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Hopf Oscillator-Based Gait Transition for A Quadruped Robot

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***Abstract*— This paper concentrates on the gait transition of a quadruped robot. Animals move with different gaits to adapt to the change of terrains or just to save energy. A biomimetic robot should achieve this process in a smooth, continuous and energy efficient way. We propose a gait transition controller that can achieve needed phase difference between every two legs based on the Hopf oscillator. The quadruped robot has achieved biomimetic gait transitions from walk to trot and trot to gallop based on the proposed controller, and the two gait transitions of the opposite direction are the same by changing the control variables reversely. All of these processes have been verified on a simulated quadruped robot. The robot can achieve a normal velocity value for the walk, trot and gallop gait respectively. Both the walk to trot and trot to gallop gait transitions take only one second. We analyze the stability and some problems existing in the simulation and two gait transitions.**

I. INTRODUCTION

The quadruped, along with other sorts of legged robots, compose today's hot topic of biomimetic robots. Each kind of robot is superior to others on some areas, so it's worthwhile to continue to pay effort to them considering their weakness that are still waiting to be improved. A quadruped robot can travel over complicated terrains easily compared to the biped one, and run faster than a hexapod one. The challenges the quadruped robot still faces limit its application in many areas. We can also have confidence that the quadruped robot will be widely used in our daily life with its improvement of performance, just like what the computer has experienced over the past few decades.

A quadruped robot often uses different gaits to adapt to different terrains or just to save energy. Both purposes are extremely important considering its performance and the limited energy storage capability it has under today's technology. On the one hand, different gaits have different strides, heights of the foot uplift and orders of the leg movement. The environment is often mutable, and a quadruped robot can't move normally with some gaits under special environment, for example, the trot gait versus the rough terrain. On the other hand, energy consumption increases with the velocity becoming larger. A different gait has a different consumption line, and there are usually some points of intersection between different lines of energy consumption [1], [2], [3], [4], [5]. This phenomenon may be caused due to the ability of velocity generation that each gait can manage. A robot can't move faster under a special gait or even becomes slower when it reaches a given value of speed, which will largely increase the energy consumption, for example, walk versus trot or trot versus gallop. All of these show the necessity and importance of gait transitions for a quadruped robot to adapt to the change of environment.

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Researchers have already done some work in gait transitions [6], [7], [8], [9], but there are still some challenges. Katsuhiko has proposed a gait transition method [10] based on special mechanical structure of the quadruped robot, which can be easily implemented. In this mechanism, two DC motors mounted on the middle of the body driving two main shafts for the opposite directions respectively. Two clutches mounted on each leg receiving these two powers respectively, and put out this power to a single common shaft for each leg. One clutch is for the standing phase and the other is for the swing phase. The main merit of this mechanism is that two legs can use a common actuator, which can keep a constant force independent of the number of the standing legs. But it's hard to control each of the legs independently at the same time for the quadruped robot, and it requires a special mechanism, which is non-universal for all kinds of robots.

Lin focuses on neural network gait learning with good examples [11], which determines at which time the leg should lift or swing, but he has not pointed out how a robot can actually achieve the gait transition. The gait transition methods in [12], [13] are achieved by modifying the gait matrix suddenly when this process is needed, which may cause impact force to legs. Although the sharp signals have modified during gait transition by replacing them with zero values or gradually changing the signals, the signals are still discontinuous. This method is easy to achieve, but requires high performance of the mechanical structure.

Matos has achieved smooth and continuous transition with a series of equations by combining the walk and trot gaits [14]. The generated trajectories are modulated by a drive signal that modifies the oscillator frequency, amplitude and the coupling parameters among the oscillators, proportionally to the drive signal length. By increasing the drive signal, locomotion can be elicited and velocity increases while switching to the appropriate gait. He has compared the stability margin and the speed changing on gradual and abrupt transition, which proves that gradual gait transition is more stable. But this method doesn't have a given model that can synthesize common situations, nor have it achieved the gait transition from trot to gallop. The locomotion speed is too slow in both walk and trot gaits, and the transition takes too much time, which is not practical in reality.

In this paper, we will propose a gait transition algorithm that synthesizes the common transitions for a quadruped robot based on the Hopf Oscillator, and the proposed method can achieve the needed phase difference.

