A Probabilistic Price Mechanism Design for Online Auctions

Jie Zhang*†, Linjing Li*, Fei-Yue Wang, *Fellow*, IEEE *†‡1

*The State Key Lab of Management and Control for Complex Systems,
Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China.
† Qingdao Academy of Intelligent Industries, Shandong 266000, China.
‡ The Research Center for Military Computational Experiments and Parallel Systems Technology,
National University of Defense Technology, Changsha 410073, China.
1. Email: feiyue.wang@ia.ac.cn (Corresponding author)

***Abstract*—Recently, there is a rapid growth of the online auctions in e-commerce platforms, in which small and medium-sized enterprises (SMEs) heavily depend on the advertising systems. We need to design flexible price mechanisms to reduce the competition of SMEs without affecting competitive large companies. In this paper, a probabilistic price mechanism design approach is investigated for online auctions. Utilizing this approach, we first introduce simple mechanisms as a tool for designing new mechanisms. Based on a simple and a classical mechanism probabilistic price mechanisms are designed for online auctions and their properties are analysed. Furthermore, two mechanism design algorithms are suggested for different online auction scenarios. Experiments are presented to demonstrate the flexibility and the effectiveness of the proposed probabilistic mechanism design approach.**

Keywords: mechanism design, online auctions, probabilistic mechanism, computational experiments, e-commerce.

I. Introduction

According to a report of eMarketer in 2016[9], the Chinese leading e-commerce platform Alibaba group generates 60% of online advertisements in China. Compared with the traditional online advertising services provided by search engines (e.g. Google AdWords [14]), online advertising auctions in e-commerce platforms have customers with much bigger purchase potential and well analyzed buying habits. Since the order of products in natural search results is very much related to the sales volume, there is very little opportunity to have effective natural impressions for SMEs. Consequently, the competition of auctions in e-commerce platforms is intense and important. However, the auction mechanisms such as the generalized first-price [11], the generalized second-price [8, 16], and the VCG [17, 5, 10] have been designed according to the classical auction theory emphasizing competition resulting in the dropout of SMEs in online applications.

In order to keep the entire industry growing it is crucial to maintain a large basis of participants regardless of their sizes by reducing the competition of SMEs while keeping the incentive of competitive large enterprises. A reasonable approach to deal with this issue is to adopt randomization [6] in determining the winner of the auction and her charge. Normally, in a standard single item auction mechanism, the object will always be awarded to the bidder with the highest bid, however, this is not guaranteed in nonstandard mechanism [13] such as randomized mechanisms[6].

The objective of our price mechanism design problem for online auctions consist of two main parts: the revenue of the seller and the winning rate of the bidders with low valuations (i.e. the SMEs). In this paper, we propose a probabilistic approach to design balanced price mechanisms for e-commerce platforms by introducing randomization. When the bids are submitted, the seller of the advertisement first chooses a mechanism randomly from a collection of mechanisms according to certain probability distribution, and then decides the winner and the corresponding price based on the selected mechanism. We develop a method to select the collection of mechanisms in order to keep rational bidders' strategies. The contributions of this paper can be summarized as follows: (1) A probabilistic price mechanism design approach is proposed for online auctions. (2) *Simple mechanisms* are introduced such that the probabilistic combination of a classical and a simple mechanism will keep the original equilibriums of bidders. (3) Two probabilistic price mechanisms are developed for online auctions in e-commerce platforms.

The rest of this paper is organized as follows. Next section reviews related works. Section III describes the mechanism design problem and gives a preliminary analysis to first-price auctions. The probabilistic mechanism design approach is presented and used to develop particular mechanisms in Section IV. Section V offers algorithms and reports experiments to evaluate the probabilistic price mechanisms for long-running auctions. Section VI concludes the paper.

II. Related Work

Auction has been widely regarded to be effective and efficient for scarce resource allocation. Traditionally, it is employed mainly to sell valuable goods like antiquities, artifacts, jewelry, etc. Recently, Internet became fantastic for selling both tangible (e.g., ebay) and virtual goods (e.g., Google AdWords , and ridesharing[18]). Online auction became one of the most successful sectors of the Internet industry and it has triggered a new wave of research on auction theory. From the point of view of scarce good allocation, efficiency is the central issue discussed in most of the classical researches based on the seminal paper of W. Vickrey [17]. Vickrey showed the equivalence of the first-price and second-price auctions in term of expected revenue. Later, R. B. Myerson [15] proved this property in a more general setting and it is now widely referred as the revenue equivalence theorem (RET) in the auction related literatures. In a standard deterministic single item auction, a bidder with highest price always wins.

