

# Neural-Network-Based Distributed Adaptive Robust Control for a Class of Nonlinear Multiagent Systems With Time Delays and External Noises

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**Abstract**—A class of nonlinear multiagent systems with time delays and external noises is investigated, and a distributed adaptive robust control protocol is developed. It is the first time for a class of multiagent systems to take both time delays and external noises into consideration. By virtue of Lyapunov–Krasovskii functional and Young’s inequality, the effects of time delay can be eliminated. Then, to exclude external noises, a robustifying term is introduced to eliminate the negative effects of these noises. Moreover, neural networks are utilized to learn the unknown nonlinear terms to adapt to the complex external environment. Finally, a numerical simulation is conducted to validate the effectiveness of our distributed control protocol.

**Index Terms**—Distributed adaptive robust control, multiagent systems, neural networks (NNs), noises, time delay.

## I. INTRODUCTION

FOR the last decade or so, multiagent systems have received visible research attention, and the advance has been notable [1]–[3]. The fundamental issues of multiagent systems including the consensus problem [4]–[9], distributed optimal control [10]–[12], formation control [13], and others [14]–[19] have been extensively studied. Among these studies, nonlinearity is ubiquitous in physical systems. However, it is difficult to obtain the complete properties

of the given systems with unknown nonlinear functions. The neural network (NN) technique is a powerful tool for approximating arbitrary functions [20]–[22], and it is reasonable to choose NNs to adapt to the unknown dynamics of multiagent systems. Polycarpou [23] proposed a stable adaptive neural control scheme to deal with nonlinear systems. Chen *et al.* [24] introduced a distributed cooperative learning control scheme with NNs to learn the unknown dynamics of the multiagent systems online. In [25], the multiagent systems were unknown and nonlinear with external disturbances. An NN-based distributed control scheme and an adaptive robust technique were designed to adapt to the unknown parts and disturbances, respectively. However, none of them take time delay into consideration. Large time delay can induce instability and increase the difficulty of NNs to learn the unknown systems. Thus, in [26], by virtue of Lyapunov–Krasovskii functional and Young’s inequality, time delays in the dynamics of multiagent systems were eliminated and the error can be reduced to an arbitrarily small domain. Noises can decrease the accuracy of the measurement and deteriorate the effectiveness of the technique used to eliminate the effects of time delays which are considered in this paper. Therefore, we investigate unknown nonlinear multiagent systems with time delays and external noises which are more practical for physical systems.

To the best of authors’ knowledge, it is the first time to include both noises and time delay in a class of nonlinear multiagent systems. First, we apply Lyapunov–Krasovskii functional and Young’s inequality to eliminate the effects of time delays. Second, a robustifying term is introduced to handle the external disturbances. Third, NNs aim to learn the unknown nonlinear terms with the norm of NN weights bounded. The simulation result shows that consensus can be achieved under our distributed protocol. The result also demonstrates the importance of introducing a robustifying term to the distributed protocol.

The remainder of this paper is organized as follows. Basic definitions and properties are given in Section II. By means of Lyapunov–Krasovskii functional and Young’s inequality, a distributed adaptive robust control protocol is designed to obtain consensus in the presence of external noises and time delays

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in Section III. In Section IV, a numerical example is conducted to demonstrate the validity of our theorem, while the conclusion is provided in Section V.

**Notations**  $(\cdot)^T$  denotes the transpose of matrix or vector.  $\text{tr}(\cdot)$  is the trace of a given matrix and  $\|\cdot\|$  is the Frobenius norm or Euclidian norm, and  $\|B\|_F = (\text{tr}(B^T B))^{1/2}$ , where  $B \in \mathbb{R}^{n \times n}$ .  $\otimes$  stands for the Kronecker product.  $\text{diag}(\cdot)$  represents diagonal matrix.

## II. PRELIMINARIES AND PROBLEM STATEMENT

### A. Problem Statement

The dynamics of the multiagent systems are as follows:

$$\begin{aligned} \dot{x}_i(t) &= f_i(x_i(t)) + g_i(x_i(t - \tau_i)) + u_i(t) + \xi_i(t) \\ i &= 1, 2, \dots, N \end{aligned} \quad (1)$$

where  $x_i(\cdot) \in \mathbb{R}^m$  is the state vector,  $f_i(\cdot): \mathbb{R}^m \rightarrow \mathbb{R}^m$  and  $g_i(\cdot): \mathbb{R}^m \rightarrow \mathbb{R}^m$  are continuous but unknown nonlinear vector functions, and  $u_i(\cdot) \in \mathbb{R}^m$  is the control vector. In addition,  $\tau_i$  and  $\xi_i(\cdot) \in \mathbb{R}^m$  represent the unknown time delay and external noises, respectively. For simplicity, in the sequel we will ignore time expression  $t$  when there is no confusion.

**Assumption 1** (See [27, Assumption 4]):  $g_i(x_i(t - \tau_i))$ ,  $i = 1, 2, \dots, N$ , are unknown smooth nonlinear functions satisfying the inequalities  $\|g_i(x_i(t))\| \leq \phi_i(x_i(t))$ ,  $i = 1, 2, \dots, N$ , where  $\phi_i(\cdot)$ ,  $i = 1, 2, \dots, N$ , are known positive smooth scalar functions. Furthermore,  $g_i(0) = 0$  and  $\phi_i(0) = 0$ ,  $i = 1, 2, \dots, N$ .

**Assumption 2** (See [27, Assumption 5]): The unknown time delays  $\tau_i$ ,  $i = 1, 2, \dots, N$ , are all bounded by a known constant  $\tau_{\max}$ , i.e.,  $\tau_i \leq \tau_{\max}$ ,  $i = 1, 2, \dots, N$ .

**Assumption 3:** The external noises  $\xi_i$ ,  $i = 1, 2, \dots, N$ , are bounded, that is

$$\|\xi_i\| \leq \alpha_i, \quad i = 1, 2, \dots, N \quad (2)$$

where  $\alpha_i$ ,  $i = 1, 2, \dots, N$ , are positive constants.

### B. Graph Theory

A triplet  $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$  is called a weighted graph if  $\mathcal{V} = \{1, 2, \dots, N\}$  is the set of  $N$  nodes,  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$  is the set of edges, and  $\mathcal{A} = (\mathcal{A}_{ij}) \in \mathbb{R}^{N \times N}$  is the  $N \times N$  matrix of weights of  $\mathcal{G}$ . Here, we denote  $\mathcal{A}_{ij}$  as the element of the  $i$ th row and  $j$ th column of matrix  $\mathcal{A}$ . The  $i$ th node in graph  $\mathcal{G}$  represents the  $i$ th agent, and a directed path from node  $i$  to node  $j$  is denoted as an ordered pair  $(i, j) \in \mathcal{E}$ , which means that agent  $i$  can directly transfer its information to agent  $j$ .  $\mathcal{A}$  is called the adjacency matrix of graph  $\mathcal{G}$  and we use the notation  $\mathcal{G}(\mathcal{A}): \mathcal{A}_{ij} \neq 0 \Leftrightarrow (j, i) \in \mathcal{E}$  to represent the fact that graph  $\mathcal{G}$  corresponds to  $\mathcal{A}$ .

