Adaptive Type-2 Fuzzy Output Feedback Control for Flexible Air-breathing Hypersonic Vehicles

Yifan Liu, Zhiqiang Pu, Jianqiang Yi

***Abstract*—In this paper, an adaptive type-2 fuzzy output feedback control scheme for the longitudinal dynamics of flexible air-breathing hypersonic vehicles is proposed. Firstly, considering possible uncertainties, an interval type-2 fuzzy logic system is employed to approximate the unknown dynamics during the flight. Then, based on the interval type-2 fuzzy logic system, a reduced-order fuzzy observer is designed to estimate the immeasurable states, namely the angle of attack and the flight path angle. Further, by the backstepping control approach, an adaptive type-2 fuzzy output feedback controller is constructed to deal with the tracking problem for commanded velocity and altitude. The stability of the closed-loop systems is explored. Simulation studies are carried out and the proposed controller is verified to be effective.**

I. INTRODUCTION

As a reliable and cost-efficient way for access to space, air-breathing hypersonic vehicles (AHVs) have been investigated by many researchers in recent decades [1]. However, the design of robust control systems for AHVs is still a challenging task due to complex coupling effects and significant uncertainties during hypersonic flight [2]. For example, strong coupling exists between propulsive and aerodynamic forces caused by the under-fuselage location of the scramjet engine.

Due to the extreme complexity of vehicle dynamics, here only the longitudinal dynamic model of flexible AHVs (FAHVs) has been considered for control design. For better description of the dynamic characteristics, the model of FAHVs was introduced by Bolender and Doman [3]. On this basis, many control strategies have been proposed for control of FAHVs during the last few years (For more details, please refer to the survey paper [1] and the references therein). Among them, backstepping control method is a powerful tool for the control design because the altitude dynamics of FAHVs can be transformed to a strict-feedback form. To improve the robustness of back-stepping control, recently, fuzzy adaptive backstepping control scheme [4], in which the fuzzy logic systems (FLS) as universal approximators were used to approximate the unknown dynamics, has attracted more attention. However, the adaptive fuzzy control addressed in current literatures is mainly based on the type-1 FLS (T1-FLS) and only few papers investigated the type-2 FLS (T2-FLS) based adaptive fuzzy control. T2-FLS, which is the extension of T1-FLS, has the potential to better deal with nonlinear systems with uncertainties because it is based on the type-2 fuzzy sets which have more design degrees of freedom than T1-FLS [5]. For reducing the computational cost, interval T2-FLS (IT2-FLS) are widely used into applications [6-7]. Hence, here, it is necessary to develop the IT2-FLS based adaptive control technique for the flight control of the FAHVs when considering the influence of uncertainties.

In practical hypersonic flight, the angle of attack (AOA) and the flight path angle are quite small, which makes their accurate measurements become costly and difficult [8]. Thus, it is of interest to address the case in which only a part of the FAHVs states are measureable. For this problem, the observer-based output feedback control is a feasible method for the flight control of FAHVs with immeasurable states. Recently, various observer designing methods have been proposed to reconstruct the AOA and flight-path angle, such as sliding mode observer [9], high-order sliding mode observer [10], and tracking differentiator based observer [11].

Motivated by the above analysis, in this paper, we propose a novel adaptive fuzzy output feedback control scheme for velocity and altitude tracking of FAHVs. Specifically, by using IT2-FLS to approximate the unknown dynamics of FAHVs, a reduced-order fuzzy state observer is designed to estimate the AOA and flight-path angle during the flight. Based on the designed state observer and backstepping approach, a new adaptive type-2 fuzzy output feedback controller is developed for FAHVs.

II. FAHV MODEL DESCRIPTION

The longitudinal dynamics of the FAHVs model, derived from Lagrange's equations, are given as below [3]:

$$\dot{V} = (T\cos\alpha - D)/m - g\sin\gamma \tag{1}$$

$$\dot{h} = V\sin\gamma \tag{2}$$

$$\dot{\gamma} = (L + T\sin\alpha)/(mV) - g\cos\gamma/V \tag{3}$$

$$\dot{\alpha} = q - \dot{\gamma} \tag{4}$$

$$\dot{q} = M / I_{yy} \tag{5}$$

$$\ddot{\eta}_i = -2\xi_i\omega_i\dot{\eta}_i - \omega_i\eta_i + N_i, \quad i = 1,2,3. \tag{6}$$

