

A Novel Model Analysis Method and Dynamic Modelling for Hybrid Structure Flexible Manipulator

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Abstract - Tip deflection and vibration have a large impact on the manipulator operation safety, and hybrid structure flexible manipulator has a larger workspace and more flexibility. In this paper, a new modeling scheme for a hybrid structure flexible manipulator is proposed. In the proposed scheme, the joint variables of hybrid structure flexible manipulator are divided in rigid joint angle and small joint angle deviations. The prismatic joint is divided into three segments according to the structure features of flexible manipulator. The three segments are the foundation segment, overlap segment and extension segment. Each segment is modeled by the theory of the Euler-Bernoulli beam, and the elastic deflection is described by assumed mode method, the kinetic energy and potential energy of every segment is analyzed respectively. Base on the previous researches of modeling for multi-link flexible manipulator, the dynamic equations of the manipulator are obtained by using the Lagrangian approach. In order to verify the correctness of model and analyze the characteristics of model, the comparative simulation experiments are designed. Tip vibration, angle deviation, vibration frequency and amplitude are observed, and the characteristics of the model are analyzed according to the experiment results.

Index Terms – model analysis, dynamic modelling, hybrid structure flexible manipulator

I. INTRODUCTION

The flexible manipulator is a preferred to tradition rigid manipulator when high-speed, low power consumption, large workspace and small actuator are required. The applications of the flexible manipulator with light weight and long arm structure becoming much wider in practice, such as free-floating flexible space manipulator, aerial ladder car, hydraulic aerial cage and large crane. However, the links and the joints have elastic deflections that can't be neglected in the modelling and controlling of the flexible manipulator. In addition, tip of the manipulator is more likely to have a large deflection and the high frequency vibration. Those deflection and vibration have large impact on the operation safety. The performance and effect of the manipulator might become unacceptable because of the low positional accuracy and tip vibration.

In the past few decades, many researchers begin to study the flexible manipulator and many methods of flexible manipulator dynamic modelling and controlling are proposed [1]. But until now, most of researches still focus on the single link or two-link rotating manipulator, and the research of the

prismatic manipulator only considers the situation that the extension link is flexible. Few researchers concentrate on the hybrid structure flexible manipulator that includes rotating joints and prismatic joints [2]. The manipulator is more complex, higher coupling and more uncertainty compare with single link or two-link rotating manipulator. But in the practical application, the hybrid structure flexible manipulator is more commonly. Therefore, research on hybrid structure flexible manipulator has a practical value. In the process of research flexible manipulator, modelling for the hybrid structure flexible manipulator is the first step for the research.

Modelling for a flexible manipulator summarized as three steps. First, a beam theory is selected to simplify the flexible manipulator. Second, a mathematical method is used to describe elastic deflection. Third, a modelling method is applied to establish system dynamic equation.

In order to analysis flexible manipulator, two types of beam theory are used, that Euler-Bernoulli beam theory and Timoshenko beam theory. Different simplified theory could be used in the different situation. It depends on whether shears and reverse should be ignored. Some of researches were used Euler-Bernoulli beam theory to simplify the curved beam because it is simpler compared with Timoshenko beam theory. But Timoshenko beam theory is more complete and the model could be more accurate.

Finite element method, finite segment method, assumed mode method and lumped mass method are the typical method to describe the elastic deflection. Guang Li introduced and analyzed the finite element method to describe elastic deflection in his doctoral dissertation [3]. J.D. Connelly and R L Huston introduced the multibody dynamic model using the finite segment method [4-5]. However, modelling by the finite segment method has large errors that make the mode not accuracy. Many flexible manipulator modelling is based on the assumed mode method to describe the elastic deflection. Alaa Shawky presented a dynamic model that used assumed mode method to truncate n order modal [6]. The elastic deflection of the manipulator can be described by the assumed mode method in Guang Li's paper [7]. By using extended Hamilton's principle and assumed mode method, E.Mirzaee obtained dynamic equations of two-link flexible arm [8]. The lumped mass method used a number of discrete points to take place the flexible link. The spatial discretization was carried out with lumped mass method in V. Feliu and R. Morales's research [9-10].

There are many modelling methods for the rotating flexible manipulator such as Newton-Euler equation, Lagrangian approach, variational principle and Kane equation. In the literature, Shibata T [11], Gamarra-Rosado V O [12] Siciliano B [13] used the Newton-Euler equation modelling for the flexible system. Lagrangian approach has been used in most of researches about the flexible manipulator. Solving a functional extremum problem instead of solving dynamic equations by using variational principle, the functional extremum problem could make the problem more simply that analyzes the motion of the flexible manipulator [14]. Kane method is a common method that establishes differential equations of motion for dynamic systems. Using D'Alembert principle to establish kinetic equations could avoid the cumbersome derivation of dynamic function calculation. In addition to the methods mentioned above, there is other dynamics modelling methods, such as Bond Graph modelling. It was used to modelling in Amit Kumar's research [15].

