STUDY ON THE LOOP CONTROL STRUCTURE OF TRAFFIC FLOW BASED ON SELF-ORGANIZATION THEORY

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Abstract Based on self-organization theory and the feature that primary parameter is slow parameter and slaves other parameters, this paper points out a traffic flow real time control loop model that includes main loop and self-adaptive loop. This model can not only assure the real time feature for traffic flow control but also assure the precise of control.

Keyword self-organization primary parameter self-adaptive loop

1 Introduction
In these days, computer-based coordinated traffic control systems are adopted in most of the large cities in the world, they are used for realizing the optimal management-control of traffic flow. However, in order to set the control goal, optimal forecasting models are always needed in project design. Unfortunately, most of the current traffic forecasting models can't be obtained to meet the desired precision. In order to improve the control precision, on one hand we can develop new forecasting method with high accuracy, on the other hand we can also try to find out new control structures. In this paper, a new double loops control structure is put forward based on slaving principle in self-organization theory.

2 a New Double Loops Control Structure Based on Self-Organization Theory

Figure 1 A new control structure

C(t) — control scheme after identification of primary parameters
U(t) — condition of traffic flow primary parameters
V(t) — traffic flow condition
V'(t) — traffic flow data derived from vehicle loop sensor
X(t) — the state of traffic flow in road net

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control strategy, and then according to the new control strategy we make a modification of the loop of the predict loop to get the final optimal control scheme.

In the following parts we'll introduce this new control system in detail. There are two loops which are primary control loops (predictive control loop) and self-adaptive control loop. The main function of the primary control loop is to ensure the real-time control through finding out the optimal control scheme based on several-steps prediction of traffic flow parameters. And the function of the self-adaptive loop is to ensure the optimal control by making modification of the control scheme of the primary control loop on the basis of the control strategy which can match the current condition of traffic primary parameters derived by on-line parameter identification.

It is obvious that in order to make control of traffic flow system with high instability and complexity we must be sure that our control scheme is exactly a precise and real-time control scheme. Here in our control structure, control precision is ensured by the self-adaptive control loop while real-time control capability is ensured by the predictive control loop.

3 Traffic Flow Analysis to Assure Primary Parameter Process

there are n variable parameters (according to actual situation, adopted as following form: flow, speed, density or flow rate of traffic flow) in a subsystem in an un-balanced traffic flow system, nth dimension vector is used to denote it:

\[ \mathbf{q} = \{q_1, q_2, \ldots, q_j, \ldots, q_n\} \]  

Generally, the motion form of \( \mathbf{q} \) can be denoted by a general Langivin equation as followings:

\[ \dot{q}_j = K_j(\mathbf{q}) + \xi_j(t) \]  

\( \xi_j(t) \) is the random fluctuation caused by control variables.

\[
K_j(\mathbf{q}) = K_j(q_1, q_2, \ldots, q_n)
\]
denotes the non-linear function of variable parameters in system.

After \( \xi_j(t) \) is omitted, formula 3.2 can be represented as:

\[
\dot{q}_j = \sum_{k=1}^{n} a_{jk} q_k + f_j(\mathbf{q}) 
\]  

(3.3)

In formula (3.3), \( f_j(\mathbf{q}) \) is non-linear function of \( \mathbf{q} \). Because the stationary state point is steady, the real part of eigenvalue of its linearization matrix is negative and matrix \( (a_{jk}) \) is negative definitive. It is definite to diagonalize it using proper linear transformation, that is:

\[
q_1 = -\gamma_1 q_1 + g_1(q_1, q_2, \ldots, q_n) 
\]

\[
q_2 = -\gamma_2 q_2 + g_2(q_1, q_2, \ldots, q_n) 
\]

\[
q_n = -\gamma_n q_n + g_n(q_1, q_2, \ldots, q_n)
\]

(3.4)

for writing convenient, the new ordination in (3.4) is also denoted by \( q_1, q_2, \ldots, q_n \).

\( \gamma_i \) — positive damp coefficient;

\( g_i(\mathbf{q}) \) — a group functions of \( \mathbf{q} \);

from the formula (3.4), we can found that the motion of parameters in system is interrelated and desultory. If the random force \( \xi_j(t) \) is considered, they only can fluctuate in disorder.

