Adaptive Type-2 Fuzzy Output Feedback Control for Flexible Air-breathing Hypersonic Vehicles

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Abstract—In this paper, an adaptive type-2 fuzzy output feedback control scheme for the longitudinal dynamics of flexible air-breathing hypersonic vehicles is proposed. Firstly, considering possible uncertainties, an interval type-2 fuzzy logic system is employed to approximate the unknown dynamics during the flight. Then, based on the interval type-2 fuzzy logic system, a reduced-order fuzzy observer is designed to estimate the immeasurable states, namely the angle of attack and the flight path angle. Further, by the backstepping control approach, an adaptive type-2 fuzzy output feedback controller is constructed to deal with the tracking problem for commanded velocity and altitude. The stability of the closed-loop systems is explored. Simulation studies are carried out and the proposed controller is verified to be effective.

I. INTRODUCTION

As a reliable and cost-efficient way for access to space, air-breathing hypersonic vehicles (AHVs) have been investigated by many researchers in recent decades [1]. However, the design of robust control systems for AHVs is still a challenging task due to complex coupling effects and significant uncertainties during hypersonic flight [2]. For example, strong coupling exists between propulsive and aerodynamic forces caused by the under-fuselage location of the scramjet engine.

Due to the extreme complexity of vehicle dynamics, here only the longitudinal dynamic model of flexible AHVs (FAHVs) has been considered for control design. For better description of the dynamic characteristics, the model of FAHVs was introduced by Bolender and Doman [3]. On this basis, many control strategies have been proposed for control of FAHVs during the last few years (For more details, please refer to the survey paper [1] and the references therein). Among them, backstepping control method is a powerful tool for the control design because the altitude dynamics of FAHVs can be transformed to a strict-feedback form. To improve the robustness of back-stepping control, recently, fuzzy adaptive backstepping control scheme [4], in which the fuzzy logic systems (FLS) as universal approximators were used to approximate the unknown dynamics, has attracted more attention. However, the adaptive fuzzy control addressed in current literatures is mainly based on the type-1 FLS (T1-FLS) and only few papers investigated the type-2 FLS (T2-FLS) based adaptive fuzzy control. T2-FLS, which is the extension of T1-FLS, has the potential to better deal with nonlinear systems with uncertainties because it is based on the type-2 fuzzy sets which have more design degrees of freedom than T1-FLS [5]. For reducing the computational cost, interval T2-FLS (IT2-FLS) are widely used into applications [6-7].

Hence, here, it is necessary to develop the IT2-FLS based adaptive control technique for the flight control of the FAHVs when considering the influence of uncertainties.

In practical hypersonic flight, the angle of attack (AOA) and the flight path angle are quite small, which makes their accurate measurements become costly and difficult [8]. Thus, it is of interest to address the case in which only a part of the FAHVs states are measurable. For this problem, the observer-based output feedback control is a feasible method for the flight control of FAHVs with immeasurable states. Recently, various observer design methods have been proposed to reconstruct the AOA and flight-path angle, such as sliding mode observer [9], high-order sliding mode observer [10], and tracking differentiator based observer [11].

Motivated by the above analysis, in this paper, we propose a novel adaptive fuzzy output feedback control scheme for velocity and altitude tracking of FAHVs. Specifically, by using IT2-FLS to approximate the unknown dynamics of FAHVs, a reduced-order fuzzy state observer is designed to estimate the AOA and flight-path angle during the flight. Based on the designed state observer and backstepping approach, a new adaptive type-2 fuzzy output feedback controller is developed for FAHVs.