Randomized mechanisms are well known for assignment problems. As a generalization of deterministic mechanisms, random serial dictatorship mechanism [1] and probabilistic serial mechanism [4] were introduced by the randomization of the ordering process. The idea is to regard each object as a continuum of probability shares [12]. V. Conitzer and T. Sandholm [6] modeled mechanism design as an optimization problem to find a randomized mechanism with a probability distribution over the outcome set in order to maximize the auctioneer's objective. In their definition of randomized mechanism with payments, the outcome is randomized and the payment selection function is deterministic. In a subsequent research [7], self-interested automated mechanism was designed to maximize the revenue of the seller. In [2], randomization is employed for double auctions. After the bids are submitted, they use the Trade reduction (TR) mechanism with probability p, and the VCG mechanism with probability $1-p$. Note that, since the equilibrium strategies for TR and VCG are the same(both truthful), the probabilistic combination will keep the equilibrium. However, if two auctions have different equilibriums, we need to investigate the new equilibrium.

III. Notations and Current Practice

In this section we provide a preliminary analysis of the traditional first-price (FP) sealed-bid auction [13]. The notations we used in this paper are listed in Table I.

Consider an auction with sealed price bids by N bidders for a single object (e.g. a keyword) for sale, where $N \geq 2$. Bidder-i assigns a value X_i to the object to represent the maximum amount[1] she wants to pay for it. It is assumed that X_i, $i = 1, 2, \ldots, N$, are independent and identically distributed on the interval $B = (0, +\infty)$ according to an distribution function F with finite expectation $\mu = E[X_i] < \infty$, furthermore, F has a continuous density $f \equiv F'$ and has full support. Bidder-i knows her actual valuation x_i. Let $\bar{b} = (b_1, \ldots, b_N) \in B^N$ denote the bidding vector of the N bidders, the allocation rule is a function $L(\bar{b}) : B^N \to B$, showing that with the bidding vector $\bar{b}$, the winner is with bid $L(\bar{b})$. For convenience, we denote by $L^{(\#m)}(\bar{b})$ the m-th largest bid among $b_1, \ldots, b_N$. Since a failed bid has no cost in current practice, we simplify the payment rule as a function $C(L^{(\#1)}(\bar{b}), \ldots, L^{(\#N)}(\bar{b})) : B^N \to B$ to denote the winner's cost. Thus, a price mechanism $\mathcal{M} = (L, C)$ can be defined with an *allocation rule* $L(\bar{b})$ and a *payment rule* $C(\bar{b})$. Note that, if in the mechanism bidder-i is charged by $C(\bar{b}) > x_i$, then the allocation will fail since a rational bidder can not pay a price higher than her valuation.

[1]Besides the real valuation of the object, budget [3, 19] is another critical factor affacts the maximum amount of money. SMEs will have smaller X.

TABLE I
List of notations

Notation	Definition
$B \subset (0, +\infty)$	bidding set & ad valuation set
$b_i \in B$	bid of bidder i
$x_i \in B$	valuation of bidder-i
Π_i	payoff of bidder i
p_i	winning rate of bidder i
m_i	expected payment of bidder i
$\beta(x) : B \to B$	symmetric equilibrium strategy
R	expected revenue of the seller
θ	entrance threshold of winning rate
$L(\bar{b}) : B^N \to B$	allocation rule
$C(\bar{b}) : B^N \to B$	payment rule
$(\mathcal{M}_1, \mathcal{M}_2; \lambda)$	a probabilistic mechanism with distribution λ
F	distribution of bidders' valuations
$F_m^{(n)}$	distribution of the m-th largest in n-th valuations
$L^{(\#m)}(\bar{b})$	the m-th largest in $b_1, \ldots, b_N$

The FP mechanism can be described as follows.

Definition 1 (First Price Mechanism, FP). *We denote the FP mechanism as* $\mathcal{M}^I = (L^I, C^I)$*, where* $L^I(\bar{b}) = L^{(\#1)}(\bar{b})$ *and* $C^I(L^{(\#1)}(\bar{b}), \ldots, L^{(\#N)}(\bar{b})) = L^{(\#1)}(\bar{b})$.

For a deterministic price mechanism $\mathcal{M} = (L, C)$, the payoff of bidder-i can be computed as

$$\Pi_i(\vec{x}, \bar{b}) = (x_i - C(L^{(\#1)}(\bar{b}), \ldots, L^{(\#N)}(\bar{b}))) \cdot \mathbb{1}_{b_i = L(\bar{b})} \quad (1)$$

where $\mathbb{1}_{b_i = A(\bar{b})}$ denotes the indicator function

$$\mathbb{1}_{b_i = A(\bar{b})} := \begin{cases} 1 & \text{if } b_i = A(\bar{b}), \\ 0 & \text{if } b_i \neq A(\bar{b}). \end{cases} \quad (2)$$

We will next review the preliminary analysis of this classic mechanism. A strategy for a rational bidder-i is a function $\beta_i : B \to B$, $b_i = \beta_i(x_i)$. We focus on the case of symmetric bidders, i.e., all of them have the same strategy β in the game.

