Abstract—Piezoelectric Actuators (PEAs) are the key components in nano-positioning. However, the inherent hysteresis nonlinearity of PEAs is seriously affected the control precision. In this paper, an active disturbance rejection controller is proposed to deal with the tracking control of PEAs. First, the hysteresis nonlinearity is reformulated as a disturbance of the closed-loop system. With this idea, a disturbance-based model is derived from the comprehensive model of PEAs. Then the so-called extended state observer is introduced to real-time estimate the hysteresis nonlinearity. With the extended state observer, the model of hysteresis or its inversion are all no longer needed. To verify the performance of the proposed method, some experiments are conducted on a commercial PEA. And the experiment results show that the proposed controller is an effective way to deal with the tracking control of PEAs.

I. INTRODUCTION

Nano-positioning has been to be the indispensable technology in modern precision manufacturing. Piezoelectric Actuators (PEAs), which has the advantages of fast response and high stiffness, are the key components in the nano-positioning applications. For instance, the PEAs are assembled to be the probe in the scanning probe microscopes [1]. In the optical disk, the PEAs are used as the manipulator to operate the disks [2]. Besides, PEAs also can be found in the applications of ultra-precision mechanisms [3]. However, the inherent hysteresis nonlinearity can seriously degrade the positioning precision of PEAs. Hysteresis nonlinearity means the current displacement of PEAs is affected by its past displacements [4]. Therefore, how to deal with the hysteresis nonlinearity is a vital and challenging topic in control of PEAs.

In the literature, hysteresis nonlinearity is usually compensated by its inverse model to result in a linear model of PEAs. This method is the so-called inversion-based method. To obtain the inverse model of hysteresis, the model of hysteresis itself is needed first, which is usually modeled by Preisach model or Prandtl-Ishlinskii (PI) model. In [5], the classical Preisach model is studied to describe the hysteresis in piezoceramic actuators. In [6], the modified PI model is used to model the asymmetry hysteresis nonlinearity, and the inverse hysteresis compensator is obtained to cancel the hysteresis. In [7], the model predictive control (MPC) method is designed for the linear model, where the hysteresis has been compensated by its inversion. Notably, the precision of inverse hysteresis model is vital to the inversion-based method. Therefore, the iterative learning control is introduced to improve the inversion’s precision, such as in [8]. Meanwhile, the adaptive identification method is also adopted to improve the compensation performance [9], [10]. Besides, the intelligent modeling method, such as the neural-network [11] and fuzzy method [12], are also adopted to compensate the hysteresis. Furthermore, some studies are focusing on the direct compensation of hysteresis [13], where the hysteresis model is not a necessity.

In the inversion-based method, the calculation of the inverse hysteresis model is always a burden in the control of PEAs. To avoid the calculation of the inverse hysteresis model, the intelligent control method, which are most belonged to the so-called inversion-free method (inversion-free means the inverse hysteresis model is no longer needed), are drawing considerable attention due to its satisfactory performance in the control of PEAs. In [14], the neural-network is adopted to model the PEA. With this model, the nonlinear MPC controller is designed and verified in experiments. Based on the study in [14], a MPC controller with a dynamic linearized neural-network model is proposed in [15], [16], [17], which is suitable for the high-speed control of PEAs. In [18], MPC is introduced to reduce the hysteresis nonlinearity in the atomic force microscope. Since the great approximation ability of fuzzy method, the fuzzy modeling and control method are also studied in the literature [19]. In [20], an adaptive fuzzy control method is investigated to realize the tracking control of PEAs. In [21], a T-S fuzzy model based predictive controller is proposed and verified by experiments. Furthermore, in [22], the adaptive MPC method is introduced to enhance the performance of the proposed controller in [21]. Although the inversion-free method do not require the inversion of hysteresis, the model of PEA is still a necessity for it.

