

# A Kinematics Analysis for a 5-DOF Manipulator

Jinyan Lu, De Xu, Peng Wang

Research Center of Precision Sensing and Control,

Institute of Automation, Chinese Academy of Sciences, Beijing 100190, P. R. China

E-mail: [jinyan.lu@ia.ac.cn](mailto:jinyan.lu@ia.ac.cn)

**Abstract:** In order to analyze a five-degree-of-freedom (DOF) manipulator for special applications, this paper establishes its kinematics model firstly, and then analyzes its workspace and characteristics, and gets its inverse kinematics solution. Finally, its advantages and disadvantages are presented. On the basis of kinematics analysis, combining with the typical operating cases, we point out the existing problems for the manipulator, and then provide important guidance for the next optimization design.

**Key Words:** 5-DOF Manipulator, Kinematics, Workspace, Simulation

## 1 INTRODUCTION

As an important equipment of modern manufacturing, industrial robot has been widely used in electronic industry, automobile manufacturing, engineering machinery, chemical industry, disaster relief, national defense military, service life, etc. At present, industrial robots mainly engage in spot welding, arc welding, spray lacquer, handling, cutting, assembly, testing and other operations. Many industrial robots have combined the specialties of humans and machines, not only can rapid response to the state of the environment, but also can continue to work for a long time with high precision and reliability, and not afraid of bad environment. The widespread use of industrial robots not only can reduce costs and improve product quality and yield, but also can reduce the operator labor intensity, and improve the work efficiency.

During the use of industrial robots, we usually have to deal with the relationship between the movement of terminal devices in the space and the movement of each joint, namely the robot kinematics problem. Accurate kinematics analysis can provide guidance for the robot's motion control, provide the basis for the trajectory planning of robot, and provide the reference for the next step optimization design of the robot. Therefore, the kinematics analysis is the foundation of the robot motion control. Generally, the kinematics mainly analyzes the movement relative to the reference frame for the robot. That is to say, it mainly focuses on the relationship between joint variables space and pose of the end-effector. The robot kinematics has a close relationship with spatial mechanism. Up to now, there are many methods concerning with the kinematics. For example, the project method based on descriptive geometry, the analytical method based on mathematical tools such as the vector analysis, matrix transformation, and quaternions [1].

Based on the actual requirements, this paper analyzes the kinematics for a 5-DOF manipulator which is used for special applications. Using the Denavit-Hartenberg methodology, Gan et al established both forward and inverse kinematics models for a 5-DOF Pioneer 2 robot arm [2]. In [3] the forward kinematics and inverse kinematics

for a 5-DOF manipulator were systematically analyzed, and an analytical solution for a 5-DOF manipulator was provided to follow a given trajectory while keeping the orientation of one axis in the end-effector frame. In order to solve the problems of the 5-DOF agricultural picking robot's trajectory planning problem, Lu et al analyzed its D-H parameters and established its kinematics model [4]. In addition, an algorithm based on the comprehensive application of analytic and geometric method was also put forward, and the inverse kinematics analytical solution was also obtained in [4]. In [5], the kinematics model for its 5-DOF cutting robot was established with the modified D-H method, and the inverse kinematics was solved using inverse transformation method. As to the question of multiple solutions of inverse kinematics, the author used flexible standard to select the solution. Shen gave the kinematics analysis to a 5-DOF Carrying Manipulator for teaching [6], which obtained solutions for the forward and inverse kinematics, and analyzed the workspace of the manipulator using graphical solutions according to the actual working condition and technology parameters. Based on the geometric model and the kinematics analysis of the 5-DOF rehabilitant manipulator, Lv et al solved the invertible matrix of five coordinate transformation matrix [7]. They also offered reliance for the actual intellectual control of the position and speed about the rehabilitating robot.

The rest of this paper is organized as follows. Section 2 presents the manipulator and coordinate system. Forward kinematics and inverse kinematics analysis are introduced in section 3. Section 4 provides simulation results to verify our analysis. Finally, the paper is concluded in section 5.

## 2 MANIPULATOR AND COORDINATE SYSTEM

A manipulator is a connecting link open loop chain mechanism which is composed by a series of joints. In this paper, the kinematics for a 5-DOF manipulator is analyzed based on the Denavit-Hartenberg methodology.

### 2.1 Characteristics of the manipulator

The 5-DOF manipulator discussed in this paper is mainly used for auxiliary installation, for example, providing power to the operators during handling and assembly.

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As shown in figure 1, 5-DOF manipulator is a serial mechanism. Its five joints are all rotary joints. Among them, the first and the fifth joint rotate in the vertical direction, and they are both electric control. The rest of the three joints rotate in the horizontal direction, and they are manual control. There is a balance mechanism between the first joint and the second joint to guarantee that the second joint always stays level when the first joint rotates.