In this paper, the assumed mode method that modified to express deflection of prismatic joint and a novel model analysis method for hybrid structure flexible manipulator are introduced in Section II. The rotating joint angles of hybrid structure flexible manipulator were decomposed into large rigid angles and small joint angle deviations. The prismatic joint was divided into three segments according to the flexible manipulator's structural features. In the Section III, dynamic model of the manipulator is established. The large rigid angles as known quantities because they could be obtained by solving out rigid link dynamics equations, and substituted the large rigid angles into the flexible dynamics equations to obtain the small joint angle deviations and modal variables. The simulation experiments are introduced and the experiment results are analyzed in the Section IV. Finally, the conclusions are summarized in Section V.

II. MODEL ANALYSIS

The object of research is a hybrid structure flexible manipulator that consists of a rotating joint and a prismatic joint. It has more complex structure, more state variables, and higher degree of coupling. But now, few researchers study the hybrid structure flexible manipulator. Establishing the dynamic model of the hybrid structure flexible manipulator is essential to the manipulator accuracy control and operational safety.

A. Assumed mode method

In our research, all the beams are considered as Euler-Bernoulli beam. The elastic deflection could be expressed as follow according to the assumed mode method.

$$w(x, t) = \sum_{i=1}^{\infty} \varphi_i(x) q_i(t). \quad (1)$$

Where $\varphi_i(x)$ is modal shape function and $q_i(t)$ is modal coordinate. In this paper, we only consider the first modal. Therefore, the elastic deflection can be expressed as

$$w(x, t) = \varphi(x) q(t). \quad (2)$$

Considering the beams are cantilever beam, so the modal variables and the modal functions could be expressed as follow according to feature of cantilever beam.

$$q(t) = A \cos \omega t + B \sin \omega t, \quad (3)$$

$$\varphi(x) = C_n [(\sin \beta x - \sinh \beta x) - \eta_n (\cos \beta x - \cosh \beta x)]. \quad (4)$$

Where

$$\eta_n = \frac{\sin \beta L + \sinh \beta L}{\cos \beta L + \cosh \beta L}. \quad (5)$$

Where ω_i is the natural frequency of vibration. Every natural frequency has its corresponding modal shape function and corresponding modal. In the model of this paper, only the first modal is considered in this paper, so $\beta L = 1.875104$.

For the hybrid structure flexible manipulator, the length of link is a variable and it leads to the change of β . That is to say,

$$\beta = \frac{\beta L}{L} = \frac{1.875}{L(t)}. \quad (6)$$

And then the modal shape function can be expressed as

$$\varphi(x) = \sin \frac{1.875}{L(t)} x - \sinh \frac{1.875}{L(t)} x - \eta_n (\cos \frac{1.875}{L(t)} x - \cosh \frac{1.875}{L(t)} x). \quad (7)$$

B. Model analysis

The traditional dynamic modelling analysis method solves the kinetic energy and potential energy of the whole system to obtain the dynamical equation directly. But the joint variables and the modal variables are mixed in the equation in this analysis method and it hardly to calculate the dynamic differential equations. In order to solve this problem, the joint variables of hybrid structure flexible manipulator are divided in rigid joint angle and small joint angle deviations. In addition, the prismatic joint is divided three segments that are foundation segment, overlap segment and extension segment. The three segments are shown in the Fig. 1. a_1, a_2 and a_3 express the segments respectively. q_1, q_2 and q_3 express the modal variables of every segment of prismatic joint.

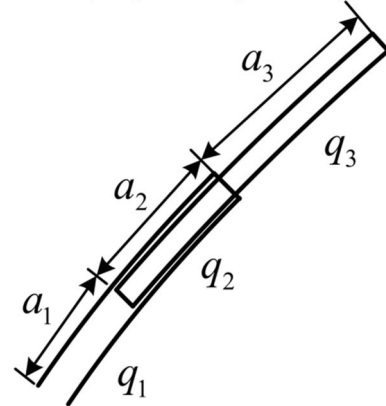


Fig. 1 three segments of prismatic joint.