In this paper, we suppose that  $\mathcal{G}$  is undirected and connected with fixed topology. Note that self-loops will be excluded in this paper, i.e.,  $\mathcal{A}_{ii} = 0$ ,  $i = 1, 2, \dots, N$ .  $\mathcal{G}$  is called connected if there is a path between any two nodes of  $\mathcal{G}$ . Let  $\mathcal{D} = \text{diag}(d_i)$  be an  $N \times N$  diagonal matrix, where  $d_i = \sum_{j \in \mathcal{N}_i} \mathcal{A}_{ij}$  and  $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$  are the set of neighboring nodes of node  $i$ ,  $i = 1, 2, \dots, N$ . The Laplacian matrix is  $\mathcal{L} = \mathcal{D} - \mathcal{A}$  corresponding to  $\mathcal{G}$ . In addition, for a

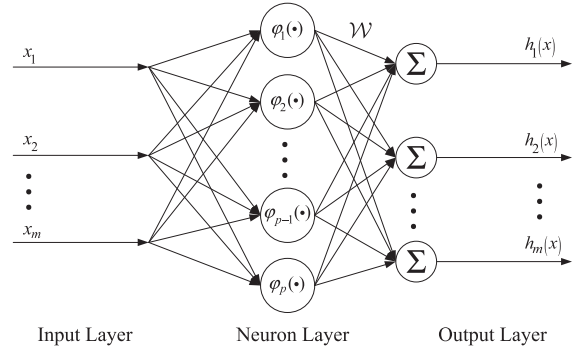


Fig. 1. Structure of the RBFNN.

connected graph,  $\mathcal{L}$  has only one single zero eigenvalue [28]. We denote  $\lambda_N(\mathcal{L}) \geq \lambda_{N-1}(\mathcal{L}) \geq \dots \geq \lambda_2(\mathcal{L}) \geq \lambda_1(\mathcal{L}) = 0$  as the eigenvalues of  $\mathcal{L}$  with  $\lambda_2(\mathcal{L}) > 0$  if  $\mathcal{G}$  is connected. We use  $\lambda_{\min}(\cdot)$  and  $\lambda_{\max}(\cdot)$  to denote the smallest nonzero eigenvalue and the largest eigenvalue of a given matrix, respectively.

### C. Function Approximation and Neural Networks

In practice, we usually employ NNs as the function approximators to model unknown functions. Radial basis function neural network (RBFNN) is a potential candidate for approximating the unknown dynamics of multiagent systems in virtue of “linear-in-weight” property. In Fig. 1,  $h(x) = [h_1(x), h_2(x), \dots, h_m(x)]^T: \mathbb{R}^m \rightarrow \mathbb{R}^m$  is a continuous unknown nonlinear function which can be approximated by an RBFNN

$$h(x) = \mathcal{W}^T \Phi(x) \quad (3)$$

where  $x \in \Upsilon_x \subset \mathbb{R}^m$  is the input vector,  $\mathcal{W} \in \mathbb{R}^{p \times m}$  is the weight matrix, and  $p$  represents the number of neurons. Additionally,  $\Phi(x) = [\varphi_1(x), \varphi_2(x), \dots, \varphi_p(x)]^T$  is the activation function vector and

$$\varphi_i(x) = \exp\left[\frac{-(x - \mu_i)^T(x - \mu_i)}{\sigma_i^2}\right], \quad i = 1, 2, \dots, p \quad (4)$$

where  $\sigma_i$  is the width of Gaussian functions and  $\mu_i = [\mu_{i1}, \mu_{i2}, \dots, \mu_{im}]^T$  is the center of the receptive field. RBFNN can approximate any continuous function over a compact set  $\Upsilon_x \subset \mathbb{R}^m$  to arbitrary precision. Therefore, for a given positive constant  $\theta_N$ , there exists an ideal weight matrix  $\mathcal{W}^*$  such that

$$h(x) = \mathcal{W}^{*T} \Phi(x) + \theta \quad (5)$$

where  $\theta \in \mathbb{R}^m$  is the approximation error with  $\|\theta\| < \theta_N$  in  $\Upsilon_x$ . It should be noted that  $\mathcal{W}^*$  is introduced for convenience of analysis. However, in real applications, we denote  $\hat{\mathcal{W}}$  as the estimation of the ideal weight matrix  $\mathcal{W}^*$ . Thus, the estimation of  $h(x)$  can be written as

$$\hat{h}(x) = \hat{\mathcal{W}}^T \Phi(x) \quad (6)$$

where  $\hat{\mathcal{W}}$  can be updated online. The online updating laws will be given in Section III.

### III. CONSENSUS DISTRIBUTED CONTROL PROTOCOL

Our aim is to design a distributed adaptive robust control protocol, which can drive the multiagent systems toward consensus. We divide the distributed controller into five parts: 1) linear feedback term; 2) neural term; 3) robustifying term; 4) state limit term; and 5) time delay eliminating term. Before proceeding, we introduce a Lyapunov–Krasovskii functional as follows:

$$L_Q(t) = \frac{1}{2} \sum_{i=1}^N \int_{t-\tau_i}^t Q_i(x_i(\zeta)) d\zeta \quad (7)$$

where  $Q_i(x_i(t)) = \phi_i^2(x_i(t))$ . The time derivative of  $L_Q(t)$  is

$$\dot{L}_Q(t) = \frac{1}{2} \sum_{i=1}^N (\phi_i^2(x_i(t)) - \phi_i^2(x_i(t - \tau_i))). \quad (8)$$

Next, we shed light on analyzing the function of each term designed in our distributed controller  $u_i$

$$u_i = -\beta_i(t)e_i - \hat{\mathcal{W}}_i^T \Phi_i(x_i) - \gamma_i \tanh\left(\frac{\kappa_i \gamma_i e_i}{\epsilon_i}\right) - \text{sgn}(e_i^T x_i) \psi_i(x_i) x_i - \frac{1}{2} \frac{e_i}{\|e_i\|^2} \phi_i^2(x_i) \quad (9)$$

where  $e_i = \sum_{j \in \mathcal{N}_i} \mathcal{A}_{ij}(x_i - x_j)$  is the total error difference of each agent  $i$  that utilizes the information from its neighborhood. Additionally,  $\text{sgn}(\cdot)$  is the sign function defined as follows:

$$\text{sgn}(b) = \begin{cases} 1, & \forall b > 0 \\ -1, & \forall b \leq 0. \end{cases} \quad (10)$$

In the sequel, for convenience of analysis, we will ignore the declaration that  $i = 1, 2, \dots, N$  and concentrate on agent  $i$ .

