Adaptive Tracking Control of Leader-Following Multi-Agent Systems

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***Abstract*— In this paper, a distributed tracking controller with an adaptive law for adjusting coupling weights between neighboring agents is designed for leader-following multi-agent systems under fixed and switching topologies. In contrast to most existing literature where agents are integrators or double integrators, the dynamics of each agent is a general linear system in this paper. To handle this problem, the controller is based on Riccati inequalities. In traditional distributed static controllers, the coupling weight depends on the communication graph. However, coupling weights associated with the feedback gain matrix in our method are updated by state errors between neighboring agents. We further present the stability analysis of leader-following multi-agent systems under switching topology. Most traditional literature requires the graph to be connected anytime, while the communication graph is only assumed to be jointly connected in this paper. The design technique is based on Riccati inequalities and algebraic graph theory. Finally, simulations are given to show the validity of our method.**

I. Introduction

DISTRIBUTED coordination control of multi-agent networks is a growing interest in the field of industrial control and automation. The consensus problem is the most fundamental problem in this area, which designs a distributed control protocol such that all the agents in the network asymptotically reach an agreement by interacting with their local neighbors as the time goes on. Although each agent has limited resources, the interconnected system as a whole can perform complex tasks in a coordinated fashion. Therefore, comparing to conventional control systems, multi-agent systems have many advantages such as reducing cost, improving system efficiency, flexibility and reliability. The applications of consensus problems cover a wide range such as spacecraft formation flying, sensor networks and cooperative surveillance. The control of multi-agent systems is a very active area of research. The consensus problem for single-integrator agents is addressed by [1]. Distributed control has been studied for the networks with and without communication delays and convergence is analyzed for directed graphs with fixed or switching topology, as well as for undirected graph. [2] considered a network of vehicles moving in a two dimensional plane and proposed a novel distributed static output feedback methodology to maintain a desired formation. Moreover the condition for consensus is relaxed by [3], where the consensus can be achieved if the union of the directed interaction graphs across some time intervals has a spanning tree frequently enough as the system evolves. Some other relevant research topics have also been addressed, such as consensus multiple agents under switching topologies [1], [4], agreement over random networks [5], coordination and consensus of networked agents with noisy measurements [6], networks with time-delays [7], [8].

In this paper, the cooperative control task under consideration is the leader-following coordination among a group of agents, where the leader is a special agent whose motion is independent of all the other agents and is followed by all the other agents. The leader-following approach has been widely used in many applications such as formation control in robotic systems and unmanned aerial vehicle formation, studied by [9], [10], [11], [12], [13], [14]. Consensus of multi-agent systems with general linear dynamic are studied by [4], [15], [16], [17], [18], [19], [20]. In particular, different static and dynamic consensus protocols are designed by [15], [16], [17], requiring the knowledge of the communication graph known by each agent to determine the bound of the coupling weights. However, the entire communication graph is global information. In other words, these consensus protocols cannot be computed and implemented by each agent in a fully distributed fashion, for example, we just need to use the local information of its own and neighbors.

In this paper, we consider the leader-following consensus problem in general linear system rather than integrators or double integrators in most existing literature. To handle the problem, we use Riccati inequalities method to design the control gain matrix for each agent. A contribution of this paper is proposing an adaptive tracking protocol for leader-following multi-agent systems. Motivated by [21], we design an adaptive tracking controller for leader-following multi-agent systems. In contrast to traditional distributed static controller, the coupling weights of adaptive controller can be adjusted by state errors between neighboring agents. It is worth to pointing out that we don't require to decide the explicit values of coupling weights beforehand, so the method is more flexible in practice.

Achieving the consensus for multi-agent systems under the fixed topology is relatively easy. However, when the interaction topologies are time-varying, this problem becomes much more difficult. The main contribution of this paper is that we discuss leader-following multi-agent consensus problem for general linear case under switching topology. The communication graph is required to be connected in

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most existing literature. However based on our method, the communication graph is only assumed to be jointly connected. Moreover, the whole interaction topologies are not required to be known. The stability of the closed-loop system is analyzed and simulations are given to illustrate the method is efficient that the consensus can be reached for leader-following multi-agent systems under time-varying topologies.

The rest of this paper is organized as follows. The problem formulation and some preliminaries about graph theory are discussed in Section II. The leader-following consensus problem under the fixed interconnection topology is discussed in Section III. The results obtained in Section III are generalized to the case of switching topology in Section IV. The simulation example is given in Section V. The conclusion is presented in Section VI.

II. Problem and preliminaries

A. Preliminaries

First we introduce some notations. Let $\mathbf{R}^{n\times n}$ be the set of $n \times n$ real matrices. I_N represents the identity matrix of dimension N. For a symmetric matrix P, the matrix inequality $P > 0 (P \geq 0)$ means that P is positive definite (positive semi-definite). $A \otimes B$ denotes the Kronecker product of matrices A and B.

