Discharge Model and Control Strategy for E-bicycle Mixed Intersections

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***Abstract*—An e-bicycle, also known as an electric bicycle, is one of the major trip modes in the third and fourth-tier small cities of China which is quite different from big cities such as Beijing and Shanghai. Traditional traffic signal control approaches are often based on the pure motor vehicles, regardless the effect of increasing e-bicycles. This paper proposed a discharge model of e-bicycle mixed traffic flows and developed an average delay formula based on that. We analysed the relationship between the average delay and cycle length, for the cases of pure motor vehicles delay and e-bicycle mixed traffic delay, respectively. Moreover, we presented a practical traffic signal control strategy for intersections with e-bicycle mixed traffic flow. It demonstrates that the optimal cycle length obtained from the proposed method is normally larger than that got from the Webster method, which complies with the field application and fits the real traffic circumstances.**

Fig. 1. E-bicycle mixed traffic flows of an intersection in Binzhou, China.

I. Introduction

The urbanization process has accelerated in many cities of China as the economy booming in recent years. It is not only the case in Beijing and Shanghai, but also in *small* cities which are called third and fourth-tier cities with a population about one million. As tens of millions of people flooding into the cities each year, traffic conditions deteriorate dramatically in both large and small cities. The amount of motor vehicles is increasing much more rapidly than the construction of roads. And therefore, traffic jam tend to occur frequently as roads are pushed beyond capacity. In small cities, more people prefer e-bicycles, as shown in Fig.1, for they are cheaper, convenient, space-saving, and environment friendly. For the signal time setting of intersections experiencing e-bicycle mixed traffic flow, we need to consider the influence of e-bicycles definitely. Otherwise, the traffic signal timing plan would be bad. The research on modelling e-bicycle mixed traffic flows, especially in third and fourth-tier cities, at present are quite few.

Traditional traffic signal setting methods are mainly for the motor vehicles [1–8] . There are not many studies in specific traffic signal setting methods for such mixed traffic flows in newly rising small cities in China, especially in average delay model and optimal cycle length calculation. We used the Webster method [9] to determine cycle length of more than 20 intersections in Binzhou, China. But field test shows that when there are e-bicycle mixed in the traffic the cycle length calculated by the Webster method is too small to clear the queue sometimes. The possible reason may be loss of considering distinctions between e-bicycle mixed traffic flow and motor vehicles traffic flow.

This paper proposed an e-bicycle mixed traffic flow discharging model from the observation of video data, in order to solve the problem of insufficient cycle length. Generally, an area (see Fig.2), which is often called waiting area, is reserved for e-bicycles in front of the stop line of motor vehicles. When the traffic signal light turns red for this approach, e-bicycles are gathering in the waiting area before it reaches the maximum capacity, and then queues of e-bicycles are formed behind the stop line and alongside the motor vehicles. The gathering e-bicycles in waiting area and queue will be discharged after the traffic signal light turns green.

The discharge model for mixed traffic flow is different from the classic discharge model of motor vehicles in references [9] and [10]. We develop an average delay equation of e-bicycle mixed traffic flow based on the classic discharge motor vehicles model. It is difficult to find an analytical solution when attempting to minimize the average delay of e-bicycle mixed traffic flow. Accordingly, we give an approximate formula of optimal cycle length.

The rest of the paper is organized as follows. The discharge model is described in section II. Average delay equation of mixed traffic flow is presented in section III. Control strategy

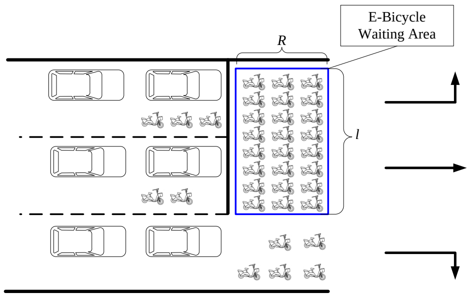

Fig. 2. Distribution of e-bicycles and vehicles near a typical intersection.