1)  $-\beta_i(t)e_i$  is the linear feedback controller utilized to drive the  $i$ th agent to the final consensus state. It contains all the information, which can be used by agent  $i$ , to guide its direction toward consensus. Moreover, if consensus can be reached, then  $-\beta_i(t)e_i$  is set to zero to release the control impact. It should be noted that  $\beta_i(t)$  controls the convergence speed of agent  $i$ , and thus it should not be too large for inducing overshoot oscillation nor too small for lacking control effect.

2) In order to model the unknown dynamics  $f_i(x_i)$  in (1),  $-\hat{\mathcal{W}}_i^T \Phi_i(x_i)$  is used to learn online the characteristics of  $f_i(x_i)$ .  $\hat{\mathcal{W}}_i \in \mathbb{R}^{p_i \times m}$  represents the RBFNN weight matrix of agent  $i$ , where  $p_i$  is the number of neurons. Due to the fact that RBFNN can approximate  $f_i(x_i)$  with arbitrarily small error,  $f_i(x_i)$  can be written as follows:

$$f_i(x_i) = \mathcal{W}_i^{*T} \Phi_i(x_i) + \theta_i \quad (11)$$

where  $\mathcal{W}_i^*$  is the optimal weight matrix and  $\|\theta_i\| \leq \theta_{N_i}$  is the approximation error where  $\theta_{N_i}$  is a given positive constant. Motivated by projection algorithm, we can derive the adaptive updating law (12) for RBFNN

weight matrix  $\hat{\mathcal{W}}_i$ ,  $i = 1, 2, \dots, N$ ,

$$\dot{\hat{\mathcal{W}}}_i = \begin{cases} a_i \Phi_i(x_i) e_i^T, & \text{if } \text{tr}(\hat{\mathcal{W}}_i^T \hat{\mathcal{W}}_i) < \mathcal{W}_i^{\max} \text{ or} \\ & \text{if } \text{tr}(\hat{\mathcal{W}}_i^T \hat{\mathcal{W}}_i) = \mathcal{W}_i^{\max} \text{ and } e_i^T \hat{\mathcal{W}}_i^T \Phi_i(x_i) < 0; \\ a_i \Phi_i(x_i) e_i^T - a_i \frac{e_i^T \hat{\mathcal{W}}_i^T \Phi_i(x_i)}{\text{tr}(\hat{\mathcal{W}}_i^T \hat{\mathcal{W}}_i)} \hat{\mathcal{W}}_i, & \\ & \text{if } \text{tr}(\hat{\mathcal{W}}_i^T \hat{\mathcal{W}}_i) = \mathcal{W}_i^{\max} \text{ and } e_i^T \hat{\mathcal{W}}_i^T \Phi_i(x_i) \geq 0; \end{cases} \quad (12)$$

where  $a_i > 0$  has effect on the adaptation rate of  $\hat{\mathcal{W}}_i$  and  $\mathcal{W}_i^{\max} > 0$  is utilized to constrain the value of  $\hat{\mathcal{W}}_i$ . It is noted that the initial RBFNN weight matrix  $\hat{\mathcal{W}}_i(0)$  should satisfy that

$$\text{tr}(\hat{\mathcal{W}}_i^T(0) \hat{\mathcal{W}}_i(0)) \leq \mathcal{W}_i^{\max}. \quad (13)$$

Thus, we let  $\hat{\mathcal{W}}_i(0)$  always be a zero matrix. Furthermore, according to [25, Lemma 2], if the updating law is based on (12), then we can obtain

$$\text{tr}(\hat{\mathcal{W}}_i^T(t) \hat{\mathcal{W}}_i(t)) \leq \mathcal{W}_i^{\max}, \quad \forall t \geq 0.$$

*Remark 1:* The projection algorithm defined in (12) is effective.  $\hat{f}_i(x_i) = \hat{\mathcal{W}}_i^T \Phi_i(x_i)$  is the output of RBFNN, then  $e_i^T \hat{\mathcal{W}}_i^T \Phi_i(x_i)$  can be seen as the value of  $\hat{f}_i(x_i)$  projected onto  $e_i$ . When  $e_i^T \hat{f}_i(x_i) < 0$ , the angle between  $e_i$  and  $\hat{f}_i(x_i)$  is greater than  $90^\circ$ . Thus, we should decrease  $\hat{\mathcal{W}}_i$ . When  $e_i^T \hat{f}_i(x_i) > 0$ , the angle between  $e_i$  and  $\hat{f}_i(x_i)$  is smaller than  $90^\circ$ , and we should increase  $\hat{\mathcal{W}}_i$ . Therefore, the final aim is to adjust the angle to  $90^\circ$ , that is,  $e_i$  is vertical to  $\hat{f}_i(x_i)$  and  $e_i$  has no impact on  $\hat{f}_i(x_i)$  with  $\hat{\mathcal{W}}_i = 0$ . This means that RBFNN weight matrix  $\hat{\mathcal{W}}_i$  has learned the unknown dynamics  $f_i(x_i)$  of agent  $i$ .

3) The function  $-\gamma_i \tanh((\kappa_i \gamma_i e_i)/\epsilon_i)$  is to eliminate negative effects of the external noises  $\xi_i$  and RBFNN approximation error  $\theta_i$ , where  $\kappa_i = 0.2785$  (more details can be found in [23]). Furthermore,  $\gamma_i$  is the robust gain satisfying

$$\gamma_i \geq \theta_{N_i} + \alpha_i \quad (14)$$

and  $\epsilon_i > 0$  is related to parameter precision. By virtue of [23, Lemma 1], we can easily get the following inequalities:

$$e_i^T \gamma_i \tanh\left(\frac{\kappa_i \gamma_i e_i}{\epsilon_i}\right) \geq 0 \quad (15a)$$

$$\gamma_i \|e_i\| - e_i^T \gamma_i \tanh\left(\frac{\kappa_i \gamma_i e_i}{\epsilon_i}\right) \leq \epsilon_i. \quad (15b)$$

4) We utilize  $-\text{sgn}(e_i^T x_i) \psi_i(x_i) x_i$  to constrain agent  $i$ 's state in a finite scope. Denote  $\psi_i(\cdot)$  as follows:

$$\psi_i(x_i) = \begin{cases} 0, & \text{if } \|x_i\| \leq c_1 \\ \frac{\|x_i\| - c_1}{c_2 - c_1} \eta_i, & \text{if } c_1 < \|x_i\| < c_2 \\ \eta_i, & \text{if } \|x_i\| \geq c_2 \end{cases} \quad (16)$$

where  $c_2 > c_1 > 0$  and  $\eta_i$  are all given positive constants.  $c_1$  satisfies

$$\|x_i(0)\| < c_1, \quad i = 1, 2, \dots, N. \quad (17)$$

It is noted that the initial state of each agent is finite in industrial application. Therefore, it is plausible to choose  $c_1$  properly when we design our distributed control protocol. Lemma 1 in the Appendix demonstrates that if the following inequality:

$$\eta_i > \frac{1}{c_2} \left( 2\sqrt{p_i} \sqrt{\mathcal{W}_i^{\max}} + (1 + \sqrt{m})\gamma_i + \phi_i^{\max} \right) \quad (18)$$

holds, then we can obtain

$$\|x_i(t)\| \leq c_2, \quad \forall t \geq 0. \quad (19)$$

Please refer to Lemma 1 for more details.