The payoff of a bidder equals her valuation minus the payment, which is determined by the payment rule. In a FP auction without reserve price ($r = 0$), with bids $b_1, \ldots, b_N$, the payoff of bidder-i is

$$\Pi^I(x_i, b_i; r = 0) = (x_i - b_i) \cdot \mathbb{1}_{b_i = L^{\#1}(\bar{b})}. \quad (3)$$

According to [13], a symmetric equilibrium strategy is

$$\beta^I(x_i; r = 0) = x_i - \int_r^{x_i} \frac{F(x)^{N-1}}{F(x_i)^{N-1}} dx, \quad (4)$$

which is a monotonic function of the valuation x_i. Suppose all the bidders employ the strategy, then the winning rate can be determined by the allocation rule and the distribution of the valuations. The probability that bidder-i wins the auction equals the probability that she has the highest valuation x_i:

$$p^I(x_i; r = 0) = F(x_i)^{N-1}. \quad (5)$$

The FP mechanism (and any standard single object auction) has an important characteristic: "first-price" wins. A rational bidder's winning rate is positively correlated with her valuation, i.e. a bidder with lower valuation has lower winning rate. For example, assume a bidder has low valuation x with $\Pr(\xi \le x) = 1/5$, her winning rate $1/5^{N-1}$ is very small when N is large. Insufficient winning rate will damage the enthusiasm of the bidders with low valuations. Assuming an entrance threshold θ as the minimum winning rate such that bidders with only $p^I(x) \ge \theta$ are willing to stay in the market, a corresponding price mechanism will exclude bidders with insufficient winning rate $p^I(x) < \theta$ from the auction.Thus, bidders with valuation $x \le F^{-1}(\theta^{1/(N-1)})$ will leave the market since their winning rates are too low to maintain the page views. In e-commerce platforms, decreasing advertisers (retailers) will reduce the advertisement company's long-term revenue. A reasonable solution is to develop mechanisms giving consideration to both the expected revenue and the winning rate of bidders with low valuations.

IV. Probabilistic Price Mechanism Design

In the following, we will introduce a probabilistic approach to design price mechanisms incorporating both the interests of the bidders with low valuations and the expected revenue of the platform. Additionally, our approach will keep the equilibrium strategy unchange.

In order to utilize standard ("first-price-win") mechanisms to incent players to bid higher, while increasing the winning rate of the bidders with low valuations, we introduce randomization to design price mechanisms. Suppose we have a mechanism $\mathcal{M}_1$ (e.g., FP), we will find a mechanism $\mathcal{M}_0$ and a discrete probability distribution $\bar{\lambda} = (\lambda_0, \lambda_1)$ over the two mechanisms. The seller first asks the bidders to provide their bids $\bar{b} = (b_1, \ldots, b_N)$, and then chooses a price mechanism randomly from the mechanisms according to the distribution $\bar{\lambda}$, and finally, selects the winner and decides the price using the selected mechanism. In the following, we focus on finding such a mechanism $\mathcal{M}_0$ and the distribution such that the new mechanism will hold the same equilibrium strategies for the bidders with $\mathcal{M}_1$.

After a bidding contract is established and the randomly selected price mechanisms is $\mathcal{M}_j$, the probabilistic price mechanism acts in the same way as the selected mechanism keeping properties such as the winner of the auction and the payment. The linearity of the conditional expectation operator implies the following properties.

Lemma 1. *Consider mechanisms $\mathcal{M}_j$, $j = 1, \ldots, M$, and assume that the bidders (according to the distribution F) bid $b_1, \ldots, b_N$. Then, bidder-i's payoff with respect to the probabilistic price mechanism $\mathcal{M} = (\mathcal{M}_1, \ldots, \mathcal{M}_M; \bar{\lambda})$ equals*

$$E[\Pi(X_i, b_i)] = \sum_{j=1}^{M} \lambda_j E[\Pi_j(X_i, b_i)]. \tag{6}$$

Hence, we can estimate the symmetric equilibrium and the expected payment for the bidders in the new mechanism. Based on the user distribution $F(x)$ and proper base mechanisms $\mathcal{M}_j$, $j = 1, \ldots, M$, we will be able to adjust parameters $\bar{\lambda}$ to design new mechanisms.