The coordinate systems of the manipulator are established as show in Fig. 2. $w_1 \sim w_3$ represent elastic deflection. θ_1 and τ represent angles and driving moment of rotating joint, u and F represent the extend length and driving force of the prismatic joint. θ_2 and θ_3 represent the joint angles of the overlap segment and extension segment. θ_e represents the tip tangential angle. XOY is the base

coordinate system and the others are the reference coordinate system. $o_1 \sim o_3$ are the reference coordinate system of every segment, and the o'_2 is the reference coordinate system of the fix overlap segment and o'_3 is the reference coordinate system of the extension segment but the origin locate in the tip of overlap segment.

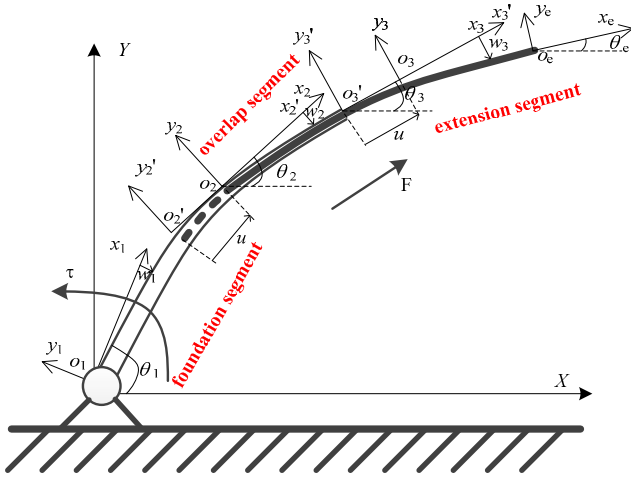


Fig. 2 hybrid structure flexible manipulator.

Assuming \mathbf{q} is a vector of the system state variable and it is expressed as

$$\mathbf{q} = \mathbf{q}_0 + \Delta\mathbf{q}, \quad (8)$$

$$\mathbf{q} = [\theta_1 \quad \theta_2 \quad \theta_3 \quad u \quad q_1 \quad q_2 \quad q_3]^T, \quad (9)$$

$$\mathbf{q}_0 = [\theta_1 \quad \theta_1 \quad \theta_1 \quad u \quad 0 \quad 0 \quad 0]^T, \quad (10)$$

$$\Delta\mathbf{q} = [0 \quad \Delta\theta_2 \quad \Delta\theta_3 \quad 0 \quad q_1 \quad q_2 \quad q_3]^T. \quad (11)$$

\mathbf{q}_0 is the vector of the rigid joint variables and $\Delta\mathbf{q}$ is the vector of the small joint angle deviations and the modal variables. The rigid joint variables and the small joint angle deviations are modeled respectively. Moreover, every segment of the prismatic joint is analyzed to obtain its kinetic energy and potential energy respectively. For the rigid joint variables, they could be obtained easily according to the research of rigid link dynamics. Then, the rigid joint variables are substituted into the dynamical equation to solve the small joint angle deviations [16].

Because all the beams are considered as Euler-Bernoulli beam, the axial displacement is neglected and arbitrary point P in the every segment could be expressed as

$$\begin{aligned} \bar{\mathbf{r}}_1 &= T_1 \begin{bmatrix} x_1 \\ w_1(x_1, t) \end{bmatrix} \\ \bar{\mathbf{r}}_2 &= \bar{\mathbf{R}}_2 + T_2 \begin{bmatrix} x_2' \\ w_2(x_2' - u, t) \end{bmatrix} \\ \bar{\mathbf{r}}_3 &= \bar{\mathbf{R}}_3 + T_2 \begin{bmatrix} x_2 \\ w_2(x_2, t) \end{bmatrix} \\ \bar{\mathbf{r}}_4 &= \bar{\mathbf{R}}_4 + T_3 \begin{bmatrix} x_3 + u \\ w_3(x_3 + u, t) \end{bmatrix} \end{aligned} \quad (12)$$

The $\bar{\mathbf{r}}_2, \bar{\mathbf{r}}_3$ both express the position vector that P in the overlap segment but $\bar{\mathbf{r}}_2$ express vector that P in the fixed link

reference to the coordinate system $o'_2, \bar{\mathbf{r}}_3$ express vector that P in the moving link reference to the coordinate system o_2 .

Where $\bar{\mathbf{R}}_i$ represents a position vector that the origin of reference coordinate system in the base coordinates system, and T_j is a transfer matrix that the reference coordinates system to the base coordinate system.