II. FAHV MODEL DESCRIPTION

The longitudinal dynamics of the FAHVs model, derived from Lagrange’s equations, are given as below [3]:

\[
\dot{V} = (T \cos \alpha - D) / m - g \sin \gamma \quad (1)
\]

\[
\dot{h} = V \sin \gamma \quad (2)
\]

\[
\dot{\gamma} = (L + T \sin \alpha) / (mV) - g \cos \gamma / V \quad (3)
\]

\[
\dot{\alpha} = q - \dot{\gamma} \quad (4)
\]

\[
\dot{q} = M / I_{xy} \quad (5)
\]

\[
\dot{\eta}_i = -2 \xi_i \omega \eta_i - \omega \eta_i + N_i, \quad i = 1, 2, 3. \quad (6)
\]

Five rigid-body states \( V, h, \gamma, \alpha, q \), which represent the vehicle velocity, altitude, flight path angle, angle of attack (AOA) and pitch rate respectively, and six flexible states \( \eta = [\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6] \) for the flexible modes are contained.

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in this model. The control inputs are the fuel equivalence ratio \( \phi \), canard deflection \( \delta_c \), and elevator deflection \( \delta_e \), which do not appear in (1)-(6) directly. Instead, they enter the aerodynamic forces and moment through the thrust \( T \), drag \( D \), lift \( L \), pitch moment \( M \), and generalized forces \( N_i \). The approximation of the forces and moments employed in the FAHVs is given by [3], which are given as:

\[
T = \overline{q}S(C_{T,\alpha}(\alpha) + C_{T,\delta}(\alpha, \delta, \eta))
\]

\[
D = \overline{q}SC_{D,\alpha}(\alpha, \delta, \eta)
\]

\[
L = \overline{q}SC_{L,\delta}(\alpha, \delta, \eta)
\]

\[
M = \overline{q}T + \overline{q}SC_{M,\delta}(\alpha, \delta, \delta, \eta)
\]

where \( \overline{q} \), \( S \), \( \bar{\tau} \) are the dynamic pressure, reference area, and mean aerodynamic chord, respectively. The corresponding coefficients in (7)-(11) are obtained using curve-fitted approximations, which can be expressed as

\[
C_{T,\alpha}(\alpha) = C_{T,0}^{\alpha} \alpha^3 + C_{T,1}^{\alpha} \alpha^2 + C_{T,2}^{\alpha} \alpha + C_{T,3}^{\alpha},
\]

\[
C_{T,\delta}(\alpha, \delta, \eta) = C_{T,0}^{\delta} \delta^3 + C_{T,1}^{\delta} \delta^2 + C_{T,2}^{\delta} \delta + C_{T,3}^{\delta},
\]

\[
C_{D,\alpha}(\alpha, \delta, \eta) = C_{D,0}^{\alpha} \alpha^3 + C_{D,1}^{\alpha} \alpha^2 + C_{D,2}^{\alpha} \alpha + C_{D,3}^{\alpha},
\]

\[
C_{D,\delta}(\alpha, \delta, \eta) = C_{D,0}^{\delta} \delta^3 + C_{D,1}^{\delta} \delta^2 + C_{D,2}^{\delta} \delta + C_{D,3}^{\delta},
\]

\[
C_{L,\delta}(\alpha, \delta, \eta) = C_{L,0}^{\delta} \delta^3 + C_{L,1}^{\delta} \delta^2 + C_{L,2}^{\delta} \delta + C_{L,3}^{\delta},
\]

\[
C_{M,\delta}(\alpha, \delta, \eta) = C_{M,0}^{\delta} \delta^3 + C_{M,1}^{\delta} \delta^2 + C_{M,2}^{\delta} \delta + C_{M,3}^{\delta},
\]

\[
C_{\eta} = [C_{\eta}^{\alpha}, 0, C_{\eta}^{\delta}, 0, C_{\eta}^{\delta}], i = T, M, L, D,
\]

\[
N_i = [N_i^{\alpha}, 0, N_i^{\delta}, 0, N_i^{\delta}], j = 1, 2, 3.
\]

And the values of the above aerodynamic coefficients vary greatly with different flight conditions.

The output to be controlled is selected as \( y = [V, h] \).

### III. CONTROL-ORIENTED UNCERTAINTY MODEL

In this section, some simplifications of the FAHVs model are made for output feedback backstepping control. First, as considered in [9], the flexible dynamics are removed during the controller design process, but their effects are taken as control-oriented modeling errors. Thus, \( F_r, \gamma, F_d, \Delta_r, F_a, \Delta_a, F_q, \Delta_q \) are denoted as unknown dynamics because they include the total perturbations caused by flexible dynamics, coefficient uncertainties, and control-oriented modeling errors. The objective of this research is to design an adaptive type-2 fuzzy output feedback control scheme to achieve robust performance against the model uncertainties as well.