The FP mechanism was originally designed to stimulate the competition. Hence, we introduce a new price mechanism without any competition to help bidder with low valuations. We call it a *simple mechanism* if both the expected values of the allocation rule and the payment rule are independent to the bids. Following the definition of the simple mechanism, we know that highest bid will not ensure the winning of a bid, thus we can combine a classical and a simple mechanism to increase the winning rate of bidders with lower valuations. For example, a simplest simple mechanism is always allocating the position to bider-1 and charge her with a constant price.

Next, we present an important property of simple mechanisms. Based on Lemma 1, it is clear that by combining an existing mechanism with a simple mechanism, the equilibrium will not change.

Theorem 1. *Consider mechanism $\mathcal{M} = (\mathcal{M}_1, \mathcal{M}_2; \bar{\lambda})$, where $\mathcal{M}_1$ is a simple mechanism. Suppose, for any N bidders, $\mathcal{M}_2$ leads to an equilibrium $\bar{b}$, then $\bar{b}$ is also an equilibrium of $\mathcal{M}$.*

Proof. Denote the equilibrium strategy of $\mathcal{M}_2$ by $\bar{\beta} = (\beta_i, \beta_{-i})$. First, suppose all but bidder i follow the original equilibrium of mechanism $\mathcal{M}_2$, then the expected payoff of bidder-i with bid b is $E[\Pi(x_i, b)] = \lambda_1 E[\Pi_1(x_i)] + \lambda_2 E[\Pi_2(x_i, b)] \le \lambda_1 E[\Pi_1(x_i)] + \lambda_2 E[\Pi_2(x_i, \beta_i(x_i))]$. The inequity holds, since the expected payoff of a simple mechanism can be determined by the allocation rule, the payment rule and the distribution of bidder valuation, which is then independent from the bids, and $\bar{\beta}$ is an equilibrium of $\mathcal{M}_2$. Hence, $\bar{\beta}$ is also an equilibrium of the probabilistic price mechanism $(\mathcal{M}_1, \mathcal{M}_2, \bar{\lambda})$ for any $\bar{\lambda}$. □

Thus, considering the probabilistic combination of FP and a simple mechanism, the new mechanism will keep the equilibrium of the original FP. This property ensures that rational bidders do not have to change their bids even if $\bar{\lambda}$ is changing and the mechanisms are not incentive compatible. Hence, we can design dynamic mechanisms during the long-term operation of the auction with a stable equilibrium. Next, we utilize this property to design static probabilistic price mechanisms based on classical ones to acquire good performance and reduce the migration cost of bidders. Here is a simple mechanism[2].

Definition 2 (Equal-Possible Mechanism, EP)**.** *Let $\mathcal{M}_j$, $j = 1, \ldots, N$ be deterministic mechanisms with $L_j = b_j$, and $C_j \equiv r$ such that mechanism $\mathcal{M}_j$ will always let bidder-j win, and charge her for reserve price r. Let $\bar{\lambda} = (1/N, \ldots, 1/N)$, then we call the probabilistic combination $\mathcal{M}^0 = (\mathcal{M}_1, \ldots, \mathcal{M}_N; \bar{\lambda})$ equal-possible (EP) mechanism.*

[2]Simple mechanisms can be useful in practice. By introducing additional parameters to the pricing rules, we can design simple mechanisms easily to adapt different scenarios. For example, in online auction settings, we can employ the advertising quality score q_i as the mechanism distribution in Definition 2, i.e. $\bar{\lambda} = (q_1/\sum_j q_j, \ldots, q_N/\sum_j q_j)$. The proposed simple mechanism will give advantage to bidders with better quality score.

$\mathcal{M}^0$ is quite different than the "first-price-win" mechanisms such as FP. Most bidders ($x_i \geq r$) will have the same rate

$$p^0(x_i; r) = \frac{1}{N} \cdot \mathbb{1}_{x_i \geq r} + \frac{1}{N} \cdot 0 \cdots = \frac{1}{N} \cdot \mathbb{1}_{x_i \geq r} \quad (7)$$

to win the auction with any bidding vector $\bar{b}$. This can be much more friendly to bidders with low valuations compared with FP, since they have disadvantages in the competition. Note that, if a rational bidder's valuation $x_i < r$, then she will not pay for her winning bid resulting in 0 payoff. Hence, r is actually a reserve price of the proposed Equal-Possible (EP) mechanism. With the purpose to reduce the difficulty for bidders with low valuations, we always assume that the reserve price r of EP is with a sufficiently small value.

