# Controller-Switching Based Gait Transition for a Quadruped Robot 

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#### Abstract

In this paper, we propose a gait transition strategy for a quadruped robot using the Hopf Oscillator model and trajectory planning. The Hopf Oscillator model is used to generate rhythmic moving pattern during trot gait, and trajectory planning based on the Stability Margin (SM) is employed to implement walk gait as well as the transition between walk gait and trot gait. Especially, we develop a moving sequence based on SM to control the transition, and the method can completes gait transition along with switches between conventional trajectory planning controller and Central Pattern Generator (CPG) based on Hopf Oscillator. So the quadruped robot can achieve the stable transition between walk gait and trot gait, and the transition approach can achieve switches between controllers. Finally, we conduct the simulation and the results are discussed. Both the walk-to-trot and trot-to-walk transitions can be achieved within less than one second.


Index Terms - Quadruped robot; Gait transition; Hopf Oscillator; Trajectory planning;

## I. INTRODUCTION

Quadruped robots, a kind of biomimetic robots, have a bright perspective because of its superiority in locomotion stability, rapid moving velocity and adaptation to rough terrains in natural environment. Self-contained quadruped robots, such as Tekken[1], BigDog[2] and AiDIN-III[3], have been developed to obtain maneuvering capability which is needed to fulfil complicated tasks and pass a variety of complex topography. Tekken[1] with bionic flexor reflex and vestibulospinal reflex is able to locomote adaptively over irregular terrain. BigDog[2] can move steadily on rough terrains with heavy load for hours. AiDIN-III[3] is able to realize gait transition on natural terrains.

A flexible walking control model is significant for a quadruped robot to improve its kinematic dexterity. Various controller models have been developed, such as traditional controller based on trajectory planning [4], [5], [6], and bioinspired controller based on pattern generators with sensory feedback [7], [8], etc. The majority of these approaches only focus on one specific moving pattern with nice performance in simple environments under constant velocity. But a robot with only a single moving pattern has limitation to adjust different topography and different velocities. For example, a quadruped robot is hard to remain stable galloping on rocky roads, and walking is hard faster than trotting on even terrains. And also,

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a suitable gait pattern can reduce the quadruped robot's energy consumption [9], [10]. Walking at high velocity will consume more energy than trotting at the same velocity. In natural world, quadruped animals can freely change their walking pattern according to the environments [11], [12], [13]. Inspired by quadruped animals, it is necessary to conduct gait transition to cope with aforementioned problem for a quadruped robot.

To improve the robustness of the gait transition, researchers have studied different kinds of method, but there are still challenges yet unsolved. Wei Xiao and Wei Wang proposed a gait transition method [14] based on Hopf Oscillator, which achieved biomimetic transition from walk to trot and trot to gallop by only changing few control variables to adjust the coupling relationship. Xiao's model achieved well performance in stability under high-velocity moving. However, in walk gait, undesirable fluctuation of the robot's body, caused by improper position of the orthocentre and its inertia under low velocity, was hard to avoid.

Jian-Nan Lin and Shin-Min Song proposed an advanced control method using CMAC neural networks [15], [16], which can generate the phase of each leg, but the method lacks practical instruction for trajectory planning of each leg. Liu An et al. focuses on transition method using rhythm control [17], which can finish the transition from walk to trot continuously and steadily, and was verified by simulation. However, this method is hard to realize dynamic stability and adaptation.

Ig Mo Koo et al. proposed a quasi-static gait transition method switching between non-periodic gait and periodic gait [18], and realized transitions between walk and trot using a real self-contained quadruped robot AiDIN-III with 16 Degree of Freedoms (DOFs) under different outdoor terrains. But this method requires high performance of the mechanical structure and at least 3 DOFs for each leg.

The gait transitions mentioned above mainly use the single controller model in the whole transition process. We propose a gait transition strategy based on the Hopf Oscillator model, combining the Stability Margin (SM)[19] and trajectory planning. We utilize Hopf Oscillator to generate rhythmic signal during trot gait for better adaptation. Hopf oscillator can also generate walking pattern. However, we argue that trajectory planning can realize more stable locomotion under low velocity, and so we develop a walk strategy based on trajectory planning, which can stabilize the center of mass. Gait transitions are accomplished using predesigned sequences according to the principles of SM. The proposed method can realize walk-to-trot and trot-to-walk transitions along with switches between trajectory planning controller and central pattern gen-
erator (CPG) based on Hopf Oscillator. So we can take advantages of conventional controller as well as CPG.

