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Allocating the transport subsidy based on the social contribution of public transport enterprises

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***Abstract*— An important problem in public transport is the allocation of transport subsidies among public transport enterprises (PTEs) fairly. The social contribution of the public transport industry is emphasized by governments because of its commonweal feature. This paper first defines the mathematical expression of the social contribution by utilizing multidimensional performance indicators, and deduces a set of constraints based on the proportional allocation concept (that is, the subsidy proportion allocated to each PTE should be equal to their social contribution proportion). The constraint set is the same as the efficient allocation set from the standpoint of data envelopment analysis. Then an allocation model is proposed by minimizing the maximum distance between the allocation plan preferred by the government administration department and our allocation plan. In general, the preferential allocation plan may be inconsistent with PTEs social contributions. Finally, a real example is presented to illustrate our approach.**

I. INTRODUCTION

To relieve the rapidly growing urban traffic congestion and transport pollution, urban public transport has been attracting significant attention. Many policies have been enacted to encourage citizens to choose the public transport as the first travel choice, such as limiting fares, keeping unprofitable suburban bus lines, improving the public transports service qualities and so on. To compensate the policy-related losses of public transport enterprises (PTEs), governments generally provide transport subsidies to them.

In the urban public transport field, the transport subsidy allocation problem is an important topic with great theoretical and practical significance. From the viewpoint of the government administration department, the losses of PTEs should be the less the better. However, the reality is always opposite. For example, from 2008 to 2010, the total losses of a city in China are 578.1305, 693.6191 and 1015.4276 millions yuan respectively. The rapid increasing loss has becoming a heavier and heavier burden to government. One of important reasons is that government lacks of scientific allocation methods. Besides, the asymmetric loss information between PTEs and their regulators is another important reason that makes the loss be hard restrained. Even though the total amount of losses can be obtained exactly, it is also hard to distinguish the true policy-related losses from it. Therefore, in order to guarantee the healthy development of the urban public transport industry, a quantitative subsidy allocation method, which can provide PTEs a motivation to not only control their input consumptions, but also improve their social contributions, is needed urgently.

Unfortunately, very few previous studies have paid attention to this important problem. [1] first presented this problem and proposed a transport subsidy allocation method based on data envelopment analysis (DEA) and the satisfaction degrees of PTEs. DEA is an important tool to evaluate the performance of a set of homogeneous decision making units (DMUs) with multidimensional inputs and outputs. These DMUs constitute a production possibility set (PPS). The PPS is enveloped by some outside DMUs. That is, the efficient frontier is comprised of the outside DMUs. The efficiency of a given DMU under evaluation is obtained by projecting it onto the efficient frontier. If the DMU is on the efficient frontier, then it is called efficient and its efficiency equals to 1. Otherwise, the DMU is called inefficient and its efficiency is not more than 1. Because DEA methods evaluate the performance of DMUs without assigning any prior weights on the performance indicators or assumptions on the production function form, it outperforms many other traditional performance evaluation methods. Since the first CCR DEA model is formally proposed by [2] in 1978, many DEA models have been extended [3-6] and widely used in many areas [7-8].

[1] treated the allocated transport subsidy as an input and obtained a unique subsidy allocation scheme with considering both the efficiency maximization principle and the maximin equity of PTEs satisfaction degrees. Therefore, the transport subsidy allocation problem can be further extended to fixed input allocation problem. In DEA, the fixed input allocation problem is an important issue. When the fixed input is considered as an independent factor, efficiency invariance [9-13] and efficiency maximization [1] [14-19] are two common allocation principles. Apart from those strategies, there exist other methods while study the problem from different perspectives [20-22]. These DEA-based allocation methods can provide some inspirations in the subsidy allocation research.

This paper proposes a transport subsidy allocation method based on the social contribution of PTEs. Considering the social contribution of each PTE is reflected in their operation data, we define the social contribution based on multidimensional performance indicators. The definition can ensure that the social contribution is directly proportional to outputs and inversely proportional to inputs, which could urge PTEs to control their input consumptions and improve their outputs.

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Then a constraint set is deduced based on the concept of the subsidy proportion allocated to each PTE should be equal to their social contribution proportion. We prove that the constraint set is the same as the efficient allocation set from the standpoint of DEA. After that, we propose our allocation model based on the constraint set. Because the government administration department may have his preferential allocation plan in practice, which may be inconsistent with PTEs social contributions, our method minimizes the maximum distance between the preferential allocation plan and our allocation plan. The optimal allocation plan is directly related to the preferential allocation plan and can reflect the social contributions of PTEs. Our method is illustrated by a real example.