5)  $-(1/2)(e_i/\|e_i\|^2)\phi_i^2(x_i)$  is introduced to eliminate the effect of time delays. Note that  $-(1/2)(e_i/\|e_i\|^2)\phi_i^2(x_i)$  has singularity at  $\|e_i\| = 0$ . Thus, we should exclude zero from control domain. At some given times  $t'$ , we choose  $k \in \mathcal{V}$ , such that  $e_k(t') = 0$ , i.e.,  $\sum_{j \in \mathcal{N}_k} \mathcal{A}_{kj}(x_k(t') - x_j(t')) = 0$ . Then,  $x_k$  can be linearly expressed by other agents' states. Therefore, at  $t'$  we can only control the states of other  $N - 1$  agents to reach the consensus except for agent  $k$  and set  $u_k(t') = 0$ . If there are more than one agent whose error differences are zero, we can use the same method. However, from a practical point of view,  $e_i = 0$  is difficult to detect due to the presence of measurement noises. Thus, intuitively it is plausible to introduce a "ball" region of errors to relax the consensus objective rather than the simple origin.

Similar to the definition in [27], we define a compact set  $\Xi_{x_i} \subset \mathbb{R}^m$ . Then,  $\forall i \in \mathcal{V}$ ,  $\Xi_{\hat{b}_i} \subset \Xi_{x_i}$ , and  $\Xi_{\hat{b}_i}^o$  are

$$\Xi_{\hat{b}_i}^o \triangleq \Xi_{x_i} - \Xi_{\hat{b}_i} = \{x \mid x \in \Xi_{x_i}, x \notin \Xi_{\hat{b}_i}\} \quad (20)$$

$$\Xi_{\hat{b}_i} \triangleq \{e_i \mid \|e_i\| < \hat{b}_i\} \quad (21)$$

where  $\hat{b}_i$  is a given positive constant and can be set arbitrarily small to satisfy control precision. According to [27, Lemma 3], we can easily know that  $\Xi_{\hat{b}_i}^o$  is a compact set.

Therefore, the distributed control protocol is as follows:

$$u_i = \begin{cases} -\beta_i(t)e_i - \hat{\mathcal{W}}_i^T \Phi_i(x_i) - \gamma_i \tanh\left(\frac{\kappa_i \gamma_i e_i}{\epsilon_i}\right) \\ -\text{sgn}(e_i^T x_i) \psi_i(x_i) x_i - \frac{1}{2} \frac{e_i}{\|e_i\|^2} \phi_i^2(x_i), & e_i \in \Xi_{\hat{b}_i}^o \\ 0, & e_i \in \Xi_{\hat{b}_i} \end{cases} \quad (22)$$

$i = 1, 2, \dots, N$ . Now, we are in position to propose the main theorem.

**Theorem 1:** A class of nonlinear multiagent systems is described by (1) with Assumptions 1–3 satisfied. The distributed control protocol is given in (22) and the linear feedback coefficients are

$$\beta_i(t) = \begin{cases} \frac{1}{k_i} \left[ \frac{1}{\|e_i\|^2} \int_{t-\tau_{\max}}^t \frac{1}{2} Q_i(x_i(\zeta)) d\zeta \right. \\ \left. + 1 + \frac{\lambda_{\max}(M)}{2} \right] + \beta_{i0} + \frac{1}{2}, & e_i \in \Xi_{\hat{b}_i}^o \\ 0, & i = 1, 2, \dots, N, \quad e_i \in \Xi_{\hat{b}_i} \end{cases} \quad (23)$$

where  $\beta_{i0} > 0$  and  $k_i > 0$ . RBFNN weight matrix updating law is given in (12) and the initial conditions satisfy (13) and (17). Then, the nonlinear multiagent systems will reach consensus.

*Proof:* First, we concentrate on the case where  $e_i \in \Xi_{\hat{b}_i}^o$ ,  $i = 1, 2, \dots, N$ . We construct a Lyapunov function

$$\begin{aligned} V(t) &= V_x(t) + L_Q(t) + \frac{1}{2} \sum_{i=1}^N \text{tr} \left( \frac{1}{a_i} \tilde{\mathcal{W}}_i^T \tilde{\mathcal{W}}_i \right) \\ &= \frac{1}{2} x^T (\mathcal{L} \otimes I_m) x + \frac{1}{2} \sum_{i=1}^N \int_{t-\tau_i}^t Q_i(x_i(\zeta)) d\zeta \\ &\quad + \frac{1}{2} \sum_{i=1}^N \text{tr} \left( \frac{1}{a_i} \tilde{\mathcal{W}}_i^T \tilde{\mathcal{W}}_i \right) \end{aligned} \quad (24)$$

where  $x = [x_1^T, x_2^T, \dots, x_N^T]^T \in \mathbb{R}^{mN}$ ,  $\tilde{\mathcal{W}}_i = \mathcal{W}_i^* - \hat{\mathcal{W}}_i$ , and  $V_x(t) = (1/2)x^T (\mathcal{L} \otimes I_m) x$ . Furthermore, the communication topology is undirected and connected. Thus, zero is an  $m$ -multiplicity eigenvalue of  $\mathcal{L} \otimes I_m$  and we define the corresponding eigenvectors of these zero eigenvalues as the following form:

$$\begin{aligned} \hat{l}_1 &= [\hat{l}_1^T, \hat{l}_1^T, \dots, \hat{l}_1^T]^T \in \mathbb{R}^{mN} \\ \hat{l}_2 &= [\hat{l}_2^T, \hat{l}_2^T, \dots, \hat{l}_2^T]^T \in \mathbb{R}^{mN} \\ &\vdots \\ \hat{l}_m &= [\hat{l}_m^T, \hat{l}_m^T, \dots, \hat{l}_m^T]^T \in \mathbb{R}^{mN} \end{aligned} \quad (25)$$

where  $\hat{l}_i \in \mathbb{R}^m$  is a column vector whose  $i$ th element is  $1/\sqrt{N}$  with all the other elements zero. Consequently, assume that  $\hat{l}_{m+1}, \hat{l}_{m+2}, \dots, \hat{l}_{mN}$  are the remaining eigenvectors of matrix  $\mathcal{L} \otimes I_m$ . Therefore, we choose  $\hat{l}_1, \hat{l}_2, \dots, \hat{l}_{mN}$  as a set of orthogonal bases of space  $\mathbb{R}^{mN}$ . With the aid of matrix theory, we can obtain that  $T = [\hat{l}_1, \hat{l}_2, \dots, \hat{l}_{mN}] \in \mathbb{R}^{mN \times mN}$  and  $T^T T = T T^T = I_{mN}$ , where  $T^T = T^{-1}$ . Hence