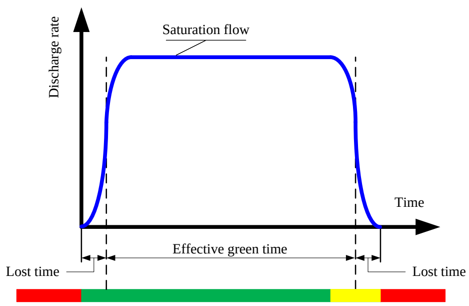

Fig. 3. Discharge rate of motor vehicles without the influence of e-bicycles.

and optimal cycle length for isolated intersections with e-bicycle mixed traffic flow is given in section IV. Finally, section V presents some conclusions and future works.

II. Discharge Model for E-bicycle Mixed Traffic Flows

We firstly introduce the discharge process of motor vehicle as the background knowledge, and then present the discharge process of e-bicycle mixed traffic flow.

A. Motor Vehicles queue discharge process

The delay is defined as the product of number of motor vehicles in the queue and the length of the time interval. For instance, if the queue is counted every Δt seconds and the queue length is n then the total delay in a specific interval is $n\Delta t$. And therefore, it is easy to obtain the total delay over a longer time. If the length of time is $m\Delta t$ and at each interval there are n_i motor vehicles then the total delay in these period is $\sum_{i=1}^{i=m} n_i \Delta t$.

Normally, the queue begins to form when traffic signal light turns red, and in this red period delay is accumulated quickly with the latest coming of motor vehicles. But even the green period starts, queues cannot discharge immediately until vehicles accelerate to a regular speed. After the acceleration, often just a few seconds, discharge rate of motor vehicle queue is almost a constant value s which is called saturation flow. When the queue is not cleaned up at the end of the green period, some motor vehicles will be slow down in the amber period.

Fig.3 describes the relationship of the discharge rate over time in a fully-saturated approach. Define amber time length A, green time length G, lost time length L_v and effective green time length G_v. Hence, $G + A = L_v + G_v$. The number of motor vehicles (defined as N_v) discharged from the queue can be obtained as the product of saturation flow and effective green time length, i.e. $N_v = sG_v$.

TABLE I. Variable Definitions

Variable	Definition
i	The subscript i means the particular approach of the intersection
j	The subscript j means the particular phase of the intersection
N	The total number of phases of the intersection
d_{v_i}	The average delay per motor vehicle
d_{e_i}	The average delay per e-bicycle
D_{e_i}	The total delay of e-bicycles
D	The total delay for all vehicles and all approaches
d_i	The average delay per vehicle including motor vehicle and e-bicycle
C	The set of all cycles in the range of minimum and maximum
c	The cycle length of the intersection
c_0	The optimal cycle length of the intersection
c_{w0}	The optimal cycle length calculated by Webster method
c_{p0}	The optimal cycle length calculated by our method
G_j	The total green time of the cycle
G_j	The green time of the phase under consideration
r	The red time of the approach under consideration
G_v	The effective green time of motor vehicles
G_e	The effective green time of e-bicycles
s	The saturation flow of motor vehicles
A	The total amber time of the cycle
A_j	The amber time of the phase under consideration
L	The total lost time of pure motor vehicles of the cycle
L_j	The lost time of pure motor vehicles flow
L_v	The lost time of motor vehicles with mixed traffic flow
L_e	The lost time of e-bicycles
q_i	The flow of motor vehicles
λ_i	The ratio of the effective green time of motor vehicles to cycle length
x_i	The degree of saturation of motor vehicles, $x_i = q/\lambda_i s$
R_{m_i}	The maximum number of rows in waiting area
l_{m_i}	The maximum number of columns in waiting area
S_i	The capacity of e-bicycle waiting area, $S_i = R_i l_i$
η_i	The ratio of the effective green time of e-bicycles to the cycle length
f_i	The flow of e-bicycles
θ_i	The saturation flow of e-bicycles
M_i	The number of e-bicycles in waiting area
R_i	The number of rows in waiting area, $R_i = \lceil M/l_{m_i} \rceil$
w_i	The intermediate variable, $w_i = (R_i - 1)/(2\theta_i)$
α_i	The weight of the delay of e-bicycles
E	The difference between optimal cycle length, $E = c_{p0} - c_{w0}$
y_j	The highest ratio of motor vehicle flow to saturation flow of phase j
y_m	The largest y_i of all phases
Y	The sum of y_i

Based on the motor vehicle discharge model, an average delay formula is given in reference [9], as shown in equation 1. Variables used in this paper are all summarized in Table I.