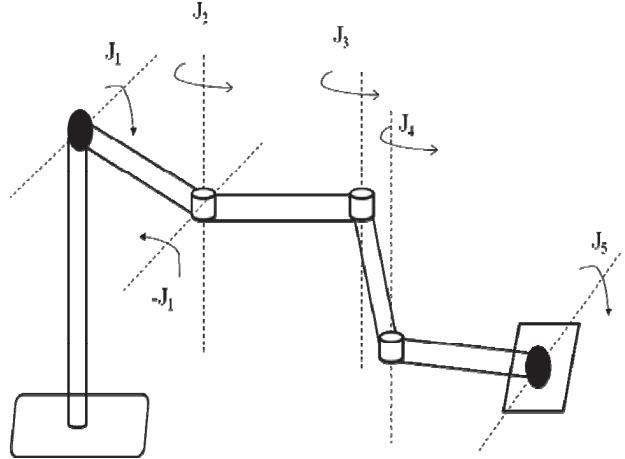


Fig 1. Sketch of the 5-DOF manipulator

## 2.2 Coordinate System of The Manipulator

To facilitate to establish the robot's kinematics equation, the coordinate system is assigned for the manipulator using the D-H convention, as shown in figure2.

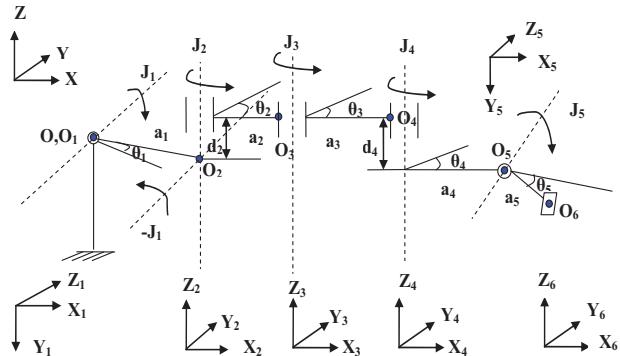


Fig 2. Coordinate system for the manipulator

The D-H parameters of the manipulator are obtained according to the coordinates given in figure 2, as shown in table 1.

Table 1 the D-H parameters of the manipulator

Link	Torsion angle (degree)	Link length (mm)	Link offset (mm)	Joint angle	Range of the joint angle (degree)
1	-90	0	0	$\theta_1$	-60~60
2	90	$a_1=500$	0	$-\theta_1$	-60~60
3	0	$a_2=400$	$d_2=100$	$\theta_2$	-120~120
4	0	$a_3=400$	0	$\theta_3$	-120~120
5	-90	$a_4=400$	$d_4=100$	$\theta_4$	-120~120
6	90	$a_5=300$	0	$\theta_5$	-60~60

## 3 KINEMATICS ANALYSIS

### 3.1 Forward Kinematics

The link transformation matrixes are obtained based on homogeneous transformation according to the D-H parameters in table 1, given as follows ( $A_i$  represents the transformation matrix from coordinate system  $O_{i-1}X_{i-1}Y_{i-1}Z_{i-1}$  to  $O_iX_iY_iZ_i$ ):

$$A_1 = \text{Rot}(X, -90^\circ) \text{Rot}(Z_1, \theta_1)$$

$$= \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin \theta_1 & -\cos \theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \text{Trans}(a_1, 0, 0) \text{Rot}(Z_1, -\theta_1) \text{Rot}(X, 90^\circ)$$

$$= \begin{bmatrix} \cos \theta_1 & 0 & -\sin \theta_1 & a_1 \\ -\sin \theta_1 & 0 & -\cos \theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \text{Rot}(Z_2, \theta_2) \text{Trans}(0, 0, d_2) \text{Trans}(a_2, 0, 0)$$

$$= \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & a_2 \cdot \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & a_2 \cdot \sin \theta_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \text{Rot}(Z_3, \theta_3) \text{Trans}(a_3, 0, 0)$$

$$= \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 & 0 & a_3 \cdot \cos \theta_3 \\ \sin \theta_3 & \cos \theta_3 & 0 & a_3 \cdot \sin \theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \text{Rot}(Z_4, \theta_4) \text{Trans}(0, 0, -d_4)$$

$$\text{Trans}(a_4, 0, 0) \text{Rot}(X_4, -90^\circ)$$

$$= \begin{bmatrix} \cos \theta_4 & 0 & -\sin \theta_4 & a_4 \cdot \cos \theta_4 \\ \sin \theta_4 & 0 & \cos \theta_4 & a_4 \cdot \sin \theta_4 \\ 0 & -1 & 0 & -d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \text{Rot}(Z_5, \theta_5) \text{Trans}(a_5, 0, 0) \text{Rot}(X_5, 90^\circ)$$

$$= \begin{bmatrix} \cos \theta_5 & 0 & \sin \theta_5 & a_5 \cdot \cos \theta_5 \\ \sin \theta_5 & 0 & -\cos \theta_5 & a_5 \cdot \sin \theta_5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

