

# An Inversion-free Fuzzy Predictive Control for Piezoelectric Actuators

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**Abstract:** Piezoelectric actuators (PEAs) are treated as the core component in the nano-positioning applications. The inherent hysteresis nonlinearity can dramatically degrade the tracking performance of PEAs. This paper presents an inversion-free fuzzy predictive controller of the parallel distributed structure. A Takagi-Sugeno (T-S) based fuzzy model of PEAs is developed first. With the aid of T-S based fuzzy model, explicit predictive control law can be obtained for each fuzzy rule. Then these predictive control laws are combined by fuzzy inference to generate the overall predictive controller. By the proposed method, the inverse hysteresis model of PEAs is no longer required. A notable feature of the proposed method is that the predictive control law can be obtained before the real-time control of PEAs. Therefore, the on-line computational burden is quite low, leading to a good tracking performance in the high frequency working conditions. Experiments are conducted on a commercial PEA to verify the proposed method. Experiment results show that the proposed method has a satisfactory tracking performance in both the low and high frequency conditions. Comparison results illustrate that the proposed method outperforms some existing approaches such as the inversion-based method and sliding mode control method.

**Key Words:** Piezoelectric actuators, hysteresis, fuzzy modeling, model predictive control.

## 1 INTRODUCTION

Recently, nano-technology has been drawn considerable attention in the nano-positioning. As a core component, piezoelectric actuators (PEAs) have been widely used in many applications, such as the micromanipulator [1], atomic force microscope [2], and ultra precision mechanism [3]. However, the undesired inherent hysteresis nonlinearity can greatly deteriorate the tracking performance of PEAs. Hysteresis is a kind of memory effect where the displacement of PEAs is affected by its historical values. Additionally, hysteresis nonlinearity is relevant to the frequency of input signal of PEAs (called *rate-dependent* property). The conventional control methods may have some difficulties in dealing with these issues. How to handle these challenges becomes an attractive task.

In the literature, inversion-based control method is commonly adopted in the high-precision control of PEAs. The model of PEAs is usually assumed to be divided into the hysteresis submodel and the dynamic submodel [4]. Among the inversion-based methods, the hysteresis submodel is usually built by Preisach model and Prandtl-Ishlinskii model. Based on these hysteresis submodels, the models' inversions are calculated to compensate the hysteresis nonlinearity [5]-[6]. Then the controller is only designed for the dynamic submodel to obtain desired performances

of PEAs. To improve the accuracy of the inverse hysteresis submodel, the iteration learning algorithm and adaptive method are introduced in the inversion-based approach [7]-[8]. Furthermore, there are some results differed from the above compensation procedure. In [9], the inverse compensation is realized by using the hysteresis model directly, which is based on the multiplicative-inverse structure. In [10], the inverse hysteresis model is directly described by the Prandtl-Ishlinskii model. This approach is based on the direct inverse hysteresis compensation concept [11]. However, the inversion-based method only has a good performance in the low frequency working conditions of PEAs. Meanwhile, the inversion-based method cannot deal with the modeling error caused by the extend disturbances.

To overcome the disadvantages of inversion-based approaches, the inversion-free method is introduced in the control of PEAs. In the inversion-free method, hysteresis nonlinearity has been seen as a bounded disturbance, then the PEA can be treated as a system affected by disturbances. Therefore, the PID-type control algorithms (such as active disturbance rejection control [12]) are used in the inversion-free method. Due to the low gain margin property of PEAs, the PID-type control algorithms are commonly applied in the low frequency working conditions of PEAs. To improve the performance of inversion-free method, the sliding mode control (SMC) is adopted and it shows a good performance to attenuate disturbances [13]-[14]. However, the SMC algorithms may involve the infinite gain problem without knowing the variation range of model's parameters. To deal with this issue, some adaptive algorithms are introduced to estimate the model parameters of PEAs, resulting

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in robust adaptive controllers [15]-[16]. Besides, some other inversion-free algorithms are also given in the literature [17]-[18], which are mainly verified by simulations.