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II. NATURAL GAIT TRANSITION

In general, four-legged animals have five different gaits, including walk, trot, pace, bound and gallop. The main differences between them are duty factor and the phase relationship between every two legs. It's easy to distinguish the former three gaits. But, there are different definitions among the bound and gallop gaits. The two front legs during the gallop gait have the same phase and are half circle ahead of the latter legs, which also have the same phase. The four legs during the bound gait have the same phase [12]. However, the WildCat of the Boston Dynamics bounds in the way of the gallop gait as mentioned above and gallops just like bound with the left legs' phase a little ahead of the right ones. We can't determine which one is proper because a different animal runs in a different way, such as deer and horse. These definitions can be put aside because we can achieve needed phase relationship with our gait transition model as we can see in chapter III.

As we mentioned above, there are three main factors we should concern during a natural gait transition, they are continuity, smoothness and energy efficiency.

A. Continuity and Smoothness

In the nature, four-legged animals usually walk before trot and trot before run. We hardly see the abrupt running, which would cause extra load to the muscles and bones. This is the same situation in the quadruped robots. On the other hand, quadrupeds are much less flexible than animals and can't run with abrupt pushing. This shows the necessity and importance of continuous gait transition.

Abrupt gait transition may add impact load to quadrupeds' foots and trunk, which could cause faster wear of joints and even damage the actuators. When gradually changing the gait, the resulting locomotion speed increase is more smooth and more controllable than for the abrupt change of gait. Also, the stability margin for the gradual gait change decreases slower than the abrupt gait change, which plummets right after the robot changes to trot [14]. So it would be better change the gait smoothly.

B. Energy Efficiency

Speed is the key factor of energy efficiency for a quadruped robot, so determining the value at which a gait transition must happen is of practical importance considering the limited energy storage capabilities of robots. The switching formula [1] can be taken as

$$v_t = \frac{1}{\pi}\sqrt{\frac{gMl^2}{nI_l}\left(\frac{1}{s_{w/t}} - \frac{1}{4s_{r/g}}\right)^{-1}}, \quad (1)$$

where v_t is the gait transition speed from trot to gallop, g is the gravity constant, M is the body mass, l is the leg length, n is the number of legs, I_l is the moment of inertia of the leg relative to the hip joint, s is the stride length, w/t and r/g represent walk (trot) and run (gallop) respectively.

The theoretical gait transition speed from (1) is closely near the experimental value [1]. We can see that trot gait consumes much more energy than gallop gait after the cross point, so we should have a gait transition when a quadruped moves faster than the switching speed in terms of energy efficiency.

III. CONTROLLER MODEL

The legs of a quadruped robot move in a rhythmic and periodic way during the normal locomotion, and they can be adjusted through the change of hip and knee rotation amplitude to cross over the rough terrain. So the normal locomotion control of a quadruped robot should be as simple as possible, and the normal controller should have an accessible interface to a higher level controller.

The traditional control method is complex, and most importantly, it can't ensure the strict phase relationship between each leg. The central pattern generator (CPG) can generate a series of fixed-phase-relationship and stable signal without complicated calculation, which can be used as the lower level controller for the legs' rhythmic motion. The CPG controller is based on the neural structure. Every oscillator is connected with each other and weakened or strengthed according to the signals' phase relationship.

Among all the CPG controllers, there are several different oscillators. Based on Van der Pol oscillator, a stable and simple one with high capacity of resisting disturbance, FitzHugh and Nagumo et al proposed FitzHugh-Nagumo oscillator [15], [16], [17], which has a high coupling property among some parameters to the signals' frequency and amplitude. Kimura et al improved the Matsuoka Oscillator to a two-mutual-affected neural structure [18], [19], but this oscillator also has a high coupling property, which is not convenient for the higher level controller. We use the Hopf oscillator in our model, whose amplitude and frequency are independent from each other, and we can change the duty factor to adapt to the walk gait's special condition.