In the plane, the T_j is expressed as

$$T_j = \begin{bmatrix} \cos \theta_j & -\sin \theta_j \\ \sin \theta_j & \cos \theta_j \end{bmatrix}. \quad (13)$$

III. DYNAMIC MODELLING

In the (12), w_i represent the elastic deflection of the i th segment, it is a multi-variable function about time and position. Because the hybrid structure flexible manipulator consist of the prismatic joint, so the x_i is also a function of time. The derivative of w_i could be expressed as

$$\frac{d}{dt} w_i(x_i, t) = \frac{\partial w_i}{\partial t} + \frac{\partial w_i}{\partial x_i} \frac{\partial x_i}{\partial t} = \dot{w}_i + w'_i \frac{\partial x_i}{\partial t}. \quad (14)$$

The velocity of P could be obtained according to the (12) and (14), so it is expressed as

$$\bar{\mathbf{v}}_i = \dot{\bar{\mathbf{r}}}_i = \dot{\bar{\mathbf{R}}}_i + \dot{T}_j \begin{bmatrix} x_j \\ w_j \end{bmatrix} + T_j \begin{bmatrix} 0 \\ \frac{dw_j}{dt} \end{bmatrix}. \quad (15)$$

In view that the transfer matrix T_j is an antisymmetric matrix, the derivative of T_j could be expressed as

$$\dot{T}_j = S T_j \dot{\theta}_j. \quad (16)$$

S is an antisymmetric unit matrix.

Small joint angle deviations exist between θ_2, θ_3 with θ_1 , so the joint angles θ_2 and θ_3 could be expressed as follow

$$\theta_2 = \theta_1 + \Delta\theta_2, \theta_3 = \theta_1 + \Delta\theta_3. \quad (17)$$

Because the $\Delta\theta_i$ is very small, we simplify the formulas as (18), and ignore the terms higher than the second order.

$$\cos \Delta\theta_i = 1, \sin \Delta\theta_i = \Delta\theta_i. \quad (18)$$

Then, the (17) and (18) are substituted into the (15). Base on the Wen Chen's research [16], we could obtain a linear expression of the small joint angle deviations and modal variables as follow.

$$\begin{aligned} \bar{\mathbf{v}}_i &= E_{i0} + E_{i1} \Delta\theta + E_{i2} \Delta\dot{\theta} + E_{i3} \mathbf{Q} + E_{i4} \dot{\mathbf{Q}} \\ &= E_{i0} + E_{i1} \begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \end{bmatrix} + E_{i2} \begin{bmatrix} \Delta\dot{\theta}_2 \\ \Delta\dot{\theta}_3 \end{bmatrix} + E_{i3} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + E_{i4} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}. \end{aligned} \quad (19)$$

The kinetic energy of the hybrid structure flexible manipulator could be obtained according to (19), it could be expressed as

$$\begin{aligned} K &= \frac{1}{2} \sum_{i=1}^4 \int_{L_{i0}}^{L_{i1}} \rho_i \bar{\mathbf{v}}_i^T \bar{\mathbf{v}}_i dx_i \\ &= \frac{1}{2} \sum_{i=1}^4 \int_{L_{i0}}^{L_{i1}} \rho_i (E_{i0}^T E_{i0} + 2E_{i0}^T E_{i1} \Delta\theta + 2E_{i0}^T E_{i2} \Delta\dot{\theta} + 2E_{i0}^T E_{i3} \mathbf{Q} + 2E_{i0}^T E_{i4} \dot{\mathbf{Q}} \\ &\quad + \Delta\theta^T E_{i1}^T E_{i1} \Delta\theta + 2\Delta\theta^T E_{i1}^T E_{i2} \Delta\dot{\theta} + 2\Delta\theta^T E_{i1}^T E_{i3} \mathbf{Q} + 2\Delta\theta^T E_{i1}^T E_{i4} \dot{\mathbf{Q}} \\ &\quad + \Delta\dot{\theta}^T E_{i2}^T E_{i2} \Delta\dot{\theta} + 2\Delta\dot{\theta}^T E_{i2}^T E_{i3} \mathbf{Q} + 2\Delta\dot{\theta}^T E_{i2}^T E_{i4} \dot{\mathbf{Q}} \\ &\quad + \mathbf{Q}^T E_{i3}^T E_{i3} \mathbf{Q} + 2\mathbf{Q}^T E_{i3}^T E_{i4} \dot{\mathbf{Q}} + E_{i4}^T E_{i4} \dot{\mathbf{Q}}) dx_i \end{aligned} \quad (20)$$

Where

$$\rho_1 = \rho_2 = \rho_m; \rho_3 = \rho_4 = \rho_c, \quad (21)$$

ρ_m, ρ_c represent the mass per unit length of fixed link and moving link respectively.