### IV. IT2-FLS

#### A. Brief Introduction of IT2-FLS

A FLS using at least one interval type-2 fuzzy set (IT2-FS) is called IT2-FLS. IT2-FLS is similar to the type-1 FLS. However, due to using IT2-FSs, the output processing block consists of type-reducer and defuzzifier in IT2-FLS. Consequently, IT2-FLS contains five parts as we can see in Fig 1, namely fuzzifier, rule base, fuzzy inference engine, type reducer, and defuzzifier.
The rule base for IT2-FLS consists of a collection of IF-THEN rules. Consider a IT2-FLS having \( p \) inputs \( x = [x_1, \ldots, x_p]^T \), one output \( y \) with \( M \) rules. Then the \( i \)th rule can be expressed as

\[
R^i: \text{IF } x = \tilde{G}_i^j \text{ and } \ldots \text{ and } x_p = \tilde{G}_i^p, \\
\text{THEN } y = \check{H}^i, i = 1, \ldots, M.
\]

where \( \tilde{G}_i^j, (j = 1, \ldots, p) \) are antecedent IT2-FSSs, \( \check{H}^i \) are consequent IT2-FSSs and they are associated with the fuzzy membership function \( \mu_{\tilde{G}_i^j}(x_p) \) and \( \mu_{\check{H}^i}(y) \), respectively.

Based on the fuzzy rules, the fuzzy inference engine gives a mapping from input IT2-FSSs to output IT2-FSSs. Each rule is interpreted as a fuzzy implication. Assuming that Mamdani implication is used, the output consequent set of the \( i \)th rule in a singleton IT2-FLS is

\[
\mu_{\check{H}^i}(y) = \mu_{\check{H}^i}(y) \cap \left[ \bigwedge_{j=1}^{p} \mu_{\tilde{G}_i^j}(x_p) \right],
\]

where symbol \( \cap \) denotes the meet operation, the firing set \( \bigwedge_{j=1}^{p} \mu_{\tilde{G}_i^j}(x) = [f^i(x), \bar{f}^i(x)] \) is an interval type-1 fuzzy set with

\[
f^i(x) = \mu_{\tilde{G}_i^j}(x_p) \cdots \mu_{\tilde{G}_i^j}(x_p), \quad \bar{f}^i(x) = \mu_{\bar{G}_i^j}(x_p) \cdots \mu_{\bar{G}_i^j}(x_p)
\]

where \( \cdots \) stands for the product operation, \( \mu_{\bar{G}_i^j}(x_p) \) and \( \mu_{\tilde{G}_i^j}(x_p) \) are the upper and lower membership grades of \( \mu_{\tilde{G}_i^j}(x_p) \), respectively.

The type-reduction acts on the output IT2-FSSs of the inference engine to generate type-1 fuzzy sets which are then defuzzified to get the crisp outputs. There are many kinds of type reduction approaches. In this paper, we will use the center-of-sets type-reduction which is given by

\[
Y_{cm} = \left[ y_1, y_2 \right] = \left[ y_1, \ldots, y_p \right] / \left[ \sum_{i=1}^{M} f_i^i(x) y_i^i \right] / \left[ \sum_{i=1}^{M} f_i^i(x) \right],
\]

where \( y_i \) is the centroid of the consequent set \( \check{H}^i \). The points \( y_1 \) and \( y_2 \) can be computed by applying the KM algorithm and written as the following form [7]

\[
y_1 = \frac{\sum_{i=1}^{M} f_i^i(x) y_i^i}{\sum_{i=1}^{M} f_i^i(x)}, \quad y_2 = \frac{\sum_{i=1}^{M} f_i^i(x) y_i^i}{\sum_{i=1}^{M} f_i^i(x)}.
\]

where \( f_i^i \) is determined by the values of \( f^i(x) \) and \( \bar{f}^i(x) \).