$$\begin{aligned} x^T (\mathcal{L} \otimes I_m) x &= x^T T^T \Lambda T x \\ &= x^T T^T \sqrt{\Lambda} \sqrt{\Lambda} T x \\ &= x^T T^T \sqrt{\Lambda} \sqrt{\Lambda} \sqrt{\Lambda}^{-1} \sqrt{\Lambda}^{-1} \sqrt{\Lambda} \sqrt{\Lambda} T x \\ &= x^T T^T \Lambda T T^T \bar{\Lambda}^{-1} T T^T \Lambda T x \\ &= x^T (\mathcal{L} \otimes I_m)^T M (\mathcal{L} \otimes I_m) x \\ &= e^T M e \end{aligned} \quad (26)$$

where

$$\begin{aligned} \Lambda &= \text{diag}(0I_m, \lambda_2 I_m, \lambda_3 I_m, \dots, \lambda_n I_m) \\ \sqrt{\Lambda} &= \text{diag}(0I_m, \sqrt{\lambda_2} I_m, \sqrt{\lambda_3} I_m, \dots, \sqrt{\lambda_n} I_m) \\ \bar{\Lambda} &= \text{diag}(\lambda_2 I_m, \lambda_2 I_m, \lambda_3 I_m, \dots, \lambda_n I_m) \\ \sqrt{\bar{\Lambda}} &= \text{diag}(\sqrt{\lambda_2} I_m, \sqrt{\lambda_2} I_m, \sqrt{\lambda_3} I_m, \dots, \sqrt{\lambda_n} I_m) \end{aligned}$$

$M = T^T \bar{\Lambda}^{-1} T$  is a positive definite matrix and  $e = [e_1^T, e_2^T, \dots, e_N^T]^T$  is a column vector. Then, we have

$$\begin{aligned} \frac{dV(t)}{dt} &= \sum_{i=1}^N e_i^T \left[ \mathcal{W}_i^{*T} \Phi_i(x_i) + \theta_i + \xi_i + g_i(x_i(t - \tau_i)) \right. \\ &\quad \left. - \beta_i e_i - \hat{\mathcal{W}}_i^T \Phi_i(x_i) - \gamma_i \tanh\left(\frac{\kappa_i \gamma_i e_i}{\epsilon_i}\right) \right. \\ &\quad \left. - \operatorname{sgn}(e_i^T x_i) \psi_i(x_i) x_i - \frac{1}{2} \frac{e_i}{\|e_i\|^2} \phi_i^2(x_i) \right] \\ &\quad + \frac{1}{2} \sum_{i=1}^N [\phi_i^2(x_i(t)) - \phi_i^2(x_i(t - \tau_i))] \\ &\quad - \sum_{i=1}^N \operatorname{tr}\left(\frac{1}{a_i} \tilde{\mathcal{W}}_i^T \dot{\mathcal{W}}_i\right) \\ &\leq \sum_{i=1}^N \|e_i\| (\theta_{N_i} + \alpha_i + \phi_i(x_i(t - \tau_i))) \\ &\quad + \sum_{i=1}^N e_i^T \left[ \tilde{\mathcal{W}}_i^T \Phi_i(x_i) - \gamma_i \tanh\left(\frac{\kappa_i \gamma_i e_i}{\epsilon_i}\right) - \beta_i e_i \right] \\ &\quad - \sum_{i=1}^N \operatorname{sgn}(e_i^T x_i) \psi_i(x_i) e_i^T x_i - \sum_{i=1}^N \frac{1}{2} \phi_i^2(x_i(t - \tau_i)) \\ &\quad - \sum_{i=1}^N \operatorname{tr}\left(\frac{1}{a_i} \tilde{\mathcal{W}}_i^T \dot{\mathcal{W}}_i\right). \end{aligned}$$

With (14), (15b), and Young's inequality, we can infer that

$$\begin{aligned} \frac{dV(t)}{dt} &\leq \sum_{i=1}^N \left[ \gamma_i \|e_i\| - \gamma_i e_i^T \tanh\left(\frac{\kappa_i \gamma_i e_i}{\epsilon_i}\right) \right] \\ &\quad - \sum_{i=1}^N \beta_i \|e_i\|^2 + \frac{1}{2} \sum_{i=1}^N \|e_i\|^2 \\ &\quad + \sum_{i=1}^N \frac{1}{2} \phi_i^2(x_i(t - \tau_i)) \\ &\quad - \sum_{i=1}^N \operatorname{tr}\left[ \tilde{\mathcal{W}}_i^T \left( \frac{1}{a_i} \dot{\mathcal{W}}_i - \Phi_i(x_i) e_i^T \right) \right] \\ &\quad - \sum_{i=1}^N \frac{1}{2} \phi_i^2(x_i(t - \tau_i)) \\ &\leq \sum_{i=1}^N \epsilon_i - \sum_{i=1}^N \left( \beta_i - \frac{1}{2} \right) \|e_i\|^2 \\ &\quad - \sum_{i=1}^N \operatorname{tr}\left[ \tilde{\mathcal{W}}_i^T \left( \frac{1}{a_i} \dot{\mathcal{W}}_i - \Phi_i(x_i) e_i^T \right) \right]. \end{aligned}$$

According to (12), we discuss the following two cases.

1) If  $\dot{\mathcal{W}}_i = a_i \Phi_i(x_i) e_i^T$ , then

$$\operatorname{tr}\left[ \tilde{\mathcal{W}}_i^T \left( \frac{1}{a_i} \dot{\mathcal{W}}_i - \Phi_i(x_i) e_i^T \right) \right] = 0.$$

2) If  $\dot{\mathcal{W}}_i = a_i \Phi_i(x_i) e_i^T - a_i (e_i^T \tilde{\mathcal{W}}_i^T \Phi_i(x_i)) / (\operatorname{tr}(\tilde{\mathcal{W}}_i^T \tilde{\mathcal{W}}_i)) \hat{\mathcal{W}}_i$ , then

$$\operatorname{tr}\left[ \tilde{\mathcal{W}}_i^T \left( \frac{1}{a_i} \dot{\mathcal{W}}_i - \Phi_i(x_i) e_i^T \right) \right] = - \frac{e_i^T \hat{\mathcal{W}}_i^T \Phi_i(x_i)}{\operatorname{tr}(\hat{\mathcal{W}}_i^T \hat{\mathcal{W}}_i)} \operatorname{tr}(\tilde{\mathcal{W}}_i^T \hat{\mathcal{W}}_i). \quad (27)$$

