinear coupled recurrent neural networks with a fixed connection topology. The proposed controller can treat the continuous and discontinuous controllers in a unified framework, and the synchronization time can be optimized by tuning certain parameters.
2) *Stability:* Zheng *et al.* investigated a class of analytic neural networks and suggested an event-triggering rule for discrete-time synaptic feedbacks. Based on the Łojasiewicz inequality, these neural networks are proved to be almost surely stable without any Zeno behaviors.

The 23 selected papers cover a broad range of topics, reflecting the present state of the art of the neurodynamic systems and their applications. Our hope is to encourage the continuous development of this field as well as the involvement of young scholars in academia and industry. Finally, we would like to thank all authors for their contributions, reviewers who helped us in reviewing these submissions, and our Editor-in-Chief, Prof. D. Liu, for his strong support and guidance throughout this process. This Special Issue would not have been possible without all their efforts.

We hope you enjoy reading this Special Issue.

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