## II. Hopf Oscillator

In neurobiology, the Central Pattern Generators (CPG) are neural networks which can generate coordinated patterns of rhythmic activity without any rhythmic inputs from sensory feedback or from higher control centres[20]. Inspired by neurobiology, CPG models have been used in locomotion of quadruped robots. For quadruped robots, CPG can produce rhythmic patterns of output signals without complicated input signals and calculation.

The CPG network is formed by mutually connected oscillators. Each oscillator affects another one according to the coupling relationship. Various kinds of oscillators can be used in the CPG network, such as the Rayleigh[21], Van del Pol[22], FitzHugh-Nagumo[22], Matsuoka[23], Hopf oscillators, etc. Hopf oscillator with phase-dependent frequency is used in our model for trot gait, and its formula [24], [25] is

$$
\begin{aligned}
{\left[\begin{array}{l}
\dot{\mathrm{x}}_{\mathrm{i}} \\
\dot{\mathrm{y}}_{\mathrm{i}}
\end{array}\right] } & =\left[\begin{array}{cc}
\alpha\left(\mu-r_{i}^{2}\right) & -\omega \\
\omega & \beta\left(\mu-r_{i}^{2}\right)
\end{array}\right]\left[\begin{array}{l}
x_{i} \\
y_{i}
\end{array}\right]+\sum_{i \neq j} R\left(\theta_{i}^{j}\right)\left[\begin{array}{l}
x_{j} \\
y_{j}
\end{array}\right],(1) \\
\omega & =\omega_{\mathrm{st}} /\left(\exp \left(-b y_{i}\right)+1\right)+\omega_{\mathrm{sw}} /\left(\exp \left(b y_{i}\right)+1\right),(2)
\end{aligned}
$$

where $i, j=1,2,3,4$ represented to the index of each leg (the left forth $\operatorname{leg}(L F)$, the right forth $\operatorname{leg}(R F)$, the left hind $\operatorname{leg}(\mathrm{LH})$ and the right hind $\operatorname{leg}(\mathrm{RH})$ respectively), and $\mathrm{r}_{\mathrm{i}}=\sqrt{x^{2}+y^{2}}$.
$\omega$ is the frequency of oscillators calculated by $\omega_{\mathrm{sw}}$ and $\omega_{\mathrm{st}}$, which are the frequencies of swing and stance phases respectively. $\mathrm{A}=\sqrt{\mu}$ specifies the amplitude of oscillators. $R\left(\boldsymbol{\theta}_{i}^{j}\right)$ is a $2 \times 2$ rotation matrix which determines the phase lag between legs. $\alpha$ and $\beta$ decide the convergence velocity for x and y respectively. b is constant.

Hopf Oscillator has a phase-dependent frequency. In another word, we can change frequencies of the stance and swing phases separately. See an example in Fig. 1 (Ignore the sum term). So the stance-swing durations can be changed for other periodic gait like bound gait $\left(\omega_{\mathrm{sw}}>\omega_{\mathrm{st}}\right)$ and trot gait $\left(\omega_{\mathrm{sw}}=\omega_{\mathrm{st}}\right)$.


Fig. 1 Output signal $x$ of Hopf Oscillator under different frequencies

Moreover, Hopf Oscillator also has the general property of limit cycle behaviour. See an example in Fig. 2 ( $\mathrm{A}=1$ and ignore the sum term), different random initial conditions can always returns to its normal rhythmic cycle. So we can finish the switch to CPG controller seamlessly from different posture steadily.


Fig. 2 Limit cycle behavior of Hopf Oscillator

## III. Gait Transition Strategy

## A. Walk Strategy

Locomotion of quadruped robots starts from walk gait. During walk gait, a quadruped robot moves slowly with only one leg off the ground [26]. Due to the low level of locomotion velocity, movement of the orthocenter based on static stability area must be highly considered. In another word, the center of mass need to be located rightly in the static stability area for every foot-step, and this is hard to realize under simple CPG controller. Otherwise the foot can't lift up in swing phase and can't step firmly in stance phase. So we present a walking strategy based on trajectory planning.