Bidder-i's payoff will be independent from her bid b_i,

$$\Pi^0(x_i, b_i; r) = \begin{cases} \max\{x_i - r, 0\}, & \text{with } 1/N \text{ prob,} \\ 0, & \text{with } (1-N)/N \text{ prob.} \end{cases} \quad (8)$$

The expected payment of a bidder-i is the product of her winning probability (i.e., $1/N$) and the constant payment (i.e., r) if $x_i \geq r$:

$$m^0(x_i; r) = \frac{1}{N} \cdot r \cdot \mathbb{1}_{x_i \geq r} + \frac{1}{N} \cdot 0 \cdots = \frac{r}{N} \cdot \mathbb{1}_{x_i \geq r}. \quad (9)$$

The expected payoff of bidder-i is

$$\begin{aligned} \pi^0(x_i; r) &= \frac{1}{N} \cdot E[x_i - m^0(x_i) | x_i \geq r] \cdot \mathbb{1}_{x_i \geq r} + \frac{1}{N} \cdot 0 \cdots \\ &= \frac{\mathbb{1}_{x_i \geq r}}{N} \cdot (1 - F(r))(x_i - r). \end{aligned} \quad (10)$$

Thus, the expected revenue of the seller becomes

$$R^0(r) = N \cdot E[m^0(v)] = r(1 - F(r)). \quad (11)$$

Since FP sell the object with much higher price than the reserve price with great probability, with the same reserve price r, the expected revenue of the seller with respect to the EP mechanism is much smaller than in the case of FP.

In the following, we will take the Equal-Possible mechanism introduced in Definition 2 and the classical FP as base mechanisms, and try to find proper distribution $\bar{\lambda}$ and reserve prices over the two mechanisms to balance the interest of both the bidders with low valuations and the advertisement seller.

Definition 3 (Probabilistic First Price Mechanism, pFP). *Let $\mathcal{M}^0$ be the EP mechanism, and $\mathcal{M}^I$ be the FP mechanism. For $\lambda \in (0, 1)$, define a probabilistic mechanism $\mathcal{M}^{pI} = \{\mathcal{M}^I, \mathcal{M}^0; \bar{\lambda}\}$, where $\bar{\lambda} = (1 - \lambda, \lambda)$.*

Based on Theorem 1, the symmetric equilibrium strategy of the rational bidders can be obtained. Then we can estimate the expected payoffs of the bidders, the winning rates of the bidders, and the expected revenue of the seller with different distributions $\bar{\lambda}$. Consider mechanism $\mathcal{M}^{pI}(r_1, r_2, \lambda)$ with $r_1, r_2 \in B$ reserve prices of the FP and the EP respectively, $r_1 \geq r_2$. Given the user valuation distribution $F(x)$, and the valuation of a bidder x, the payoff of this bidder with $\mathcal{M}^{pI}(r_1, r_2, \lambda)$ is $\max\{x - r_2, 0\}$ with λ/N probability, and $(x - L^{\#1}(\bar{b})) \cdot \mathbb{1}_{b = L^{\#1}(\bar{b}) \geq r_1}$ with $1 - \lambda$ probability.

According to Theorem 1, if $x \geq r_1$, the symmetric equilibrium bidding strategy of $\mathcal{M}^{pI}$ is $\beta^{pI}(x; r_1, r_2, \lambda) = \beta^I(x; r_1)$. Her winning rate is

$$\begin{aligned} &p^{pI}(x; r_1, r_2, \lambda) = p^0(x; r_2) \Pr(\mathcal{M}^0) + p^I(x; r_1) \Pr(\mathcal{M}^I) \\ &= \begin{cases} \lambda/N + (1 - \lambda)F(x)^{N-1}, & \text{if } x \in [r_1, \infty), \\ \lambda/N, & \text{if } x \in [r_2, r_1), \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (12)$$

Assume the bidder valuations are uniformly distributed in $[0, 1]$, we set the reserve prices $r_1 = 0.2$, $r_2 = 0.1$ to the two FPs respectively, and set pFP's parameters $(0.1, 0.2; 0.01)$. As we can ses in Figure 1, pFP with certain parameters will increase the winning rate of bidders with low valuations.

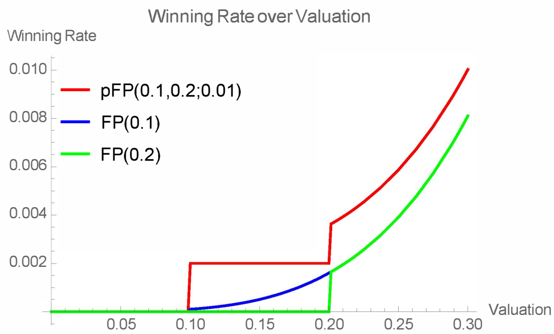

Fig. 1. Winning rate of bidders with low valuations.