Then, by denoting \( \theta = [y_1, y_2, \ldots, y_M]^T \), \( \phi(x) = [\xi_1(x), \xi_2(x), \ldots, \xi_M(x)] \), \( \xi_1(x) = \frac{[\xi_1^1, \xi_1^2, \ldots, \xi_1^M]^T}{2} \), \( \xi_2(x) = \frac{[\xi_2^1, \xi_2^2, \ldots, \xi_2^M]^T}{2} \), and \( \xi_3 = \frac{[\xi_3^1, \xi_3^2, \ldots, \xi_3^M]^T}{2} \), the defuzzfied crisp value is obtained as

\[
y = \frac{y_1 + y_2}{2} = \frac{\theta^T (\xi_1(x) + \xi_3(x))}{2} = \theta^T \phi(x).
\]
### A. State Observer Design

Note that the states $\gamma$ and $\alpha$ in the FHAVs are not available during the flight; therefore, a state observer should be designed to estimate the immeasurable states. By only using the information of outputs, IT2-FLS based reduced-order fuzzy state observer is designed as follow:

$$\dot{\hat{\gamma}} = \hat{F}_\gamma(X_\gamma \mid \theta_\gamma) + g_\gamma \hat{\alpha} + k_\gamma(h_V - \hat{\gamma}),$$

$$\dot{\hat{\alpha}} = \hat{F}_\alpha(X_\alpha \mid \theta_\alpha) - g_\alpha \hat{\alpha} + g_\alpha q,$$

where $\hat{\alpha}, \hat{\gamma}$ are the estimates of $\alpha, \gamma$ respectively and $k_\gamma$ is the design parameter. Note that in contrast with the full-order observer designed in [8-9], the proposed reduced order observer offers the advantage of reducing the order of the closed-loop system and the burden of computation.

Let the observer error be described as $\hat{\gamma} = \gamma - \hat{\gamma}$ and $\hat{\alpha} = \alpha - \hat{\alpha}$. Then using (24)-(25) and (27)-(28), the dynamics of the observation errors are given by

$$\begin{align*}
\dot{\hat{\gamma}} &= g_\gamma \hat{\alpha} - k_\gamma \hat{\gamma} + \delta_\gamma, \\
\dot{\hat{\alpha}} &= -g_\alpha \hat{\alpha} + \delta_\alpha,
\end{align*}$$

(29) and (30)

Consider the following Lyapunov candidate

$$W_o = \frac{1}{2} \hat{\gamma}^2 + \frac{1}{2} \hat{\alpha}^2.$$  

(31)

The time-derivative of $W_o$ along (29)-(30) is

$$\dot{W}_o = -k_\gamma \hat{\gamma}^2 - g_\gamma \hat{\alpha} \hat{\gamma} + g_\gamma \hat{\alpha} \hat{\gamma} + \delta_\gamma \hat{\gamma} + \delta_\alpha \hat{\alpha}.$$  

(32)

Note that during the flight the coefficients and parameters in $g_\gamma, g_\alpha, \delta_\gamma, \delta_\alpha, k_\gamma$ are bounded and satisfy that $\bar{g}_\gamma > 0, S_\gamma > 0, C_\gamma > 0, m > 0, V > 0$. Hence, there exist positive constants $\bar{g}_\gamma$ and $\bar{g}_\alpha$ such that $\bar{g}_\gamma \geq g_\gamma \geq \bar{g}_\gamma > 0$. Then, we have

$$\dot{W}_o \leq -k_\gamma \hat{\gamma}^2 - g_\gamma \hat{\alpha} \hat{\gamma} + g_\gamma \hat{\alpha} \hat{\gamma} + \delta_\gamma \hat{\gamma} + \delta_\alpha \hat{\alpha}.$$  

(33)