The active disturbance rejection control (ADRC) theory [24] has been drawing considerable attention in recent years, and it shows promising performance in nano-positioning applications [23]. With the ADRC method, there are some studies focusing on the hysteresis compensation. In [25], [26], the hysteresis nonlinearity is regarded as a general disturbance for the plant. And some theoretical results are given in these studies. However, there are only simulations to verify the ADRC method to deal with hysteresis. In [27], the ADRC method is used to operate a PEA-driven nanopositioner. And the experiment results show that the ADRC method is an effective way for the precision positioning. However, only the setpoint control is verified.
This paper proposes an active disturbance rejection controller. First, based on the comprehensive model of PEAs, a disturbance-based model is illustrated. With this model structure, the hysteresis nonlinearity is transferred to a disturbance of the closed-loop system. This idea is distinct from the common view of hysteresis in the literature. Then the so-called extended state observer (ESO) is introduced to estimate the disturbance in an on-line way. Therefore, the modeling of hysteresis or its inverse model are all no needed for the controller’s design. After that, with the estimation from ESO, the closed-loop system can be compensated to be a double integral equivalent system. Then this equivalent system can be easily controlled by the PID-type controllers. Meanwhile, the so-called tracking differentiator is also introduced in the proposed method, which is used to improve the performance of the transient response. To verify the proposed method, experiments are conducted on a commercial PEA. Both the setpoint control and the tracking control are investigated, and the experiment results suggest that the proposed method is an effective control method to deal with the positioning of PEAs.

The rest of this paper is organized as follows: Section II gives a new model structure of PEAs, which is reformulated from the comprehensive model of PEAs; Based on this new model structure, an active disturbance rejection controller is designed in Section III; To verify the proposed method, some experiments are conducted on a commercial PEA in Section IV; Finally, this paper is concluded in Section V.

II. FORMULATIONS OF THE PEA’S DISTURBANCE-BASED MODEL

In practical applications, the PEAs are usually consisted by piezoelectric materials and a positioning mechanism. If the mass of the positioning mechanism is much larger than the piezoelectric materials, the mechanical dynamics of PEA can be approximate as a second-order system. Then a comprehensive dynamic model of PEAs can be obtained, as shown in Fig. 1. And this comprehensive model can be expressed in the following [1]:

\[ R_0 q'(t) + v_h(t) + v_A(t) = k_{amp} v_{in}(t), \]

\[ v_h(t) = H(q), \]

\[ q(t) = q_e(t) + q_p(t), \]

\[ v_A(t) = q_e(t)/C_A, \]

\[ q_p(t) = T_{em} x(t), \]

\[ F_A = T_{em} v_A(t), \]

\[ m \ddot{x}(t) + b_s \dot{x}(t) + k_s x(t) = F_A, \]

where

\[ R_0 \] is the equivalent resistance of the driven circuit;

\[ q \] and \[ \dot{q} \] are the total charge and the charge flowing in the PEA;

\[ v_h \] is the voltage which is generated by hysteresis nonlinearity;

\[ v_A \] is transduced voltage;

\[ v_{in}(t) \] is the control input to the voltage amplifier;

\[ k_{amp} \] is the amplification ratio of the voltage amplifier;

\[ q_e \] is the charge in the capacitance \( C_A \);

\[ q_p \] is the transduced charge due to the piezoelectric effect;

\( C_A \) is the total capacitance of the PEA;

\( T_{em} \) represents the piezoelectric effect as a mechanical transducer;

\( F_A \) is the transduced force;

\( x \) is the displacement of the positioning mechanism;

\( m, b_s, \) and \( k_s \) are the equivalent mass, damping coefficient, and stiffness of the positioning mechanism, respectively.