According to the matrix  $A_1, A_2, A_3, A_4, A_5, A_6$ , the pose matrix T of end-effector in the base coordinate system is gotten. In other words, the kinematics is obtained.

$$T = A_1 A_2 A_3 A_4 A_5 A_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where

$$n_x = \cos \theta_5 \cos(\theta_2 + \theta_3 + \theta_4) \quad (1)$$

$$n_y = \cos \theta_5 \sin(\theta_2 + \theta_3 + \theta_4) \quad (2)$$

$$n_z = -\sin \theta_5 \quad (3)$$

$$o_x = -\sin(\theta_2 + \theta_3 + \theta_4) \quad (4)$$

$$o_y = \cos(\theta_2 + \theta_3 + \theta_4) \quad (5)$$

$$o_z = 0 \quad (6)$$

$$a_x = \sin \theta_5 \cos(\theta_2 + \theta_3 + \theta_4) \quad (7)$$

$$a_y = \sin \theta_5 \sin(\theta_2 + \theta_3 + \theta_4) \quad (8)$$

$$a_z = \cos \theta_5 \quad (9)$$

$$p_x = a_5 \cos \theta_5 \cos(\theta_2 + \theta_3 + \theta_4) + a_4 \cos(\theta_2 + \theta_3 + \theta_4) + a_3 \cos(\theta_2 + \theta_3) + a_2 \cos \theta_2 + a_1 \cos \theta_1 \quad (10)$$

$$p_y = a_5 \cos \theta_5 \sin(\theta_2 + \theta_3 + \theta_4) + a_4 \sin(\theta_2 + \theta_3 + \theta_4) + a_3 \sin(\theta_2 + \theta_3) + a_2 \sin \theta_2 \quad (11)$$

$$p_z = -a_5 \sin \theta_5 + d_2 - d_4 - a_1 \sin \theta_1 \quad (12)$$

### 3.2 Inverse kinematics

Inverse kinematics mainly analyzes the mapping relationship from the pose of the end-effector to the coordinate in the link coordinate system for the manipulator. In other words, it solves the value of joint angle for each joint after the pose matrix T is given.

According to figure 1, the manipulator has five rotary joints, among them there are two joints rotate along Y axis, and three joints rotate along Z axis. It can translate along X axis, Y axis and Z axis, and rotate about Y axis and Z axis. The pose of end-effector is denoted as  $(p_x, p_y, p_z, \theta_x, \theta_y, \theta_z)$ , so the value of  $\theta_y$  depends on  $\theta_5$ , the value of  $p_z$  depends on  $\theta_1$  and  $\theta_5$ ,  $\theta_2$ ,  $\theta_3$  and  $\theta_4$  jointly determine the value of  $\theta_z$ , the value of  $\theta_x$  is constant. So we obtain the following expression:

$$\theta_y = \theta_5 \quad (13)$$

$$\theta_z = \theta_2 + \theta_3 + \theta_4 \quad (14)$$

Based on expressions from (1) to (14), the values of  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$  are gotten easily.

(1) Solution for  $\theta_5$

From (3) and (9), we can get  $\frac{n_z}{a_z} = -\tan \theta_5$ , so the solution for  $\theta_5$  is obtained:

$$\theta_5 = \arctan(-n_z, a_z) \quad (15)$$

(2) Solution for  $\theta_1$

Substituting (5) into (12), we get

$p_z = a_5 \cdot n_z + d_2 - d_4 - a_1 \cdot \sin \theta_1$ , so the solution for  $\theta_1$  is obtained:

$$\theta_1 = \arcsin \frac{a_5 n_z + d_2 - d_4 - p_z}{a_1} \quad (16)$$

(3) Solution for  $\theta_2$

Substituting (5) and (9) into (10), we get the following expression:

$$\cos(\theta_2 + \theta_3) = (p_x - a_4 o_y - a_5 a_z o_y - a_2 \cos \theta_2 - a_1 \cos \theta_1) / a_3 \quad (17)$$

Substituting (3) and (4) into (11), we get the following expression:

$$\sin(\theta_2 + \theta_3) = (p_y + a_5 a_z o_x + a_4 o_x - a_2 \sin \theta_2) / a_3 \quad (18)$$