In the literature, the multi-agent system is always represented as a graph. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a undirected graph of order N, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is a finite set and a finite set of arcs $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges of the graph. An arc of $\mathcal{G}$ is denoted by (i, j), which starts from i and ends on j and represents the information flow from agent j to agent i. A path in $\mathcal{G}$ is a sequence $i_0, i_1, \ldots, i_q$ of distinct vertices such that (i_{j-1}, i_j) is an arc for $j = 1, \ldots, q$. If there exists a path from vertex i to vertex j, we say that vertex j is reachable from vertex i. Furthermore, if there exists a path from every vertex to vertex j, then vertex j is a globally reachable vertex of $\mathcal{G}$. The graph is undirected which means that the edges (i, j) and (j, i) in $\mathcal{E}$ are considered to be the same. Two nodes i and j are neighbors to each other if $(i, j) \in \mathcal{E}$. The set of the neighbors of node i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}, j \neq i\}$. A graph is connected if there exists a path between each pair of the nodes. A component of the graph $\mathcal{G}$ is a connected subgraph that is maximal. A nonnegative matrix $\mathscr{A} = [a_{ij}] \in \mathbf{R}^{N\times N}$ is called an adjacency matrix of graph $\mathcal{G}$ if the element a_{ij} associated with the edge (i, j) is positive, i.e., $a_{ij} = 1$ if $(i, j) \in \mathcal{E}$. Moreover, we assume that $a_{ii} = 0$ for all $i \in \mathcal{V}$. Note that the adjacency matrix $\mathscr{A}$ is a symmetric matrix for an undirected graph. A diagonal matrix $\mathscr{D} \in \mathbf{R}^{N\times N}$ is called the degree matrix whose ith diagonal element is defined as $\sum_{j=1}^{N} a_{ij}$. Then the Laplacian matrix of the graph $\mathcal{G}$ is defined as $L = \mathscr{D} - \mathscr{A}$.

When the graph $\mathcal{G}$ is used to describe the interconnection topology of a multi-agent system consisting of one leader and N followers, the leader can be represented by vertex 0 and information is exchanged between the leader and the agents which are in the neighbors of the leader. We define a diagonal matrix $D = \text{diag}\{d_1, d_2, \ldots, d_N\} \in \mathbf{R}^{N\times N}$ to be a leader adjacency matrix, i.e., $d_i = 1$ if ith follower is connected to the leader across the communication link $(i, 0)$, otherwise $d_i = 0$. Let $\overline{\mathcal{G}}$ be the graph defined on the vertices $\{0, 1, 2, \ldots, N\}$.

We define a new matrix $H = L + D \in \mathbf{R}^{N\times N}$, and the following lemma plays an important role in the sequel.

Lemma 1: The following statements are equivalent: 1)

1) Vertex 0 is a globally reachable vertex for all vertices $i \in \mathcal{V}$.
2) The matrix H is symmetric positive define.

Remark 1: If the vertex 0 is a global reachable vertex for all vertices $i \in \mathcal{V}$, the undirected graph $\overline{\mathcal{G}}$ is connected. According to the Lemma proposed by [4], H is symmetric positive define.

To analyze the time-varying topologies $\mathcal{G}$ of the leader-following multi-agent system, we give the following general assumptions:

1) There exists a switching signal $\sigma : [t_0, \infty) \to \mathcal{P}$, which is piecewise constant. $\mathcal{P}$ is a finite set of all possible interconnection topologies of the multi-agent system and t_0 is the initial time. We denote all the possible graphs defined on the vertices $\{0, 1, 2, \ldots, N\}$ by $\{\overline{\mathcal{G}}_p : p \in \mathcal{P}\}$, and use $\{\mathcal{G}_p : p \in \mathcal{P}\}$ to denote subgraphs defined on vertices $\{1, 2, \ldots, N\}$.
2) The time-interval $[t_0, \infty)$ is constituted by an infinite sequence of bounded, non-overlapping, contiguous time-intervals $[t_j, t_{j+1})$ for $j = 0, 1, \ldots$ with $t_0 = 0$. For each interval $[t_k, t_{k+1})$, there is a sequence of nonoverlapping subintervals
$$[t_k^0, t_k^1), [t_k^1, t_k^2), \ldots, [t_k^{m_k-1}, t_k^{m_k}), t_k = t_k^0, t_{k+1} = t_k^{m_k}$$
satisfying $t_k^{j+1} - t_k^j \geq \tau$, $0 \leq j \leq m_k - 1$ for some integers $m_k \geq 1$ and a given constant $\tau \geq 0$, such that during each of such subintervals, the interconnection topology is fixed. Therefore, during each subinterval $[t_k^j, t_k^{j+1})$, the graph denoted by $\mathcal{G}_{\sigma(t)}$ is fixed and we denote it by $\mathcal{G}_k^j$.

B. Leader-following problem

In this paper, we consider the multi-agent systems consisting an active leader and N following agents. The dynamics of each agent is represented as

$$\dot{x}_i = Ax_i + Bu_i, \tag{1}$$

where $x_i \in \mathbf{R}^n$ is the state of ith agent and $u_i \in \mathbf{R}^m$ is the control input of the ith agent which can only use the local information of its neighbors and itself. The leader indexed as 0 is described as

$$\dot{x}_0 = Ax_0, \tag{2}$$

where x_0 represents the state of the leader. The control input of the leader is zero. That is, the leader's dynamic is independent of others. We take the system matrices for all the agents and leader to be identical because this case has practical background such as group of birds. In order to satisfy that the followers can track the leader with the feedback control, the following assumption is proposed.