Since the charge control of PEAs is hard to realize in real applications, the voltage control is the main control method of PEAs. With the comprehensive dynamic model (1), the voltage control form of (1) can be written as follows:

\[ R_0 C_A \dot{q}(t) + q(t) - T_{em} x(t) = C_A k_{amp} \left[ v_{in}(t) - \frac{H(q)}{k_{amp}} \right], \]

\[ m \ddot{x}(t) + b_s \dot{x}(t) + (k_s + \frac{T_{em}^2}{C_A}) x(t) = \frac{T_{em}}{C_A} q(t). \]

In the literature, if the external loads are not considered, it is reasonable to assume that \( R_0 = 0 \) for PEAs [1]. Then the comprehensive dynamic model is reduced to

\[ m \ddot{x}(t) + b_s \dot{x}(t) + k_s x(t) = T_{em} [k_{amp} v_{in}(t) - H(q)]. \]

Based on this model (3), there are two common model paradigms [29] for modeling and control of PEAs: the cascade model and the parallel model. The cascade model assumes that the term \( \left[ k_{amp} v_{in}(t) - H(q) \right] \) is a new hysteresis term \( H(v_{in}(t)) \), which is to govern the transduced force. Then the model of PEAs can be represented as a linear second-order system cascaded by a hysteresis submodel. The parallel model is directly derived from (3), and \( H(q) \) is assumed that it is affected by the output of PEAs due to the inverse piezoelectric effect \( q_p \) in the transducer). These two models are shown in Fig. 2, and \( u(t) = k_{amp} v_{in}(t) \).

In the cascade model, the hysteresis nonlinearity can be seen as the induction between the input voltage and the transduced force of PEA. That means the hysteresis nonlinearity deteriorates the linear relationship between the input voltage and the transduced force, i.e., the hysteresis is
a disturbance between the input voltage and the transduced force. In the parallel model, the hysteresis property can be regarded as a disturbance from the output of PEA to the input voltage. Moreover, based on the ADRC theory, we can further reduce the model into an equivalent double integral system with a general disturbance. This general disturbance is combined not only the hysteresis nonlinearity, but also the other dynamics from the mechanical part. Based on these ideas, the comprehensive model structure of PEAs can be reformulated into a general disturbance-based model structure, which is given in (4). And the schematic of this model is shown in Fig. 3.

\[
\ddot{x}(t) = f(\cdot) + bu(t),
\]

where \( f(\cdot) \) is the general disturbance, and \( b \) is a coefficient to adjust the control effort.

With this disturbance-based model structure, the model of hysteresis or its inversion are all no needed. And if we can real-time estimate the disturbance term \( f(\cdot) \), the system can be compensated to be a double integral system, which can be easily controlled by PID-type controllers.

### III. An Active Disturbance Rejection Controller for PEA

The proposed active disturbance rejection controller is combined by three parts: the extended state observer (ESO), the tracking differentiator (TD), and the control law. First, the ESO is used to real-time estimate the hysteresis nonlinearity and other dynamics. Then the TD is adopted to arrange a transient phase of the desired trajectory (namely transient reference). Finally, with the estimation of ESO, the control law is designed to achieve the compensation of hysteresis and the control of PEAs. The schematic of the proposed method is shown in Fig. 4.

#### A. The extended state observer

In Section II, the hysteresis nonlinearity and other dynamics are reformulated as a general disturbance \( f(\cdot) \) of the closed-loop system. In the active disturbance rejection control theory, the so-called extended state observer (ESO) [24] is used as an estimator to real-time obtain the general disturbance. However, the original ESO is usually a nonlinear one, which has some limitations in practical applications. Therefore, the linear extended state observer (LESO) [28] is adopted due to its ability to avoid chattering phenomenon in the original ESO.

In the LESO, the general disturbance is regarded as an extended state variable of the system. Then the state space model of PEAs (4) can be rewritten as follows:

\[
\begin{align*}
\dot{x} &= Ax + Bu + Sh \\
y &= Cx,
\end{align*}
\]

where \( A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \), \( B = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} \), \( S = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \). The state variables are defined as \( x = [x_1, x_2, x_3]^T = [x, \dot{x}, f]^T \). Then for this model, the extended state variables can be estimated by the state observer in the following:

\[
\begin{align*}
\dot{z} &= Az + Bu + L(y - \hat{y}) \\
\hat{y} &= Cz,
\end{align*}
\]

where \( L = [\beta_1, \beta_2, \beta_3]^T \) is the gain vector of the observer.