In a non-linear phase-variable system, system parameters include primary parameter \( u \) and dissipation parameter \( A_D \). When the system is
close to critical point, the damp coefficient of primary parameters change to zero and the damp coefficient of other parameters is a non-zero bounded number. then, the primary parameter \( u \) will become critical slow-changing and the evolution of whole system will be dominated by \( u \) which will slave and dominate other parameters. For universality, we set:

\[
\begin{align*}
    u &= q_{j=1}; \quad A_D = q_{j=n} \\
    (3.5)
\end{align*}
\]

In formula (3.5), \( a \in \{2, 3, \ldots, n\} \), the formula (2.4) can be written as:

\[
\begin{align*}
    u &= -\gamma_1 u + g_1(u, q_2, \ldots, A_D, \Lambda, q_j, \Lambda, q_n) \\
    q_2 &= -\gamma_2 q_2 + g_2(u, q_2, \ldots, A_D, \Lambda, q_j, \Lambda, q_n) \\
    \ldots \ldots \\
    A_D &= -\gamma_n A_D + g_n(u, q_2, \ldots, A_D, \Lambda, q_j, \Lambda, q_n) \quad (3.6)
\end{align*}
\]

\[
\begin{align*}
    q_n &= -\gamma_n q_n + g_n(u, q_2, \ldots, A_D, \Lambda, q_j, \Lambda, q_n)
\end{align*}
\]

when the traffic flow of the system is close to the critical point, it is can be known from Synergetics theory that \( \gamma_1 \) tends towards zero and other \( \gamma_j > 0 \) (\( j = 2, 3, \ldots, n \)). So, other variables are quick-changing parameters except \( u \) that is slow-changing parameter. According to slaving principle, we set:

\[
\begin{align*}
    q_2 &= q_{a-1} = A_D = q_{a+1} = \ldots = q_n = 0 \quad (3.7)
\end{align*}
\]

It is can be deduced from formula (3.7) and (3.6):

\[
\begin{align*}
    -\gamma_2 q_2 + g_2(u, q_2, \Lambda, A_D, \Lambda, q_j, \Lambda, q_n) &= 0 \\
    \ldots \ldots \\
    -\gamma_n A_D + g_n(u, q_2, \Lambda, A_D, \Lambda, q_j, \Lambda, q_n) &= 0 \quad (3.8)
\end{align*}
\]

\[
\begin{align*}
    -\gamma q_2 + g(u, q_2, \Lambda, A_D, \Lambda, q_j, \Lambda, q_n) &= 0
\end{align*}
\]

Solve the (n-1) equations in (3.8), we get:

\[
\begin{align*}
    q_2 &= h_2(u) \\
    \ldots \ldots \\
    A_D &= h_n(u) \quad (3.9)
\end{align*}
\]

\[
\begin{align*}
    q_n &= h_n(n)
\end{align*}
\]

Hence, quick-changing parameters \( q_2, \Lambda, A_D, \Lambda, q_j, \Lambda, q_n \) are controlled by primary parameter \( u \). Submit (3.9) into the first equation of (2.6), the primary parameter equation is obtained:

\[
\begin{align*}
    u &= -\gamma u + g_1(u, q_2, \Lambda, A_D, \Lambda, q_j, \Lambda, q_n) \\
    &= -\gamma u + g_1(u, h_2(u), \Lambda, h_2(u), \Lambda, h_j(u), \Lambda, h_n(u)) \\
    &= -\gamma u + G(u) \quad (3.10)
\end{align*}
\]

it can be seen from (3.10) that system status of traffic flow can be determined by primary parameter which is far less than the total parameters and other parameters are dominated by primary parameter. So, the dimension of system decreases and self-organization can be realized.

Now we will try to find out what is the real primary parameter of traffic flow system.

Now consider a section R that can be divided into \( n \) segments. Now propose the maximal expected mean speed is \( \hat{V}_i(t) \) and actual mean speed is \( V_i(t) \). Here we define a parameter \( x_i(t) \), which can be used to distinguish organized traffic flow and disorganized traffic flow.

\[
\begin{align*}
    x_i(t) &= \frac{V_i(t)}{\hat{V}_i(t)} \xi \quad 0 < \xi < 1
\end{align*}
\]

We can choose a proper value for \( \xi \)
according to the actual traffic condition. Thus, we can get the conclusion: if \( x_i(t) \geq 1 \), then traffic flow is in organized condition, otherwise, it's in disorganized condition.

Let \( S_i(t) \) stands for the traffic flow saturation of segment \( i \) at time \( t \),

\[
S_i(t) = \frac{F_i(t)}{N_i}
\]

\( F_i(t) \) = traffic volume passing segment \( i \) at time \( t \)

\( N_i \) = traffic capacity of segment \( i \)

Now it is proved that mean vehicle speed will change unexpectedly if traffic saturation arrives at a certain value.

