# Design and Kinematics Analysis of a 4-DOF Articulated Steering Mechanism 

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#### Abstract

Articulated tracked vehicles possess outstanding traveling capability due to the special articulated steering mechanism (ASM), which makes them be widely used in many application areas. In view of the importance of ASM, we design a new structure of such mechanisms. The designed 4-DOF ASM can perform yaw and pitch movement actively while roll movement passively. To work efficiently, the ASM is designed to operate in three different modes: "float" mode, "lock" mode, and "active control" mode. Then we deduce the kinematics of articulated vehicle and analyze the workspace of the front vehicle with respect to the rear vehicle to demonstrate the motion performance of the designed ASM. With the elongation and shortening of hydraulic cylinders, the maximum steer angle is beyond 0.4 rad , the range of the rear pitch angle is $(-0.22,0.27)$ rad, and the range of the front pitch angle is $(-0.45,0.5) \mathrm{rad}$.


Key Words: Articulated steering mechanism, articulated vehicle, kinematics, workspace

## 1 Introduction

The articulated tracked vehicle (ATV, also called all terrain tracked carrier, ATTC) is a twin chassis multi-purpose vehicle with all four tracks powered and fully articulated steering. It can traverse any terrain with ease and dexterity, such as mud, swamp, snow, and etc. With outstanding cross-country performance, the articulated tracked vehicle can be adapted for different applications, eg. , troop carrier, command post, ambulance, fuel resupply vehicles, fire engines.

Different from the conventional tracked vehicles, ATV is fully articulated steering instead of skip steering. So the design of steering mechanism has been the subject of a great deal of research for many years. And many achievements have been patented. A typical articulating mechanism was designed by [1], which mounted hydraulic means in the form of rams for articulating the front unit relative to the rear unit. The configuration of the mechanism was generally tubular, thereby permitting passage of a universal jointed driveline through its center. The patent [2] introduced a ball and socket type coupling for connecting the sections of the tractor-trailer type vehicle. And the coupling mechanism included a spring loaded dog clutch to permit the trailer to be quickly disconnected from or connected to the tractor by loosening and removing a clamp. This type of coupling mechanism was used by CL-91 Dynatrac of Canadair Ltd. ${ }^{[3]}$. The patent [4] presented a coupling mechanism composed of a two-member ring and hydraulic actuators. The two-member ring assembly was for coupling draft members on each of the vehicle elements and for providing independent pitch and roll movement of the elements. And the hydraulic actuators pivotally connected to the ring assembly accomplish steering of the vehicle. The patent [5] by BAE Systems Alvis Hägglunds AB presented a steering

[^0]arrangement which could couple and uncouple two tracked units by a rapid and simple means. The steering mechanism comprised uprights, fixtures, hydraulic ram cylinder units, steering link units and etc., and permitted the tracked vehicle units to rotate relative to one another about a longitudinal, horizontal axis, and to be coupled or uncoupled with the steering unit remaining fixed to one of the tracked units. The patent [6] by Singapore Technologies Kinetics Ltd. presented an articulation device which allowed pitch, yaw and roll rotations for the two bodies relative to each other actively controlled via hydraulic, electrical or mechanical means or they could be passive. In particular the articulation device coupled and decoupled the two bodies quickly. The Japanese Forestry and Forest Products Research Institute designed a hydraulically powered joint with four degrees of freedom. The joint assembly could be controlled and operated in "float" mode, "lock" mode and "active control" mode ${ }^{[7]}$. The reference [8] designed an articulated mechanism that could achieve the pitching motion between the front and the rear units independently. On the consideration that mining vehicle should adapt the rough terrain and have high obstacle performance, an active joint mechanism was proposed by [9], and another structure comprised four hydraulic cylinders was designed by [10]. Reference [11] designed an articulated mechanism having three freedoms, achieving yaw, pitch and roll motions without mutual interferences. In some applications such as recreational vehicles, the advantages of great maneuverability of articulated vehicles has been lost, because hydraulic rams may not be adaptable to the size of the structure or the demands for durability. To address the aforesaid issues, the patent [12] disclosed a steering joint which had a control unit and a trailing unit driven by gears and sprockets. The patent [13] presented a central multi-direction transmission system that works by activating each of its mechanisms or gears with electric motors or hydraulic injection motors. More kind of articulated mechanisms can refer to $[14,15,16,17,18,19]$.