By using Young’s inequality, we have

$$g_\gamma \hat{\alpha} \hat{\gamma} \leq \frac{g_\gamma \hat{\alpha}^2}{2} \leq \frac{g_\gamma \hat{\alpha}^2}{2},$$

$$\delta_\gamma \hat{\gamma} \leq \frac{\delta_\gamma^2}{2},$$

where $\epsilon_1 > 0, \epsilon_2 > 0$ are parameters to be determined. Substituting the above inequalities into (33) results in

$$\dot{W}_o \leq (-k_\gamma + \frac{\bar{g}_\gamma}{2\epsilon_1} + \frac{\epsilon_1}{2})\hat{\gamma}^2 + (-g_\gamma + \frac{\bar{g}_\gamma}{2\epsilon_2} + \frac{\epsilon_2}{2})\hat{\alpha}^2 + \frac{\delta_\gamma^2}{2} + \frac{\delta_\alpha^2}{2}.$$  

(34)

From (34), it can be seen that by selecting appropriate design parameters, the fuzzy state observer can guarantee the convergence of the observer errors. Hence, in the next section, observer-based adaptive fuzzy output controller is designed to make the resulting closed-loop system stable.

### B. Altitude Controller Design

By using the backstepping technique, the altitude controller is designed and the whole procedure is completed in four steps as follow.

Step1. Define altitude tracking error as

$$e_\alpha = h - h_\alpha,$$

where $h_\alpha$ is the tracking reference trajectory of the altitude.

Step2. Define $\hat{e}_\alpha = \hat{\gamma} - \hat{\gamma}$.

Step3. Define $\hat{\gamma}_d = \frac{1}{V}(-k_\epsilon e_\gamma + h_\gamma)$

Step4. Define $q = q - q_d$. Differentiating $e_\gamma$, we have

$$\dot{e}_\gamma = (\beta_\gamma + \omega_\gamma)\varphi(x) + \eta_\gamma - \delta_\gamma + g_\gamma \hat{\alpha} + k_\gamma(h_V - \hat{\gamma}) - \dot{\hat{\gamma}}_d.$$

By viewing $\dot{\hat{\gamma}}$ as a virtual control input, the virtual feedback control law can be designed as

$$\dot{\hat{\gamma}}_d = \frac{1}{V}(-k_\epsilon e_\gamma + h_\gamma)$$  

(35)

where $k_\epsilon$ is a positive design constant to be specified later.

where $\beta_\gamma, \omega_\gamma, \eta_\gamma, \delta_\gamma, \theta_\gamma, \theta_\gamma'$ are the design parameters. The adaptive law of $\theta_\gamma$ is designed as

$$\dot{\hat{\theta}}_\gamma = \beta_\gamma \varphi_\gamma(x) - \sigma_\gamma (\theta_\gamma - \theta_\gamma'),$$

(37)

where $\beta_\gamma > 0, \sigma_\gamma > 0$ and $\theta_\gamma'$ are the design parameters.

Step3. Define $e_\alpha = \hat{\alpha} - \alpha_d$. Differentiating $e_\alpha$, we obtain

$$\dot{e}_\alpha = \theta_\alpha \varphi_\alpha(x) + \omega_\alpha \varphi_\alpha(x) + \eta_\alpha - \delta_\alpha - g_\alpha \hat{\alpha} + g_\alpha \dot{\hat{\alpha}}.$$  

(38)

where $k_\alpha > 0, \kappa > 0$ are the design parameters. The adaptive law of $\theta_\alpha$ is designed as

$$\dot{\hat{\theta}}_\alpha = \beta_\alpha \varphi_\alpha(x) - \sigma_\alpha (\theta_\alpha - \theta_\alpha').$$

(39)

where $\beta_\alpha > 0, \sigma_\alpha > 0$ and $\theta_\alpha'$ are the design parameters.