$$d_{v_i} = \frac{c(1-\lambda_i)^2}{2(1-\lambda_i x_i)} + \frac{x_i^2}{2q_i(1-x_i)} + 0.65(\frac{c}{q_i^2})x_i^{(2+5\lambda_i)} \quad (1)$$

B. E-bicycle mixed queue discharge process

In contrast with pure motor vehicle flows, an e-bicycle mixed queue discharge process seems a little bit more complicated. E-bicycles will firstly gather in the waiting area while the traffic signal light is red and at the same time motor vehicles will form a queue behind the stop line. If the waiting area reaches maximum capacity, the e-bicycles will form a queue alongside the motor vehicles behind the stop line (see Fig.2).If the capacity of the waiting area is S_i, the number of rows in the area is R_i, the number of columns is l_i, and $S_i = R_i l_i$ obviously.

In the specific approach i, motor vehicles arrive with inflow q_i, and e-bicycles arrive with the inflow f_i. Thus, motor vehicles arrive at intervals of $1/q_i$ and e-bicycles arrive at intervals of $1/f_i$. Saturation flow of e-bicycles is θ and saturation flow of motor vehicles is s. When the traffic signal light turns green, the discharge process begins in the waiting area first. Clearly, the e-bicycles in the first row of waiting area leaves at the same time, but they are delayed by an additional time δ due to their acceleration. Since the saturation flow of e-bicycles is θ, $1/\theta$ is the period when e-bicycles are following at minimum headway. After the first row, e-bicycles of next rows follow at an equal interval of $1/\theta$ until no e-bicycles are waiting in the area. If the saturation flow of e-bicycles is defined as the maximum rate of discharge of the e-bicycles in one column, the e-bicycles in the waiting area have a higher discharge rate because more e-bicycles can move ahead side by side, i.e. in a parallel way.

The discharge process of e-bicycle mixed traffic flow is demonstrated in Fig.4. The yellow curve and the blue curve represents the discharge rate of e-bicycles and motor vehicles respectively in the case of e-bicycle mixed traffic flow, respectively. When green period starts, the discharge rate of e-bicycles (the yellow curve) jump to a rather high value due to the waiting area starts to clean up. After waiting area cleaned up, the discharge rate of e-bicycles fall back to the saturation flow. Some e-bicycles may cross an intersection during the amber period, while others may not. The lost time of e-bicycles is defined as L_e and the effective green time of e-bicycles is defined as G_e, so $G + A = L_e + G_e$. Since motor vehicles are delayed by the e-bicycles in the waiting area, it is easy to find that $L_v > L_e$, i.e. $G_v < G_e$.

The blue curve represents the discharge rate of pure motor vehicles without the influence of e-bicycles. Obviously, the acceleration process of motor vehicles is extended, when comparing the red curve and the blue curve. In other words, with the influence of e-bicycles the effective green time of motor vehicles is reduced. This model explained that why the optimal cycle calculated by Webster method is not long enough sometimes when there are many e-bicycles gathered in the waiting area.

III. Modelling the Average Delay of E-bicycle Mixed Traffic Flow

In this section, we attempt to model the average delay of e-bicycle mixed traffic flow based on the discharge process described in section II. We assume that the arrival rate and the discharge rate of e-bicycles are relatively stable and the e-bicycle flow is not over saturated. This assumption is based on our observation that e-bicycles will not overflow the waiting area at the beginning of the green period normally, and even if sometimes there are overflowed e-bicycles, those e-bicycles have low impact on motor vehicles. Accordingly, modelling the delay of e-bicycles in waiting area is necessary part to obtain the average delay of mixed traffic flow.

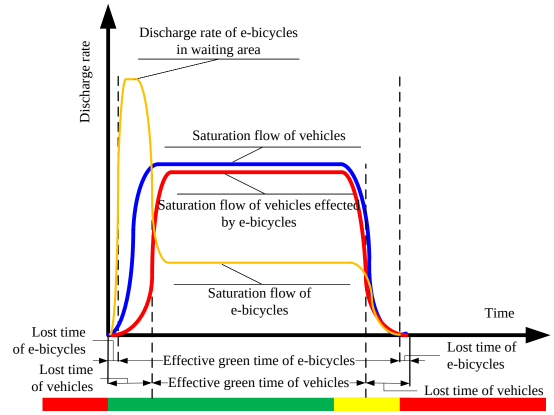

Fig. 4. Comparison of discharge rate of mixed vehicles and motor vehicles.