Those articulated mechanisms mentioned above are designed for specific applications. And the structures are too complicated for achieving such functions.

This paper will present a new articulated steering mechanism with a simple structure to achieve the same functions. To verify the kinematics performance of the designed mechanism, we will give the kinematic model of the new designed tracked vehicle and derive the critical parameters.

The paper is organized as follows. Section 2 describes the structure of the designed articulated steering mechanism. The kinematic model of the twin chassis vehicle is presented in Section 3. The critical parameters are derived in Section 4. Section 5 analyses the kinematics performance of the designed mechanism by simulation. Section 6 concludes the paper.

## 2 Articulated Steering Mechanism

The articulated steering mechanism (ASM) is designed to make the articulated vehicle to negotiate essentially all kinds of terrain, including mountainous and rugged ground, short vertical walls, steep slopes, forest, snow, swamp, and other commonly encountered surfaces.

The articulated tracked vehicle consists of two bodies or units, the front unit and the rear unit, hinged together by an articulated device, as shown in Fig. 1.


Fig. 1: The structure of articulated steering mechanism
In addition to the front vehicle 2 and the rear vehicle 1 , the designed articulated device comprises a yaw and pitch block 3 , a rear roll block 4, a front roll block 5, two rear brackets 6 , two front brackets 7, two rear pitch brackets 8, a front bracket 12 , two rear pitch cylinders 9 , two steer cylinders 10 and a front pitch cylinder 11 .

The above arrangement of the articulated device provides two axes of pitch movement, together with an orthogonal axis of yaw movement and a further orthogonal axis of roll movement.

The yaw and pitch block 3 is rotatably mounted on the rear brackets 6 and a pair of lugs of the rear roll block 4. A pair of steer hydraulic cylinders 10, disposed horizontally, is each pivotally connected at one end to one lug of the rear roll block 4 and at the other end to the lug of 3 . The steer cylinders 10 control the relative yaw between the yaw and pitch block 3 and the rear roll block 4 . And the yaw (steer) motion is performed by the relative rotation between the
front unit and the rear unit about the steer axis, caused by extension and retraction of the steer cylinders 10 .

Above the rear brackets 6 , the rear pitch brackets 8 are also mounted on the front of the rear unit 1 . The cylinder ends of the rear pitch cylinders 9 are rotatably mounted within the rear pitch brackets 8 , and the front piston ends are pivotally mounted on the external pitch spindles of the yaw and pitch block 3. This rear pitch cylinders 9 control the relative pitch motion between the rear unit 1 and the yaw and pitch block 3 . And with the shortening and lengthening of the rear pitch cylinders 9 , the pitch motion of the rear unit 1 about the rear pitch axis is achieved.

The outer piston rod end of the front pitch cylinder 11 is rotatably mounted on pitch control lugs on the top of the front roll block 5, and the other end is fixed by the front pitch bracket 12 which can pivot relative to the front unit 2 . The front roll block 5 is pivotally mounted on the front brackets 7 rigidly attached to the rear of the front vehicle 2 . This front pitch cylinder 11 controls the relative pitch between the front unit 2 and the front roll block 5. And the front pitch motion about the front pitch axis is executed by elongation and shortening of the hydraulic cylinder 11.

The rear roll block 4 and the front roll block 5 are internal joined together by a needle bearing subassembly. The roll motion is non-active controlled for better capability in curve negotiation.

Each of the yaw and pitch block 3, the rear roll block 4 and the front roll block 5 has a hole through which articulation shafts can pass without hindrance. The holes are at the neutral axes of the articulation portions and are sized to reduce or ensure no interference between any of the components and the articulation shafts, whatever the angle and configuration.

According to the aforesaid work process, it can be seen that the articulated steering mechanism has four degrees of freedom and performs motion pivotally about four axes: steer axis, front pitch axis, rear pitch axis and roll axis. And the articulated steering mechanism is active, with single steer (yaw) control using two hydraulic actuators and two pitch controls using separate hydraulic actuators. Furthermore, the steer, pitch and roll movement are mutually independent, which permits the vehicle to conform to different types of terrain to create a favorable ground pressure distribution so that their respective loads are transmitted to the ground and not to a structural member as would be the case where there is no freedom of movement between the front and rear unit.