$$\begin{aligned} L_{B1} &= 0; L_{E1} = l_1 + u \\ L_{B2} &= u; L_{E2} = l_2 \\ L_{B3} &= 0; L_{E3} = l_2 - u \\ L_{B4} &= -u; L_{E4} = l_3 \end{aligned} \quad (22)$$

Simplify (20), and we obtain the expression of the kinetic energy as follow.

$$\begin{aligned} K &= \frac{1}{2} (E_0^T E_0 + 2E_0^T E_1 \Delta \theta + 2E_0^T E_2 \Delta \dot{\theta} + 2E_0^T E_3 \mathbf{Q} + 2E_0^T E_4 \dot{\mathbf{Q}} \\ &+ \Delta \theta^T E_1^T E_1 \Delta \theta + 2\Delta \theta^T E_1^T E_2 \Delta \dot{\theta} + 2\Delta \theta^T E_1^T E_3 \mathbf{Q} + 2\Delta \theta^T E_1^T E_4 \dot{\mathbf{Q}} \\ &+ \Delta \dot{\theta}^T E_2^T E_2 \Delta \dot{\theta} + 2\Delta \dot{\theta}^T E_2^T E_3 \mathbf{Q} + 2\Delta \dot{\theta}^T E_2^T E_4 \dot{\mathbf{Q}} \\ &+ \mathbf{Q}^T E_3^T E_3 \mathbf{Q} + 2\mathbf{Q}^T E_3^T E_4 \dot{\mathbf{Q}} + \dot{\mathbf{Q}}^T E_4^T E_4 \dot{\mathbf{Q}}) \end{aligned} \quad (23)$$

The potential energy of arbitrary point P may be calculated as

$$\begin{aligned} P_i &= g \int_{L_{Bi}}^{L_{Ei}} \rho_i [0 \quad 1] \tilde{r}_i dx_i + \frac{1}{2} \int_{L_{Bi}}^{L_{Ei}} EI_i \left(\frac{\partial^2 w_i}{\partial x_i^2} \right)^2 dx_i \\ &= H_{i0} + H_{i1} \Delta \theta + H_{i2} \mathbf{Q} + \frac{1}{2} \mathbf{Q}^T H_{i3} \mathbf{Q} \end{aligned} \quad (24)$$

The potential energy of the overall system is obtained by adding the potential energy of all the segments, it is expressed as

$$\begin{aligned} P &= g \sum_{i=1}^4 \int_{L_{Bi}}^{L_{Ei}} \rho_i [0 \quad 1] \tilde{r}_i dx_i + \frac{1}{2} \sum_{i=1}^4 \int_{L_{Bi}}^{L_{Ei}} EI_i \left(\frac{\partial^2 w_i}{\partial x_i^2} \right)^2 dx_i \\ &= H_0 + H_1 \Delta \theta + H_2 \mathbf{Q} + \frac{1}{2} \mathbf{Q}^T H_3 \mathbf{Q} \end{aligned} \quad (25)$$

Using the Lagrangian approach and the equations of motion of the manipulator could be obtained,

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \Delta \dot{\mathbf{q}}} \right) - \frac{\partial L}{\partial \Delta \mathbf{q}} = \Delta \mathbf{T} \quad (26)$$

Where

$$\begin{aligned} L &= K - P \\ &= \frac{1}{2} (E_0^T E_0 + 2E_0^T E_1 \Delta \theta + 2E_0^T E_2 \Delta \dot{\theta} + 2E_0^T E_3 \mathbf{Q} + 2E_0^T E_4 \dot{\mathbf{Q}} \\ &+ \Delta \theta^T E_1^T E_1 \Delta \theta + 2\Delta \theta^T E_1^T E_2 \Delta \dot{\theta} + 2\Delta \theta^T E_1^T E_3 \mathbf{Q} + 2\Delta \theta^T E_1^T E_4 \dot{\mathbf{Q}} \\ &+ \Delta \dot{\theta}^T E_2^T E_2 \Delta \dot{\theta} + 2\Delta \dot{\theta}^T E_2^T E_3 \mathbf{Q} + 2\Delta \dot{\theta}^T E_2^T E_4 \dot{\mathbf{Q}} \\ &+ \mathbf{Q}^T E_3^T E_3 \mathbf{Q} + 2\mathbf{Q}^T E_3^T E_4 \dot{\mathbf{Q}} + \dot{\mathbf{Q}}^T E_4^T E_4 \dot{\mathbf{Q}}) \\ &- H_0 - H_1 \Delta \theta - H_2 \mathbf{Q} - \frac{1}{2} \mathbf{Q}^T H_3 \mathbf{Q} \\ \Delta \mathbf{T} &= [\Delta \tau_2 \quad \Delta \tau_3 \quad 0 \quad 0 \quad 0]^T \end{aligned} \quad (27)$$