Step4. Define $q = q - q_d$. Differentiating $e_\gamma$, we have

$$\dot{e}_\gamma = \theta_\gamma \varphi_\gamma(x) + \omega_\gamma \varphi_\gamma(x) + \eta_\gamma + g_\gamma \delta_\gamma - \dot{\hat{\gamma}}_d.$$  

(40)
Finally, the feedback control law \( \delta_c \) can be designed as
\[
\delta_c = \frac{1}{g_\beta}(-k_eq - \varepsilon_\beta \phi_\beta(x_i) + \eta_i - \tau_i \tanh(\tau_i e_i / \kappa) - g_\alpha e_\alpha)
\]
where \( k_\gamma > 0, \kappa > 0 \) are the design parameters. The adaptive law of \( \theta_\gamma \) is designed as
\[
\dot{\theta}_\gamma = \beta'e_\gamma \phi_\gamma(x_i) - \sigma_\gamma (\theta_\gamma - \theta'_\gamma)
\]
(41)
where \( \beta'_\gamma > 0, \sigma_\gamma > 0 \) and \( \theta'_\gamma \) are the design parameters.

### C. Velocity Controller Design

The velocity controller is developed. Define velocity tracking error as
\[
e_\gamma = V - V_d
\]
The time-derivative of \( e_\gamma \) is
\[
\dot{e}_\gamma = \theta'_\gamma \phi_\gamma(x_i) + \delta_\gamma \phi_\gamma(x_i) + \eta_i + g_\alpha (\alpha_\gamma - V)
\]
Then the velocity controller is designed as
\[
\phi = \frac{1}{g_\gamma} (-k_eq - \varepsilon_\gamma \phi_\gamma(x_i) + \eta_i - \tau_i \tanh(\tau_i e_i / \kappa))
\]
(42)
where \( k_\gamma > 0, \kappa > 0 \) are the design parameters. The adaptive law of \( \theta_\gamma \) is designed as
\[
\dot{\theta}_\gamma = \beta'e_\gamma \phi_\gamma(x_i) - \sigma_\gamma (\theta_\gamma - \theta'_\gamma)
\]
(43)
where \( \beta'_\gamma > 0, \sigma_\gamma > 0 \) and \( \theta'_\gamma \) are the design parameters.

### VI. STABILITY ANALYSIS

**Theorem 1:** Consider the closed-loop system consisting of the model (12)-(16) with control laws (35), (36), (38), (40), (42) and adaptive laws (37), (39), (41), (43). Then all the signals involved are bounded and the output tracking error converges to a small neighborhood of the origin.

**Proof:** Consider the following Lyapunov function:
\[
W = W_x + W_y + W_i + W_a + W_q + W_q
\]
where
\[
W_x = \frac{1}{2} \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2, \quad W_y = \frac{1}{2} \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2, \quad W_i = \frac{1}{2} \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2, \quad W_a = \frac{1}{2} \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2, \quad W_q = \frac{1}{2} \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2.
\]

Differentiating \( W_i \) and combining (35)-(43), we obtain
\[
\dot{W}_i = \frac{1}{2} \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2
\]
(45)
\[
\dot{W}_i = \frac{1}{2} \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2
\]
(46)
\[
\dot{W}_a = \frac{1}{2} \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2
\]
(47)
\[
\dot{W}_q = \frac{1}{2} \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2
\]
(48)
\[
\dot{W}_q = \frac{1}{2} \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2
\]
(49)
where \( A = \frac{S_0}{m} [C_{\beta'} (\alpha^2 + \alpha \beta + \beta^2) + C_{\phi'} (\alpha^2 + \alpha \beta + C_{\beta'})]. \)

Here, assume \( A \) is bounded during the flight, i.e. \( |A| < \overline{A} \).

Note the following inequality holds:
\[
\frac{\sigma_\beta}{2 \beta_\beta} \dot{\theta}_\beta ^2 - \frac{\sigma_\alpha}{2 \beta_\alpha} \dot{\theta}_\alpha ^2 - \frac{\sigma_\delta}{2 \beta_\delta} \dot{\theta}_\delta ^2 - \frac{\sigma_\gamma}{2 \beta_\gamma} \dot{\theta}_\gamma ^2 \leq -\frac{\sigma_\delta}{2 \beta_\delta} \dot{\theta}_\delta ^2 - \frac{\sigma_\gamma}{2 \beta_\gamma} \dot{\theta}_\gamma ^2
\]
(42)
\[
\dot{W}_q = \frac{1}{2} \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2
\]
(43)
where \( \delta_\gamma > 0 \) is a design parameter.