Once the general disturbance \( f(\cdot) \) is obtained, the closed-loop system (4) can be real-time compensated to an equivalent double integral system.

#### B. The tracking differentiator

In the design of conventional control method, the reference signal is usually used directly. However, for a step reference, this case will lead to some problems such as the setpoint jump (the reference changes abruptly due to the initial large

![Fig. 2. The paradigms of the model structure: the cascade model and the parallel model.](image1)

![Fig. 3. The disturbance-based model structure.](image2)

![Fig. 4. The schematic of the proposed method.](image3)
error). To avoid the setpoint jump problem, it is necessary to introduce a transient reference. This transient reference is a new desired trajectory based on the original reference signal. With the transient reference, the transient response of PEA can have a better performance.

In this paper, the so-called tracking differentiator [24] is adopted to obtain the transient reference. The tracking differentiator can arrange a transient reference, whose convergence speed to the original reference is adjustable. The tracking differentiator can be written as:

\[
\dot{g}_1 = g_2 \\
\dot{g}_2 = fhan(g_1 - v(t), g_2, r, h),
\]

where \(r\) and \(h\) are used to adjust the acceleration of tracking the signal \(v(t)\), and the function \(fhan(e_1, e_2, r, h)\) is defined as

\[
\begin{align*}
d &= rh, \\
d_0 &= hd, \\
d_1 &= e_1 + he_2, \\
a_0 &= \sqrt{d^2 + 8r|d_1|}, \\
a_2 &= a_0 + \text{sign}(d_1)(a_1 - d)/2, \\
s_d &= (\text{sign}(d_1 + d) - \text{sign}(d_1 - d))/2, \\
a &= (a_0 + d_1 - a_2)s_d + a_2, \\
s_a &= (\text{sign}(a + d) - \text{sign}(a - d))/2, \\
fhan &= -(\frac{a}{d} - \text{sign}(a))s_a + \text{sign}(a)).
\end{align*}
\]

As pointed in [24], (7) is a time-optimal solution that guarantees the fastest convergence from \(g_1\) to \(v(t)\). Meanwhile, the tracking differentiator can also estimate the differential signal of \(g_1\) (which is \(g_2\) here).

C. The control law

With the estimations of LESO and the transient reference from TD, the control law can be obtained as follows:

\[
u = u_0(z_1, z_2, g_1, g_2) - \frac{z_3}{b},
\]

then the system (4) will reduce to be:

\[
\dot{x} = (f - z_3) + bu_0(z_1, z_2, g_1, g_2) \approx bu_0(z_1, z_2, g_1, g_2). 
\]

Notably, this control law reduce the closed-loop system to a double integrator equivalent system. Then it can be easily designed \(u_0(z_1, z_2, g_1, g_2)\) to achieve the desired control performance. In this paper, we choose the linear PID controller as the specific form of \(u_0(z_1, z_2, g_1, g_2)\). That is:

\[
u_0 = K_p(g_1 - z_1) + K_i \int (g_1 - z_1)dt + K_d(z_2 - g_2),
\]

where \(K_p, K_i,\) and \(K_d\) are the gains of the PID controller.

IV. EXPERIMENTAL RESULTS AND DISCUSSIONS

To verify the performance of the proposed method, the setpoint and tracking control experiments are conducted on a commercial PEA (P-753.1CD, Physik Instrumente, Karlsruhe, Germany), which is connected to a voltage amplifier (E-665.CR, Physik Instrumente, Karlsruhe, Germany). With this voltage amplifier, the PEA is driven to generate a horizontal movement up to 12 \(\mu\)m. Meanwhile, a built-in capacitive displacement sensor, which has a high resolution of 0.05nm, is used to measure the movement of the PEA. The communication card in the host computer (with MATLAB/SIMULINK environment) and the voltage amplifier are both wired to an I/O data acquisition board (PCI-1716, Advantech, Beijing, China). All of the experimental equipments are placed on a vibration-isolation platform. The proposed method is realized by MATLAB/SIMULINK with the toolbox of the Real-Time Windows Target. The experiment setup is given in Fig. 5. The sampling interval in the following experiments is set to 0.0001s.