A. Low Level CPG Controller

In order to achieve interlimb coordination, we rotate each oscillator onto each other. The formula [14] is coupled as

$$\begin{bmatrix}\dot{x}_i \\ \dot{z}_i\end{bmatrix} = \begin{bmatrix}\alpha(\mu - r_i^2) & -\omega \\ \omega & \alpha(\mu - r_i^2)\end{bmatrix}\begin{bmatrix}x_i \\ z_i\end{bmatrix} + \sum_{i\neq j} R(\theta_i^j)\begin{bmatrix}x_i \\ z_i\end{bmatrix} \quad (2)$$

$$\omega = \frac{\omega_{\mathrm{st}}}{e^{-az_i}+1} + \frac{\omega_{\mathrm{sw}}}{e^{az_i}+1}, \quad (3)$$

where i and j means the i_{th} and j_{th} leg, r_i=sqrt $(x_i^2 + z_i^2)$, A=sqrt (μ) determines the amplitude, ω specifies the frequency of oscillators, ω_{st} and ω_{sw} represent the frequency of stance and swing durations, R (θ_i^j) is a rotation matrix, θ_i^j is the required relative phase among the i_{th} and j_{th} oscillators to perform the gait, α and a are constants.

From the above controller, we can get the hip joint and knee joint control signals as

$$\begin{cases} A_{hi} = x_i \\ A_{ki} = \begin{cases} \pm|z_i| & , z_i > 0 \\ 0 & , z_i < 0 \end{cases} \end{cases}, \quad (4)$$

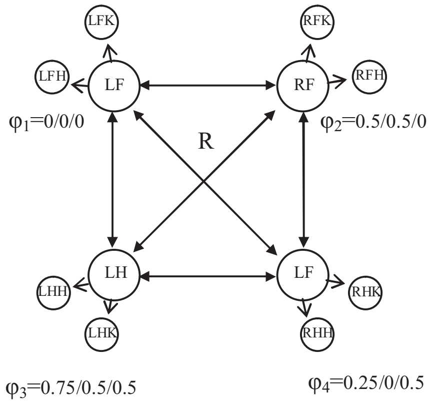

Fig. 1. The coupling relationship of oscillators and the phase value for walk, trot and gallop respectively.

where A_{hi} and A_{ki} represent the signal amplitude of the hip and knee joint for the i_{th} leg respectively. A_{ki} is positive for the fore legs and reverses to negative for the hind legs. A_{ki} is set as 0 when z_i<0.

All of (2), (3) and (4) compose the low level controller. Fig. 1 shows the detailed coupling relationship of the oscillators and the phase values for each leg under different gait. R represents the rotation matrix, corresponding to the R (θ_i^j).

B. Gait Transition Controller

The low level controller, working automatically, offers several accessible interfaces for the gait transition controller. These interface parameters are the determinants of gait pattern. A quadruped can achieve gait transition by varying these parameters, and the gait transition of the opposite direction can be achieved by changing them reversely.

The phase difference between every two legs is different according to a quadruped's special requirement. So the generalization of this controller should be considered.

In (2), θ_i^j is the key factor of gait pattern. We define θ_i^j as a controllable one

$$\theta_i^j = 2\pi \times (\varphi_a \times \varphi_c + \varphi_{nj} - \varphi_{ni} - \varphi_{nc} \times \varphi_a) \tag{5}$$

$$\varphi_a = \frac{\varphi_{mj} - \varphi_{nj} + \varphi_{ni} - \varphi_{mi}}{\varphi_{mc} - \varphi_{nc}}, \tag{6}$$

where φ_c is the control variable, φ_{mc} and φ_{nc} are the original and final phase value from gait m to gait n respectively, φ_{mi}, φ_{mj} and φ_{ni}, φ_{nj} represent the i_{th}, j_{th} leg phase value for gait m and gait n respectively.

From walk to trot gait transition, we take φ_3 as the control variable. The duty factor varies with φ_3 at the same time, which satisfies the wave gait rule. This rule can improve the locomotion stability [20], [21], [22]. We take φ_2 as the control variable during the trot-to-gallop gait transition to satisfy the synchronism between this process and the change of leg phase.