Substituting the above inequalities into (45)-(49) and considering (34), one has
\[
\dot{W} \leq (\frac{g_\beta}{2 \epsilon_\beta} + 1) \dot{\delta}^2 + \sigma_\delta \dot{\theta}_\delta ^2 + \sigma_\gamma \dot{\theta}_\gamma ^2 + \frac{4 \dot{\alpha}^2 + (\epsilon_\delta + 1) \dot{\delta}^2}{2 \dot{\alpha}^2} + \dot{\alpha}^2 - \epsilon_\delta \dot{\theta}_\delta ^2
\]
(46)
\[
\dot{W} \leq (\frac{g_\beta}{2 \epsilon_\beta} + 1) \dot{\delta}^2 + \sigma_\delta \dot{\theta}_\delta ^2 + \sigma_\gamma \dot{\theta}_\gamma ^2 + \frac{4 \dot{\alpha}^2 + (\epsilon_\delta + 1) \dot{\delta}^2}{2 \dot{\alpha}^2} + \dot{\alpha}^2 - \epsilon_\delta \dot{\theta}_\delta ^2
\]
(47)
\[
\dot{W} \leq (\frac{g_\beta}{2 \epsilon_\beta} + 1) \dot{\delta}^2 + \sigma_\delta \dot{\theta}_\delta ^2 + \sigma_\gamma \dot{\theta}_\gamma ^2 + \frac{4 \dot{\alpha}^2 + (\epsilon_\delta + 1) \dot{\delta}^2}{2 \dot{\alpha}^2} + \dot{\alpha}^2 - \epsilon_\delta \dot{\theta}_\delta ^2
\]
(48)
\[
\dot{W} \leq (\frac{g_\beta}{2 \epsilon_\beta} + 1) \dot{\delta}^2 + \sigma_\delta \dot{\theta}_\delta ^2 + \sigma_\gamma \dot{\theta}_\gamma ^2 + \frac{4 \dot{\alpha}^2 + (\epsilon_\delta + 1) \dot{\delta}^2}{2 \dot{\alpha}^2} + \dot{\alpha}^2 - \epsilon_\delta \dot{\theta}_\delta ^2
\]
(49)
\[
\dot{W} \leq (\frac{g_\beta}{2 \epsilon_\beta} + 1) \dot{\delta}^2 + \sigma_\delta \dot{\theta}_\delta ^2 + \sigma_\gamma \dot{\theta}_\gamma ^2 + \frac{4 \dot{\alpha}^2 + (\epsilon_\delta + 1) \dot{\delta}^2}{2 \dot{\alpha}^2} + \dot{\alpha}^2 - \epsilon_\delta \dot{\theta}_\delta ^2
\]
(50)

Choose the appropriate design parameters \( k_\alpha, \epsilon_1, \epsilon_2, \epsilon_3, i(k = h, \gamma, \alpha, q, V) \), \( \sigma_\delta (k = h, \gamma, \alpha, q, V) \), \( \beta_\delta (k = h, \gamma, \alpha, q, V) \) such that \( (\frac{g_\beta}{2 \epsilon_\beta} + 1) \dot{\delta}^2 + \sigma_\delta \dot{\theta}_\delta ^2 + \sigma_\gamma \dot{\theta}_\gamma ^2 + \frac{4 \dot{\alpha}^2 + (\epsilon_\delta + 1) \dot{\delta}^2}{2 \dot{\alpha}^2} + \dot{\alpha}^2 - \epsilon_\delta \dot{\theta}_\delta ^2 \leq 0 \), \( k_\gamma > 0 \), \( k_\alpha > 0 \), \( \beta_\gamma > 0 \), \( \beta_\delta > 0 \) (51)