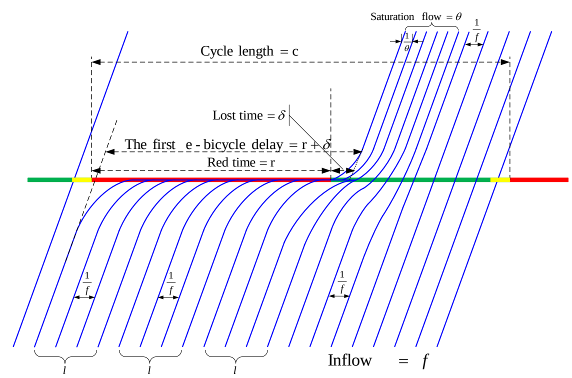

Fig. 5. Distance-time diagram of e-bicycles in a signalized intersection.

A. Delay model of e-bicycles in waiting area

It is assumed that all of the e-bicycles have a constant arrival rate $1/f$ and when green period begins those staying in the waiting area will leave in a constant discharge rate $1/\theta$. If the first e-bicycle arrives at the waiting area at the beginning of the red period, the delay of it is $r + \delta$. The parameter δ is a period of time of acceleration for bicycles of the first row in the waiting area.

Apparently, e-bicycles of the first row have the same period of acceleration and arriving at a fixed interval of $1/f$ sequentially. Hence, the delay of the second e-bicycle of the first row is $r + \delta - 1/f$ and the delay of the third one is $r + \delta - 2/f$ and so on. It's easy to find that all the e-bicycles of the same row in the waiting area discharge at the some time and arriving sequentially at an interval $1/f$. For instance, if the M-th e-bicycle arrives in row R in the red period, the delay of this e-bicycle is $r + \delta + (R-1)/\theta - (M-1)/f$. All of the delays of e-bicycles in waiting area are shown in Table II.

The whole procedure is demonstrated in Fig.5. Obviously, while e-bicycles in the waiting area are leaving, some e-bicycles called "the last wave" are still arriving at the interval $1/f$. If the waiting area is saturated at the beginning of green time, the last wave e-bicycles will be delayed because the queue still exists. But the delay of the last wave are very small compared to the delay of e-bicycles in the waiting area arriving in the red period. It is negligible when we calculate the total delay, and therefore, often be ignored. In summary, the total delay of e-bicycles can be calculated as equation 2.

$$D_e = \sum_{R=1}^{R=M} \{r + \delta + \frac{R-1}{\theta} + \frac{M-1}{f}\} \tag{2}$$

B. Average delay of e-bicycles

As mentioned above, if the e-bicycles do not overflow the waiting area when traffic signal light turns green, the total delay of e-bicycles can be replaced by the delay of those in the waiting area as a reasonable approximation. The variation of e-bicycles' delay with the order of arrival is demonstrated in Fig.6. The area of light green shadow portion surrounded by green, red and blue solid lines represents the total delay of e-bicycles. In other word, the area of light green shadow portion is a graphical representation of equation 2.

The area of light green shadow can approximate as a trapezoid composed of green solid lines and yellow dashed line. To obtain the area of the trapezoid is much easier than to get the area of an irregular polygon. The area of the trapezoid can be calculated as the product of the arithmetic mean of the two parallel sides ($r+\delta$ and $r+\delta+(R-1)/\theta+(M-1)/f$) and the perpendicular distance between these sides $M-1$ (see equation 3).

$$D_e \approx (M-1)\frac{(r+\delta) + (r+\delta+(R-1)/\theta+(M-1)/f)}{2} \tag{3}$$

As e-bicycles are assumed as a constant arrival rate, $M-1$ can be approximate to M, i.e. $(r+\delta)f$. Since $r+\delta = c - G_e$ and $\eta = G_e/c$, $r+\delta$ can replaced by $c(1-\eta)$. Hence the approximate total delay can be obtained as equation 4.

$$D_e \approx \frac{(2f+1)(1-\eta)^2c^2 + (1-\eta)(R-1)fc/\theta}{2} \tag{4}$$

The total number of e-bicycles arriving druing an entire cycle are cf, so that average delay of e-bicycles d_e is $D_e/(cf)$, and this can be written as:

$$d_e = (1+\frac{1}{2f})(1-\eta)^2c + (1-\eta)w \tag{5}$$

where $w = \frac{R-1}{2\theta}$, and c is the cycle length of the intersection.