As we can see in the formulas (5) and (6), they can achieve needed phase relationship between every two legs when given the proper original and final conditions. We set the original and final conditions as shown in Fig. 1 for the two transition processes. After arrangement, the terminal equations all the way from walk to trot and trot to gallop are

$$\begin{aligned} \theta_1^2 &= \theta_3^4 = -\theta_2^1 = -\theta_4^3 = \pi(1-\tau) + 2\pi\varphi_2\tau \\ \theta_1^3 &= \theta_2^4 = -\theta_3^1 = -\theta_4^2 = 2\pi\varphi_3(1-\tau) + \pi\tau \\ \theta_1^4 &= \theta_2^3 = -\theta_4^1 = -\theta_3^2 = (2\pi\varphi_3 - \pi)(1-\tau) + (\pi - 2\pi\varphi_2)\tau \end{aligned},$$

where τ equals 0 when gait transits from walk (φ_3 = 0.75) to trot (φ_3 = 0.5) and equals 1 from trot (φ_2 = 0.5) to gallop (φ_2 = 0). The duty factor changes with φ_3 from walk to trot and keeps constant during the trot-to-gallop gait transition.

The formulas (5) and (6) are just a module of the gait transition controller, not the controller itself. The above equations stretch out three control variable, φ_2, τ and φ_3. We get the additional controller formulas after considering the change process as

$$\beta = \varphi_3 = 0.75 - \frac{0.25(t - t_{wts})}{\eta_{wtl}} \tag{7}$$

$$\varphi_2 = \frac{0.5(t_{tge} - t)}{\eta_{tgl}} \tag{8}$$

$$\omega_{st} = \frac{1-\beta}{\beta}\omega_{sw} \tag{9}$$

$$\mu = \begin{cases} -A_0^2 & ,t < t_s \\ A_0^2 & ,t_s < t < t_{as} \\ (A_0 + at)^2 & ,t > t_{as} \end{cases}, \tag{10}$$

where β is the duty factor, t_{wts} and t_{tge} mean the start time of walk-to-trot transition and the end time of trot-to-gallop transition respectively, η_{wtl} and η_{tgl} represent the lasting time of walk-to-trot and trot-to-gallop transition respectively. The general locomotion speed is controlled by ω_{sw}. t_s is the start time of oscillation, *i.e.*, locomotion; t_{as} is the start time of amplitude's acceleration, the lasting time is determined by the acceleration value a and the final amplitude.

The control signals of the hip (x_i) and knee (z_i) joints equal zero when $\mu < 0$, and oscillate around zero for $\mu > 0$. The ascending (swing) phase and descending (stance) phase durations can be controlled by β and ω_{sw}.

We usually need a continuous gait transition, so t_{tge}-η_{tgl} needs to be greater than t_{wts}. Energy efficiency should also be considered in case of the limited energy storage capabilities of robots. In the whole process, the quadruped robot starts to walk after time t_s, transits to trot during t_{wts} and t_{wts}+η_{wtl}, accelerates to the velocity threshold, starts to transit to gallop gait triggered by the threshold and gallops forward after completing gait transition. The control flow diagram shows in Fig. 2. The gait transition controller passes several parameters to the CPG controller, and the CPG controller passes the hip and knee signals to each leg.

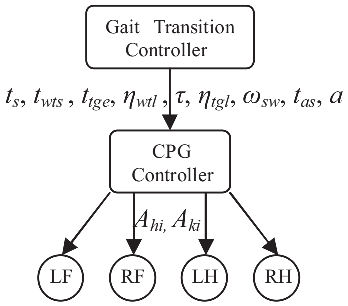

Fig. 2. The parameters transmission and the relationship between each controller and the quadruped robot.

IV. SIMULATION AND DISCUSSION

The combination of the CPG controller and the gait transition controller produces the desired control signals for the quadruped's joints. We use the signals as position control for the quadruped robot's hip and knee joints.

A. The Simulated Robot

The simulation environment is in the MD Adams 2010 software platform with Matlab R2009A. All simulations are performed on a Intel Double Core 2.1 GHz PC.

The virtual prototype of the quadruped robot used in simulation, shown in Fig. 3, is composed of trunk, thigh, shank and foot. Each leg has two degrees of freedom, including hip pitch and knee pitch. The virtual prototype is 1050 mm long, 720 mm wide and 660mm high. Both the thigh and shank are 350 mm long. The original leg posture is with γ_0 (30 degree) and δ_0 (60 degree) for the hip joint and knee joint respectively. The detailed parameters of the virtual prototype are shown in Table I.