So, Pitch-Folk momentum equation can be adopted to describe each segment.

\[
\frac{dV_i(t)}{dt} = k_1(S_i(t) - S_0)(V(t) - V_i) - k_2(V(t) - V_i)^3
\]

\[
\frac{dS_i(t)}{dt} = \frac{k_3(S_i(t) - S(t)) - 3k_2(V(t) - V_i)^2}{k_1V(t) - k_2S(t) - k_4} P_{\&}(t)
\]

From the equation we can see: relax-tense coefficient of \( V_i \) is positive while that of \( S_i \) is negative. Here, we propose \( V_i = f_i(S_i) \). Of course, \( f_i \) is different for each segments. So, consider

\[
x_i(t) = \frac{f_i(S_i)}{V_i(t) \xi}
\]

Since \( f_i(S_i) \) is a decreasing function, we get that if \( S_i(t) < a_i \), then traffic flow is organized, and if \( S_i(t) > a_i \), then traffic flow is disorganized.

Here, we can describe the traffic flow system as following:

\[
P_{\&}(t) = -r_i(S)V_i(t) + h_i(S, V, t)
\]

\[
S'_i(t) = -\lambda_i(V)S_i(t) + k_i(S, V, t)
\]

We can see the first item in each equation is proportional to \( V_i(t) \) or \( S_i(t) \) and the second item is nonlinear.

Through analysis of the equation group by cooperation theory we get:

\( S_i \), which is a slow variable in comparison with \( V_i \), can decide the value of \( \{ r_i \} \) and \( V_i \), and its change can bring the changes of state variables. And if \( S_i \) arrives at a certain value, \( V_i \) will change abruptly and thus the whole traffic condition will change unexpectedly.

Now, we can see \( S_i \) not only describe condition of traffic flow system, but also decide developing direction of the traffic flow system. So, we can say \( S_i \) is the primary parameter of the traffic flow system.

4 The Process of Identifying the Primary Parameter

1) The Advantage of Using Primary Parameter as the Identified Parameter

The common method is to combine the various situation, such as, the vehicles arriving, queuing in the intersection, and passing intersection, into a traffic status \( x \in X \), \( X \) is a subset in \( n \) dimension space, let

\[
x = [x_1, x_2, \ldots, x_n] = (x_{1, \min}^{\max} \times x_{2, \min}^{\max} \times \ldots \times x_{n, \min}^{\max})
\]

(4.1)

where \( (x_i^{\min, \max}) \) denotes the feasible scope of the \( i \)th status variable, \( i = 1, 2, \ldots, n \) is up to the layout of intersection.

For identifying the concrete status rapidly and accurately, every status \( (x_i^{\min, \max}) \) is
divided into $m_i$ subsets, which are non-intersecting each other. We get:

$$(\chi_i^\text{min} : \chi_i^\text{max}) \cap (\chi_i^\text{min} : \chi_i^\text{max}) = \phi, 1 \leq j, k \leq n, j \neq k$$

\(4.2\)

where

$$(\chi_i^\text{min} : \chi_i^\text{max}) \cap (\chi_i^\text{min} : \chi_i^\text{max}) = \phi, 1 \leq j, k \leq n, j \neq k$$

So, every subset $(\chi_i^\text{min} : \chi_i^\text{max})$ can be denoted by a characteristic value $j$, the status $X$ can be represented by $m_i$ characteristic value.

Traffic status space can be simplified as a point set that consists of $N$ n-dimension points, where $N = \prod_{i=1}^{n} m_i$. $S$ is called as standard traffic status space and point $s$ that store is called as standard traffic status vector. Point $s$ belongs to $S$.

The points set $S$ correspond to an intersection. When a region includes $M$ intersections, the number of points in points set $S$ is

$N = \prod_{j=1}^{M} \prod_{i=1}^{n} m_j$ which is very large. So, when the number of controlled intersections is not large, this method is practicable. But, if the controlled road net includes many intersections, the system status $X_i$ needed to identified will be very large and the dimension of characteristic vector is also large, it will influence the real-time performance of control system.

We can use self-organization theory to find the primary parameter $U$. First, it is necessary to realize the transformation from $X(t) = \{x_1(t), x_2(t), \ldots, x_n(t)\}$ to $U(t) = \{u_1(t), u_2(t), \ldots, u_j(t)\}$, of course, $j < n$. Then, the dimension of characteristic vector decreases, it is more convenient to identify primary parameter. Let the number of primary parameters in the whole controlled zone is $k$, then constructed points set include $N = \prod_{i=1}^{k} m_i$ points. So, the number of control strategies need to match decrease largely and the real-time characteristics of system increase.