Model predictive control is a promising approach in the control of PEAs due to its good robustness and disturbance rejection properties [19]. MPC has been combined with the inversion-based method and the inversion-free method, and shows good tracking performances. In [20], the Duhem hysteresis model is adopted to compensate the hysteresis nonlinearity, then the MPC is designed for the dynamic submodel of PEAs. In [21], MPC is used to alleviate the inherent chattering problem in SMC. In [22], a nonlinear model predictive controller which is obtained by solving nonlinear optimization problem is designed based on a neural network model of PEAs. In [23]-[24], the on-line linearization technique is used to linearize the neural network based model of PEAs during the control procedure, and an explicit MPC law is obtained in each sampling interval by solving a quadratic programming problem.

This paper presents an inversion-free fuzzy predictive control for PEAs. The great approximation ability of fuzzy approach has been reported in the control of PEAs [25]-[26]. In this paper, the Takagi-Sugeno (T-S) based fuzzy model is used to approximate the dynamics of PEAs. With this T-S based fuzzy model, the MPC follows a parallel distributed control structure [27], which leads to an explicit control law for each fuzzy rule. Then in each sampling interval, these predictive control laws are combined to obtain the overall control law by fuzzy inference procedure. Since the inversion hysteresis model is not required, the drawbacks of the inversion-based method can be avoided. Although neural network based PEAs' model is first determined off-line, some on-line linearization procedure which costs extra computation resources is still required in our previous work [22]-[24]. A notable feature of the inversion-free fuzzy predictive control is that the design of predictive control law can be accomplished in a fully off-line way. The only on-line computation requirement is the fuzzy inference which can be calculated in a fast manner. Therefore, the inversion-free fuzzy predictive control can be applied in the high frequency working conditions of PEAs. The experiments are conducted on a commercial PEA (P-753, Physik Instrumente). The T-S based fuzzy model is verified first. Then the tracking performance of inversion-free fuzzy predictive control are shown in the experiments. The comparison with existing methods in the literature are also given.

## 2 A T-S based Fuzzy Model of PEAs

In this paper, the model of PEAs is constructed in an integrated way, without dividing it into the hysteresis submodel and the dynamic submodel. It is reasonable to use the “nonlinear auto-regressive moving average with exogenous inputs” (NARMAX) model structure to approximate the dynamics of PEAs, since the displacements of PEAs are effected by its historical values. The structure of the NARMAX model is

$$y(t) = \mathcal{F}(\mathbf{x}(t)), \quad (1)$$

where  $\mathbf{x}(t) = [y(t-1), \dots, y(t-n_y), u(t), \dots, u(t-n_u)] \in \mathbb{R}^p$ ,  $y(t)$  and  $u(t)$  are the displacement and input voltage of PEAs, respectively. Integers  $n_y$  and  $n_u$  are the corresponding maximum lags for  $y(t)$  and  $u(t)$ . In this paper, the T-S based fuzzy approach is used to approximate the nonlinear function  $\mathcal{F}(\cdot)$ .

### 2.1 T-S Based Fuzzy Modeling of PEAs

Based on the model structure in (1), the T-S based fuzzy model can be expressed as follows:

$$\begin{aligned} R_i : & \text{If } \mathbf{x}(t) \text{ is } A_i \\ & \text{then } y_i = f_i(\mathbf{x}(t)), i = 1, 2, \dots, K, \end{aligned} \quad (2)$$

where  $R_i$  denotes the  $i$ th rule and  $K$  is the number of rules;  $\mathbf{x}(t)$  defined in (1) is used as the antecedent variable and  $y_i$  is the consequent variable.  $A_i$  is the antecedent fuzzy set of the  $i$ th rule.