To obtain good performance and work efficiently, the designed articulated steering mechanism can also operate in three modes like [7]: "float" mode, "lock" mode, and "active control" mode.
(1) The "float" mode. The hydraulic liquid can flow into and out of the five cylinders freely. The cylinders apply no force to the articulation portions, which gets these portions to rotate freely about the steer axis, front pitch axis, rear pitch axis and roll axis. In this mode, the front unit and the rear unit can adapt to different types of terrain to the highest extent and get the biggest grounding area, especially when walking through the flat surface.
(2) The "lock" mode. The hydraulic liquid movement is completely stopped. The elongation and shortening of
five hydraulic cylinders is forbidden, and the pitch and steer movement are not allowed. In this mode, the front unit and the rear unit work as a rigid body. The ability of crossing obstacles is improved, especially when crossing a large gap, such as a crevasse or trench.
(3) The "active control" mode. The hydraulic circuit works and the extension and retraction of hydraulic cylinders can be controlled by hydraulic valves. The vehicle can performs active steer and pitch movements. For instance, when retracting the front pitch cylinder, the front vehicle is raised and attains a certain height on the obstacle. Thus, the vehicle climbs over an obstacle or a vertical wall more easily.

## 3 Kinematics Modeling of Articulated Vehicle

The purpose of kinematics modeling is to derive the kinematic relationships among the front unit, the rear unit, and the articulated steering mechanism.

To analysis the kinematics, the coordinate systems are assigned for the vehicle as show in Fig. 2 and Fig. 3. For simplicity and computational efficiency, the front vehicle and the rear vehicle are replaced by cuboids respectively, and we assume that: (1) the centre and the barycenter of the front unit are identical, in the plane determined by the front pitch axis and the front brackets; (2) the centre and the barycenter of the rear unit are identical, in the plane determined by the rear pitch axis and the rear brackets.


Fig. 2 Coordinate Systems of Articulated Vehicle


Fig. 3 Coordinate Systems of Articulated Steering Mechanism
As a reference, a fixed world coordinate system $O_{0}-x_{0} y_{0} z_{0}$ is set. The mobile coordinate system $O_{1}-x_{1} y_{1} z_{1}$ is fixed to the rear unit, with its origin $O_{1}$ on the centre of the rear unit and $y$-axis directing from the vehicle's right side to left, x -axis along the heading direction, z -axis determined right-handed. In like manner, the coordinate system $O_{5}-x_{5} y_{5} z_{5}$ is set on the front unit, with its origin $O_{5}$ on the centre of the front unit. Then, as shown in Fig. 3, the coordinate systems $O_{2}-x_{2} y_{2} z_{2}, O_{3}-x_{3} y_{3} z_{3}$ and $O_{4}-x_{4} y_{4} z_{4}$ are attached to the proper locations of the articulated steering mechanism.

The homogeneous transformation of a simple rotation about an axis is denoted Rot such that a rotation of $\theta$ about $z$-axis is $\operatorname{Rot}(z, \theta)$, while the same rotation about the $y$-axis is $\operatorname{Rot}(y, \theta)$, and about the $x$-axis is $\operatorname{Rot}(x, \theta)$. Similarly, the homogeneous transformation of a simple translation along an axis is denoted Trans such that a translation of $d$
along an x -axis is $\operatorname{Trans}(d, 0,0)$, and the same translation along y -axis is $\operatorname{Trans}(0, d, 0)$, the translation along z -axis is Trans $(0,0, d)$.
The position and orientation of the rear unit can be denoted by a vector $\left(x_{1}, y_{1}, z_{1}, \psi_{1}, \theta_{1}, \varphi_{1}\right)$, where, $\left(x_{1}, y_{1}, z_{1}\right)$ is the coordinate of the origin $O_{1}, \theta_{1}$ is the pitch angle of the rear unit relative to the world coordinate system, $\varphi_{1}$ the yaw angle, and $\psi_{1}$ the roll angle. Then, the transformation matrix ${ }^{0} T_{1}$ of $O_{1}-x_{1} y_{1} z_{1}$ with respect to the world coordinate system $O_{0}-x_{0} y_{0} z_{0}$ is given by