(2) the Process of Identifying Primary Parameter

In the course of describing external things, people always use some fuzzy language to depict the property or character of things. It is difficult to distinguish the status space and strategy space strictly and accurately, and the strict partition is not scientific in most cases. Take the status space of primary parameter as example and let the certain status variable of primary parameter is a continuous variable, we get the result of decomposition as followings.

$$(\chi_j^\text{max} : \chi_j^\text{max}) \cap (\chi_j^\text{min} : \chi_j^\text{max}) \cap (\chi_j^\text{min} : \chi_j^\text{min})$$

\(4.3\)

Where, $U_{j}^{\text{max}} = U_{j+1}^{\text{min}}, 1 \leq j \leq m-1$.

The distance of two arbitrary points $a, b$ in field $\mathcal{O}(U_j^{\text{max}}, \varepsilon)$, which is an adjacent field of field $U_j^{\text{max}}$, is less than $\varepsilon$. But if $a, b$ point belongs to the left adjacent field or the right adjacent field of $U_j^{\text{max}}$ respectively, the different standard status vectors will be obtained. Then, it is necessary to adopt different control strategies. The reason for this situation is too strict partition to status space. So, instead of certain continuous group of the state space, fuzzy group is selected, instead of certain rules, fuzzy inferential rules is adopted. Based on fuzzy group and fuzzy inference, identification of traffic flow situation and inference from state space to strategy space can be effectively implemented.
5 Simulation

We make two systems for simulation. One is a single intersection system (Figure 2) and the other is a system including three intersections (Figure 2). Here we assume that each of in-approaches in NS or SN direction includes two lanes and each of in-approaches in WE or EW direction includes only one lane. The phase configuration is show in Figure 2. We put three sensors at the entrance, exit and 150m before exit of in-approach lane so that we can get the value of traffic saturation and vehicle delay through computation more easily. Author finishes simulation program with VB. Here author chooses 10 seconds as an interval and 1500 seconds as the simulation time for every time, then make simulation for 10 times. the simulation result is showed in the table 1 and table 2.

In order to do some comparison, author make simulation of the method in reference [6] under the same condition, the simulation result is showed in table 1 and table 2.

![Figure 2](image)

Table 1: simulation result of single intersection

<table>
<thead>
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<th>Simulation number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<th>7</th>
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<th>9</th>
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<tr>
<td>delay (s/veh)</td>
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</tr>
<tr>
<td>method in this paper</td>
<td>10.8</td>
<td>8.2</td>
<td>10.0</td>
<td>7.9</td>
<td>10.0</td>
<td>11.3</td>
<td>7.8</td>
<td>10.9</td>
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<td>8.0</td>
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<tr>
<td>method in reference [6]</td>
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<td>8.3</td>
<td>10.4</td>
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<td>10.5</td>
<td>11.6</td>
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<td>7.1</td>
<td>8.6</td>
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relative 4.5 1.2 3.8 2.5 4.7 2.5 3.7 0 4.2 6.9
amelioration degree

<table>
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<th>3</th>
<th>4</th>
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<th>7</th>
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<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>vehicle method</td>
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<td>11.2</td>
<td>12.0</td>
<td>12.2</td>
<td>13.8</td>
<td>11.0</td>
<td>8.3</td>
<td>10.3</td>
<td>11.8</td>
<td>14.5</td>
</tr>
</tbody>
</table>
| average delay     | in this paper (with identifying (s/veh) primary parameter)
| method            | 16.4 | 12.9 | 13.9 | 14.0 | 15.3 | 11.8 | 9.4 | 13.1 | 13.9 | 16.9 |
| (without identifying primary parameter) |
| relative amelioration degree | 13.4 | 13.1 | 19.4 | 12.8 | 9.8 | 6.7 | 11.7 | 21.3 | 15.1 | 14.2 |

From the simulation result, we can find that the average delay of the method without identifying parameter is 9.47s/veh and the average delay of our method is 9.15s/veh (decreasing 3.4%) when simulating in single intersection. We can also find that the average delay decreases more (13.8s/veh→11.9s/veh : 13.7%) with our method when the number of intersection increases to three. So we can say that the improvement on traffic flow control with the method in this paper will be obvious when the number of intersections is more than three.

6 Summary
Applying system self-organization theory to traffic flow management-control is a important innovate. From the control structure of traffic flow system and based on system self-organization theory , this paper puts forward a double loops control structure which consist of main control loop and self-adaptive loop, then analyses how to assure primary parameter and identify primary parameter. From theoretic angle this control structure is reasonable and feasible and is able to ensure the optimal and real time character of control. What should be done later is to apply this structure to practical traffic flow management-control indeed.

References