For pFP mechanism, according to the previous analysis, a rational bidder with valuation $x < r_2$ will never have a chance to win a bid. A bidder with valuation $r_2 \leq x < r_1$ will bid $r_2 \leq b \leq x$, since lower bid (smaller than r_2) will cause her miss the λ/N probability to win. Moreover, if $r_2 \leq b \leq x < r_1$, then her bid will not influence her payoff or her winning rate. Thus, a bidder's expected payment is

$$\begin{aligned} &m^{pI}(x; r_1, r_2, \lambda) = m^0(x; r_2)P(\mathcal{M}^0) + m^I(x; r_1)P(\mathcal{M}^I) \\ &= \begin{cases} \lambda r_2/N + (1 - \lambda)[r_1 G(r_1) + \int_{r_1}^{x} y g(y) dy], & x \in [r_1, \infty) \\ \lambda r_2/N, & x \in [r_2, r_1) \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (13)$$

where $g(y) = (N - 1)F(y)^{N-2} f(y)$, $G(y) = F(y)^{N-1}$ is the distribution of Y_1^{N-1} (the highest value among the $N - 1$ remaining bidders).

The expected revenue of the seller is similarly

$$\begin{aligned} &R^{pI}(r_1, r_2, \lambda) = R^0(r_2)P(\mathcal{M}^0) + R^I(r_1)P(\mathcal{M}^I) \\ &= \lambda r_2(1 - F(r_2)) + (1 - \lambda)N\Big[r_1(1 - F(r_1))F(r_1)^{N-1} \\ &\quad + \int_{r_1}^{\infty} y \left[1 - F(y)\right] g(y) dy\Big]. \end{aligned} \quad (14)$$

Now, we have constructed a probabilistic price mechanism without the "first-price-win" allocation rule. The equilibrium strategies are the same as the original FP. The proposed mechanisms have three parameters.

(1) The *competitive reserve price* r_1, i.e. the reserve price of FP. Increasing it (smaller than the optimal reserve price $\bar{r}$ as [13]) in FP raises the seller's revenue. However, it also has some drawbacks. First, it may have a detrimental effect on efficiency; Second, it introduces deadweight social loss; It is more important that in real-world auctions, excluding bidders with low valuations will reduce the coverage of the targeted market, which is not healthy in considering long-term selling. Hence, in most of the online auctions, the competitive reserve price has a very small value.

(2) The *subsidy reserve price* r_2, i.e. the reserve price of EP. Since EP is employed to increase the winning rate of bidders with low valuations and to decrease the entrance threshold of the game, r_2 also should have a small value. Rational bidders with valuation smaller than r_2 will never have the chance to win the auction. But, when $r_2 \leq x < r_1$, the bidder can still have positive winning rate. Thus, with the help of subsidy reserve price, the seller can try to increase r_1 and decrease r_2 to get better revenue without the coverage drawbacks.

(3) The mechanism distribution $\bar{\lambda}$ can be used to adjust the level of competition. For the special case of $\bar{\lambda} = (1, 0)$, the probabilistic mechanism is actually FP, which is the most competitive; as if $\bar{\lambda} = (0, 1)$, then it degenerates an EP, which has no competition.

V. Algorithms and Experiments

In this section, we employ the probabilistic approach to design two price mechanisms which ensure the bidders with low valuations can get better winning rate. Assume a player in an e-commerce platform will leave the market if her winning rate is lower than the entrance threshold θ. The first mechanism is designed to increase the winning rate (not less than θ) of bidders with low valuations (more than the reserve price). The second mechanism is to demostrate that we can develop probabilistic mechanisms without loss of the seller's expected revenue. Then, we conduct a computational experiment to simulate senarios with θ-entrance threshold and compare the performances of different mechanisms.

In order to analyze the algorithms, we consider an auction with $N = 5$ rational bidders whose valuations uniformly distributed in $[0, 1]$, set reserve price of the base-line FP mechanism with $r = 0.1$ and the entrance threshold $\theta = 0.1\%$. We will take the expected winning rate and the expected revenue two major performance indexes.