$$
\begin{align*}
& { }^{0} T_{1}=\operatorname{Trans}\left(x_{1}, y_{1}, z_{1}\right) \operatorname{Rot}\left(z, \varphi_{1}\right) \operatorname{Rot}\left(y, \theta_{1}\right) \operatorname{Rot}\left(x, \psi_{1}\right) \\
& =\left[\begin{array}{cccc}
1 & 0 & 0 & x_{1} \\
0 & 1 & 0 & y_{1} \\
0 & 0 & 1 & z_{1} \\
0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{cccc}
\mathrm{c} \varphi_{1} & -\mathrm{s} \varphi_{1} & 0 & 0 \\
\mathrm{~s} \varphi_{1} & \mathrm{c} \varphi_{1} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{cccc}
\mathrm{c} \theta_{1} & 0 & \mathrm{~s} \theta_{1} & 0 \\
0 & 1 & 0 & 0 \\
-\mathrm{s} \theta_{1} & 0 & \mathrm{c} \theta_{1} & 0 \\
0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & \mathrm{c} \psi_{1} & -\mathrm{s} \psi_{1} & 0 \\
0 & \mathrm{~s} \psi_{1} & \mathrm{c} \psi_{1} & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{1}\\
& =\left[\begin{array}{cccc}
\mathrm{c} \varphi_{1} \mathrm{c} \theta_{1} & \mathrm{c} \varphi_{1} \mathrm{~s} \theta_{1} \mathrm{~s} \psi_{1}-\mathrm{s} \varphi_{1} \mathrm{c} \psi_{1} & \mathrm{c} \varphi_{1} \mathrm{~s} \theta_{1} \mathrm{c} \psi_{1}+\mathrm{s} \varphi_{1} \mathrm{~s} \psi_{1} & x_{1} \\
\mathrm{~s} \varphi_{1} \mathrm{c} \theta_{1} & \mathrm{~s} \varphi_{1} \mathrm{~s} \theta_{1} \mathrm{~s} \psi_{1}+\mathrm{c} \varphi_{1} \mathrm{c} \psi_{1} & \mathrm{~s} \varphi_{1} \mathrm{~s} \theta_{1} \mathrm{c} \psi_{1}-\mathrm{c} \varphi_{1} \mathrm{~s} \psi_{1} & y_{1} \\
-\mathrm{s} \theta_{1} & \mathrm{c} \theta_{1} \mathrm{~s} \psi_{1} & \mathrm{c} \theta_{1} \mathrm{c} \psi_{1} & z_{1} \\
0 & 0 & 0 & 1
\end{array}\right]
\end{align*}
$$

Where, abbreviations are used, $\mathrm{c} *=\cos *$ and $\mathrm{s} *=\sin *$.
Accordingly, we can obtain those corresponding transformation matrixes of the established coordinate systems relative to their preceding.

$$
{ }^{1} T_{2}=\operatorname{Trans}\left(l_{1}, 0,0\right) \operatorname{Rot}\left(y, \theta_{r}\right)=\left[\begin{array}{cccc}
c \theta_{r} & 0 & s \theta_{r} & l_{1}  \tag{2}\\
0 & 1 & 0 & 0 \\
-s \theta_{r} & 0 & c \theta_{r} & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

$$
\begin{gather*}
{ }^{2} T_{3}=\operatorname{Trans}\left(l_{2}, 0,0\right) \operatorname{Rot}(z, \varphi)=\left[\begin{array}{cccc}
\mathrm{c} \varphi & -\mathrm{s} \varphi & 0 & l_{2} \\
\mathrm{~s} \varphi & \mathrm{c} \varphi & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{3}\\
{ }^{3} T_{4}=\operatorname{Trans}\left(l_{3}, 0,0\right) \operatorname{Rot}(x, \psi)=\left[\begin{array}{cccc}
1 & 0 & 0 & l_{3} \\
0 & \mathrm{c} \psi & -\mathrm{s} \psi & 0 \\
0 & \mathrm{~s} \psi & \mathrm{c} \psi & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{4}\\
{ }^{4} T_{5}=\operatorname{Rot}\left(y, \theta_{f}\right) \operatorname{Trans}\left(l_{4}, 0,0\right)=\left[\begin{array}{cccc}
\mathrm{c} \theta_{f} & 0 & \mathrm{~s} \theta_{f} & l_{4} \mathrm{c} \theta_{f} \\
0 & 1 & 0 & 0 \\
-\mathrm{s} \theta_{f} & 0 & \mathrm{c} \theta_{f} & -l_{4} \mathrm{~s} \theta_{f} \\
0 & 0 & 0 & 1
\end{array}\right] \tag{5}
\end{gather*}
$$