By assumption 3, \( \dot{\delta}_\gamma \) and \( \dot{\alpha}_\gamma \) are bounded which imply \( \dot{\lambda} \) in (53) is bounded. Hence, for (51), all the signals involved

where \( c = 2 \times \max \{(-k_\gamma + \frac{g_\beta}{2 \epsilon_\beta} + 1),(-g_\beta + \frac{g_\beta}{2 \epsilon_\beta} + \epsilon_\delta \dot{\theta}_\delta ^2)\} \).

\[
\dot{W} \leq c \dot{W} + \lambda
\]
(52)

\[
\dot{W} = \frac{1}{2} \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2
\]
(53)

\[
\dot{W} = \frac{1}{2} \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2
\]
(54)
in the closed-loop system are bounded and the output tracking error converges to a small neighborhood of the origin.

VII. SIMULATION

In this simulation study, the adaptive type-2 fuzzy output feedback control schemes are applied to the tracking control of the FAHVs.

Simulations are conducted on the model of the FAHVs with flexible states described as (1)-(6). The initial flying states are chosen as \( [V, h, \gamma, \alpha, Q] = [7846.4 \text{ ft/s}, 85000 \text{ ft}, 0 \text{ rad}], \) 
\( 0.0219 \text{ rad}, 0 \text{ rad/s} \), \( \eta = [0.594, 0, -0.0976, 0, -0.0335], \) and the initial control inputs are chosen as \( [\phi, \delta] = [0.12, 0.12 \text{ rad}] \).

The control objective is to control the states \( V, h \) to track their reference trajectories. The reference trajectory is generated by filtering the given 1000 ft/s step in velocity and a 10000 ft change in altitude channel through second-order prefilters.

The premise variables of IT2-FLS are selected as \( x_r = [V^\alpha, \gamma^\alpha, \alpha^\alpha, Q^\alpha] \), \( x = [V, \gamma, \alpha, Q] \), \( x_r = [V^\alpha, \gamma^\alpha, \alpha^\alpha, Q^\alpha] \). The consequent IT2-FSs are chosen to be T1-FSs, their values are initially set as random numbers between -1 and 1. Specifying the design parameters \( \beta_i = 1 (k = V, \gamma, \alpha, Q) \), \( \sigma_i = 0.1 (k = \gamma, \alpha) \) and \( \sigma_i = 0.01 \), hence the adaptive laws in (37), (39), (41) and (43) are constructed.

The rest parameters of the proposed controllers are chosen as \( k_v = 10 \), \( k_i = h, \gamma, \alpha, Q, V \), \( k_p = 2 \), \( k_s = 2.5 \), \( k_v = 4 \), \( k_u = 1 \), \( k_q = 40 \) and \( \kappa = 1 \). In usual backstepping control design, the analytic computation of the virtual control signal derivatives is tedious. Here in simulation, we use command filters to obtain the derivatives of the virtual control signals as well as reference command signals.

To illustrate the performance of the proposed output feedback control system, the control laws are conducted on FAHVs model with 0% and 30% uncertainties added in all aerodynamic coefficients in (7)-(10). The trajectories of the FAHVs model with 0% and 30% uncertainties are showed in Fig.3. It can be seen that the velocity \( V \) and altitude \( h \) can track their desired trajectories. Other flight states remain bounded as shown in Fig.4. Hence, the proposed adaptive type-2 fuzzy output feedback controller does provide good tracking performance, despite the presence of incomplete state measurement and the model uncertainties.

VIII. CONCLUSION

In this paper we propose a new adaptive fuzzy output feedback control scheme for the tracking control of the flexible air-breathing vehicles. This control scheme involves IT2-FLS, which contributes to approximating the unknown dynamics of FAHVs. Based on these results, the approach of designing reduced-order fuzzy states observer is presented, which gives the estimates of the unmeasured states. Under the framework of the backstepping control method, an observer-based adaptive type-2 fuzzy controller is constructed to deal with the tracking problem for FAHVs. Furthermore, it is proven that the proposed controller guarantees that all signals in the closed-loop systems are bounded and the tracking error converges to a small neighborhood of the origin. Simulation results of the tracking control have been shown to demonstrate the effectiveness of the proposed results.

REFERENCES