The antecedent proposition “ $\mathbf{x}(t)$  is  $A_i$ ” is always expressed as a logical combination of simple propositions with univariate fuzzy sets. The univariate fuzzy sets are constructed for the individual components of  $\mathbf{x}(t)$ , which are denoted by  $x_j$ . Then the antecedent part of the fuzzy model in (2) can be modified in the conjunctive form:

$$\begin{aligned} R_i : & \text{If } x_1 \text{ is } A_{i1} \text{ and } x_2 \text{ is } A_{i2} \text{ and } \dots \text{ and } x_p \text{ is } A_{ip} \\ & \text{then } y_i = f_i(\mathbf{x}(t)), i = 1, 2, \dots, K. \end{aligned} \quad (3)$$

The structure of consequent functions  $f_i(\mathbf{x}(t))$  remains equal in all rules. According to the T-S fuzzy modeling principle and (1),  $f_i(\mathbf{x}(t))$  are chosen as

$$y_i(t) = f_i(\mathbf{x}(t)) = \sum_{j=1}^{n_y} -a_{ij}y(t-j) + \sum_{j=0}^{n_u} b_{ij}u(t-j) + \zeta_i, \quad (4)$$

where  $a_{ij}$  and  $b_{ij}$  are constant parameters,  $\zeta_i$  is a bias term in the consequent functions. Since  $f_i(\cdot)$  contains the terms of the historical values of  $u(t)$ , it is reasonable to assumed that  $f_i(\cdot)$  includes the proportional terms of  $(u(t) - u(t-1))$ ,  $(u(t-1) - u(t-2))$ ,  $\dots$ ,  $(u(t+n_u) - u(t-n_u))$ . Therefore, the *rate-dependent* property is inherently implemented in  $f_i(\cdot)$ .

To obtain the real output of the T-S based fuzzy model, the degree of fulfillment  $\beta_i(\mathbf{x}(t))$  of the antecedent should be calculated first. Since the antecedent of (3) is a logic combination of  $x_j$ ,  $\beta_i(\mathbf{x}(t))$  is calculated as a combination of  $\mu_{A_{ij}}(x_j)$ . That is

$$\beta_i = \mu_{A_{i1}}(x_1) \wedge \mu_{A_{i2}}(x_2) \wedge \dots \wedge \mu_{A_{ip}}(x_p), \quad (5)$$

where  $\mu_{A_{ij}}(\cdot)$  is the membership function of  $A_{ij}$ . Then the real output of the T-S based fuzzy model can be expressed by the following defuzzification formula

$$y = \frac{\sum_{i=1}^K \beta_i(\mathbf{x}) y_i}{\sum_{i=1}^K \beta_i(\mathbf{x})} = \sum_{i=1}^K \tilde{\beta}_i(\mathbf{x}) y_i. \quad (6)$$

To obtain the T-S based fuzzy rules in (3), clustering the experimental data for model identification is the first step. In this paper, the Gustafson-Kessel algorithm is adopted to

cluster the experimental data and calculate the fuzzy partition matrix. Then the antecedent fuzzy sets  $A_{ij}$  can be extracted from the fuzzy partition matrix. Within each cluster, the parameters in the corresponding local model (4) can be identified by the least-square algorithm. Details about the identification can be found in [28].

### 3 Inversion-Free Fuzzy Predictive Control

The inversion-free fuzzy predictive control is implemented with the T-S based fuzzy model of PEAs. It can be seen that the T-S based fuzzy model could be used directly to design the model predictive control law, resulting in a nonlinear predictive law. However, the nonlinear predictive controller has to solve a complicated nonlinear optimization problem during each sampling interval. It is a great computational burden when the PEAs are working at a high frequency condition. To overcome these difficulties, the inversion-free fuzzy predictive control proposed in this paper is designed in the parallel distributed control structure. The predictive control law are designed for each rule in the T-S based fuzzy model first. After that, a fuzzy inference module is introduced to obtain the overall controller.