Where, $2 l_{1}$ is the length of the rear unit, and $2 l_{4}$ the length of the front unit; $l_{2}$ is the distance measured along a line that is mutually perpendicular to both the rear pitch axis and the steer axis; $l_{3}$ is the distance measured along a line that is mutually perpendicular to both the steer axis and the front pitch axis; $\theta_{r}$ is the pitch angle of the rear unit relative to the yaw and pitch block 3 , and $\theta_{f}$ the pitch angle of the front unit relative to the front roll block 5; $\varphi$ is the steer (yaw) angle; $\psi$ is the roll angle.

The transformation matrix of the coordinate system $O_{5}-x_{5} y_{5} z_{5}$ relative to the world coordinate system $O_{0}-x_{0} y_{0} z_{0}$ is given by

$$
\begin{equation*}
{ }^{0} T_{5}={ }^{0} T_{1}{ }^{1} T_{2}{ }^{2} T_{3}{ }^{3} T_{4}{ }^{4} T_{5} \tag{6}
\end{equation*}
$$

## 4 Deriving Key Parameter

According to the kinematics derived in section 3, it can be seen that some parameters play an important role in the kinematics. And they are steering angle, front pitching angle, and rear pitching angle. The steer movement and the pitch movement are active controlled by hydraulic actuating cylinders while the roll movement is passive. So we will deduce the relationships between those angles and the elongation of cylinders.

### 4.1 Steer Angle

The vertical view of the articulated mechanism is shown in Fig. 4(a), and the schematic drawing of steering mechanism is shown in Fig. 4(b). The steer angle $\varphi$ changes with the movement of hydraulic cylinder piston.


Fig. 4: Vertical View of Articulated Mechanism

The length of elongated cylinder is $l_{B C^{\prime}}$, and the length of shortened cylinder is $l_{A D^{\prime}}$. According to the geometry of Fig. 4(b), we can deduce that

$$
\begin{gather*}
l_{H^{\prime} C^{\prime}}=\left(\frac{1}{2} l_{C D}-l_{G E} \tan (\varphi)\right) \sin (\varphi)+l_{G F}+\frac{l_{G E}}{\cos (\varphi)}  \tag{7}\\
l_{B H^{\prime}}=\frac{1}{2} l_{A B}-\left(\frac{1}{2} l_{C D}-l_{G E} \tan (\varphi)\right) \cos (\varphi)  \tag{8}\\
l_{D^{\prime} J}=l_{G F}+\frac{l_{G E}}{\cos (\varphi)}-\left(\frac{1}{2} l_{C D}+l_{G E} \tan (\varphi)\right) \sin (\varphi)  \tag{9}\\
l_{A J}=\frac{1}{2} l_{A B}-\left(\frac{1}{2} l_{C D}+l_{G E} \tan (\varphi)\right) \cos (\varphi) \tag{10}
\end{gather*}
$$