According to (20), (25) and (26), the equations of motion could be computed as

$$\mathbf{M} \Delta \ddot{\mathbf{q}} + \mathbf{C} \Delta \dot{\mathbf{q}} + \mathbf{K} \Delta \mathbf{q} = \Delta \mathbf{T} + \mathbf{N} \quad (28)$$

In order to simplify the equations of motion, (11) could be rewrite as follow,

$$\Delta \mathbf{q} = [\Delta \theta^T \quad \mathbf{Q}^T]^T = [\Delta \theta_2 \quad \Delta \theta_3 \quad q_1 \quad q_2 \quad q_3]^T \quad (29)$$

Where

$$\mathbf{M} = \begin{bmatrix} M_\theta & M_{\theta Q} \\ M_{Q\theta} & M_Q \end{bmatrix}; \mathbf{C} = \begin{bmatrix} C_\theta & C_{\theta Q} \\ C_{Q\theta} & C_Q \end{bmatrix}; \mathbf{K} = \begin{bmatrix} K_\theta & K_{\theta Q} \\ K_{Q\theta} & K_Q \end{bmatrix} \quad (30)$$

$$\mathbf{N} = [N_1^T \quad N_2^T]^T \quad (31)$$

A linearized dynamics model of the hybrid structure flexible manipulator is obtained through above equations. In the process of the modelling, the joint variables of hybrid structure flexible manipulator are divided in rigid joint angle and small joint angle deviations. We ignore the terms of $\Delta \theta_i$ higher than second order and simplify the dynamic equations. This model could be expressed as (29), \mathbf{M} , \mathbf{C} , \mathbf{K} are the function of the θ_i and u . According to the dynamic model of the rigid link, the θ_i and u are the known quantity. Through this modelling method, we could obtain more complex model that considering smaller joint angle deviation. The n-link hybrid structure flexible manipulator could be analyzed and obtained the dynamic model theoretically.

IV. EXPERIMENTAL AND NUMERICAL SIMULATION

In order to verify the proposed model, the comparative simulation experiments are designed. According to the model proposed, we design the hybrid structure flexible manipulator with hypothetical parameter values are shown in the Table I.

TABLE I
HYPOTHETICAL PARAMETER VALUES

Parameter	Symbol	Value	Unit
Density of link 1	ρ_m	0.3	kg/m
Density of link 2	ρ_c	0.25	kg/m
Length of first segment	l_1	1	m
Length of second segment	l_2	0.8	m
Length of third segment	l_3	1	m
Flexural rigidity of link 1	EI_1	0.227	N.m ²
Flexural rigidity of link 2	EI_2	0.3	N.m ²

The typical trajectories are given to the model, then the small joint angle deviations and tip deflection are calculated according to the above equations of motion and the characteristic of the model is analyzed. Different joint trajectories are applied to the manipulator, and the tip vibration, angle deviation, vibration frequency and amplitude are observed.

Firstly, the telescopic motion is only considered in the experiment. The begin tip position coordinate is $P_{t=3}(0, -2.8)$ and the target position coordinate is $P_{t=7}(0, -3.2)$. The joint position could be obtained as follow.

$$u(3) = 0, u(7) = 0.4, \theta_i(t) = 0 \quad (32)$$

Assumed the velocity of telescopic is zero when the $t = 3$ and $t = 7$, so

$$u'(3) = 0, u'(7) = 0 \quad (33)$$

Rotary motion is only considered in the second experiment. The tip position is given as follow. When $t = 3$ s, the manipulator began to rotation, the tip coordinate is $P_{t=3}(0, -2.8)$. The manipulator arrive the target position when $t = 7$ s, and the coordinate is $P_{t=7}(2.8, 0)$. According to kinematic dynamics of the rigid manipulator, the joint position could be solved as follow.

$$\theta_1(3) = -\frac{\pi}{2}, \theta_1(7) = 0, u(t) = 0 \quad (34)$$

Considering the velocity of the rotary joint are zero when $t = 3$ s and $t = 7$ s, so

$$\theta_1'(3) = -\frac{\pi}{2}, \theta_1'(7) = 0. \quad (36)$$

In our research, the third order polynomial trajectory is selected. According to the above initial conditions, the rotary joint trajectory and the prismatic joint trajectory are expressed as follow.

$$u(t) = -\frac{t^3}{80} + \frac{3t^2}{16} - \frac{63t}{80} + \frac{81}{80}, \quad (37)$$

$$\theta_1(t) = -\frac{\pi}{64}t^3 + \frac{15\pi}{64}t^2 - \frac{63\pi}{64}t + \frac{49\pi}{64}. \quad (38)$$

Substituting the rigid joint trajectories to the dynamic model, the small joint angle deviations and tip deflection are observed through simulation experiments. The experiment results are shown in the Fig. 3 and Fig. 4.