#### 3.1 The predictive control law of each rule

The local linear model (4) is used as the displacement predictor of PEAs. For convenience, we temporarily drop the rule index  $i$  in (4). Then the  $-a_{ij}$  and  $b_{ij}$  are changed to  $-a_j$  and  $b_j$ , respectively. To eliminate the constant bias term  $\zeta_i$ , the adjacent differential form of (4) is employed, leading to the displacement predictor as follows:

$$\begin{aligned}\hat{y}(t+N) = & (1-a_1)\hat{y}(t+N-1) + (a_1-a_2)\hat{y}(t+N-2) \\ & + \cdots + a_{n_y}\hat{y}(t+N-n_y-1) + b_0\Delta u(t+N) \\ & + \cdots + b_{n_u}\Delta u(t+N-n_u),\end{aligned}\quad (7)$$

where  $\hat{y}(t)$  is the predicted displacement of PEAs. Define

$$\begin{aligned}G' &= \begin{bmatrix} b_0 & 0 & 0 & \cdots & 0 & \cdots & \cdots & 0 \\ b_1 & b_0 & 0 & \cdots & 0 & \cdots & \cdots & 0 \\ b_2 & b_1 & b_0 & \cdots & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \ddots & \vdots \\ b_{n_u} & b_{n_u-1} & b_{n_u-2} & \cdots & b_0 & \cdots & \cdots & 0 \\ 0 & b_{n_u} & b_{n_u-1} & \cdots & b_1 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & b_{n_u-1} & \cdots & \cdots & b_0 \\ b_1 & b_2 & \cdots & b_{n_u} \\ b_2 & b_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ b_{n_u} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix} \in \Re^{N \times N}, \\ H' &= \begin{bmatrix} 1-a_1 & a_1-a_2 & a_2-a_3 & \cdots & a_{n_y} \\ a_1-a_2 & a_2-a_3 & a_3-a_4 & \cdots & 0 \\ a_2-a_3 & a_3-a_4 & a_4-a_5 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ a_{n_y-1}-a_{n_y} & a_{n_y} & 0 & \cdots & 0 \\ a_{n_y} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \in \Re^{N \times n_u}, \\ S' &= \begin{bmatrix} a_{n_y-1}-a_{n_y} & a_{n_y} & 0 & \cdots & 0 \\ a_{n_y} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \in \Re^{N \times (n_y+1)},\end{aligned}$$

and  $T$  (in the top of next page). Since the entries in the main diagonal of  $T$  are all one,  $T$  is invertible. Then from the first step to the  $N$ -th step, the displacement predictor is

given as follows:

$$\begin{aligned}\begin{bmatrix} \hat{y}(t+1) \\ \hat{y}(t+2) \\ \vdots \\ \hat{y}(t+N) \end{bmatrix} &= G \begin{bmatrix} \Delta u(t+1) \\ \Delta u(t+2) \\ \vdots \\ \Delta u(t+N) \end{bmatrix} \\ &+ H \begin{bmatrix} \Delta u(t) \\ \Delta u(t-1) \\ \vdots \\ \Delta u(t-n_u+1) \end{bmatrix} + S \begin{bmatrix} y(t) \\ y(t-1) \\ \vdots \\ y(t-n_y) \end{bmatrix},\end{aligned}\quad (8)$$

where  $G = T^{-1}G'$ ,  $H = T^{-1}H'$ , and  $S = T^{-1}S'$  are constant matrices. For convenience, (8) can be written in a compact form:

$$\hat{Y}(t) = G\Delta U(t) + H\Delta U'(t) + SY'(t),\quad (9)$$

where  $\Delta U(t) = [\Delta u(t+1), \Delta u(t+2), \dots, \Delta u(t+N)]^T$ ,  $\Delta U'(t) = [\Delta u(t), \Delta u(t-1), \dots, \Delta u(t-n_u+1)]^T$ , and  $Y'(t) = [y(t), y(t-1), \dots, y(t-n_y)]^T$ . Here the control horizon is equal to the predictive horizon  $N$ .