A. Algorithms and Analysis

1) Algorithm 1(Mechanism with Small Revenue-Loss): In FP with $r = 0.1$, 17.8% bidders will have winning rate less than $\theta = 0.1\%$. On the other hand, in the first algorithm, we let the support of winning rate $\lambda/N = \theta$, the subsidy reserve price $r_2 = r$, and the competitive reserve price r_1 solving $p(r_1-; r_1, r_2, \lambda) = p(r_2; r_1, r_2, \lambda)$. It is easy to verify that the mechanism generated by the above algorithm will guarentee that bidders with valuation less than r_2 have increased winning rate compared with FP. As is shown in Figure 2, bidders with low valuation (e.g., satisfying $F(x) \leq 30\%$ in this experiment) always have better winning rate compared with FP. Moreover, 99.5% players (compared with 82.2% of FP(0.1).) are with winning rate higher than the entrance threshold $\theta = 0.1\%$.

Then, we increase the reserve price r of the base-line FP from 0 to 0.5 to evaluate the seller's expected revenue. Figure 3 shows that the expected revenue of the seller will decrease for less than 1% by using the new algorithm. This will be the tradeoff the e-commerce platform persuades the 17.7% players with low valuation in this experiment setting.

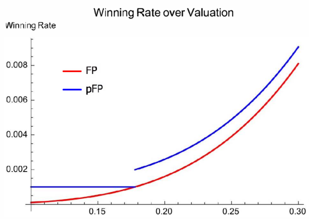

Fig. 2. Algorithm 1: winning rate over bidder's valuation of FP(0.1) and pFP.

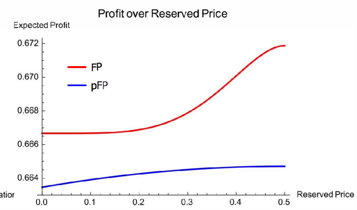

Fig. 3. Algorithm 2: seller's expected revenue over different reserve price.

2) Algorithm 2(Mechanism without Revenue-Loss): The seller does not have to lose the revenue. Algorithm 2 constructs a probabilistic price mechanism to keep the same expected revenue as FP, and increase winning rate of bidders with the lowest valuation. For FP with reserve price r, we let the subsidy reserve price $r_2 = r$, let λ solves $\theta = N \cdot \min_{x \geq r_2} p(x; r_1, r_2, \lambda)$ according to Eq-(12), and let r_1 solves $R(r_1, r_2, \lambda) = R(r_2)$ according to Eq-(14).

As is shown in Figure 4, it increases the winning rate of bidders with very small valuations. When using the proposed pFP, all the bidders with valuation higher than the reserve price r will surpass the entrance threshold θ. In this case, the seller will have the same expected revenue compared with FP. Apparrently, bidders with valuation between $[0.178, 0.380]$ pays for the seller's additional market. Although this algorithm is not always suggested, it shows probabilistic mechanisms with a simple mechanism does not always hurt the sellers.

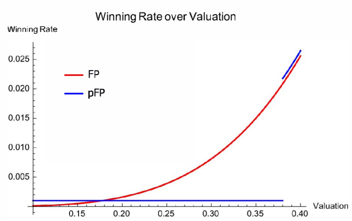

Fig. 4. Algorithm 2: winning rate over bidder's valuation of FP(0.1) and pFP.

B. Computational Experiments

We conduct computational experiments to evaluate the performances of second-price (SP) mechanism and the corresponding probabilistic second-price (pSP) mechanism without reserve prices. Suppose there are 1000 initial players in the market. Averagely, $N = 5$ of them bid for one user-click. For simplicity, we assume a bidder will assign the same amount of money to her interested user-clicks (bid=b_i or 0), and she will leave the market if she did not win any bid in 1000 trials. We simulate for 100000 user-clicks.

In Figure 5 and 6, we can see the revenues and remaining players of SP and pSP. In the first $19k$ trials, the revenues both increase linearly. Later, the revenue of SP increases rapidly since the dropout of bidders with low valuations. However, after about $85k$ trials, the remaining players of SP vanish because of the intense long-term competition. The proposed pSP outperforms SP with both factors in the computational experiments. Different distributions of valuation and different base mechanisms (FP/pFP) lead to similar results.

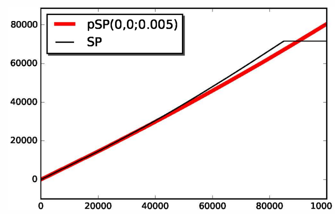

Fig. 5. Revenues comparison

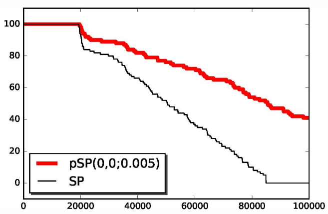

Fig. 6. Remaining players comparison

C. Discussion and Managerial Insights

From the experiments, it is clear that utilizing the proposed probabilistic approach to design price mechanism will brings better performance. In all cases above, the probabilistic price mechanism outperforms the original one. When facing different requirements, the decision makers can implement the algorithms devised above to find proper probabilistic mechanism by combining classical and our simple mechanisms.