Assumption 1: The pair (A, B) is stabilizable.

In this paper, we consider design the distributed control for the leader-following multi-agent system under fixed or time-varying topology so that each follower can track the leader using the local information. Therefore the leader-following consensus is said that closed-loop system should satisfy

$$\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0,$$

for any initial condition $x_i(0)$, $i = 1, 2, \ldots, N$.

III. Leader-following consensus under fixed topology

In this section, we focus on designing distributed control protocol for the leader-following multi-agent systems under fixed topology so that the closed-loop system can be stabilized. We discuss the static controller, whose coupling weight is a priori given. Then we purpose an adaptive controller for the leader-following multi-agent systems. In contrast with static controller, the coupling weights of the adaptive controller are updated according to neighboring agents.

The following assumption is assumed throughout this section.

Assumption 2: The vertex 0 associated with the leader is a global reachable vertex in the undirected graph $\overline{\mathcal{G}}$.

A. Tracking with distributed static controller

Based on the relative states between neighboring agents, the distributed control protocol for leader-following multi-agent systems described by (1) and (2) can be designed as

$$u_i = cK\left[\sum_{j=1}^{N} a_{ij}(x_i - x_j) + d_i(x_i - x_0)\right], i = 1, 2, \ldots, N \tag{3}$$

where $c > 0$ is the coupling weight among neighboring agents and $K \in \mathbf{R}^{m\times n}$ is a feedback gain matrix to be determined. Let a_{ij} be (i, j)-th entry of the adjacency matrix $\mathscr{A}$ associated with graph $\mathcal{G}$ and d_i be i-th diagonal entry of the leader adjacency matrix D. Under the Assumption 1, there exists a solution $P > 0$ to the following Riccati inequality

$$PA + A^{\mathsf{T}}P - 2PBB^{\mathsf{T}}P + \beta I_n \leq 0, \tag{4}$$

where $\beta > 0$ is a tuning parameter. The matrix K in (3) is designed as

$$K = -B^{\mathsf{T}}P. \tag{5}$$

Theorem 1: Consider the multi-agent systems described by (1) and (2). Let Assumption 1 and 2 hold. Let $P > 0$ and K be the solutions of (4) and (5). Then under the distributed control protocol (3) with $c \geq 1/\lambda_1$, where $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$ are the eigenvalues of H, the leader-following consensus problem can be solved.

Proof: For the lack of space, the details are not given in this paper. Select a Lyapunov function candidate, then the theorem is proved as long as time derivative of the function is negative. ■

B. Tracking with distributed adaptive controller

In Subsection A, the coupling weight c is dependent on the minimal eigenvalue λ_1 of H. However it is not easy to compute λ_1 when the multi-agent network is of large scale. Moreover, λ_1 is the global information of multi-agent systems. In this section, we attempt to design an adaptive controller without requiring explicit λ_1. The coupling weights of adaptive controller are updated according to neighboring agents. The distributed adaptive controller is presented as

$$\begin{aligned} u_i &= c_iK\left[\sum_{j=1}^{N} a_{ij}(x_i - x_j) + d_i(x_i - x_0)\right] \\ \dot{c}_i &= \tau_i\left[\sum_{j=1}^{N} a_{ij}(x_i - x_j) + d_i(x_i - x_0)\right]^{\mathsf{T}} \\ &\quad \times \Gamma\left[\sum_{j=1}^{N} a_{ij}(x_i - x_j) + d_i(x_i - x_0)\right], i = 1, 2, \ldots, N, \end{aligned} \tag{6}$$

where c_i is the time-varying coupling weight associated with agent i, $K \in \mathbf{R}^{m\times n}$ is the feedback gain matrix and a_{ij} is the (i, j)-th entry of adjacency matrix $\mathscr{A}$ associated with graph $\mathcal{G}$. Let d_i be the i-th diagonal entry of leader adjacency matrix D and τ_i be a positive scalar. Γ is a constant gain matrix.

Let $\varepsilon_i = x_i - x_0$. Using (6), the dynamics of state error ε_i can be obtained as

$$\dot{\varepsilon}_i = A\varepsilon_i + c_iBK\left[\sum_{j=1}^{N} a_{ij}(\varepsilon_i - \varepsilon_j) + d_i\varepsilon_i\right], i = 1, 2, \ldots, N. \tag{7}$$

Denote $\varepsilon = (\varepsilon_1^{\mathsf{T}}, \varepsilon_2^{\mathsf{T}}, \ldots, \varepsilon_N^{\mathsf{T}})^{\mathsf{T}}$. Then (7) can be rewritten in a compact form

$$\dot{\varepsilon} = [(I_N \otimes A) + CH \otimes BK]\,\varepsilon, \tag{8}$$

where matrix $C = \mathrm{diag}(c_1, c_2, \ldots, c_N)$, and H is defined in Section II.