According to the Pythagorean theorem, the length of each steer cylinder is

$$
\begin{align*}
& l_{B C^{\prime}}^{2}=l_{H^{\prime} C^{\prime}}^{2}+l_{B H^{\prime}}^{2} \\
&= \sin (\varphi) l_{A B} l_{G E}+\sin (\varphi) l_{C D} l_{G F}-\frac{1}{2} l_{A B} l_{C D} \cos (\varphi)  \tag{11}\\
&+2 \cos (\varphi) l_{G E} l_{G F}+\frac{1}{4} l_{A B}^{2}+\frac{1}{4} l_{C D}^{2}+l_{G E}^{2}+l_{G F}^{2} \\
& l_{A D^{\prime}}^{2}=l_{D^{\prime} J}^{2}+l_{A J}^{2} \\
&= \frac{1}{2} l_{A B} l_{C D} \cos (\varphi)+2 \cos (\varphi) l_{G E} l_{G F}-\sin (\varphi) l_{A B} l_{G E}  \tag{12}\\
&-\sin (\varphi) l_{C D} l_{G F}+\frac{1}{4} l_{A B}^{2}+\frac{1}{4} l_{C D}^{2}+l_{G E}^{2}+l_{G F}^{2} \\
& l_{A D}^{2}= l_{B C}^{2}=\frac{1}{4} l_{A B}^{2}-\frac{1}{2} l_{A B} l_{C D}+\frac{1}{4} l_{C D}^{2}+l_{G E}^{2}+2 l_{G F} l_{G E}+l_{G F}^{2} \tag{13}
\end{align*}
$$

We obtain the relationship between the steer angle $\varphi$ and the elongation of hydraulic cylinder by subtracting (13) from (11).

$$
\begin{align*}
\Delta l_{B C}\left(\Delta l_{B C}+l_{B C}\right)= & \sqrt{\left(l_{A B} l_{G E}+l_{C D} l_{G F}\right)^{2}+\frac{1}{4}\left(l_{A B} l_{C D}-4 l_{G F} l_{G E}\right)^{2}} \sin \left(\varphi+\beta_{1}\right)  \tag{14}\\
& +\frac{1}{2} l_{A B} l_{C D}-2 l_{G F} l_{G E}
\end{align*}
$$

Where, $\Delta l_{B C}$ is the elongation of hydraulic cylinder, and $\beta_{1}$ satisfies the following equation (15).

$$
\begin{equation*}
\tan \left(\beta_{1}\right)=\frac{4 l_{G F} l_{G E}-l_{A B} l_{C D}}{2\left(l_{A B} l_{G E}+l_{C D} l_{G F}\right)} \tag{15}
\end{equation*}
$$

Accordingly, the relation between the steer angle $\varphi$ and the shortening distance $\Delta l_{A D}$ of hydraulic cylinder is obtained.

$$
\begin{align*}
\Delta l_{A D}\left(\Delta l_{A D}+l_{A D}\right)= & \sqrt{\frac{1}{4}\left(l_{A B} l_{C D}-4 l_{G F} l_{G E}\right)^{2}+\left(l_{A B} l_{G E}+l_{C D} l_{G F}\right)^{2}} \sin \left(\varphi+\beta_{2}\right)  \tag{16}\\
& -\frac{1}{2} l_{A B} l_{C D}+2 l_{G F} l_{G E}
\end{align*}
$$

The paramter $\beta_{2}$ satisfies

$$
\begin{equation*}
\tan \left(\beta_{2}\right)=\frac{2 l_{A B} l_{G E}+2 l_{C D} l_{G F}}{l_{A B} l_{C D}-4 l_{G F} l_{G E}} \tag{17}
\end{equation*}
$$

### 4.2 Rear Pitch angle

The lateral view of articulated mechanism is shown in Fig. 5. The pitch device is consisted of two parts: the rear pitch mechanism and the front pitch mechanism jointed by the roll device.


Fig. 5 Lateral View of Articulated Mechanism


Fig. 6 Schematic Drawing of Rear Pitch Mechanism
The schematic drawing of rear pitch mechanism is show in Fig. 6, and the rear pitch angle is $\theta_{r}$. It can be obtained that

$$
\begin{gather*}
l_{G F^{\prime}}=l_{B F} \sin \left(\theta_{r}+\arctan \left(\frac{l_{A F}}{l_{A B}}\right)\right)  \tag{18}\\
l_{B G}=-l_{B F} \cos \left(\theta_{r}+\arctan \left(\frac{l_{A F}}{l_{A B}}\right)\right)  \tag{19}\\
l_{G C}=l_{B C}+l_{B F} \cos \left(\theta_{r}+\arctan \left(\frac{l_{A F}}{l_{A B}}\right)\right) \tag{20}
\end{gather*}
$$

The length of the rear pitch cylinder is $l_{E F}$ and $l_{E F}$, respectively, before and after the rear pitch movement.