C. Average delay of mixed traffic flow

The average delay formulas of motor vehicles and e-bicycles have been derived in equation 1 and equation 5, respectively. Assuming motor vehicles and e-bicycles have the same weight, the average delay for mixed traffic flow can be written as:

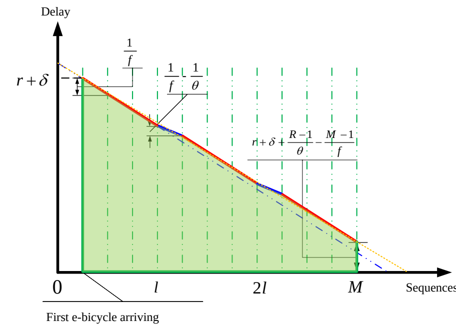

Fig. 6. Variation of e-bicycles' delay with the order of arrival.

$$\begin{aligned} d_i = \frac{1}{q_i+f_i}\{q_i[\frac{c(1-\lambda_i)^2}{2(1-\lambda_i x_i)} + \frac{x_i^2}{2q_i(1-x_i)} \\ +0.65(\frac{c}{q_i^2})x_i^{(2+5\lambda_i)}] + f_i[(1+\frac{1}{2f_i})(1-\eta_i)^2c \\ +(1-\eta_i)w_i]\} \end{aligned} \tag{6}$$

where q_i and f_i is the flow of motor vehicles and e-bicycles in the approach i of an intersection, respectively. Generally, a motor vehicle occupies a much larger space than an e-bicycle and a larger weight should be given to the motor vehicles. Hence, equation 6 can be rewritten as:

$$\begin{aligned} d_i = \frac{c(1-\lambda_i)^2}{2(1-\lambda_i x_i)} + \frac{x_i^2}{2q_i(1-x_i)} + 0.65(\frac{c}{q_i^2})x_i^{(2+5\lambda_i)} \\ +\alpha_i[(1+\frac{1}{2f_i})(1-\eta_i)^2c + (1-\eta_i)w_i] \end{aligned} \tag{7}$$

where α_i is the weight for e-bicycles delay. In this paper, motor vehicles have a greater weight so that $0 \le \alpha_i \le 1$.

Equation 7 demonstrates the average delay in the specific approach i. The total delay of motor vehicles and e-bicycles for each approach is the product of average delay for mixed vehicles d_i and the weighted mixed flow $(1+\alpha_i)q_i$. Hence, the total delay of mixed vehicles for each approach is given as follow:

$$D = \sum_{i=1}^{i=N}[d_i(1+\alpha_i)q_i] \tag{8}$$

IV. Practical E-bicycle Mixed Traffic Signal Control Strategy

According to the delay model of e-bicycle mixed traffic flow developed in section III, we can see that the delay is mainly determined by the cycle length when the traffic flow and green time ratio are fixed. Conversely, the least average delay has the corresponding optimal cycle length. The optimal cycle length is obtained by solving equation 9.