Proposed foot trajectory of forth legs and hind legs are shown in Fig.3, marked with different numbers regard of its relative location (denoted at the bottom of the figure). Arrows numbered zero point to the original postures. The blue dash lines are feet's track at the start of walk. The blue solid lines represent the moving track of a steady walking foot. The red dash lines and solid lines are leg postures corresponding to different foot points. We can see from Fig. 3 that all feet in stance phase are well set at the surface paralleled to its trunk.


Fig. 3 Foot trajectories: foot trajectory of hind legs (left), and foot trajectory of forth legs (right).

As illustrated in Fig.4-a, in each phase, three legs step on the ground and promote the center of mass forward with constant velocity when the last one leg swings forward. (We assume that the center of mass is coincident with the geometrical center of robot's trunk.) The center of mass are always placed within the triangular stability area, and this satisfy the principle of Stability Margin (SM)[19]. Specific poses of each leg in the whole sequence are shown in Table I. Walk gait starts from still pose (Step 1), then repeats from Step 3 to Step 6. The time gap between two adjacent steps is constant, same in the following.

TABLE I
Specific Leg-posea of Walk Gait

| Leg | Step |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |
| RF | 0 | -1 | 2 | 0 | -2 | -4 | Go to 3 |  |
| LH | 0 | -1 | -2 | 4 | 2 | 0 |  |  |
| RH | 0 | 3 | 2 | 0 | -2 | 4 |  |  |
| LF | 0 | -1 | -2 | -4 | 2 | 0 |  |  |

${ }^{\text {an Refer to Fig. } 3}$

## B. Trot Strategy

During trot gait, diagonal legs of quadruped move in pairs with much higher velocity and locomotion efficiency [27]. In our model, the CPG controller is used to generate periodic signals for revolute joints of each leg in trot gait. With the characteristic of simple calculation, CPG controller can easily realize faster and more adaptive locomotion of a quadruped robot. For Hopf Oscillators used in CPG, we determine these parameters

$$
\begin{gathered}
\theta_{1}^{2}=\theta_{3}^{4}=-\theta_{2}^{1}=-\theta_{4}^{3}=\pi \\
\theta_{1}^{3}=\theta_{2}^{4}=-\theta_{3}^{1}=-\theta_{4}^{2}=\pi \\
\theta_{1}^{4}=\theta_{2}^{3}=-\theta_{4}^{1}=-\theta_{3}^{2}=0 \\
\alpha=\beta=500, \quad \mathrm{~b}=100, \mathrm{~A}=0.285
\end{gathered}
$$

which determine the coupling relationship between each legs (assume that LF leg is the reference leg). We obtain input signals for each joint using the equations

$$
\begin{align*}
& A_{h i}=x_{i}, \\
& A_{k i}=\left\{\begin{array}{cl} 
\pm k\left|y_{i}\right| & , y_{i}>0 \\
0 & , y_{i} \leq 0
\end{array}\right. \tag{3}
\end{align*}
$$

where positive for the fore knees and negative for the hind knee joints. We use $X_{i}$ as the input signals for hip joints, and $\pm k\left|y_{i}\right|$ as the input signals for knee joints, and $k$ is a positive constant. Here, we use $\mathrm{k}=1.18$.
(1), (2) and (3) express the CPG controller model used in trot gait. With the advantage of independence between each leg, we can also achieve gait such as gallop and bound by changing $\theta_{i}^{j}$ and the duty factor.

## C. Walk-to-Trot Gait Transition

As shown in Fig. 4-b, we assume that transition starts after Step 6 of walk gait (see Step 1). First, the LH leg swings forward half of the former distance in walk gait, when other three legs follow its former schedule (see Step 2). Second, the RF leg and the LH leg remain stance status to the backmost pose of trot gait, when the RH leg and the LF leg swing to the opposite (see Step 3). Finally, walk gait can be seamlessly transited to trot gait controlled by CPG controller with the LH leg and the RF leg start to swing (see Step 4). Initial conditions of the oscillators are

$$
\begin{aligned}
& x_{2}=x_{3}=-A, x_{1}=x_{4}=A \\
& y_{i}=0, i=1,2,3,4
\end{aligned}
$$

and then the trot gait will repeats from Step 4 to Step 5. The whole transition process is shown in Table II with specific pose of each leg.