The rest of this paper is organized as follows: Section 2 proposes the allocation method. The proposed approach is applied to allocate the transportation subsidy among seven PTEs in Section 3. Conclusions are given in the end.

II. ALLOCATION METHOD

Suppose there are n rational PTEs (DMUs) to be allocated the transport subsidy. The jth $(j = 1, 2, \ldots, n)$ PTE yields s outputs y_{rj} $(r = 1, 2, \ldots, s)$ by consuming m inputs x_{ij} $(i = 1, 2, \ldots, m)$. The outputs include satisfaction degrees of passengers, transporting passengers, operating mileages, incomes and so on. The inputs include the number of standard operating buses, cost expenses, employees and so on.

Usually, the abilities of PTEs for using of inputs and generating outputs and the scales of PTEs are not all the same. That is to say, the social contributions of PTEs are not all the same. Considering the social contribution can be reflected in various performance indicators, to describe the social contribution more comprehensively and perfectly, this paper selects multidimensional performance indicators. As a result, we can get the following definition:

Definition 1. The social contribution of the dth $(d \in 1, 2, \ldots, n)$ PTE is given as follows:

$$S_d = \sum_{r=1}^{s} \mu_r y_{rd} - \sum_{i=1}^{m} \upsilon_i x_{id}, \upsilon_i, \mu_r \geq 0, \forall i, r \quad (1)$$

where S_d is the social contribution of DMU_d. υ_i and μ_r are unknown weights attaching the ith input and the rth output respectively. It can be found from equation (1) that *the social contribution is directly proportional to outputs and inversely proportional to the input*. Thus it would be useful to encourage the PTEs to control the input consumptions and improve the outputs. Based on a common set of weights, different indicators can be transferred into the same platform so as to make further evaluations and comparisons. The measurement units of input and output do not affect the result of S_d as,

$$\begin{aligned} S_d &= \sum_{r=1}^{s} \mu_r y_{rd} - \sum_{i=1}^{m} \upsilon_i x_{id} \\ &= \sum_{\substack{r=1 \\ r \neq r'}}^{s} \mu_r y_{rd} + \mu_{r'} a y_{r'd} - \sum_{\substack{i=1 \\ i \neq i'}}^{m} \upsilon_i x_{id} - \upsilon_{i'} b x_{i'd} \end{aligned}$$

where $a > 0$ and $b > 0$, and they are the scaled factors attaching to the r'th output and the i'th input respectively.

Setting $\mu'_{r'} = \mu_{r'} a$ and $\upsilon'_{i'} = \upsilon_{i'} b$, we can get

$$S_d = \sum_{\substack{r=1 \\ r \neq r'}}^{s} \mu_r y_{rd} + \mu'_{r'} y_{r'd} - \sum_{\substack{i=1 \\ i \neq i'}}^{m} \upsilon_i x_{id} - \upsilon_{i'} x_{i'd}, \quad (2)$$

where the weights of $\upsilon_i (\forall i \neq i')$, $\upsilon'_{i'}$, $\mu_r (\forall r \neq r')$ and $\mu'_{r'}$ are unknown and no less than 0. That is, the feasible region of equation (2) is the same as that of equation (1), Therefore, the result of S_d will be not affected by the input and output measurement units.

We believe that a fair transport subsidy allocation plan should ensure the subsidy proportion allocated to each PTE according with their social contribution proportion. The rule reflects the proportional fairness [5]. Accordingly, we can get the following equation:

$$\begin{aligned} &\Omega = \{(P_1, ..., P_n) | P_j = M/N, \\ &M = (\textstyle\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \upsilon_i x_{ij}), \\ &N = \textstyle\sum_{k=1}^{n} (\sum_{r=1}^{s} \mu_r y_{rk} - \sum_{i=1}^{m} \upsilon_i x_{ik}), \\ &\quad P_j, \mu_r, \upsilon_i \geq 0, \forall r, i, j\} \end{aligned} \quad (3)$$

where P_j is the subsidy proportion allocated to DMU_j, $(j = 1, 2, ..., n)$. Thus, it can be found that the total amount of transport subsidies do not affect the optimal allocation plan.