After obtaining the displacement predictor, a performance index is designed to obtain the predictive control law. The major purpose of the tracking control is to minimize the error between the real displacement of PEAs and the desired displacement, which results in the following index:

$$J = [R(t) - \hat{Y}(t)]^T[R(t) - \hat{Y}(t)] + \rho\Delta U^T(t)\Delta U(t),\quad (10)$$

where  $R(t) = [r(t), \dots, r(t+N)]^T$  denotes the desired displacement signal of PEAs. Parameter  $\rho > 0$  is a penalty term to limit  $\Delta U(t)$ . Since (10) is a convex quadratic programming problem, the optimal solution can be obtained by solving the following equation

$$\frac{\partial J}{\partial \Delta U(t)} = 0.\quad (11)$$

This results in

$$\Delta U(t) = (G^T G + \rho I)^{-1} G^T (R(t) - H\Delta U'(t) - SY'(t)).\quad (12)$$

The first entry of  $\Delta U(t)$  is used as the control increment for the next sampling interval. Then we add the subscript  $i$  in  $\Delta u(t+1)$ , leading to  $\Delta u_i(t+1)$  to denote the control predictive control law for each fuzzy rule of the proposed model.

#### 3.2 The overall control law by fuzzy inference

After obtaining the predictive control law, a fuzzy inference module is used to generate the overall control law. Since the predictive control laws is based on the rules of T-S based fuzzy model, the overall control law is obtained similarly like the defuzzification formula (6). That is

$$\Delta u_f(t+1) = \sum_{i=1}^K \tilde{\beta}_i(\mathbf{x}) \Delta u_i(t+1).\quad (13)$$

Then the control output is determined by the following equation

$$u(t+1) = u(t) + \Delta u_f(t+1).\quad (14)$$

The fuzzy predictive control scheme is shown in Fig. 1.

$$T = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ a_1 - 1 & 1 & 0 & \cdots & 0 & \cdots & 0 \\ a_2 - a_1 & a_1 - 1 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{n_y} - a_{n_y} - 1 & a_{n_y} - 1 - a_{n_y} - 2 & a_{n_y} - 2 - a_{n_y} - 3 & \cdots & \cdots & \cdots & 0 \\ -a_{n_y} & a_{n_y} - a_{n_y} - 1 & a_{n_y} - 1 - a_{n_y} - 2 & \cdots & \cdots & \cdots & 0 \\ 0 & -a_{n_y} & a_{n_y} - a_{n_y} - 1 & \cdots & \cdots & a_{1-1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_{n_y} - 1 - a_{n_y} - 2 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{N \times N},$$

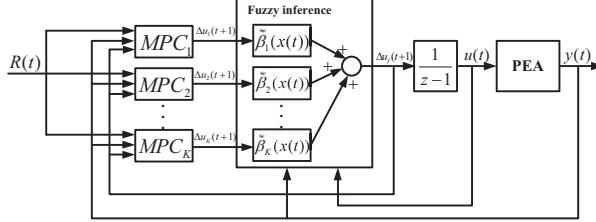


Figure 1: The structure of the fuzzy predictive control.

## 4 Experiments and Discussions

To verify the proposed model and the control scheme, experiments are conducted on a commercial PEA (P-753, Physik Instrumente, Karlsruhe, Germany). The horizontal movement of P-753 is up to  $15 \mu\text{m}$ , and it has a built-in capacitive sensor to measure its displacement. The host computer and the amplifier of P-753 are wired via a data acquisition board (PCI-1716, Advantech, Beijing, China). The proposed model and control scheme are implemented in MATLAB/SIMULINK. In the following experiments, the sampling time is set to be 0.05 ms.

### 4.1 Verification of T-S based fuzzy model

To obtain the T-S based fuzzy model, some basic parameters should be chosen first. The number of rules in the proposed model is set to be 3. According to [29], the structure of each local model (4) is chosen that  $n_y = 2$  and  $n_u = 1$ . With this model structure, a mixed sinusoid excited signal is used to drive the PEA. The amplitude of this input signal is 90 V, and the frequency is between 1 Hz to 100 Hz. Then the displacement of PEA is measured. The clustering algorithm, Gustafson-Kessel, is adopted to obtain the optimal  $A_{ip}$  of each rule. With the clustered data, the parameters  $a_{ij}$  and  $b_{ij}$  are obtained by the least-square algorithm.

The fitting ability of the T-S based fuzzy model is shown in Fig. 2. The T-S based fuzzy model has a good ability to approximate the behavior of PEA, under the input signal with different amplitude and frequencies. This suggests that the *rate-dependent* property is inherent implemented by the proposed model.