TABLE II
Specific Leg-POSE ${ }^{\text {a }}$ OF WALK-TO-Trot Transition

| Leg | Step |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 |
| RF | 2 | 0 | $\mathrm{~A}_{\mathrm{h}}=-\mathrm{A} ; \mathrm{A}_{\mathrm{k}}=0$ |  |
| LH | switch to CPG |  |  |  |
| LH | -2 | 0 |  | $\mathrm{~A}_{\mathrm{h}}=\mathrm{A} ; \mathrm{A}_{\mathrm{k}}=0$ |

${ }^{\text {a Refer to Fig. } 3}$

(b)

Fig. 4 The black circles represent feet in stance phase, and red circles represent feet in the begin of swing phase; the black doted open circles represent the next supporting phase of the swinging feet; the black boxes represent trunk position of the quadruped robot, a Walk gait, $\mathbf{b}$ Gait transition process

In another case, if the gait transition happens after Step 4, firstly, the RH leg swings forward half distance instead of the LH leg, and other three legs remain their former schedule. And then, the LF leg and the RH leg remain stance status to the backmost, when the LH leg and the RF leg swing to the opposite. It can still seamless transit to CPG controlled trot gait.

The quadruped robot can choose one of the case mentioned previously in this section according to the status when it receives a gait transition command. If one of the right legs is in the swing phase, the robot should follow the first strategy. On the contrary, if one of the left legs lifts off the ground, the robot should follow the latter one.

## D. Trot-to-Walk Gait Transition

As illustrated in Fig. 4-b, we assume that transition begins when Step 5 is over. First, the RF leg and the LH leg swing to the origin pose when the RH leg and the LF leg remain their former schedule (see Step 6). Second, the RH leg swings forward when others stand and promote the center of mass (see Step 7, which is the same as the Step 1 in walk gait). The whole transition sequence is shown in Table III with specific pose information.

TABLE III
Specific Leg-pose ${ }^{\text {a }}$ OF Trot-to-walk Transition

| Leg | Step |  |  |
| :---: | :---: | :---: | :---: |
|  | 5 | 6 | 7 |
| RF | $A_{h}=-A ; A_{k}=0$ |  |  |
| LH | 0 | switch to trajectory planning <br> RH <br> controller for walk gait |  |
| LF |  |  |  |

${ }^{\text {a }}$ Refer to Fig. 3
If the gait transition happens after Step 4, firstly, the LF leg and the RH leg swing to the origin pose when the other legs remain the former track. And then, the LH leg swings forward when others stand and promote the orthocenter (the process is symmetric with the Step 1 in walk gait).

Just like walk-to-trot, if the LF leg is in the swing phase, the robot should follow the former strategy. On the contrary, if the RF leg lifts off the ground, the robot should follow the latter one.

## IV. Simulation Results and Discussion

## A. Simulated Robot

The simulation environment is in the MD Adams 2013 software platform with Matlab R2012a.

The simulated quadruped robot is composed of trunk, and 4 pairs of thigh, shank and foot. Each leg has two DOFs, the hip joint and the knee joint. The virtual prototype is 1044 mm long, 720 mm wide and 656 mm high. The thigh is 353.6 mm long, and the combination of shank and foot is 380.8 mm long. The original posture is shown in Fig. 5.The related parameters of the virtual robot are shown in Table IV

TABLE IV
Parameters of Virtual Prototype

| PARAMETERS OF VIRTUAL PROTOTYPE |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Part | Volume <br> $\left(\mathrm{mm}^{3}\right)$ | Density <br> $\left(\mathrm{kg} / \mathrm{mm}^{3}\right)$ | Mass <br> $(\mathrm{kg})$ | Z-inertia <br> $\left(\mathrm{kg} . \mathrm{mm}^{2}\right)$ |
| Trunk | $3.393 \mathrm{E}+005$ | $5.895 \mathrm{E}-005$ | 20.002 | $4.131 \mathrm{E}+005$ |
| H-link | $2.441 \mathrm{E}+006$ | $6.145 \mathrm{E}-007$ | 1.500 | $5.036 \mathrm{E}+002$ |
| Thigh | $1.381 \mathrm{E}+007$ | $2.534 \mathrm{E}-007$ | 3.500 | $2.738 \mathrm{E}+003$ |
| Shank | $4.403 \mathrm{E}+006$ | $4.622 \mathrm{E}-007$ | 2.035 | $4.017 \mathrm{E}+002$ |
| Foot | $8.482 \mathrm{E}+005$ | $5.895 \mathrm{E}-007$ | 0.500 | $1.461 \mathrm{E}+002$ |



The quadruped robot starts to walk following planned foot trajectories at the very beginning of the simulation. The robot remains walking until 3.04 second. After that, the gait begins to change with the LH leg steps forward a little step for 0.32 second. The whole walk-to-trot transition only takes $0.62 \mathrm{sec}-$ ond, finished at 3.66 second. The robot trots from 3.66 to 7.26 second using CPG controller. At 7.26 second, the robot slows down and begins to change its gait by making its steps shorter. The whole trot-to-walk transition only takes 0.49 second, finished at 7.75 second. Finally, the robot begins to walk after 7.75 second.