Furthermore

$$\begin{aligned} \operatorname{tr}(\tilde{\mathcal{W}}_i^T \hat{\mathcal{W}}_i) &= \operatorname{tr}(\tilde{\mathcal{W}}_i^T \mathcal{W}_i^*) - \operatorname{tr}(\tilde{\mathcal{W}}_i^T \tilde{\mathcal{W}}_i) \\ &= \frac{1}{2} \left[ \operatorname{tr}(\tilde{\mathcal{W}}_i^T \mathcal{W}_i^*) + \operatorname{tr}(\mathcal{W}_i^{*T} \tilde{\mathcal{W}}_i) \right] - \operatorname{tr}(\tilde{\mathcal{W}}_i^T \tilde{\mathcal{W}}_i) \\ &= \frac{1}{2} \left[ \operatorname{tr}(\mathcal{W}_i^{*T} \mathcal{W}_i^*) - \operatorname{tr}(\hat{\mathcal{W}}_i^T \mathcal{W}_i^*) \right] \\ &\quad + \frac{1}{2} \left[ \operatorname{tr}(\hat{\mathcal{W}}_i^T \tilde{\mathcal{W}}_i) + \operatorname{tr}(\tilde{\mathcal{W}}_i^T \hat{\mathcal{W}}_i) \right] - \operatorname{tr}(\tilde{\mathcal{W}}_i^T \tilde{\mathcal{W}}_i) \\ &= \frac{1}{2} \operatorname{tr}(\mathcal{W}_i^{*T} \mathcal{W}_i^*) - \frac{1}{2} \operatorname{tr}(\hat{\mathcal{W}}_i^T \hat{\mathcal{W}}_i) - \frac{1}{2} \operatorname{tr}(\tilde{\mathcal{W}}_i^T \tilde{\mathcal{W}}_i) \\ &\leq 0 \end{aligned}$$

where

$$\operatorname{tr}(\tilde{\mathcal{W}}_i^T \tilde{\mathcal{W}}_i) \geq 0$$

and

$$\operatorname{tr}(\hat{\mathcal{W}}_i^T \hat{\mathcal{W}}_i) = \mathcal{W}_i^{\max} \geq \operatorname{tr}(\mathcal{W}_i^{*T} \mathcal{W}_i^*).$$

Then along with

$$\operatorname{tr}(\hat{\mathcal{W}}_i^T \hat{\mathcal{W}}_i) = \mathcal{W}_i^{\max} > 0 \quad \text{and} \quad e_i^T \hat{\mathcal{W}}_i^T \Phi_i(x_i) \geq 0$$

we can obtain that

$$\operatorname{tr}\left( \tilde{\mathcal{W}}_i^T \left( \frac{1}{a_i} \dot{\mathcal{W}}_i - \Phi_i(x_i) e_i^T \right) \right) \geq 0$$

holds. In terms of Assumption 2, we have

$$\frac{1}{2} \sum_{i=1}^N \int_{t-\tau_i}^t Q_i(x_i(\zeta)) d\zeta \leq \frac{1}{2} \sum_{i=1}^N \int_{t-\tau_{\max}}^t Q_i(x_i(\zeta)) d\zeta.$$

Consequently

$$\begin{aligned} \frac{dV(t)}{dt} &\leq - \sum_{i=1}^N \beta_{i0} \|e_i\|^2 - \sum_{i=1}^N \frac{1}{k_i} \int_{t-\tau_{\max}}^t \frac{1}{2} Q_i(x_i(\zeta)) d\zeta \\ &\quad - \sum_{i=1}^N \frac{1}{k_i} \left[ 1 + \frac{\lambda_{\max}(M)}{2} \right] \|e_i\|^2 + \sum_{i=1}^N \epsilon_i \\ &\leq - \frac{1}{k} V_x(t) - \frac{1}{k} L_Q(t) + \epsilon \\ &\quad - \sum_{i=1}^N \frac{2\mathcal{W}_i^{\max}}{ka_i} + \sum_{i=1}^N \frac{2\mathcal{W}_i^{\max}}{ka_i} \\ &\leq - \frac{1}{k} V_x(t) - \frac{1}{k} L_Q(t) - \frac{1}{2k} \sum_{i=1}^N \operatorname{tr}\left( \frac{1}{a_i} \tilde{\mathcal{W}}_i^T \tilde{\mathcal{W}}_i \right) \\ &\quad + \sum_{i=1}^N \frac{2\mathcal{W}_i^{\max}}{ka_i} + \epsilon \\ &\leq - \frac{1}{k} V(t) + \sum_{i=1}^N \frac{2\mathcal{W}_i^{\max}}{ka_i} + \epsilon \end{aligned}$$

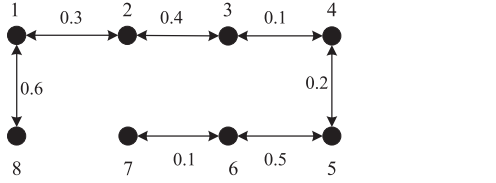


Fig. 2. Topology of example with eight agents.

where  $k = \max_{i \in \mathcal{V}} k_i$  and  $\epsilon = \sum_{i=1}^N \epsilon_i$ . Then according to [29, Lemma 1.1], we have

$$V(t) \leq V(0)e^{-\frac{1}{k}t} + v(1 - e^{-\frac{1}{k}t}) \quad (28)$$

where  $v = \sum_{i=1}^N (2\mathcal{W}_i^{\max})/a_i + k\epsilon$ .

Therefore, as  $t \rightarrow \infty$ ,  $V(t)$  is bounded. Next, we demonstrate that  $e_i(t)$  can be reduced to an arbitrarily small domain.

With (24) and (28), we obtain

$$\frac{\lambda_{\min}(M)}{2} \sum_{i=1}^N \|e_i(t)\|^2 \leq V_x(t) \leq v + V(0)e^{-\frac{1}{k}t}.$$

That is

$$\sum_{i=1}^N \|e_i(t)\|^2 \leq \frac{2}{\lambda_{\min}(M)} v + \frac{2}{\lambda_{\min}(M)} V(0)e^{-\frac{1}{k}t}. \quad (29)$$

Thus, there exists  $t_0 > 0$  such that if  $t > t_0$ , then we can obtain

$$\|e_i(t)\| < \sqrt{\frac{2}{\lambda_{\min}(M)}} v, \quad i = 1, 2, \dots, N.$$

Choosing  $\mathcal{W}_i^{\max}$ ,  $a_i$ ,  $k_i$ , and  $\epsilon_i$  properly, we can eventually derive  $e_i(t) \in \Xi_{\hat{b}_i}$ ,  $i = 1, 2, \dots, N$ , where

$$\hat{b}_i \geq \sqrt{\frac{2}{\lambda_{\min}(M)}} v, \quad i = 1, 2, \dots, N.$$

As we have stated before while talking about  $-(1/2)(e_i/\|e_i\|^2)\phi_i^2(x_i)$  in (9),  $\exists k \in \mathcal{V}$  such that  $e_k \in \Xi_{\hat{b}_k}$ , we can suppose that  $x_k$  has achieved the consensus state of its own and hold  $x_k$  still. We set  $u_k = 0$  and drive other  $N - 1$  agents toward consensus. Moreover, we utilize the same trick for the case where there are more than one agent who come up with the similar conditions above. Finally, when  $e_i(t) \in \Xi_{\hat{b}_i}$ ,  $\forall i$ , the consensus will be achieved. ■

#### IV. SIMULATION EXAMPLE

In this example, a multiagent system containing eight agents is shown in Fig. 2. Each agent can represent a robot that moves on the plane with  $x$ - and  $y$ -axis.  $x_i = [x_{i1}, x_{i2}]^T$  is the position of agent  $i$ . In addition, the dynamics of the multiagent system is described by the following equations:

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \end{bmatrix} &= \begin{bmatrix} x_{i2}(t) \sin(p_{i1}x_{i1}(t)) \\ x_{i1}(t) \cos(p_{i2}x_{i2}(t)) \end{bmatrix} + u_i + \xi_i \\ &+ \begin{bmatrix} s_{i1}x_{i1}^2(t - \tau_i) \\ s_{i2}x_{i2}(t - \tau_i) \sin(x_{i1}(t - \tau_i)) \end{bmatrix} \end{aligned} \quad (30)$$

TABLE I  
COEFFICIENT VALUES OF AGENT  $i$

$i$	1	2	3	4	5	6	7	8
$p_{i1}$	0.4	-0.65	8	1	-10	1.5	0.5	-1
$p_{i2}$	0.5	0.45	-6	11	11	9	2	5

TABLE II  
COEFFICIENT VALUES OF TIME DELAY OF AGENT  $i$

$i$	1	2	3	4	5	6	7	8
$s_{i1}$	0.9	1.2	-1.1	-0.7	0.6	0.3	0.2	0.6
$s_{i2}$	1.2	0.8	0.6	0.3	0.8	0.4	-0.1	-0.4

TABLE III  
TIME DELAY OF AGENT  $i$

$i$	1	2	3	4	5	6	7	8	$\tau_{\max}$
$\tau_i$	0.1	0.05	0.15	0.08	0.18	0.1	0.01	0.11	0.2

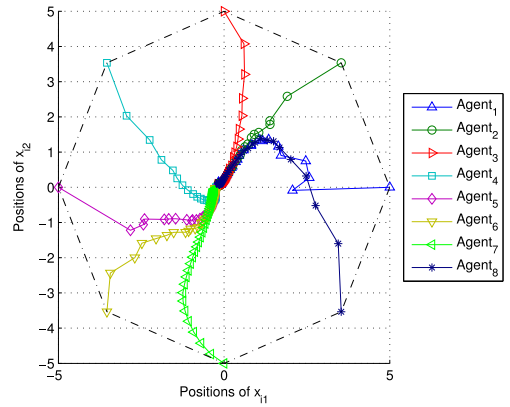


Fig. 3. Trajectories of eight agents on the plane.

where  $p_{i1}, p_{i2}$  and  $s_{i1}, s_{i2}$  are the corresponding constant coefficients given in Tables I and II. We choose  $\phi_i(x_i) = \sqrt{(s_{i1}x_{i1}^2)^2 + (s_{i2}x_{i2})^2}$ .  $\tau_i$  and  $\tau_{\max}$  are time delays shown in Table III.  $\xi = [\xi_1^T, \xi_2^T, \xi_3^T, \xi_4^T, \xi_5^T, \xi_6^T, \xi_7^T, \xi_8^T]^T$  is the external noise vector added to control input, where

$$\begin{bmatrix} \xi_1^T \\ \xi_2^T \\ \xi_3^T \\ \xi_4^T \\ \xi_5^T \\ \xi_6^T \\ \xi_7^T \\ \xi_8^T \end{bmatrix} = \begin{bmatrix} \exp(-2t) & \sin(2t^2) \\ \exp(-3t) & \cos(0.5t^2) \\ \sin(t^2) & \sin(t) \exp(-3t) \\ -\cos(t^2) & \cos(t) \exp(-2t) \\ -\cos(t^2) \sin(t^2) & \sin(-t) \exp(-3t) \\ \sin(t^2) \exp(-5t) & \sin(2t) \cos(t^2) \\ -\cos(t) & \exp(-2t) \\ \exp(-5t) & \cos(-0.1t^2) \end{bmatrix}.$$

We suppose that the initial states of the multiagent system are on a circle with radius 5. We suppose that all the eight agents have the same parameters.  $\beta_{i0} = 50$ ,  $\beta_i = 50$ ,  $\mathcal{W}_i^{\max} = 100$ ,  $a_i = 100$ ,  $\hat{b}_i = 4 \times 10^{-3}$ ,  $\gamma_i = 2$ ,  $\kappa_i = 0.2785$ ,  $\epsilon_i = 0.01$ ,  $c_1 = 7$ ,  $c_2 = 10$ , and  $\eta_i = 50$ . The number of neurons for each RBFNN is 16 and  $\sigma_i^2 = 2$ .  $\mu_i$ s are distributed uniformly among the range  $[-5, 5] \times [-5, 5]$ .

Our control objective is to drive the eight robots on the plane to the consensus states, i.e., to the same position. Fig. 3 illustrates that the eight agents can reach consensus with the distributed control protocol (22) designed in Section III.

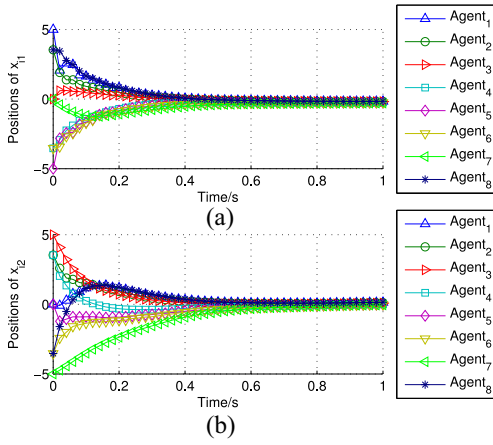


Fig. 4. Trajectories of two dimensions of eight agents on the plane. (a) First dimension trajectories of eight agents. (b) Second dimension trajectories of eight agents.

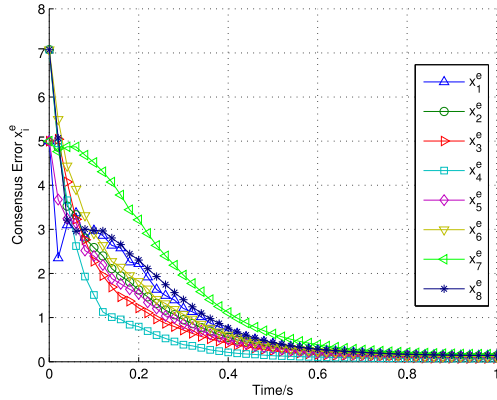


Fig. 5. Consensus error trajectories of eight agents.

Additionally, Fig. 4 shows the trajectories of two dimensions which further demonstrates that all the eight agents approach the consensus states eventually. To show the consensus error, we define the measurement of consensus error of each agent

$$x_i^e = \left| x_{i1} - \frac{1}{8} \sum_{j=1}^8 x_{j1} \right| + \left| x_{i2} - \frac{1}{8} \sum_{j=1}^8 x_{j2} \right|, \quad i = 1, 2, \dots, 8. \quad (31)$$

In Fig. 5, it shows that the consensus error of each agent can gradually approach the given ball-region  $\Xi_{\hat{b}_i}$ .