### 4.2 Verification of fuzzy predictive control

The main parameter of the fuzzy predictive control law is the predictive horizon  $N$  and  $\rho$  in (10), and they are set to be 7 and 80, respectively, in the following experiments. To verify the tracking performance in the low frequency working conditions of PEAs, the sinusoid signal with frequencies of 1Hz and 5Hz are used as the references of the PEA. Experiments results are illustrated in Fig. 3. It can be seen

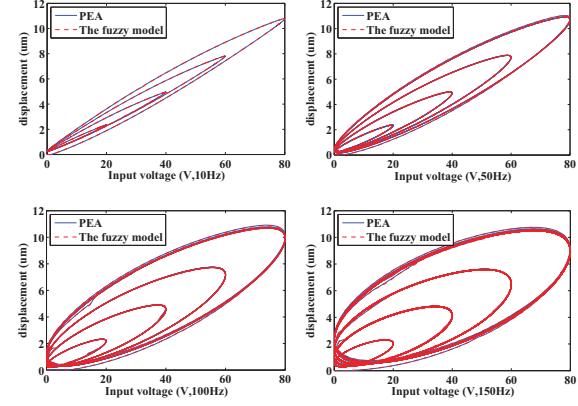


Figure 2: The displacement of the proposed model and PEA under varied sinusoid input signal.

that the proposed method has a satisfactory tracking performance in the low frequency case. Besides, two mixed signals is used to verify the tracking performance. Each mixed signal is combined by four different sinusoid signals. The experimental results are shown in Fig. 4. Under the references with varied amplitudes, the proposed method can also have a good tracking performance. It suggests that the proposed method is an effective control scheme to deal with varied references in the low frequency working conditions.

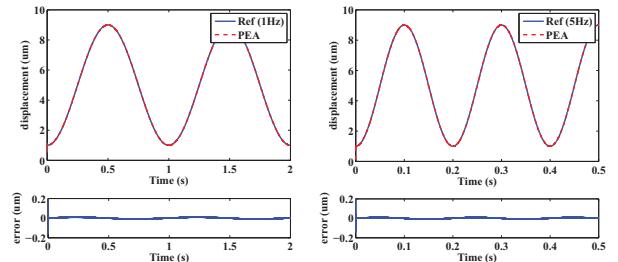


Figure 3: The tracking performance of the proposed method in the low frequency working conditions.

To verify the control performance of proposed method in the high frequency working conditions, three high frequency sinusoid signals are adopted as the references. The frequency of these signals are 100Hz and 200Hz. The experimental results are given in Fig. 5. Notably, the proposed method gives a satisfactory tracking performance. Although the references' frequency is increased, the tracking error still keeps in an acceptable range. The experi-

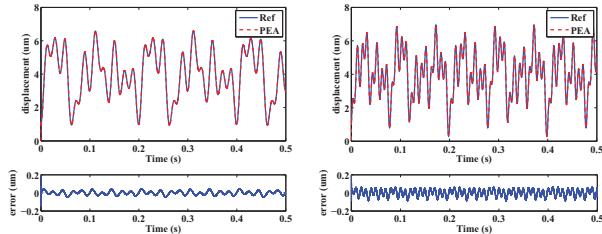


Figure 4: The displacement of PEAs under varied references.

ments results suggest that the proposed method could deal with the tracking problem in the high frequency case.

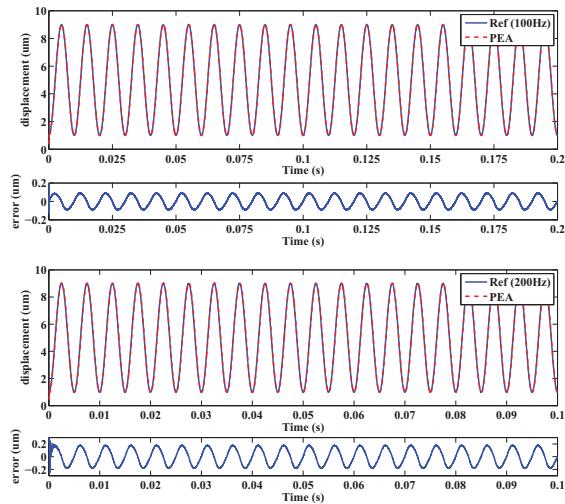


Figure 5: The tracking performance of the proposed method in the high frequency working conditions.