Letting $t = 1/\sum_{k=1}^{n} (\sum_{r=1}^{s} \mu_r y_{rk} - \sum_{i=1}^{m} \upsilon_i x_{ik})$, $u_r = t\mu_r$ and $v_i = t\upsilon_i$, then we can rewrite equation (3) as:

$$\begin{aligned} &\Omega = \{(P_1, ..., P_n) | P_j = \textstyle\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij}, \\ &\textstyle\sum_{k=1}^{n} P_k = 1, P_j, u_r, v_i \geq 0, \forall r, i, j\} \end{aligned} \quad (4)$$

From the perspective of DEA, the optimal subsidy allocation plan generated by our approach has another good nature as given in the following theorem:

Theorem 1. All PTEs must be efficient based on the subsidy allocation plan generated in equation (4).

This is because that our equation (4) is also an efficient allocation set in DEA methodology [18-19]. The relevant proof process can be referred to Theorem 2 in [18]. Thus the allocation plan generated under the constraint of equation (4) must ensure all PTEs efficient. As mentioned above, the efficiency maximization is an important fixed input allocation principle in DEA, such as [1] and [14-19]. Therefore, *the equation (4) provides a bridge between the efficiency and the profit, which can ensure the theoretical and practical values*.

In practice, the government administration department (central decision maker) may often have his preferential allocation plan. However, this allocation plan may be inconsistent with PTEs social contributions. That is, we can not find a common set of weights in equation (4) for all PTEs. Therefore, the preferential allocation plan may be unfair for some PTEs who have good social contribution performance, and it would go against the improvement of PTEs performance.

The allocation plan preferred by the central decision maker is denoted as $(P'_j, \forall j)$, which is a known allocation scheme, then we can propose the following subsidy allocation model:

$$\begin{aligned} &\underset{u,v}{Min} \max_{1\le d\le n} \quad |P_d - P'_d| \\ &s.t. P_j = \sum\nolimits_{r=1}^{s} u_r y_{rj} - \sum\nolimits_{i=1}^{m} v_i x_{ij}, \forall j \\ &\sum\nolimits_{k=1}^{n} P_k = 1, \\ &P_j, u_r, v_i \ge 0, \forall r, i, j \end{aligned} \tag{5}$$

The objective function of model (5) means minimizing the maximal distance between the preferential allocation plan and our proposed allocation plan. The constraint set of model (5) is equation (4).

The objective function of model (5) is the absolute value form. To deal with it, we can rewrite model (5) by setting $a_j = [|P_j - P'_j| + P_j - P'_j]/2$, $b_j = [|P_j - P'_j| - P_j + P'_j]/2$:

$$\begin{aligned} &\underset{u,v}{Min} \max_{1\le d\le n} \quad (a_d + b_d) \\ &s.t. \sum\nolimits_{r=1}^{s} u_r y_{rj} - \sum\nolimits_{i=1}^{m} v_i x_{ij} - a_j + b_j = P'_j, \forall j \\ &\sum\nolimits_{k=1}^{n} (a_k - b_k) = 0 \\ &a_j - b_j + P'_j \ge 0, \forall j \\ &u_r, v_i, a_j, b_j \ge 0, \forall r, i, j \end{aligned} \tag{6}$$

To solve multi-objective mathematical programming, we can rewrite model (6) by setting $\max_{1\le d\le n}(a_d + b_d) = \beta$:

$$\begin{aligned} &\underset{u,v}{Min} \quad \beta \\ &s.t. a_j + b_j \le \beta, \forall j \\ &\sum\nolimits_{r=1}^{s} u_r y_{rj} - \sum\nolimits_{i=1}^{m} v_i x_{ij} - a_j + b_j = P'_j, \forall j \\ &\sum\nolimits_{k=1}^{n} (a_k - b_k) = 0 \\ &a_j - b_j + P'_j \ge 0, \forall j \\ &u_r, v_i, a_j, b_j \ge 0, \forall r, i, j \end{aligned} \tag{7}$$