Let $P > 0$ and K be the solutions to (4) and (5). Γ can be designed as

$$\Gamma = PBB^{\mathsf{T}}P. \tag{9}$$

According to Assumption 2 and Lemma 1, we derive that the symmetric matrix H is positive definite. Since $\Gamma \geq 0$, the coupling weight c_i is nondecreasing. Obviously, the leader-following consensus problem is solved by adaptive controller (6) if the state error ε converges to zero.

Theorem 2: Consider the multi-agent systems represented by (1) and (2). Let Assumption 1 and 2 hold. The distributed tracking problem can be solved under the controller (6) whose $P > 0$, K and Γ are the solutions to (4), (5) and (9). Moreover, each coupling weight c_i converges to some finite steady-state.

Proof: Consider the Lyapunov function candidate

$$V_2 = \varepsilon^\mathsf{T}(H \otimes P)\varepsilon + \sum_{i=1}^{N} \frac{1}{\tau_i}(c_i - \alpha)^2.$$

Taking time derivative of (III-B along the trajectory of system (8), we obtain that

$$\begin{aligned}
\dot{V}_2 &= 2\varepsilon^\mathsf{T}(H \otimes P)\left[(I_N \otimes A) + CH \otimes BK\right]\varepsilon \\
&\quad + \sum_{i=1}^{N} \frac{2}{\tau_i}(c_i - \alpha)\dot{c}_i \\
&= 2\varepsilon^\mathsf{T}\left[(H \otimes PA) + HCH \otimes PBK\right]\varepsilon \\
&\quad + \sum_{i=1}^{N} \frac{2}{\tau_i}(c_i - \alpha)\dot{c}_i. \qquad (10)
\end{aligned}$$

Using $K = -B^\mathsf{T}P$ and (6), we have

$$\begin{aligned}
&2\varepsilon^\mathsf{T}\left(HCH \otimes PBK\right)\varepsilon + \sum_{i=1}^{N} \frac{2}{\tau_i}(c_i - \alpha)\dot{c}_i \\
=& -2\sum_{i=1}^{N} c_i \left(\sum_{j=1}^{N} a_{ij}(\varepsilon_i - \varepsilon_j) + d_i\varepsilon_i\right)^\mathsf{T} PBB^\mathsf{T}P \\
&\times \left(\sum_{j=1}^{N} a_{ij}(\varepsilon_i - \varepsilon_j) + d_i\varepsilon_i\right) + \sum_{i=1}^{N} \frac{2}{\tau_i}(c_i - \alpha)\dot{c}_i \\
=& -2\alpha\varepsilon^\mathsf{T}(HH \otimes PBB^\mathsf{T}P)\varepsilon. \qquad (11)
\end{aligned}$$

Substituting (11) into (10), $\dot{V}_2$ can be rewritten as

$$\dot{V}_2 = \varepsilon^\mathsf{T}\left[H \otimes (PA + A^\mathsf{T}P)\right]\varepsilon - \varepsilon^\mathsf{T}\left(2\alpha HH \otimes PBB^\mathsf{T}P\right)\varepsilon. \qquad (12)$$

Let $H = U^\mathsf{T}\Lambda U$, where U is an orthogonal matrix. Denote $\delta = (U \otimes I_n)\varepsilon$. Then (12) becomes

$$\begin{aligned}
\dot{V}_2 &= \delta^\mathsf{T}\left[\Lambda \otimes (A^\mathsf{T}P + PA)\right]\delta - \delta^\mathsf{T}\left(2\alpha\Lambda^2 \otimes PBB^\mathsf{T}P\right)\delta \\
&= \sum_{i=1}^{N} \lambda_i\delta_i^\mathsf{T}(A^\mathsf{T}P + PA - 2\alpha\lambda_i PBB^\mathsf{T}P)\delta_i. \qquad (13)
\end{aligned}$$

As long as $\alpha\lambda_i \geq 1, i = 1, 2, \ldots, N$ hold, by applying (4), (13) becomes

$$\dot{V}_2 \leq -\sum_{i=1}^{N} \lambda_i\beta\delta_i^\mathsf{T}\delta_i. \qquad (14)$$

Thus, for any $\delta \neq 0, V_2 < 0$, which implies that for any $\varepsilon \neq 0, V_2 < 0$. Therefore, $V_2(t)$ is bounded and so are c_i. Note $\Lambda \geq 0$, so it is obvious that c_i are nondecreasing. Then c_i converge to some finite steady-state constants respectively. That is, system (8) is globally stable and all the agents can follow the leader. ■

Remark 2: It is worth to mentioning that since c_i are nondecreasing, the steady-state constant α can be sufficiently large to satisfy $\alpha\lambda_i \geq 1$, $i = 1, 2, \ldots, N$.

IV. Leader-following consensus under switching topology

In this section, we discuss the consensus of leader-following multi-agent systems under switching topology. To ensure leader-following consensus under switching topology, there is another constraint on the dynamics of agents. We suppose that

Assumption 3: The internal dynamics matrix A has no positive real part eigenvalues.