Five rigid-body states V, h, γ, α, q , which represent the vehicle velocity, altitude, flight path angle, angle of attack (AOA) and pitch rate respectively, and six flexible states $\eta = [\eta_1, \dot{\eta}_1, \eta_2, \dot{\eta}_2, \eta_3, \dot{\eta}_3]$ for the flexible modes are contained

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in this model. The control inputs are the fuel equivalence ratio ϕ, canard deflection δ_c, and elevator deflection δ_e, which do not appear in (1)-(6) directly. Instead, they enter the aerodynamic forces and moment through the thrust T, drag D, lift L, pitch moment M, and generalized forces N_i. The approximation of the forces and moments employed in the FAHVs is given by [3], which are given as:

$$T = \bar{q}S[C_{T,\phi}(\alpha)\phi + C_T(\alpha) + C_T^{\eta}\eta] \tag{7}$$

$$D = \bar{q}SC_D(\alpha,\delta_e,\delta_c,\eta) \tag{8}$$

$$L = \bar{q}SC_L(\alpha,\delta_e,\delta_c,\eta) \tag{9}$$

$$M = z_T T + \bar{q}SC_M(\alpha,\delta_e,\delta_c,\eta) \tag{10}$$

$$N_i = \bar{q}S(N_i^{\alpha^2}\alpha^2 + N_i^{\alpha}\alpha + N_i^{\delta_e}\delta_e + N_i^{\delta_c}\delta_c + N_i^0 + N_i^{\eta}\eta),\ i=1,2,3 \tag{11}$$

where $\bar{q}$, S, $\bar{c}$ are the dynamic pressure, reference area, and mean aerodynamic chord, respectively. The corresponding coefficients in (7)-(11) are obtained using curve-fitted approximations, which can be expressed as

$$C_{T,\phi}(\alpha) = C_T^{\phi\alpha^3}\alpha^3 + C_T^{\phi\alpha^2}\alpha^2 + C_T^{\phi\alpha}\alpha + C_T^{\phi},$$
$$C_T(\alpha) = C_T^3\alpha^3 + C_T^2\alpha^2 + C_T^1\alpha + C_T^0,$$
$$C_M(\alpha,\delta_e,\delta_c,\eta) = C_M^{\alpha^2}\alpha^2 + C_M^{\alpha}\alpha + C_M^{\delta_e}\delta_e + C_M^{\delta_c}\delta_c + C_M^0 + C_M^{\eta}\eta,$$
$$C_L(\alpha,\delta_e,\delta_c,\eta) = C_L^{\alpha}\alpha + C_L^{\delta_e}\delta_e + C_L^{\delta_c}\delta_c + C_L^0 + C_L^{\eta}\eta,$$

$$\begin{aligned} C_D(\alpha,\delta_e,\delta_c,\eta) &= C_D^{\alpha^2}\alpha^2 + C_D^{\alpha}\alpha + C_D^{\delta_e^2}\delta_e^{\,2} + C_D^{\delta_e}\delta_e \\ &\quad + C_D^{\delta_c^2}\delta_c^{\,2} + C_D^{\delta_c}\delta_c + C_D^0 + C_D^{\eta}\eta, \\ C_j^{\eta} &= [C_i^{\eta 1}, 0, C_i^{\eta 2}, 0, C_i^{\eta 3}, 0],\ i = T, M, L, D, \\ N_i^{\eta} &= [N_i^{\eta 1}, 0, N_i^{\eta 2}, 0, N_i^{\eta 3}, 0],\ j = 1,2,3. \end{aligned}$$

And the values of the above aerodynamic coefficients vary greatly with different flight conditions.

The output to be controlled is selected as $y = [V, h]$.

III. CONTROL-ORIENTED UNCERTAINTY MODEL

In this section, some simplifications of the FAHVs model are made for output feedback backstepping control. First, as considered in [9], the flexible dynamics are removed during the controller design process, but their effects are taken as perturbations and will be evaluated in simulation. Second, in order to eliminate the nonminimum phase behavior, the canard deflection and elevator deflection are ganged together and described as $\delta_c = k_{ec}\delta_e$, where $k_{ec} = -C_L^{\delta_e}/C_L^{\delta_c}$ [10]. Then there are only two control inputs left to be determined, namely the fuel equivalence ratio ϕ and elevator deflection δ_e.