Algorithm 1. Obtain a unique subsidy allocation scheme

Step 1. Initialization. Let $t = 1$ and the optimal solution of model (7) is denoted as $\left(a^*_{1j}, b^*_{1j}, u^*_{1r}, v^*_{1i}, \beta^*_1, \forall r, i, j\right)$. Then the PTE set $J_0 = \{1, ..., n\}$ can be divided into the following two subsets:

$$\begin{aligned} J_1 &= \{j |\, a_j + b_j = \beta^*_1, \forall j \in J_0\}, \\ J_2 &= \{j |\, a_j + b_j < \beta^*_1, \forall j \in J_0\} \end{aligned}$$

where $J_0 = J_1 \cup J_2$. The number of PTEs in J_1 is denoted as n_1, and their subsidy allocation plan is $\left(P^*_{1j} = a^*_{1j} - b^*_{1j} + P'_j, \forall j \in J_1\right)$. If $n_1 = m + s$, then stop and the optimal solution $\left(a^*_{1j}, b^*_{1j}, u^*_{1r}, v^*_{1i}, \beta^*_1, \forall r, i, j\right)$ is unique. If $n_1 < m + s$, then go to Step 2;

Step 2. Let $t = t+1$ and solve the general model as follows:

$$\begin{aligned} &\beta^*_k = \underset{u,v}{Min} \quad \beta \\ &s.t. a_j + b_j = \beta^*_1, j \in J_1 \\ &a_j + b_j = \beta^*_2, j \in J_3 \\ &\cdots \\ &a_j + b_j = \beta^*_{t-1}, j \in J_{2t-3} \\ &a_j + b_j \le \beta, j \in J_{2t-2} \\ &\sum\nolimits_{r=1}^{s} u_r y_{rj} - \sum\nolimits_{i=1}^{m} v_i x_{ij} - a_j + b_j = P'_j, \forall j \\ &\sum\nolimits_{k=1}^{n} (a_k - b_k) = 0 \\ &a_j - b_j + P'_j \ge 0, \forall j \\ &u_r, v_i, a_j, b_j \ge 0, \forall r, i, j \end{aligned} \tag{8}$$

The optimal solution of model (8) is denoted as $\left(a^*_{tj}, b^*_{tj}, u^*_{tr}, v^*_{ti}, \beta^*_t, \forall r, i, j\right)$, and the subset J_{2t-2} can be further divided into two sub-subsets:

$$\begin{aligned} J_{2t-1} &= \{j |\, a_j + b_j = \beta^*_t, \forall j \in J_{2t-2}\}, \\ J_{2t} &= \{j |\, a_j + b_j < \beta^*_t, \forall j \in J_{2t-2}\} \end{aligned}$$

where $J_{2t-2} = J_{2t-1} \cup J_{2t}$ and the number of PTEs in J_{2t-1} is denoted as n_t, and their subsidy allocation plan is $P^*_{tj} = a^*_{tj} - b^*_{tj} + P'_j, \forall j \in J_{2t-1}$. If $n_1 + n_2 + \cdots + n_t = m + s$, then stop and the optimal solution $\left(a^*_{tj}, b^*_{tj}, u^*_{tr}, v^*_{ti}, \beta^*_t, \forall r, i, j\right)$ is unique. If $n_1 + n_2 + \cdots + n_t < m + s$, then repeat Step 2.

Theorem 2. The optimal transport subsidy allocation scheme obtained by Algorithm 1 must be unique.

Proof: Combining $P_j = \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij}$ and $P^*_{1j} = a^*_{1j} - b^*_{1j} + P'_j$, we can get

$$\sum\nolimits_{r=1}^{s} u_r y_{rj} - \sum\nolimits_{i=1}^{m} v_i x_{ij} - a^*_{1j} + b^*_{1j} = P'_j \tag{9}$$

If $n_1 = m + s$, (9) has m plus s equations including m plus s variables $(u_r, v_i, \forall r, i)$. Because the source data $X_j = (x_{1j}, x_{2j}, \cdots, x_{mj})^T$ and $Y_j = (y_{1j}, y_{2j}, \cdots, y_{sj})^T$ are independent, the subsidy allocation would be uniquely determined and the procedure stops.

If $n_1 < m + s$, the number of equations in (9) is less than that of the variables $(u_r, v_i, \forall r, i)$, thus the procedure should be continued. If $n_1 + n_2 + \cdots n_t = m + s$, then the solution $(u_r, v_i, \forall r, i)$ would be unique and stop, else do until there are m plus s equations uniquely determining the variables $(u_r, v_i, \forall r, i)$.