$$c_0 = \arg\min_{c \in C} D(c) \tag{9}$$

TABLE II. DELAY OF ARRIVING E-BICYCLES

Ordinal	**Row** 1	**Row** 2	$\cdots$	**Row** R	$\cdots$	**Row** R_m
1	$r+\delta$	$r+\delta+\frac{1}{\theta}-\frac{l}{f}$	$\cdots$	$r+\delta+\frac{R-1}{\theta}-\frac{(R-1)l}{f}$	$\cdots$	$r+\delta+\frac{(R_m-1)}{\theta}-\frac{(R_m-1)l}{f}$
2	$r+\delta-\frac{1}{f}$	$r+\delta+\frac{1}{\theta}-\frac{l+1}{f}$	$\cdots$	$r+\delta+\frac{R-1}{\theta}-\frac{(R-1)l+1}{f}$	$\cdots$	$r+\delta+\frac{(R_m-1)}{\theta}-\frac{(R_m-1)l+1}{f}$
2	$r+\delta-\frac{2}{f}$	$r+\delta+\frac{1}{\theta}-\frac{l+2}{f}$	$\cdots$	$r+\delta+\frac{R-1}{\theta}-\frac{(R-1)l+2}{f}$	$\cdots$	$r+\delta+\frac{(R_m-1)}{\theta}-\frac{(R_m-1)l+2}{f}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
l_{m_i}	$r+\delta-\frac{l-1}{f}$	$r+\delta+\frac{1}{\theta}-\frac{2l-1}{f}$	$\cdots$	$r+\delta+\frac{R-1}{\theta}-\frac{Rl-1}{f}$	$\cdots$	$r+\delta+\frac{(R_m-1)}{\theta}-\frac{R_m l-1}{f}$

To obtain the optimal cycle length, we can take the derivative of $D(c)$. Notice that parameters λ_i and η_i are related to cycle length c. The differential equation $\frac{\partial D(c)}{\partial c}=0$ is an equation of higher degree, it is difficult to find an analytical solution. Considering the particularity of the traffic time setting problem, there is no need to worry about. Because the cycle length in practice must be an integer within the range between c_{min} and c_{max} and the computing power is good enough to find the best solution very quickly.

A. Influence of e-bicycles on cycle length and delay

The major concern is the optimal cycle length when the delay achieves a minimum value rather than the specific value of delay. Hence, how e-bicycles effect the optimal cycle is the issue to be explored. The parameters α_i, R_i and f_i are studied in this part. It is assumed that in a symmetrical two-phase four-approach intersection, both motor vehicles flow and e-bicycles flow are symmetrical. For convenience, the delay of e-bicycle mixed traffic flow is displayed as a half of the real value in Fig.7, Fig.8 and Fig.9.

Firstly, the effect of weight α_i is studied. Fig.7 demonstrated the relationship between the cycle length and the delay with different weights α. The red curve is the reference curve obtained by Webster's method. We can see from Fig.7 that the larger value of α is, the smaller the optimal cycle length is.

Subsequently, the influence of the number of rows R_i is explored (see Fig.8). The red curve in Fig.8 is the reference curve as mentioned above. Interestingly, the relationship between the cycle length (the blue curves) and the delay does not seem not sensitive to R_i at all. In other words, the value of R_i hardly affects the calculation of optimal cycle.

Finally, the impact of the e-bicycles flow f_i is investigated. The blue curves Fig.9 presented relationship between the delay and the cycle length with different f_i. When keeping other parameters unchanged, the higher flow results in larger optimal cycle length which is consistent with the intuition.

B. A Practical traffic signal control strategy

Fig.7 to Fig.9 show that the optimal cycle length of mixed traffic flow is always larger than the pure motor traffic flow obtained by Webster's method, and when the cycle length is smaller than the optimal cycle, the delay will shoot up. In practical engineering, small changes in the cycle length like increasing or decreasing one or two seconds, does not make much sense to the traffic condition of an intersection. Hence, if the e-bicycle mixed traffic flow has a minor impact on the optimal cycle length, a practical method for engineering application can be obtained.

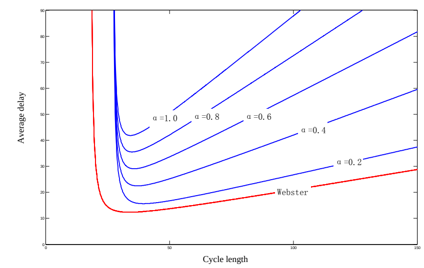

Fig. 7. The relationship between the cycle length and the delay with different α .

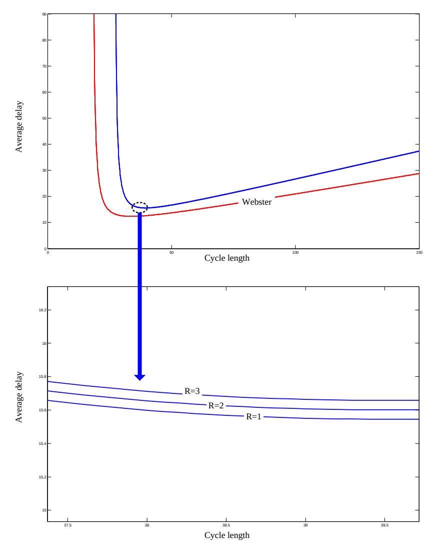

Fig. 8. The relationship between cycle length and delay with different R.