Assumption 1: The flight-path angle γ and angle of attack α are quite small during the hypersonic flight, which gives the following approximations:

$$\sin\gamma \approx \gamma,\ \cos\gamma \approx 1,\ \sin\alpha \approx \alpha,\ \cos\alpha \approx 1$$

Assumption 2: The thrust term $T\sin\alpha$ in (3) can be neglected because it is generally much smaller than L, i.e., $T\sin\alpha << L$.

After the above steps of simplifications and applying the Assumption 1 and 2, (1)-(5) can be rewritten as the following form:

$$\dot{V} = F_V + g_V(\alpha)\phi \tag{12}$$

$$\dot{h} \approx V\gamma \tag{13}$$

$$\dot{\gamma} = F_{\gamma} + g_{\gamma}\alpha \tag{14}$$

$$\dot{\alpha} = F_{\alpha} + g_{\alpha}q \tag{15}$$

$$\dot{q} = F_q + g_q\delta_e \tag{16}$$

where

$$F_V = f_V + \Delta_V,\ F_{\gamma} = f_{\gamma} + \Delta_{\gamma},\ F_{\alpha} = f_{\alpha} + \Delta_{\alpha},\ F_q = f_q + \Delta_q,$$

with

$$\begin{aligned} f_V &= \frac{\bar{q}S}{m}[C_T(\alpha) - (C_D^{\alpha^2}\alpha^2 + C_D^{\alpha}\alpha + C_D^{\delta_e^2}\delta_e^2 \\ &\quad + C_D^{\delta_e}\delta_e + C_D^{\delta_c^2}\delta_c^2 + C_D^{\delta_c}\delta_c + C_D^0)] - g\gamma, \\ g_V(\alpha) &= \frac{\bar{q}S}{m}C_{T,\phi}(\alpha),\ f_{\gamma} = \frac{\bar{q}S}{mV}[C_L^{\delta_e}\delta_e + C_L^{\delta_c}\delta_c + C_L^0] - \frac{g}{V}, \\ g_{\gamma} &= \frac{\bar{q}S}{mV}C_L^{\alpha},\ f_{\alpha} = -(f_{\gamma} + g_{\gamma}\alpha + \Delta_{\gamma}),\ g_{\alpha} = 1, \\ f_q &= \frac{\bar{q}S}{I_{yy}}\{z_T(C_T^{\phi\alpha^3}\phi + C_T^3)\alpha^3 + [z_T(C_T^{\phi\alpha^2}\phi + C_T^2) + \bar{c}C_M^{\alpha^2}]\alpha^2 \\ &\quad + [z_T(C_T^{\phi\alpha}\phi + C_T^1) + \bar{c}C_M^{\alpha}]\alpha + [z_T(C_T^{\phi}\phi + C_T^0) + \bar{c}C_M^0]\}, \\ g_q &= \frac{\bar{q}\bar{c}S}{I_{yy}}(C_M^{\delta_e} + k_{ec}C_M^{\delta_c}) \end{aligned}$$

and $\Delta_k, (k = V,\gamma,\alpha,q)$ denote the total perturbations caused by flexible dynamics, coefficient uncertainties, and control-oriented modeling errors. Thus, F_V, F_{γ}, F_{α}, F_q are denoted as unknown dynamics because they include the total perturbations $\Delta_k, (k = V,\gamma,\alpha,q)$.

The objective of this research is to design an adaptive type-2 fuzzy output feedback control scheme to achieve accurate tracking for the velocity and altitude commands with robust performance against the model uncertainties as well.

IV. IT2-FLS

A. *Brief Introduction of IT2-FLS*

A FLS using at least one interval type-2 fuzzy set (IT2-FS) is called IT2-FLS. IT2-FLS is similar to the type-1 FLS. However, due to using IT2-FSs, the output processing block consists of type-reducer and defuzzifier in IT2-FLS. Consequently, IT2-FLS contains five parts as we can see in Fig.1, namely fuzzifier, rule base, fuzzy inference engine, type reducer, and defuzzifier.