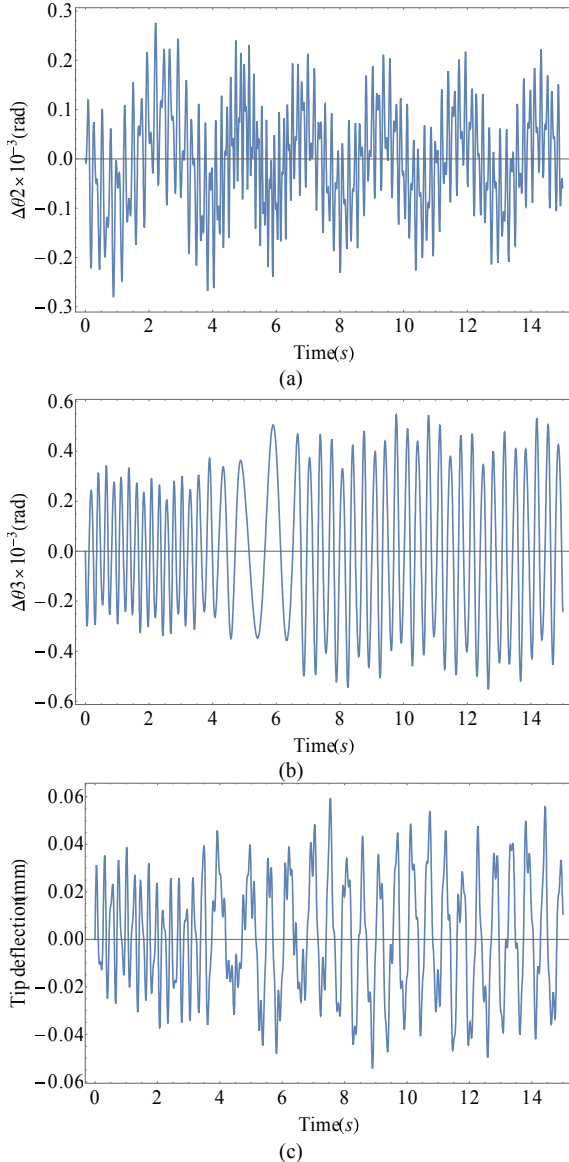


Fig. 3 (a) variation of $\Delta\theta_2$, (b) variation of $\Delta\theta_3$, (c) tip deflection of flexible manipulator when the telescopic motion is considered.