To compare with other inversion-free methods and inversion-based methods, some experiments are also conducted. The inversion-free PID-based SMC method in [14] is used first. For this comparison experiment, the reference signal is defined as  $r(t) = 5\sin(2\pi ft - \frac{\pi}{2}) + 5$ , where  $f$  varies from 1Hz to 150Hz. The root mean square (RMS) of the tracking error is used as the performance index, and the experimental results are given in Table 1. It can be seen that the proposed method has a better tracking effect than the inversion-free PID-based SMC method. To compare with the inversion-based method, the inversion-based MPC approach in [20] is used in the experiment. A reference signal called SW in [20] is set as  $SW = 0.5(\frac{30}{7}\sin[\frac{2\pi f_{max}}{20}(t-0.1)+\pi]+\frac{25}{7}\sin[\frac{2\pi f_{max}}{5}(t-0.1)+0.5\pi]+\frac{5}{7}\sin[\frac{2\pi f_{max}}{2}(t-0.1)+0.2\pi]+\frac{5}{7}\sin[2\pi f_{max}(t-0.1)]) + 5$ , where  $f_{max}$  is the maximum frequency. The comparison experiments results are given in Table 1. Since the inverse hysteresis model is avoided, the proposed method has a better tracking performance.

## 5 Conclusions and Future works

In this paper, an inversion-free fuzzy predictive control is proposed. A T-S based fuzzy model of PEAs is obtained

Table 1: The comparison experiments between the inversion-free fuzzy predictive control and the methods in [14] or [20]: the root mean square (RMS) tracking error.

	References	the proposed method	method in [14] or [20]
Sinusoid references from [14]	$f = 1 \text{ Hz}$	$0.0062 \mu\text{m}$	$0.008 \mu\text{m}$
	$f = 5 \text{ Hz}$	$0.0063 \mu\text{m}$	$0.012 \mu\text{m}$
	$f = 10 \text{ Hz}$	$0.0125 \mu\text{m}$	$0.018 \mu\text{m}$
	$f = 50 \text{ Hz}$	$0.0316 \mu\text{m}$	$0.051 \mu\text{m}$
	$f = 100 \text{ Hz}$	$0.0631 \mu\text{m}$	$0.086 \mu\text{m}$
	$f = 150 \text{ Hz}$	$0.0943 \mu\text{m}$	$0.138 \mu\text{m}$
SW in [20]	$f_{max}=10\text{Hz}$	$0.0051 \mu\text{m}$	$0.0091 \mu\text{m}$
	$f_{max}=50\text{Hz}$	$0.0184 \mu\text{m}$	$0.0250 \mu\text{m}$

first. After that, the predictive control law can be calculated for each fuzzy rule of the T-S based fuzzy model. Notably, the predictive control law is obtained before the real-time control of PEAs. Then, a fuzzy inference module is used to obtain the overall controller by combining all predictive control laws. Comparison experiments results suggest that the proposed method has a better tracking performance than some existing methods in the literature.

Another way to obtain an explicit control law is to use the on-line linearization algorithm to linearize the T-S based fuzzy model in each sampling interval. This method may be an alternative to the proposed method in this paper. Besides, the physical constraints on the control input and output of PEAs should be considered. These constraints are significantly practical requirements in the control of PEAs. In MPC, the constrained tracking problem is transformed into an optimization problem. And the methods in [30]-[34] can be used to solve this optimization problem effectively. Furthermore, the uncertainties and disturbances of the control system are not considered in this paper. To deal with this case, the methods in [35]-[37] can be introduced to design the on-line adaptive controller for tracking control of PEAs. The authors will focus on these ideas in near future.

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