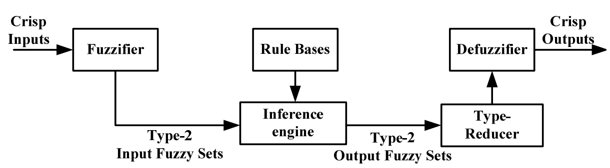

Figure.1 The structure of IT2-FLS

The rule base for IT2-FLS consists of a collection of IF-THEN rules. Consider a IT2-FLS having p inputs $x=[x_1,\ldots,x_p]^T$, one output y with M rules. Then the ith rule can be expressed as

$$R^i: \text{IF } x_1 \text{ is } \tilde{G}_1^i \text{ and} \cdots \text{and } x_p \text{ is } \tilde{G}_p^i,$$
$$\text{THEN } y \text{ is } \tilde{H}^i \;\; i=1,\ldots,M \;.$$

where $\tilde{G}_j^i,(j=1,\ldots,p)$ are antecedent IT2-FSs, $\tilde{H}^i$ are consequent IT2-FSs and they are associated with the fuzzy membership function $\mu_{\tilde{G}_p^s}(x_p)$ and $\mu_{\tilde{H}^s}(y)$, respectively.

Based on the fuzzy rules, the fuzzy inference engine gives a mapping from input IT2-FSs to output IT2-FSs. Each rule is interpreted as a fuzzy implication. Assuming that Mamdani implication is used, the output consequent set of the ith rule in a singleton IT2-FLS is

$$\mu_{\tilde{B}^i}(y)=\mu_{\tilde{H}^i}(y)\sqcap[\mathop{\sqcap}_{j=1}^{p}\mu_{\tilde{G}_j^i}(x_j)],$$

where symbol $\sqcap$ denotes the meet operation, the firing set $\mathop{\sqcap}_{j=1}^{p}\mu_{\tilde{G}_j^i}(x_j)\equiv[\underline{f}^i(x),\overline{f}^i(x)]$ is an interval type-1 fuzzy set with

$$\underline{f}^i(x)=\underline{\mu}_{\tilde{G}_1^i}(x_1)*\cdots*\underline{\mu}_{\tilde{G}_p^i}(x_p),\quad \overline{f}^i(x)=\overline{\mu}_{\tilde{G}_1^i}(x_1)*\cdots*\overline{\mu}_{\tilde{G}_p^i}(x_p)$$

where $*$ stands for the product operation, $\overline{\mu}_{\tilde{G}_j^i}(x_j)$ and $\underline{\mu}_{\tilde{G}_j^i}(x_j)$ are the upper and lower membership grades of $\mu_{\tilde{G}_j^i}(x_j)$, respectively.

The type-reduction acts on the output IT2-FSs of the inference engine to generate type-1 fuzzy sets which are then defuzzified to get the crisp outputs. There are many kinds of type reduction approaches. In this paper, we will use the center-of-sets type-reduction which is given by

$$Y_{\cos}=[y_l,y_r]=\int_{y^1}\cdots\int_{y^M}\int_{f^1}\cdots\int_{f^M}1\Big/\frac{\sum_{i=1}^{M}f^i(x)y^i}{\sum_{i=1}^{M}f^i(x)},$$

where y^i is the centroid of the consequent set $\tilde{H}^i$. The points y_l and y_r can be computed by applying the KM algorithm and written as the following form [7]

$$y_l=\frac{\sum_{i=1}^{M}f_l^i(x)y^i}{\sum_{i=1}^{M}f_l^i(x)},\quad y_r=\frac{\sum_{i=1}^{M}f_r^i(x)y^i}{\sum_{i=1}^{M}f_r^i(x)},$$

where f_l^i is determined by the values of $\underline{f}^i(x)$ and $\overline{f}^i(x)$. Then, by denoting $\theta=[y^1,y^2,\ldots,y^M]^T$, $\varphi(x)=(\xi_l(x)+\xi_r(x))/2$, $\xi_l(x)=[\xi_l^1,\xi_l^2,\ldots,\xi_l^M]^T$, $\xi_r(x)=[\xi_r^1,\xi_r^2,\ldots,\xi_r^M]^T$, where $\xi_l^i=f_l^i/\sum_{i=1}^{M}f_l^i$ and $\xi_r^i=f_r^i/\sum_{i=1}^{M}f_r^i$, the defuzzfied crisp value is obtained as

$$y=\frac{y_l+y_r}{2}=\frac{\theta^T(\xi_l(x)+\xi_r(x))}{2}=\theta^T\varphi(x). \tag{17}$$

B. *IT2-FLS Approximation*

By [12], T2-FLS is a universal approximator, which means that it can approximate any real continuous function on a compact set. Based on the analysis of IT2-FLS and its approximation capability, we can assume that the unknown dynamics in (12)-(16) can be approximated as

$$\hat{F}_k(x_k\mid\theta_k)=\theta_k^T\varphi_k(x_k),(k=V,\gamma,\alpha,q), \tag{18}$$

where $x_k,(k=V,\gamma,\alpha,q)$ are the input vector of IT2-FLS and composed of the measurement outputs V,h,q and $\hat{\alpha},\hat{\gamma}$, where $\hat{\alpha},\hat{\gamma}$ are the estimates of α,γ respectively and defined in the next section.