By applying a square sum to equation (17) and (18), a new equation that contains  $\theta_2$  can be obtained as follow:

$$(p_x - a_4 o_y - a_5 a_z o_y - a_2 \cos \theta_2 - a_1 \cos \theta_1)^2 + (p_y + a_5 a_z o_x + a_4 o_x - a_2 \sin \theta_2)^2 = a_3^2 \quad (19)$$

For the sake of convenience, we denote the coefficient of  $\sin \theta_2$  as  $B$ , and denote the coefficient of  $\cos \theta_2$  as  $C$ , and denote the constant term in equation (19) as  $D$ . Then (19) is rewritten as (20).

$$B \sin \theta_2 + C \cos \theta_2 = D \quad (20)$$

So two solutions of  $\theta_2$  are obtained:

$$\theta_2 = \arctan\left(-\frac{D}{\sqrt{B^2 + C^2}}, \pm\sqrt{1 - \frac{D^2}{B^2 + C^2}}\right) \quad (21)$$

$$-\arctan\left(\frac{C}{\sqrt{B^2 + C^2}}, \frac{B}{\sqrt{B^2 + C^2}}\right)$$

(4) Solution for  $\theta_3$

From equation (17) and (18), the solution of  $\theta_3$  is obtained.

$$\theta_3 = \arctan((p_y + a_5 \cdot a_z \cdot o_x + a_4 \cdot o_x - a_2 \cdot \sin \theta_2), (p_x - a_4 \cdot o_y - a_5 \cdot a_z \cdot o_y - a_2 \cdot \cos \theta_2 - a_1 \cdot \cos \theta_1)) - \theta_2 \quad (22)$$

(5) Solution for  $\theta_4$

From (4) and (5), we can get

$$\frac{o_x}{o_y} = -\tan(\theta_2 + \theta_3 + \theta_4) = -\tan \theta_z, \text{ so the solution for } o_y$$

$\theta_4$  is obtained:

$$\theta_4 = \theta_z - \theta_2 - \theta_3 \quad (23)$$

The Inverse kinematics of the manipulator has two group solutions, the structure of solutions is as shown in figure 3.

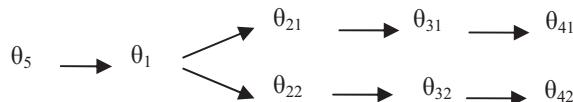


Fig 3 the structure of solutions for the inverse kinematics

In practice, according to the workspace of the manipulator and its current movement situation, we can forsake inappropriate solution, and get the right solution for inverse kinematics.

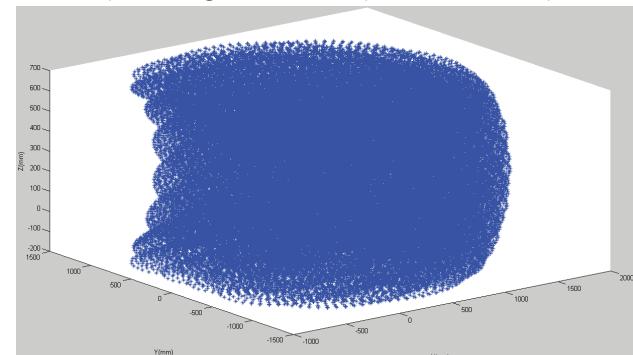
## 4 SIMULATION AND RESULTS

According to the kinematics model established in section 3, the simulation with MATLAB was conducted. Firstly, the workspace of the manipulator was simulated by the forward kinematics model. Secondly, the correctness of the inverse kinematics model was verified by data in the workspace. Finally, several typical operating cases were simulated, and the existing problems were analyzed.

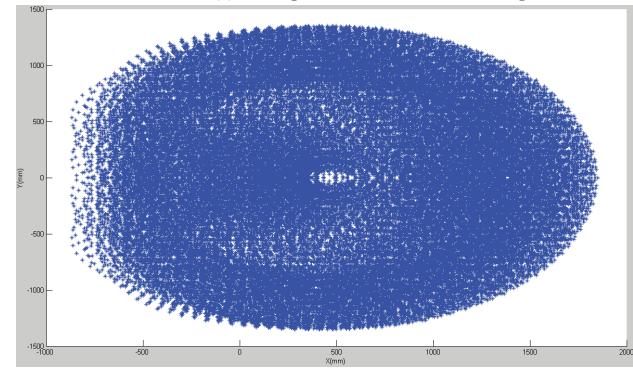
### 4.1 Workspace

There are many methods to solve the workspace, such as analytical method, graphical method, and numerical method. Because the manipulator in this paper is 5-DOF, it is difficult and not intuitive to solve the workspace by analytical method or graphical method. So the numerical method was used to solve the workspace. In the simulation, the step interval is 5 degree for each joint. The joint value was increased from its lower to upper limitation for each joint to form the workspace. Based on the forward kinematics model, the workspace of the manipulator was obtained, as shown in figure 4.