In switching topology case, the controller of agent i is designed as

$$\begin{aligned}
u_i &= K\left[\sum_{j=1}^{N} c_{ij}a_{ij}(t)(x_i - x_j) + c_i d_i(t)(x_i - x_0)\right], \\
\dot{c}_{ij} &= \eta_{ij}a_{ij}(t)(x_i - x_j)^\mathsf{T}\Gamma(x_i - x_j), \\
\dot{c}_i &= \eta_i d_i(t)(x_i - x_0)^\mathsf{T}\Gamma(x_i - x_0), \quad i = 1, 2, \ldots, N,
\end{aligned} \qquad (15)$$

where $K \in \mathbf{R}^{m\times n}$ is the feedback gain matrix and a_{ij} is the (i, j)-th entry of the adjacency matrix $\mathscr{A}$ associated with graph $\mathcal{G}$. d_i is the connection weight between agent i and the leader, and Γ is a constant gain matrix. c_{ij} is the time-varying coupling weight between agent i and agent j, and c_i the time-varying coupling weight between agent i and the leader, $i = 1, 2, \ldots, N$. Let $\eta_{ij} = \eta_{ji}$, η_i be positive constants and $c_{ij}(0) = c_{ji}(0)$. Comparing to the controller of fixed topology, it is important to note that the neighbors of each agent $a_{ij}(t)$ and $d_i(t)$ describing the neighbors of the leader vary with time. We use $L_{\sigma(t)}$ and $D_{\sigma(t)}$ to describe the time-dependent of graph topology, where σ is a switching signal defined in Section II. To present the stability, we consider an infinite sequence of nonempty, bounded and contiguous time interval $[t_k, t_{k+1})$ defined as in Section II. Over each time interval $[t_k, t_{k+1})$, some or all of $\mathcal{G}_k^j$ are permitted to be disconnected. We only require the graph to be jointly connected which is defined as follows.

Definition 1: The union of a collection graph is a graph whose vertex and edge sets are the unions of the vertex and edge sets of the graphs in the collection. we say that such a collection is jointly connected if the union of its members is a connected graph. The graphs are said to be jointly connected across the time interval $[t, t+T], T > 0$ if the union of the graphs $\overline{\mathcal{G}}_{\sigma(s)} : s \in [t, t+T]$ is jointly connected.

To ensure leader-following consensus under switching topology, we propose another assumption.

Assumption 4: The graphs are jointly connected across each interval $[t_k, t_{k+1}), k = 0, 1, \ldots$.

Let $P > 0$ be the solution of the following inequalities

$$\begin{aligned}
PA + A^\mathsf{T}P - 2PBB^\mathsf{T}P &< 0, \\
PA + A^\mathsf{T}P &\leq 0. \qquad (16)
\end{aligned}$$

Then the feedback gain matrix K can be designed as

$$K = -B^\mathsf{T}P. \qquad (17)$$

The constant gain matrix Γ can be designed as

$$\Gamma = PBB^{\mathsf{T}}P. \tag{18}$$

Theorem 3: Consider the multi-agent systems represented by (1) and (2). Let Assumption 1 and 3 hold, and switching signal σ satisfies Assumption 4. The distributed tracking problem can be solved under the controller (15) whose $P > 0$, K and Γ are the solutions to (16), (17) and (18). Moreover, each coupling weight c_{ij} and c_i converge to some finite steady-state.

Proof: Let $\varepsilon_i = x_i - x_0, i = 1, 2, \ldots, N$. According to (1) and (2), the dynamics of ε_i can be derived as

$$\dot{\varepsilon_i} = A\varepsilon_i + BK\left[\sum_{j=1}^{N} c_{ij}a_{ij}(t)(\varepsilon_i - \varepsilon_j) + c_i d_i(t)\varepsilon_i\right]. \tag{19}$$

Consider Lyapunov function candidate

$$V_3 = \sum_{i=1}^{N} \varepsilon_i^{\mathsf{T}} P\varepsilon_i + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} \frac{(c_{ij}-\alpha)^2}{2\eta_{ij}} + \sum_{i=1}^{N}\frac{(c_i-\alpha)^2}{\eta_i}, \tag{20}$$

where $\alpha > 0$ is the positive constants to be determined. The time derivative of (20) can be obtained as

$$\dot{V}_3 = \sum_{i=1}^{N} 2\varepsilon_i^{\mathsf{T}} P\dot{\varepsilon}_i + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} \frac{(c_{ij}-\alpha)}{\eta_{ij}}\dot{c}_{ij} + \sum_{i=1}^{N}\frac{2(c_i-\alpha)}{\eta_i}\dot{c}_i. \tag{21}$$