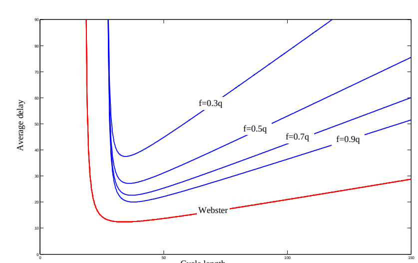

Fig. 9. The relationship between cycle length and delay with different f.

Consider a symmetrical two-phase four-approach intersection, there are waiting areas in each approach. When motor traffic flow varies, different values of optimal cycle length are obtained both in our method and Webster's method, as shown in Fig.10. Fig.10 shows the differences of optimal cycle length E between our method and Webster's method with different lost time L, $E = c_{p0} - c_{w0}$. Obviously, we can see from Fig.10 that for a certain lost time L, $E(y_i)$ is approximately a linear function. For example, when $L = 10s$, $\alpha_i = 0.3$, $f = q$, the approximation of $E(y_i)$ is $E(y_i) = \lceil 0.01 y_i s + 1 \rceil$. That is to say, the optimal cycle of mixed traffic flow can be approximate to $c_0 = c_{w0} + \lceil 0.01 y_i s + 1 \rceil$. And in this case $3 \le E(y_i) \le 10$ seconds.

Thus, the optimal cycle can be inferred as equation 10.

$$c_0 = \frac{1.5L + 5}{1 - Y} + E(y_m) \tag{10}$$

Where the first term is the Webster formula, and the second term is a linear function of y_m which can be easily obtained from Fig.10.

For configuring the signal timing plan of a single intersection, the next step is to calculate the green time of each phase. A simple way to practice is use the ratio of y_i to obtain the ratio of effective green time (see equation).

$$G_j = \frac{y_j}{Y}(c_0 - L - A) + L_j \tag{11}$$

Algorithm 1 Control algorithm in e-bicycles mixed traffic flow

Require: Historical traffic flow data of motro vehicles and e-bicycles, intersection geometry.

initialization
model the discharge process e-bicycles
model the discharge process e-bicycle mixed traffic flow
obtain the delay of e-bicycles d_e
obtain the delay of motor vehicles d_v
compute mean delay of e-bicycle mixed traffic flow d
calculate the optimal cycle length c_0
if the optimal cycle length is out of range **then**
    adjust the optimal cycle
end if
calculate the green time G_j of each phase

V. Conclusion

This paper presents an e-bicycle mixed traffic flow discharge model which can be used to analyse the impact of e-bicycles on the delay in a single intersection. And with this model, a traffic delay model for average delay is developed to obtain the optimal cycle length under e-bicycle mixed traffic conditions. A practical traffic signal control strategy for e-bicycle mixed traffic flow is proposed with the foundation of the mixed delay model. And a time setting method for engineering is given in a simple form. This paper is aimed at configuring traffic signal timing plans for a single intersection, and sequentially future work will concentrate on the exploration of coordinating multiple intersections with e-bicycle mixed traffic flow.

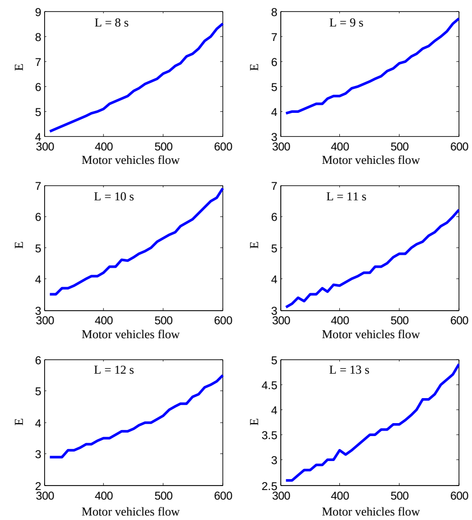

Fig. 10. The difference between Optimal cycle length, $E = c_{p0} - c_{w0}$.

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