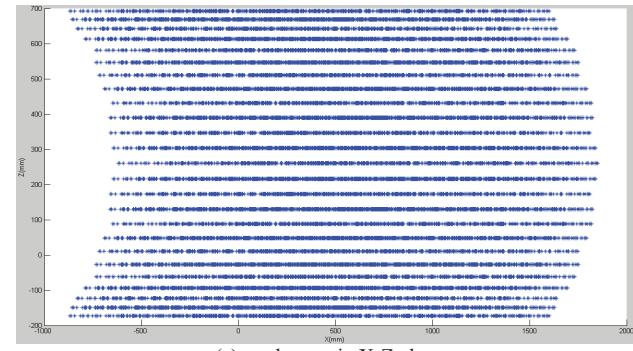
From figure 4, we can see that the workspace is a similar ellipsoid, and the end-effector can reach any point on the ellipsoid. In the workspace, the range of X axis is (-1000mm, 2000mm), the range of Y axis is (-1500mm, 1500mm), the range of Z axis is (-200mm, 700mm).



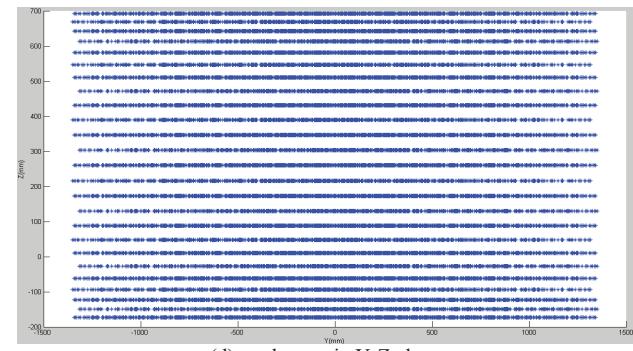
(a) workspace in three-dimensional space



(b) workspace in X-Y plane



(c) workspace in X-Z plane



(d) workspace in Y-Z plane

Fig 4 workspace of the manipulator

## 4.2 Typical Operating Cases

According to the requirements in practice, the rationality of tasks based on the above kinematics model was analyzed, and then guidance to operators was provided.

Operating case 1: some joints are fixed, and then the specified pose of the end-effector is obtained by adjusting the remaining joints.

According to the structure of the manipulator, we can see that, if some joints are fixed, some poses in the workspace cannot be reached. The more fixed joints, the more limited poses that can be reached, the more possible no solution. Based on the inverse kinematics model for the manipulator, we know that, for any pose in the workspace, there are only two group solutions. In fact, by the verification of MATLAB simulation, there is only one group appropriate solution in most case. For the specified pose, it is difficult for operators to manually adjust all the joints to the unique solution.

Operating case 2: move the end-effector to the installation location along a straight line, and adjust the posture of the end-effector to meet the installation requirements.

To meet the requirements in case 2, the end-effector can be adjusted in two manners. In the first manner, it can be moved to the installation location along a straight line, and then be adjusted to the installation posture. In the second manner, it can be adjusted to the installation posture, and then be moved to the installation location along a straight line trajectory. If the location is first adjusted, from the structure of the manipulator, we know that, the last joint can only rotate about Y axis, the remaining rotary freedom cannot be adjusted, so it is difficult to adjust the end-effector to the exact installation posture. If the posture is first adjusted, from the above kinematics analysis, we know that the manipulator can be moved along a straight line. However, there are at most two group solutions for each location in the workspace. In order to move the end-effector to the installation location, operators have to repeatedly test different joints values.

Based on the above analysis, a new joint is needed to provide more flexibility of the manipulator to meet the

requirements of installation tasks as case 1 and 2. In addition, electric control joint should be added for operators. For example, joint 2, 3, or 4 can be changed to electric control.

## 5 CONCLUSION AND FUTURE WORK

In this paper, the kinematics equation for the 5-DOF manipulator is established, and the forward and the inverse kinematics solutions are presented. The workspace is simulated with the kinematics model. In addition, by simulation and analysis, we point out the existing problems for the manipulator. In the future, in order to better meet the actual requirement, we plan to improve the structure of the manipulator. The next work is to simulate and analyze the new manipulator, and verify whether it can solve the existing problems.

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