Observing (15), we can obtain that $c_{ij}(t) = c_{ji}(t), \forall t \geq 0$. Using (15) and (18), we get

$$\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} \frac{(c_{ij}-\alpha)}{\eta_{ij}}\dot{c}_{ij}\\
&= \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} (c_{ij}-\alpha)a_{ij}(t)(\varepsilon_i-\varepsilon_j)^{\mathsf{T}} PBB^{\mathsf{T}}P(\varepsilon_i-\varepsilon_j)\\
&= 2\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} (c_{ij}-\alpha)a_{ij}(t)\varepsilon_i^{\mathsf{T}} PBB^{\mathsf{T}}P(\varepsilon_i-\varepsilon_j).
\end{aligned} \tag{22}$$

Substituting (22) and (15) into (21), (21) becomes

$$\begin{aligned}
\dot{V}_3 &= 2\sum_{i=1}^{N}\varepsilon_i^{\mathsf{T}} PA\varepsilon_i - 2\alpha\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(t)\varepsilon_i^{\mathsf{T}} PBB^{\mathsf{T}}P(\varepsilon_i-\varepsilon_j)\\
&\quad - 2\alpha\sum_{i=1}^{N} d_i(t)\varepsilon_i^{\mathsf{T}} PBB^{\mathsf{T}}P\varepsilon_i\\
&= 2\sum_{i=1}^{N}\varepsilon_i^{\mathsf{T}} PA\varepsilon_i - 2\alpha\sum_{i=1}^{N}\sum_{j=1}^{N} H_{ij}(t)\varepsilon_i^{\mathsf{T}} PBB^{\mathsf{T}}P\varepsilon_j,
\end{aligned} \tag{23}$$

where $H_{ij}(t)$ denotes the (i,j)-th entry of the matrix $H_{\sigma(t)} = L_{\sigma(t)} + D_{\sigma(t)}$. Let $\varepsilon = (\varepsilon_1^{\mathsf{T}}, \varepsilon_2^{\mathsf{T}}, \ldots, \varepsilon_N^{\mathsf{T}})$. Then (23) can be written in a compact form

$$\dot{V}_3 = \varepsilon^{\mathsf{T}}\left[I_N \otimes (PA + A^{\mathsf{T}}P) - 2\alpha H \otimes PBB^{\mathsf{T}}P\right]\varepsilon. \tag{24}$$

Let $H = U^{\mathsf{T}}\Lambda U$, where U is an orthogonal matrix. By denoting $\delta = (U \otimes I_n)\varepsilon$, (24) becomes

$$\begin{aligned}
\dot{V}_3 &= \delta^{\mathsf{T}}\left[I_N \otimes (PA + A^{\mathsf{T}}P) - 2\alpha\Lambda \otimes PBB^{\mathsf{T}}P\right]\delta\\
&\leq \sum_{i\in l(\sigma(t))} \delta_i^{\mathsf{T}}(PA + A^{\mathsf{T}}P - 2\alpha\lambda_i PBB^{\mathsf{T}}P)\delta_i\\
&\leq -\sum_{i\in l(\sigma(t))} \delta_i^{\mathsf{T}}\rho\delta_i\\
&\leq 0,
\end{aligned} \tag{25}$$

where $-\rho < 0$ is the maximum eigenvalue of the matrix $PA + A^{\mathsf{T}}P - 2PBB^{\mathsf{T}}P$ and $\alpha\lambda_i > 1$. Because the coupling weights c_{ij} and c_i are nondecreasing, the steady-constant α can be sufficiently large to make $\alpha\lambda_i > 1, \forall i \in l(\sigma(t))$. Therefore, V_3 has the limit as time goes on. Next we prove that $\varepsilon(t)$ converges to 0. Consider the infinite sequences $V_3(t_i), i = 0, 1, \ldots$. Because V_3 has the limit, V_3 satisfy the Cauchy's convergence criteria. So we can obtain that for any $\epsilon > 0$, there exists a positive number M, such that for $\forall k \geq M$

$$|V_3(t_{k+1}) - V_3(t_k)| < \epsilon. \tag{26}$$

Therefore, considering $\varepsilon(t)$ we can get

$$\rho\left[\int_{t_k^0}^{t_k^1} \sum_{i\in l\left(\sigma(t_k^0)\right)} \delta_i^{\mathsf{T}}(t)\delta_i(t)dt + \cdots + \int_{t_k^{m_k-1}}^{t_k^{m_k}} \sum_{i\in l\left(\sigma(t_k^{m_k-1})\right)} \delta_i^{\mathsf{T}}(t)\delta_i(t)dt\right] \leq \epsilon$$

Thus, for $\forall k > M$, we have

$$\rho\left[\int_{t_k^0}^{t_k^0+\tau} \sum_{i\in l\left(\sigma(t_k^0)\right)} \delta_i^{\mathsf{T}}(t)\delta_i(t)dt + \cdots + \int_{t_k^{m_k-1}}^{t_k^{m_k-1}+\tau} \sum_{i\in l\left(\sigma(t_k^{m_k-1})\right)} \delta_i^{\mathsf{T}}(t)\delta_i(t)dt\right] \leq \epsilon,$$

which implies

$$\lim_{t\to\infty}\int_t^{t+\tau} \sum_{i\in l(\sigma(t))} \delta_i(s)^{\mathsf{T}}\delta_i(s)ds = 0.$$