$$
\begin{align*}
& l_{E F}^{2}=\left(l_{A C}-l_{D E}\right)^{2}+\left(l_{A F}-l_{C D}\right)^{2}  \tag{21}\\
& l_{E F^{\prime}}^{2}=\left(l_{G C}-l_{D E}\right)^{2}+\left(l_{G F^{\prime}}-l_{C D}\right)^{2} \tag{22}
\end{align*}
$$

We define that the the elongation of rear pitch cylinder is $\Delta l_{E F}$. According to (18)-(22), we obtain that

$$
\begin{align*}
\Delta l_{E F}\left(2 l_{E F}\right. & \left.-\Delta l_{E F}\right)=l_{A C}^{2}+l_{A F}^{2}-l_{B C}^{2}-l_{B F}^{2}-2 l_{A C} l_{D E}-2 l_{A F} l_{C D} \\
& +2 l_{B F} \sqrt{l_{C D}^{2}+\left(l_{B C}-l_{D E}\right)^{2}} \sin \left(\theta_{r}+\arctan \left(\frac{l_{A F}}{l_{A B}}\right)+\alpha_{1}\right) \tag{23}
\end{align*}
$$

Where, the paramter $\alpha_{1}$ satisfies

$$
\begin{equation*}
\tan \alpha_{1}=\frac{-l_{B C}+l_{D E}}{l_{C D}} \tag{24}
\end{equation*}
$$

### 4.3 Front Pitch Angle

The schematic drawing of the front pitch mechanism is shown in Fig. 7. The front pitch angle is $\theta_{f}$, and the length of the elongated front pitch cylinder is

$$
\begin{align*}
l_{C^{\prime} D}^{2} & =\left(l_{A B}-l_{A D} \sin \left(\theta_{f}\right)\right)^{2}+\left(l_{B C}-l_{A D} \cos \left(\theta_{f}\right)\right)^{2}  \tag{25}\\
& =l_{A B}{ }^{2}+l_{A D}{ }^{2}+l_{B C}{ }^{2}-2 l_{A B} l_{A D} \sin \left(\theta_{f}\right)-2 l_{A D} l_{B C} \cos \left(\theta_{f}\right)
\end{align*}
$$

The initial length of the front pitch cylinder is

$$
\begin{equation*}
l_{C D}^{2}=l_{A B}^{2}+\left(l_{B C}-l_{A D}\right)^{2} \tag{26}
\end{equation*}
$$

Defining $\Delta l_{C D}$ the elongation distance of cylinder, we obtain that

$$
\begin{align*}
\Delta l_{C D}\left(2 l_{C D}+\Delta l_{C D}\right) & =2 l_{A D}\left(l_{A B} \sin \left(\theta_{f}\right)+l_{B C} \cos \left(\theta_{f}\right)-l_{B C}\right) \\
& =2 l_{A D}\left(\sqrt{l_{A B}^{2}+l_{B C}^{2}} \sin \left(\theta_{f}+\alpha_{2}\right)-l_{B C}\right) \tag{27}
\end{align*}
$$

Where, $\alpha_{2}$ satisfies


Fig. 7 Schematic Drawing of Front Pitch Mechanism

## 5 Simulation

To simplify the designing process, the five hydraulic cylinders used in the articulated steering mechanism are of the same type, with the stroke length 53 mm and mounting distance 150 mm .

We substitute actual parameters into the expression (14) and (16), and obtain the relationship (shown in Fig. 8 ) between the steer angle $\varphi$ and the length variation of the steer cylinder.


Fig. 8 Relationship between Steer Angle and Length Variation of Steer Cylinder

Fig. 8 suggests that: (1) when the steer angle $\varphi$ is small, the elongation and the shortening distance are approximately the same; (2) the "elongation" curve and the "shortening" curve are symmetrical about the origin; (3) the part of the
"elongation" curve less than zero means that the piston rod of steer cylinder moves inward, similarily, the part of the "shortening" curve less than zero means that the piston rod has an outward movement; (4) the maximum steer angle is more than 0.4 rad ; (5) when the steer angle $\varphi$ is certain, the elongation length and the shorten length of steer cylinders are different, which is mainly because of the trapezoidal structure of the steer mechanism as shown in Fig. 4(b).

Likewise, we can obtain the the relationship (shown in Fig.