The quadruped robot stands still until 0.5 second. After that, the oscillators begin to output stable signals, which are used in the hip and knee joints. At about 4 second, the gait transition controller passes on the gradual change of φ_3 and β in (7) to the CPG controller, which makes the gait begin to change from walk to trot. This transition takes only 1 second and ends at about 5 second with the period changing to 0.5 second. The duty factor β and the third leg phase φ_3 ends at 0.5 at the same time. After about 1 second's trot, the robot begins to accelerate by increasing the frequency of signals. This is achieved by increasing the value of ω_{sw} in (9) from 4π to 6π linearly. The speed of

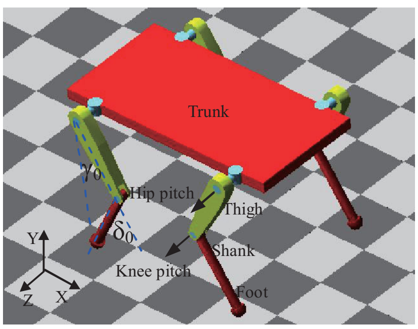

Fig. 3. The simulated quadruped robot.

TABLE I

PARAMETERS OF VIRTUAL PROTOTYPE

Name	Volume (mm^3)	Density (kg/mm^3)	Mass (kg)	Z-inertia ($kg.mm^2$)
Trunk	2.6E+007	7.7E-007	20	2.2E+006
H-link	2.4E+005	6.2E-006	1.5	2554.058
Thigh	1.4E+006	2.5E-006	3.5	3.7E+004
Shank	4.3E+005	4.7E-006	2.0	2.0E+004
Foot	8.5E+004	5.9E-006	0.5	195.012

the robot locomotion reaches the threshold derived from (1) at about 8 second. Then, gait transition is stimulated by changing the value of φ_2 in (8). To better satisfy running, the amplitude of the leg's swing also needs to increase linearly, which is achieved in (10). This process takes only 1 second, and the robot begins to gallop after 9 seconds.

B. The Simulation Result and Analysis

The snapshots of the simulation are shown in Fig. 4, and all the parameters' switching procedure is listed in Fig. 5. φ_3 and β use the same line considering their synchronism. τ is a line in bold and purple with green line of φ_2 in the second frame. The frames, from the third to the sixth, present the four legs' signals. The red solid lines are hip signals, while the blue dashed lines are knee signals. The last frame shows the velocity of the quadruped's center of mass (COM) during the simulation.

The walk gait signals in Fig. 5 increase slowly from zero after μ is given a positive value, which can decrease the impact to the joints. This feature makes the Hopf oscillator a better choice than the Matsuoka oscillator. The ascending phase, lasting about a quarter of one period, corresponds to a leg's swing movement. This can make sure that there are at least three legs supporting the trunk at the same time. So the walk gait has static stability. During the trot gait, there are only two legs supporting the body. The robot needs to utilize the inertia of its own in order to sustain a dynamically stable gait. This requires the COM is on the supporting line as near as possible at all times. A gallop gait is completely different in that there is a moment that all legs are in the air, which requires the mechanism to have a high performance of shock absorption.

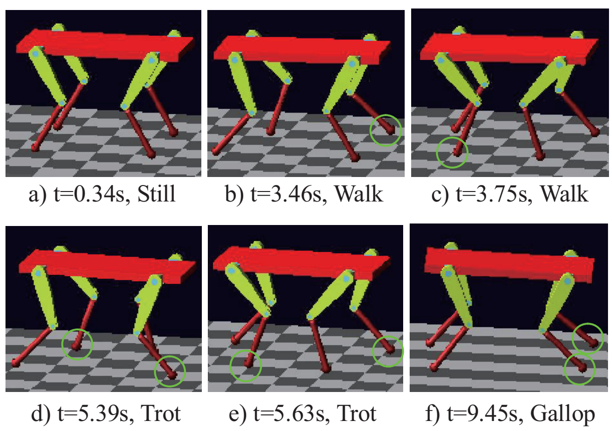

Fig. 4. Snapshots of the simulation (the green solid circles mean that the selected legs are in the swing phase). The detailed gait transition process refers to the attached video of this paper.