Therefore,

$$\lim_{t\to\infty}\int_t^{t+\tau}\left[\sum_{i\in l(\sigma(t_k^0))} \delta_i^{\mathsf{T}}(s)\delta_i(s) + \cdots + \sum_{i\in l(\sigma(t_k^{m_k-1}))} \delta_i^{\mathsf{T}}(s)\delta_i(s)\right]ds = 0 \tag{27}$$

According to Lemma 5 in [4] and Assumption 4, (27) can be written as

$$\lim_{t\to\infty}\int_t^{t+\tau}\left[\sum_{i=1}^{N} r_i\delta_i^{\mathsf{T}}(s)\delta_i(s)\right]ds = 0,$$

where $r_1, \ldots, r_N$ are positive numbers. Because V_3 is nonincreasing, $\varepsilon(t)$ is bounded and so is $\dot{\varepsilon}(t)$. Therefore $\sum_{i=1}^{N} r_i \delta_i^{\mathsf{T}}(s)\delta_i(s)$ is uniformly continuous. According to Barbalat's Lemma, we can obtain that $\lim_{s\to\infty} \sum_{i=1}^{N} r_i \delta_i^{\mathsf{T}}(s)\delta_i(s) = 0$. Therefore δ_i converges to 0, so does the ε_i, $i = 1, 2, \ldots, N$.

Moreover, due to $\dot{V}_3(t) \le 0$, c_{ij}, c_i are bounded, $i = 1, 2, \ldots, N, j = 1, \ldots, N$. Note that c_{ij} and c_i are nondecreasing, so all of them converge to some steady constants. ■

V. Simulation study

Consider the multi-agent system under switching topologies. Suppose there are four followers and a leader described by

$$A = \begin{bmatrix} -1.9978 & -0.0325 & 1.8869 \\ 5.3221 & -1.9177 & -4.8528 \\ -4.9833 & 1.8477 & 3.4155 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.2853 & 0.1536 \\ 0.1634 & 0.1525 \\ 0.2720 & 0.1502 \end{bmatrix}.$$

It is easy to check that (A, B) is stabilizable and the matrix A has no positive real part eigenvalues. Let all the possible interaction graphs be $\{\overline{\mathcal{G}}_1, \overline{\mathcal{G}}_2, \overline{\mathcal{G}}_3, \overline{\mathcal{G}}_4, \overline{\mathcal{G}}_5, \overline{\mathcal{G}}_6\}$ shown in Figure 1. The interaction topologies are switched in order of

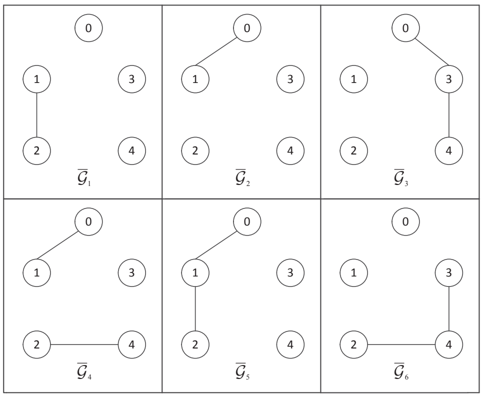

Fig. 1. Possible interaction topologies in the example

$\overline{\mathcal{G}}_1 \to \overline{\mathcal{G}}_2 \to \overline{\mathcal{G}}_3 \to \overline{\mathcal{G}}_4 \to \overline{\mathcal{G}}_5 \to \overline{\mathcal{G}}_6 \to \overline{\mathcal{G}}_1 \to \cdots$, and each graph is kept for $1/3$ s. Because the graphs $\overline{\mathcal{G}}_1 \cup \overline{\mathcal{G}}_2 \cup \overline{\mathcal{G}}_3$ and $\overline{\mathcal{G}}_4 \cup \overline{\mathcal{G}}_5 \cup \overline{\mathcal{G}}_6$ are connected, Assumption 4 is satisfied. Using Matlab, we can obtain a solution to (16), (17) and (18) as

$$P = \begin{bmatrix} 52.9921 & -19.2742 & -41.8210 \\ -19.2742 & 10.8752 & 10.9239 \\ -41.8210 & 10.9239 & 38.6169 \end{bmatrix},$$

$$K = \begin{bmatrix} -0.5939 & 0.7506 & -0.3572 \\ 1.0813 & -0.3387 & -1.0424 \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} 1.5218 & -0.8120 & -0.9150 \\ -0.8120 & 0.6782 & 0.0850 \\ -0.9150 & 0.0850 & 1.2143 \end{bmatrix}.$$

Under the control law (15), the states error trajectories of $e_i = x_i - x_0, i = 1, 2, \ldots, 4$ are shown in Figure 2. In addition the coupling weights trajectories of $c_{ij}, i = 1, 2, \ldots, 4, j = 1, 2, \ldots, 4$ and $c_i, i = 1, 2, \ldots, 4$ are shown is Figure 3 and Figure 4 converging to some constants.