9 ) between the rear pitch angle $\theta_{r}$ and the length variation $\Delta l_{E F}$ of the rear pitch cylinder, as well as the the relationship (shown in Fig. 10 ) between the front pitch angle $\theta_{f}$ and the length variation $\Delta l_{C D}$ of the front pitch cylinder.

Fig. 9 and Fig. 10 sugest that: (1) the relationsip between the pitch angle and the lenthen variation of pitch cylinders is nonlinear; (2) the range of the rear pitch angle is $(-0.22,0.27)$ rad; (3) the range of the front pitch angle is $(-0.45,0.5) \mathrm{rad}$.


Fig. 9 Relationship between Rear Pitch Angle and Length Variation of Rear Pitch Cylinder


Fig. 10 Relationship between Front Pitch Angle and Length Variation of Front Pitch Cylinder


Fig. 11 Workspace of Front Unit Relative to Rear Unit

We assume that the rear unit is fixed on the origin $O_{0}$ of world coordinate system $O_{0}-x_{0} y_{0} z_{0}$, with its centre $O_{1}$ and the origin $O_{0}$ in superposition completely and its pose vector
$(0,0,0,0,0,0)$. According to eqution (6), we can get the workspace of the front unit relative to the rear unit. As shown in Fig. 11, (a) shows the workspace; (b) is the top view of the workspace; (c) is the front view from x -axis direction; and (d) is the left view from $y$-axis direction. We can see that, (1) the workspace is a part of the arched surface, but with some varying thinkness; (2) the workspace is symmetrical about the plane $\mathrm{y}=0$; (3) the depression angle of the vehicle is bigger than elevation angle.

## 6 Conclusion

This article introduced a 4-DOF articulated steering mechanism, which provides independent yaw, pitch and roll movement controlled via hydraulic means. The designed mechanism operated in three modes to make the articulated vehicle obtain good performance. Then we established the kinematics model determined by some key parameters which were deduced later. At the end, the workspace of the front unit relative to the rear unit was analyzed, which demonstrated the motion performance of the designed mechanism. In addition to articulated tracked vehicles, the designed articulated steering mechanism can be used in other applications, such as loader, snow-truck, underground mining vehicle, and so on.

## References

[1] B. Harvey M, H. Kummen, N. Jr Clifford J, Articulated tracked vehicles, 1958.
[2] Thomas Ian A, Articulated joint assembly, 1963.
[3] http://www.casr.ca/bg-army-mosv-origins-dynatrac.htm.
[4] F. John P, F. Robert W, Articulated vehicle, 1963.
[5] M. Vigren, M. Thoren, Steering arrangement for articulated tracked vehicles, 1999.
[6] F. L. Heng, S. Jun, J. T. Shi, et al., Articulated vehicle, an articulation device and a drive transmission, 2003.
[7] S. Sasaki, T. Yamada, E. Miyata, Articulated tracked vehicle with four degrees of freedom, Journal of Terramechanics, 28(2-3): 189-199, 1991.
[8] Y. Luo, L.J. Chen, et al., Trench crossing analysis of uneven terrain articulated tracked vehicles, Journal of Machine Design, 29(10): 50-54, 2012.
[9] J. T. Chen, L. Li, J. J. Wu, Research on Stationary Turning Performance of Articulated Tracked Vehicles, Computer Simulation, 24(12): 155-158, 2007.
[10] Liang, Z., L. Li and L. Xiao-fei, Design and optimization of a multi-degree-of-freedom articulated mechanism, Modern Manufacturing Engineering, (9): 120-123,111, 2009.
[11] Y. Li, Research on riding performance of articulated tracked vehicle, 2011, Ji Lin University.
[12] W.I. Van, Snow machine, 1974.
[13] F.J. Lares, Central multi directional transmission system, 2010.
[14] A. R. Kenneth, H. R. Alan, All terrain vehicle and method of operating same, 1985.
[15] Bernard Baillargeon, All-terrain vehicle, 1988.
[16] Björn Nordberg, Protective device for an articulated vehicle, 2010.
[17] Sverker Svärdby, Patrik Forsberg, Steering device for articulated vehicle, 2010.
[18] M. Thorén, M. Carlsson, Steering device for articulated vehicle, 2012.
[19] C. Audet, All-Terrain Vehicle, 2012.


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