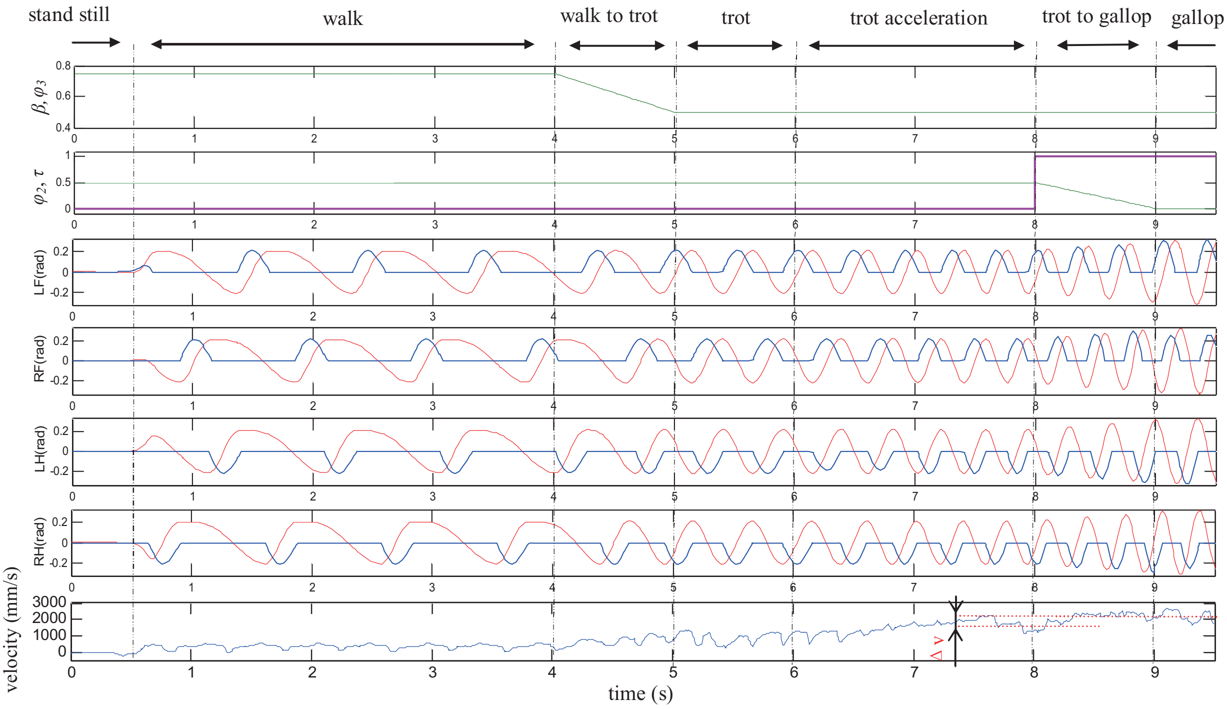

Fig. 5. The whole gait transition process from still to gallop of the quadruped robot.

The last frame in Fig. 5 shows that the locomotion speed of the robot fluctuates during the simulation. This is the same situation in real animals. The quadruped robot walks about 300 millimeters per second, much slower than the trot's 1000 mm/s under the same situation, which shows that the trot is much more efficient than walk. However, the trot's speed fluctuates greater than the walk gait, and the speed amplitude of the gallop is even bigger.

As we can notice that the speed of locomotion has a temporary slow down at about 8 second. This just happens during the process of trot acceleration, which proves that the trot has an upper speed limitation. When the speed is over the ability that the trot can manage, it decreases rather than increases continuously. The speed increases again after the gait transition from trot to gallop is stimulated. It shows that the gallop gait is much more suitable in high speed locomotion. The robot gallops stably at a high speed of about 2200mm/s after the transition is completed. The marker in Fig. 5 shows a Δv speed difference between the gallop and trot gait under the same condition, which proves that the gallop is more energy efficient than the trot after the speed is over the threshold corresponding to (1).

The stability of a quadruped robot locomotion is influenced by many factors, including the mechanical structure of the legs, distribution of the COM, interaction with the environment and the control signals assigned to each leg. In our simulation, the former three factors are all bounded in the Adams environment, which may be quiet different from the real world. The effective way of analyzing its stability is to consider the source of signals, leaving the interface of the signals and actuators to the real robot.