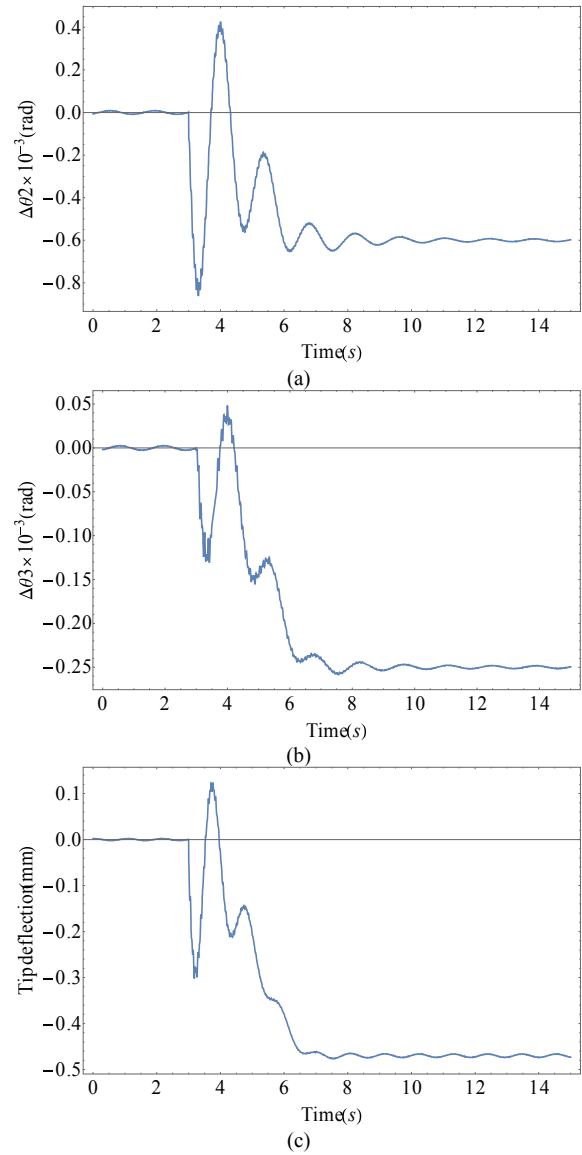


Fig. 4 (a) variation of $\Delta\theta_2$, (b) variation of $\Delta\theta_3$, (c) tip deflection of flexible manipulator when the rotary motion is considered.

In the Fig. 3(a) and Fig. 3(b), the small joint angle deviations are described. When $t < 3s$ or $t > 7s$, flexible manipulator is a stationary state, the small joint angle deviations are obvious. The fluctuations of $\Delta\theta_2$ is superimposed by the vibrations from multiple frequencies. Overlap segment and extension segment both have an impact on $\Delta\theta_2$. The vibration of $\Delta\theta_3$ is mainly determined by extension segment. Because every segment could generate a different frequency vibration, the vibration of tip deflection is also superimposed by the vibrations from multiple frequencies. The tip vibration is shown in the Fig. 3(c) and it shows the similar result with Wen Chen's research [16]. Observing the whole curve in the Fig. 3(a) and Fig. 3(b), we could find difference of the vibration frequency and amplitude. In the variety of the u , the increase of the u leads to the decrease of the overlap segment and the increase of the extension segment. At the same time, the vibration frequency of $\Delta\theta_3$ and the vibration amplitude of $\Delta\theta_2$ is decrease. While

the vibration frequency of $\Delta\theta_2$ and the vibration amplitude of $\Delta\theta_3$ is increase. Therefore, we get a result, that the vibration frequency of the manipulator decreases and the amplitude of manipulator increase by increasing length of manipulator. This result is the same as the Mete Kalyoncu's research[17].

The small joint angle deviations are shown in the Fig. 4(a) and Fig. 4(b) and the tip vibration is shown in the Fig. 4(c) when the joint angle θ_1 rotates as the (38). Due to the impact of gravity, $\Delta\theta_2$ and $\Delta\theta_3$ are the negative value when the $t > 7s$. The tip of manipulator also has a large deflection and it is accompanied by the multiple frequencies vibration.

According to all simulation results, there are severe problems could be found. First, the tip vibration couldn't be neglected in the process of modelling and controlling for the flexible manipulator. In our model, the lengths of link are far smaller than the actual manipulator, so the tip vibration has a large impact in the practical application of flexible link. Therefore, the vibration suppression control is a most important work for the process of controlling. Second, the tip vibration is consists of multiple frequencies vibrations. In addition, the first order modal is only considered in the process of modelling, and the actual flexible manipulator system include n-order modal and the high order not be expressed in the model. Third, the frequency and amplitude of vibration will be changed as the prismatic joint's motion. So the vibration suppression control will be very difficult. Fourth, the tip deflection is a problem for accurate positioning. In order to improving location accuracy, tip deflection and vibration of the flexible manipulator is a key problem and that must be solved.

V. CONCLUSIONS

This paper proposed a modeling method to obtain the hybrid structure flexible manipulator dynamic equations. The process of modelling could be summarized as five steps. Step 1: according to the characteristics of hybrid structure flexible manipulator, joint variables are decomposed large rigid angles and small angle deviations. Step 2: the prismatic joint is divided into three segments that are foundation segment, overlap segment and extension segment. Step 3: every segment of the flexible link is assumed Euler-Bernoulli beam and using assumed mode method to describe the elastic deflection. Step 4: the kinetic energy and potential energy of every segment are obtained by infinitesimal method respectively. Step 5: according to the model analysis, the dynamic modeling of the hybrid structure flexible manipulator is optimized and simplified.

In order to verify this model, the simulation experiments are designed. The typical trajectories are considered in the simulation experiments and the small joint angle deviations and tip vibration are observed. According to the results, the phenomenon of the tip vibration of the hybrid structure flexible manipulator is illustrated and we could get the conclusion that the length of link connect with the vibration frequency and amplitude. In addition, multivariable and strong coupling is typical feature of hybrid structure flexible manipulator. Designing a suitable control algorithm to

suppress tip vibration and compensate tip deflection is very important to the flexible manipulator's application in the ongoing research.

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