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III. AN EMPIRICAL EXAMPLE

There are seven PTEs (A-G) located in a city in Jiangsu province. As shown in Table I, we select two inputs and three outputs in 2010 to measure their social contributions. Input 1 and 2 are the number of standard operating buses and the total of expended costs, respectively. Outputs from 1 to 3 are the satisfaction degrees of passengers, the number of transporting passengers and the total kilometers, respectively. The income is not selected as the commonweal feature of the public transport industry. The second column is the initial allocation plan proposed by the government administration

TABLE I
STATISTICAL DATA

	Input 1	Input 2	Output 1	Output 2	Output 3
Avg	835.14	32805.10	59.86	13851.54	5768.99
Std. Dev.	872.10	39614.21	4.81	17138.73	6009.86

TABLE II
RELEVANT RESULTS

PTE	P'_j	$\underline{P_j}$	$\overline{P_j}$	P^*_j	SD_j	E^*_j
A	0.1669	0.0738	0.4835	0.2211	0.3595	1
B	0.0794	0.0000	0.3789	0.0778	0.2053	1
C	0.0114	0.0055	0.1834	0.0112	0.0320	1
D	0.0412	0.0000	0.5769	0.0281	0.0487	1
E	0.6014	0.0000	0.6064	0.5883	0.9702	1
F	0.0622	0.0000	0.2380	0.0492	0.2067	1
G	0.0374	0.0136	0.2754	0.0243	0.0409	1

department. As mentioned above, it may be inconsistent with the social contributions of PTEs.

By using $Max \quad P_d$ (or $Min \quad P_d$) as the objective function and equation (4) as the constraints, we can obtain the dth PTEs maximum allocation proportion $\overline{P_d}$ (or the minimum allocation proportion $\underline{P_d}$). When the subscript d is set from 1 to 7 respectively, we can get all PTEs allocation proportion ranges. They are given in the third and fourth column of Table II. It can be found that the allocation proportion ranges of PTEs are different from each other. The optimal allocation plan would be generated in this range.

Based on model (6), we can obtain the optimal allocation plan $(P^*_j, \forall j)$ as given in the fifth column of Table II. It is different with the allocation plan proposed by the government administration department. The optimal allocation plan is calculated under a common set of weights, thus it not only approaches the preference of the government administration department, but also accords with the scales of PTEs and their social contributions. It would be helpful to encourage PTEs to improve their performance on social contribution and decrease input consumptions.

The sixth column of Table II is the satisfaction degrees of PTEs to the optimal allocation plan. Here the satisfaction degree of the dth PTE is defined as $SD_d = (P_d - \underline{P_d})/(\overline{P_d} - \underline{P_d})$ [18]. The definition can well describe the attitude of each PTE to the subsidy allocation plan. In general, a rational PTE would selfishly pursue his maximal possible allocation proportion. The satisfaction degree of a PTE who is allocated a large number of subsidies would be high. When $P_d = \overline{P_d}$, his satisfaction degree reaches the maximum $(SD_d = 1)$. When $P_d = \underline{P_d}$, his satisfaction degree reaches the minimum $(SD_d = 0)$. Affected by the initial allocation plan $(P'_j, \forall j)$, the difference of PTEs satisfaction degrees is quite large. A trend is that a PTE with large scale may have a relative large satisfaction degree, such as PTE E, A, F and B.

The last column of Table II is the efficiencies of all PTEs based on the optimal allocation plan. It is calculated by CCR DEA model [2] with treating the allocation plan as an independent input. The efficiency result shows that all PTEs would be efficient after the allocation, which accords with Theorem 1.

IV. CONCLUSIONS

This paper proposes a method to allocate the transport subsidy among a set of PTEs. Because the social contribution of a PTE is reflected in the operating process, we describe the social contribution of each PTE by the form of the weighted sum of multidimensional outputs minus the weighted sum of multidimensional inputs. The input and out measurement units do not affect the result of the social contribution. Based on the proportional allocation concept, we can deduce a set of constraints, which is also the same as the efficient allocation set from the viewpoint of DEA. Considering the government administration department may always have his preferential allocation plan, which may be inconsistent with PTEs social contributions, we propose our subsidy allocation model. The model is a minimax programming with an absolute value objective function. The optimal allocation scheme is reflected by the preference of the government administration department. Besides, the allocation method can also be helpful to encourage PTEs to improve their social contribution and decrease input consumptions. The example results illustrate our allocation method is valid.

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