The limit cycle of the hip joint signal is shown in Fig. 6. There are three stable cycles, corresponding to the walk, trot and gallop cycle respectively deflected in the figure. The cycle starts at a zero point, and then runs into the stable walk cycle. The lower side begins to extend after the gait transition from walk to trot starts and stabilizes at the symmetrical trot cycle. The signal continues to extend evenly at both sides after the robot begins to accelerate and diverges outside during the process of transition from trot to gallop. The limit cycle reaches at the stable gallop cycle finally and goes into an infinite loop. All this process is just like a satellite starting from the ground, running through the scheduled orbits and reaching the goal orbit finally. This feature proves that the controller has a stable source of signals.

We present the contact force of the RH foot (Y-direction) during the simulation process in Fig. 7. The contact force stabilizes at about 0.18 KN (about one third of the total weight) at the beginning, and it becomes two or three times larger during the trot gait and gallop gait. There are two force fluctuations corresponding to the two gait transition

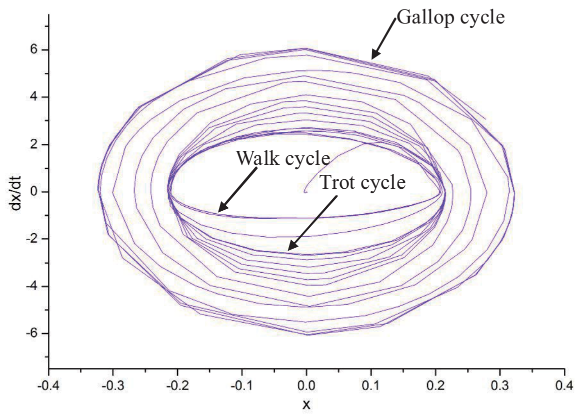

Fig. 6. The limit cycle of the hip joint signal.

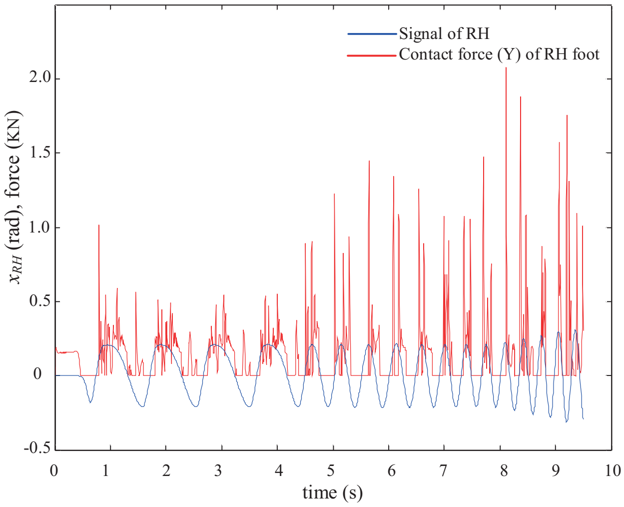

Fig. 7. Hip control signal and contact force of RH

processes. The maximum contact force is within the acceptable range (less than 2.0 KN). The contact force is zero during the swing phase, which proves that the hind leg can lift up during this process, *i.e.*, this is a stable gait transition process. The sudden increase of the contact force is caused by the leg's advanced landing when the leg is still during the swing phase as shown in Fig. 7, we will study this phenomenon in our future work.

V. CONCLUSION

This paper has proposed a general gait transition control model, which has been verified on a simulated quadruped robot. The simulation result shows that the proposed gait transition controller has achieved continuous and stable gait transitions all the way from walk to trot and trot to gallop. The energy efficiency has also been taken into account. The gait transition controller can achieve needed phase difference between every two legs based on the Hopf oscillator. This feature can make it possible that the controller can be used in other forms of state transition. The walk, trot and gallop gaits are just special states of the controller after the three control variables in the arranged equations are set as special values. However, the proposed method fully depends on the Hopf oscillator, and whether this thought can be used on other oscillators still needs to be studied in the future work.

The whole gait transition process has just been proved in the simulation, which may be insufficient because some actual factors have not been considered. We will apply this controller on a real robot that is flexible enough to achieve a gallop gait in our future work. The real robot's stability during the gait transition will also be studied from the view of velocity and fluctuation of the COM. The foot-ground contact force is another problem we will get to address, especially the impact force during the ending of swing phase. The study in [23] deeply analyzes the impact force during each gait, which is a good reference for this problem.

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