## B. Simulation Results and Analysis

All inputs as well as the velocity of the robot are shown in Fig. 6. The first to the fourth frames represent the input signals of four legs respectively. The blue solid lines are hip signals, while the red solid lines are knee signals. The last frame shows the horizontal velocity of the geometrical center as well as the orthocenter of the quadruped's trunk. Nine representative snapshots of the simulation are shown in Fig. 7.

As illustrated in Fig 6, the walk gait starts with two small steps before acceleration. All the walking signals are planned for steady movement. We can see from Fig. 7-b, c that the robot's body can maintain approximate horizontal while walking, and there are always at least three supporting legs. This can ensure that the robot can move steadily getting rid of redundant energy lost.


Fig. 6 Gait transition process


Fig. 7 Snapshots of the simulation (the yellow dotted circles mean that the circled legs are in the swing phase).

Walk-to-trot transition begins after about 3 second. The robot accelerates from around $300 \mathrm{~mm} / \mathrm{s}$ to $800 \mathrm{~mm} / \mathrm{s}$ and changes its foot pattern. Postures of the robot in this transition are shown in Fig. 7-d, e. The number of supporting legs changes from three to two. And the robot begins trotting and its velocity reaches a stable level at about $1000 \mathrm{~mm} / \mathrm{s}$ after 4 second. Snapshots of trotting robot are shown in Fig. 7-f, g. We can see clearly in Fig. 7-g that the LF leg and the RH leg are suspending in the air while the other two legs supporting the trunk.

Trot-to-walk transition begins after about 7.3 second. The robot decelerates from around $1000 \mathrm{~mm} / \mathrm{s}$ to $300 \mathrm{~mm} / \mathrm{s}$ and changes its walking pattern. As shown in Fig. 7-h, the robot slows down with the two fore legs stand on the ground to neutralize the inertia force. After that, the robot walks about 300 millimeters per second. As illustrated in Fig. 7-i, the robot walks steady with three supporting legs after about 7.8 second.

time (s)
Fig. 8 Hip control signal and contact force of LH
The contact force of the LH foot (straight up direction) during the simulation process is shown in Fig. 8. Refer to Table IV, the total weight is about 50 kg . The contact force is about 0.2 kN during the stance phase in walk gait, which is less than half of the weight. It increases to around 0.5 kN during the trot gait. The largest contact force of the LH foot is 1.42 kN . The force becomes zero when the LH foot lifts off the ground. This proves that in most of the time, the LH foot can normally lifts off during the swing phase, and can supports its body during the stance phase. The sudden pulses of the contact force are cause by improper landing during swing phase especially during gait transition. In the aspect of mechanical structure, additional passive ankle or other shock absorptive mechanism may protect the robot from the shock and realize better performance. For the algorithm, sensory feedback or a more specific 3D model for trajectory planning will improve our work.

## V. Conclusions and Future Work

We made an effort to use the Hopf Oscillator model and trajectory planning to implement the stable transition between walk gait and trot gait for the quadruped locomotion. In the proposed approaches, the Hopf Oscillator is employed to generate trot gait for better adaptivity, and we design foot trajectory sequences according to the SM principles for walk gait and gait transition process, such that the center of mass can be stabilize for better efficiency and stability of locomotion. Meanwhile, this strategy could switch the control model between trajectory planning controller and CPG controller model, and this can make the robot walk steadily and agilely, while the robot can trot environment-adaptively.

We carried out our transition strategy on a simulated quadruped robot. The result shows that the proposed method has realized stable and continuous gait transition from walk to trot and vice versa.

Future work will be aimed at realizing more flexible and more stable switches between various gait patterns, such as trot-to-gallop after walk-to-trot transition. Furthermore, we will run this controller on a real quadruped being developed at our lab. Stability and adaptivity will be tested on the real robot. And the impact force caused by improper touch down need to be deeply studied.

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