VI. Conclusion

In this paper, we consider leader-following multi-agent consensus problem for general linear case. In contrast to traditional distributed static controller which requires the knowledge of entire communication graph, we propose an adaptive protocol based on Riccati inequalities with an adaptive law for adjusting coupling weights between neighboring agents. We discuss multi-agent consensus problems under fixed topology and switching topology, respectively. The consensus can be reached for multi-agent systems under jointly connected graph in our method and the stability is demonstrated using Lyapunov's method. Finally, simulation is given to show our adaptive controller is efficient.

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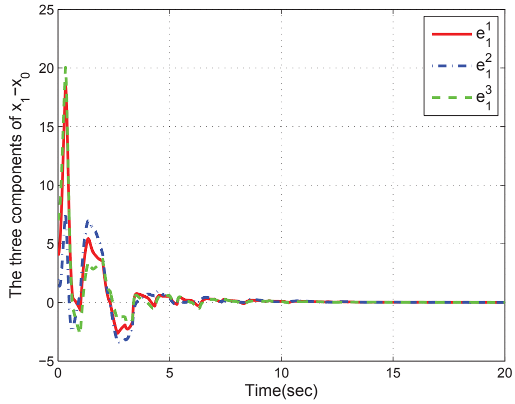
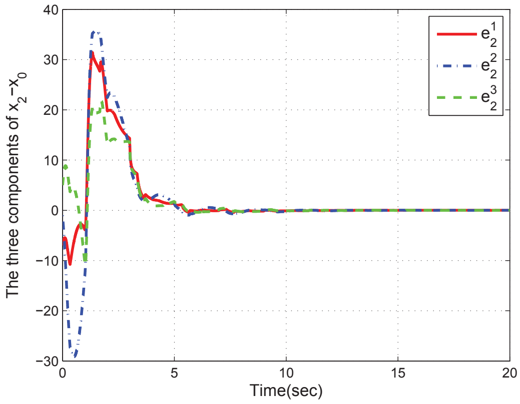
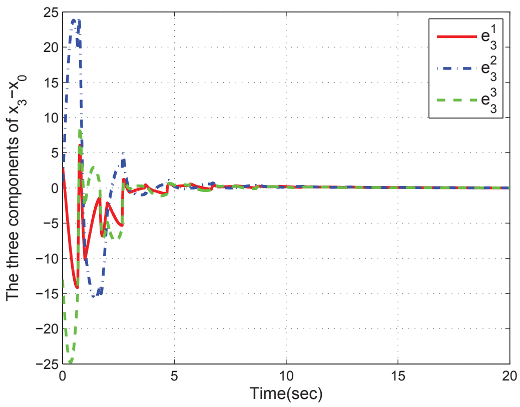


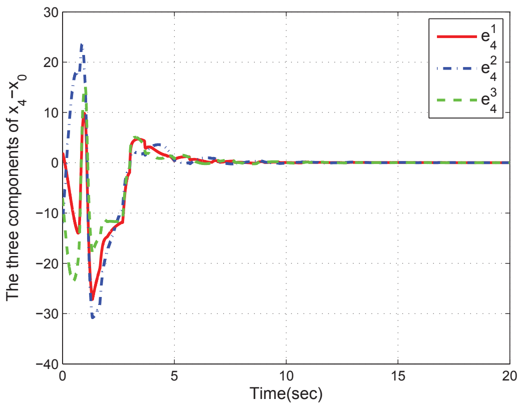
(a) The error trajectories of e_1 in the example



(b) The error trajectories of e_2 in the example



(c) The error trajectories of e_3 in the example



(d) The error trajectories of e_4 in the example

Fig. 2. The error trajectories in the example

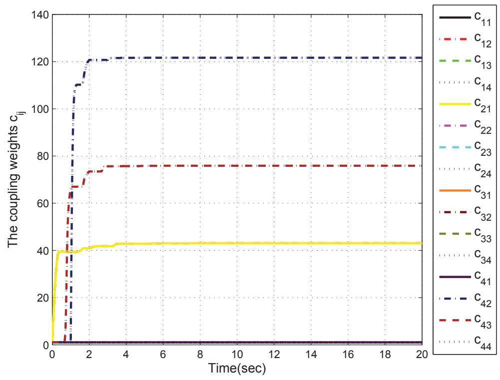


Fig. 3. The coupling weights trajectories of c_{ij} in the example

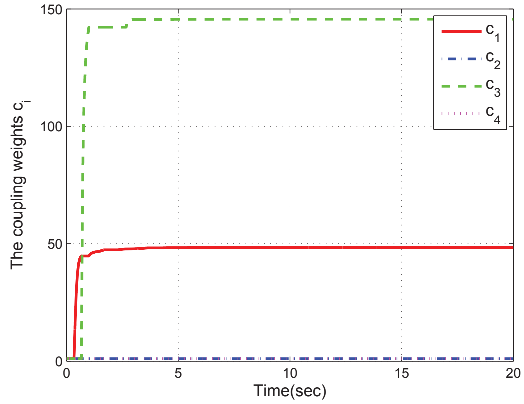


Fig. 4. The coupling weights trajectories of c_i in the example

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