*Remark 2:* In Fig. 6, we remove the robustifying term from the control input. Then we can see that at  $t = 10$  s, it is difficult to obtain consensus comparing with what we see in Fig. 5. Feedback control term can decrease the error at the beginning. However, when the effect of feedback term becomes weak, the external noises dominate the signal which are difficult to be excluded. Therefore, comparing with [26], external noises are negative factors for control performance which cannot be neglected and it is of great importance to investigate the nonlinear multiagent systems especially including noises.

*Remark 3:* Actually, RBFNN needs a period of time to learn the unknown dynamics of the multiagent systems. Thus, the linear feedback control plays a dominant role at the beginning time. From the view of engineering and practical systems,

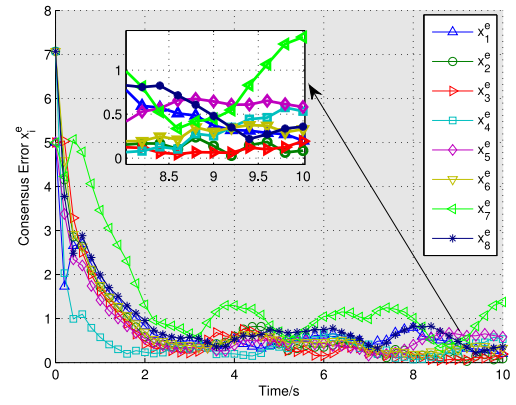


Fig. 6. Consensus error trajectories of eight agents without robustifying term.

the form of Lyapunov–Krasovskii functional (7) needs to specify the value of state when  $t < \tau_i$ . Here we assume that  $x_i(t) \equiv 0, i = 1, 2, \dots, N, \forall t < 0$ .

## V. CONCLUSION

We study a class of unknown nonlinear multiagent systems with external disturbances and time delays. With the help of NN techniques, we can approximate the unknown nonlinear dynamics of multiagent systems. Moreover, we use Lyapunov–Krasovskii functional and Young’s inequality to eliminate time delay effects. Meanwhile, we add a robustifying term to the control protocol so as to limit the negative effects of external disturbances. Our future work will concentrate on directed communication topology, leader–follower collision avoidance, and unknown coefficients of the states.

## APPENDIX

*Lemma 1:* If the distributed adaptive robust control protocol is defined by (9), then we can derive that

$$\|x_i(t)\| \leq c_2, \quad i = 1, 2, \dots, N, \quad \forall t \geq 0. \quad (32)$$

*Proof:* First, let  $P_{xi} = \frac{1}{2} x_i^T x_i = \frac{1}{2} \|x_i\|^2$ . Then

$$\begin{aligned} \frac{dP_{xi}(t)}{dt} &= x_i^T \dot{x}_i \\ &= x_i^T \left[ \mathcal{W}_i^{*T} \Phi_i(x_i) + \theta_i + \xi_i + g_i(x_i(t - \tau_i)) \right. \\ &\quad \left. - \beta_i e_i - \hat{\mathcal{W}}_i^T \Phi_i(x_i) - \gamma_i \tanh\left(\frac{\kappa_i \gamma_i e_i}{\epsilon_i}\right) \right. \\ &\quad \left. - \operatorname{sgn}(e_i^T x_i) \psi_i(x_i) x_i - \frac{1}{2} \frac{e_i}{\|e_i\|^2} \phi_i^2(x_i) \right] \\ &\leq \|x_i\| \|\tilde{\mathcal{W}}_i\|_F \|\Phi_i(x_i)\| + \|x_i\| \alpha_i + \|x_i\| \theta_{N_i} \\ &\quad + \|x_i\| \phi_i(x_i(t - \tau_i)) - \beta_i e_i^T x_i \\ &\quad + \gamma_i \|x_i\| \left\| \tanh\left(\frac{\kappa_i \gamma_i e_i}{\epsilon_i}\right) \right\| - \frac{1}{2} \frac{e_i^T x_i}{\|e_i\|^2} \phi_i^2(x_i) \\ &\quad - \operatorname{sgn}(e_i^T x_i) \psi_i(x_i) \|x_i\|^2 \\ &\leq - \left( \beta_i + \frac{\phi_i^2(x_i)}{2 \|e_i\|} \right) e_i^T x_i - \operatorname{sgn}(e_i^T x_i) \psi_i(x_i) \|x_i\|^2 \\ &\quad + \left( 2\sqrt{p_i} \sqrt{\mathcal{W}_i^{\max}} + (1 + \sqrt{m}) \gamma_i \right) \|x_i\| \\ &\quad + \|x_i\| \phi_i(x_i(t - \tau_i)) \end{aligned}$$

where  $p_i$  is the number of neurons of agent  $i$  and  $m$  is the dimension of  $x_i$ . The initial state of agent  $i$  is bounded in the range of  $\|x_i(0)\| < c_1$ . With the aid of contradiction, suppose that there exists  $t_1$  such that

$$\begin{cases} \|x_k(t_1)\| = c_2, \frac{dP_{xk}(t)}{dt}\Big|_{t=t_1} > 0 \\ \|x_i(t_1)\| \leq c_2, 0 \leq t \leq t_1, i \in \mathcal{V}/k. \end{cases} \quad (33)$$

Then

$$e_k^T x_k \geq d_k \|x_k\|^2 - \sum_{j \in \mathcal{N}_i} \mathcal{A}_{kj} \|x_j\| \|x_k\| \geq c_2^2 \left( d_k - \sum_{j \in \mathcal{N}_i} \mathcal{A}_{kj} \right) = 0.$$

According to Assumption 1 and (33) where  $\|x_i\| \leq c_2$  when  $t \leq t_1$ , we can obtain that  $\phi_i(x_i(t))$  has an upper bound  $\phi_i^{\max} = \max_{i \in \mathcal{V}, t \leq t_1} \phi_i(x_i(t))$ . Therefore

$$\begin{aligned} \frac{dP_{xk}(t)}{dt}\Big|_{t=t_1} &\leq \left( 2\sqrt{p_k} \sqrt{\mathcal{W}_k^{\max}} + (1 + \sqrt{m})\gamma_k \right) c_2 \\ &\quad + \phi_k^{\max} c_2 - \eta_k c_2^2. \end{aligned} \quad (34)$$

Subsequently, if

$$\eta_k > \frac{1}{c_2} \left( 2\sqrt{p_k} \sqrt{\mathcal{W}_k^{\max}} + (1 + \sqrt{m})\gamma_k + \phi_k^{\max} \right) \quad (35)$$

holds, then

$$\frac{dP_{xk}(t)}{dt}\Big|_{t=t_1} \leq 0$$

which is a contradiction with (33).

Therefore, if  $\|x_i(0)\| \leq c_1$ , then  $\|x_i(t)\| \leq c_2$ ,  $i = 1, 2, \dots, N$ ,  $\forall t \geq 0$ . ■

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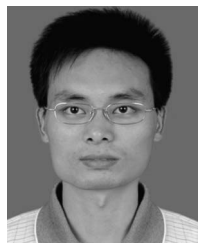
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