A. The tuning method of the proposed controller

The LESO is the key component of the ADRC method, and its observer gain \(L = [\beta_1, \beta_2, \beta_3]^T\) should be tuned first. By a trial-and-error procedure, we finally choose \(L = [101, 1027, 2019]^T\).

For the tracking differentiator, the parameter \(r\) is the main parameter to adjust the transient reference. And its effect will be analyzed in the following experiments. Another parameter \(h\) can be chosen as the sampling time, i.e., \(h = 0.0001\).

When the parameters of LESO and TD have been determined, the control law \(u_0\) can be obtained by an on-line adjustment. We finally choose the gains of PID controller as \(K_p = 1.1, K_i = 0.01,\) and \(K_d = 0.035\).

B. The setpoint control of the proposed controller

First, the performance of the setpoint control is verified. The parameter \(r\) is chosen to be 100. The tracking differentiator is used to generate the transient reference of the step response. The experiments results are shown in Fig. 6. It can be seen that the proposed ADRC controller has a satisfactory control performance, and the overshoot of the step response has a very small value.

Second, the tracking differentiator with different values of \(r\) are studied. With different values of \(r\), the control performance of the proposed ADRC controller are shown in Fig. 7. It can be seen that the parameter \(r\) could adjust
the profile of the transient reference, and the proposed ADRC controller can also have a good tracking performance with different transient references. Relatively, if the tracking differentiator is not used, an obvious oscillation will occur in the transient response. This phenomenon is mainly caused by the large initial tracking error, which results in a large control effort in the initial phase. These experiment results suggest that the tracking differentiator could improve the performance of transient response. And the practitioner can arbitrarily adjust the tracking differentiator to achieve the desired control performance.

The tracking control of the proposed controller

Fig. 6. The setpoint control of the proposed ADRC controller.

Fig. 7. The setpoint control with different transient references.

C. The tracking control of the proposed controller

First, the tracking control performance of periodic signal is verified. The sinusoid signal with 1Hz is adopted as the desired trajectory. The experiment results are shown in Fig. 8. It can be seen that the output of the PEA achieves the reference in a fast manner, and the overshoot is in the acceptable range. Meanwhile, the steady-state error is only between $[-0.0269, 0.0234] \mu m$. These results suggest that the proposed ADRC controller is an effective method for the tracking control of PEAs, and the proposed method can also deal with the hysteresis nonlinearity very well.

Second, the tracking performance of non-periodic signal is studied. The experiment results are illustrated in Fig. 10. For the non-periodic reference, the proposed ADRC controller can also have a satisfactory tracking performance. And the steady-state error is only between $[-0.0632, 0.0771] \mu m$. Meanwhile, as shown in Fig. 11, the hysteresis nonlinearity of the closed-loop system is effectively compensated by the proposed controller. These experiment results suggest that the proposed ADRC controller is a promising method to deal with the tracking control of PEAs.

V. CONCLUSIONS

This paper proposed an active disturbance rejection controller for PEAs. The hysteresis nonlinearity in PEAs is regarded as a disturbance of the closed-loop system. Compared with the common view of hysteresis, the disturbance is a relative novel method of dealing with hysteresis in PEAs. Then the extended state observer is introduced to estimate the hysteresis and other dynamics of the mechanical part. With the extended state observer, the model of hysteresis or its inversion are all no longer needed. This is an obvious advantage for real-time control of PEAs. Based on the extended state observer and tracking differentiator, an active disturbance rejection controller is designed. And the verifica-