We also noted that the winning rate of bidders with low valuations and the expected revenue of the seller are the two most important factors in e-commerce platforms. In order to balance these two conflicting intertests, only the reserve price can be adjusted in classical first-price and second-price based mechanisms. This constraint is highly relaxed in our method since bringing more flexibility through additional parameters.

In addition, the simple mechanisms used in the above algorithms can be regarded as fine tunings of the classical one. When proper parameters are used (e.g., with small λ), the probabilistic price mechanism will, in a sense, keep some qualities of the classical one. Fortunately, we can design complex mechanisms utilizing the probabilistic approach in much more complex scenarios without changing the equilibriums. It will be practical for real-world applications.

VI. Conclusions

In this paper, we propose a probabilistic approach to design auction mechanisms especially for e-commerce platforms. We designed probabilistic mechanisms pFP based on the classical FP. Properties pFP are then investigated. Furthermore, we also propose two algorithms to help designing optimal price mechanisms for different application scenarios. Experimental results demonstrate the flexibility and performance of the proposed probabilistic approach for price mechanism design.

Moreover, the proposed probabilistic approach for price mechanism design is not limited to a generalization of FP. Our analysis suggests new probabilistic price mechanisms based on existing and known ones for multiple purposes.

Acknowledgements

This work was supported in part by the National Natural Science Foundation of China under Contracts 71402178, 71232006, 61233001, 71202169, 71472174 and the Early Career Development Award of SKLMCCS.

References

[1] A. Abdulkadiroğlu and T. Sönmez. Random serial dictatorship and the core from random endowments in house allocation problems. *Econometrica*, pages 689–701, 1998.
[2] M. Babaioff and N. Nisan. Concurrent auctions across the supply chain. *Journal of Artificial Intelligence Research*, pages 595–629, 2004.
[3] J.-P. Benoit and V. Krishna. Multiple-object auctions with budget constrained bidders. *The Review of Economic Studies*, 68(1):155–179, 2001.
[4] A. Bogomolnaia and H. Moulin. A new solution to the random assignment problem. *Journal of Economic Theory*, 100(2):295–328, 2001.
[5] E. H. Clarke. Multipart pricing of public goods. *Public Choice*, 11(1):17–33, 1971.
[6] V. Conitzer and T. Sandholm. Complexity of mechanism design. In *Proceedings of the Eighteenth conference on Uncertainty in artificial intelligence*, pages 103–110. Morgan Kaufmann Publishers Inc., 2002.
[7] V. Conitzer and T. Sandholm. Self-interested automated mechanism design and implications for optimal combinatorial auctions. In *Proceedings of the 5th ACM conference on Electronic commerce*, pages 132–141. ACM, 2004.
[8] B. Edelman, M. Ostrovsky, and M. Schwarz. Internet advertising and the generalized second-price auction: Selling billions of dollars worth of keywords. *American Economic Review*, 97(1):242–259, 2007.
[9] eMarketer. Programmatic ad spending in china is growing rapidly, February 3 2016.
[10] T. Groves. Incentives in teams. *Econometrica*, 41(4):617–631, 1973.
[11] B. J. Jansen and T. Mullen. Sponsored search: An overview of the concept, history, and technology. *International Journal of Electronic Business*, 6(2):114–131, 2008.
[12] F. Kojima and M. Manea. Incentives in the probabilistic serial mechanism. *Journal of Economic Theory*, 145(1):106–123, 2010.
[13] V. Krishna. *Auction Theory, Second Edition*. Academic press, 2010.
[14] S. Muthukrishnan. Internet ad auctions: Insights and directions. In *Proceedings of the 35th international colloquium on Automata, Languages and Programming, Part I*, volume 5125, pages 14–23, 2008.
[15] R. B. Myerson. Optimal auction design. *Mathematics of Operations Research*, 6(1):58–73, February 1981.
[16] H. R. Varian. Online ad auctions. *American Economic Review*, 99(2):430–434, 2009.
[17] W. Vickrey. Counterspeculation, auction, and competitive sealed tender. *Journal of Finance*, 16(1):8–37, 1961.
[18] J. Zhang, D. Wen, and S. Zeng. A discounted trade reduction mechanism for dynamic ridesharing pricing. *IEEE Transactions on Intelligent Transportation Systems*, 17(6):1586–1595, 2016.
[19] J. Zhang, Y. Yang, X. Li, R. Qin, and D. Zeng. Dynamic dual adjustment of daily budgets and bids in sponsored search auctions. *Decision support systems*, 57:105–114, 2014.