Define the optimal parameter vectors $\theta_k^*(k=V,\gamma,\alpha,q)$ as

$$\theta_k^*=\arg\min_{\theta_k\in\Omega_k}\left[\sup_{x_k\in U_k}\left|F_k-\hat{F}_k(x_k\mid\theta_k)\right|\right],\;\; k=V,\gamma,\alpha,q, \tag{19}$$

where Θ_k and χ_k are compact regions for θ_k and x_k , respectively. Hence, the approximation errors δ_k and fuzzy minimum approximation errors η_k can be defined as

$$\delta_k=F_k-\hat{F}_k(x_k\mid\theta_k),\,k=V,\gamma,\alpha,q, \tag{20}$$

$$\eta_k=F_k-\hat{F}_k(x_k\mid\theta_k^*)\,,k=V,\gamma,\alpha,q, \tag{21}$$

Assumption 3. There exist known upper bounds $\overline{\eta}_k,\overline{\delta}_k,k=V,\gamma,\alpha,q$, satisfying that $|\eta_k|\le\overline{\eta}_k,|\delta_k|\le\overline{\delta}_k$.

By the definition of δ_k in (20), the system (12)-(16) can be rewritten as

$$\dot{V}=\hat{F}_V(x_V\mid\theta_V)+g_V\phi+\delta_V, \tag{22}$$

$$\dot{h}\approx V\gamma, \tag{23}$$

$$\dot{\gamma}=\hat{F}_\gamma(x_\gamma\mid\theta_\gamma)+g_\gamma\alpha+\delta_\gamma, \tag{24}$$

$$\dot{\alpha}=\hat{F}_\alpha(x_\alpha\mid\theta_\alpha)-g_\gamma\alpha+g_\alpha q+\delta_\alpha, \tag{25}$$

$$\dot{q}=\hat{F}_q(x_q\mid\theta_q)+g_q\delta_e+\delta_q, \tag{26}$$

V. CONTROL SCHEME DESIGN

In this section, the adaptive type-2 fuzzy output feedback control scheme for FAHVs is proposed. First, a state observer is designed to estimate the immeasurable states $\hat{\alpha},\hat{\gamma}$. Then, as it can be seen from (12)-(16), the model of FAHVs can be divided into two functional subsystems, namely the altitude subsystem including (13)-(16) and the velocity subsystem (12). By using the system outputs V,h,q and the observed states $\hat{\alpha},\hat{\gamma}$, the altitude controller and velocity controller are designed separately for the corresponding altitude and velocity subsystems. Fig.2 illustrates the structure of the proposed control scheme.

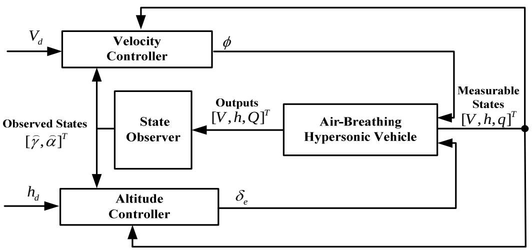

Figure2. Structure of the proposed control scheme

A. State Observer Design

Note that the states γ and α in the FHAVs are not available during the flight; therefore, a state observer should be designed to estimate the immeasurable states. By only using the information of outputs, IT2-FLS based reduced-order fuzzy state observer is designed as follow:

$$\dot{\hat{\gamma}} = \hat{F}_\gamma(X_\gamma \mid \theta_\gamma) + g_\gamma \hat{\alpha} + k_o(\dot{h}/V - \hat{\gamma}), \tag{27}$$

$$\dot{\hat{\alpha}} = \hat{F}_\alpha(X_\alpha \mid \theta_\alpha) - g_\gamma \hat{\alpha} + g_\alpha q, \tag{28}$$

where $\hat{\alpha}, \hat{\gamma}$ are the estimates of α, γ respectively and k_o is the design parameter. Note that in contrast with the full-order observer designed in [8-9], the proposed reduced order observer offers the advantage of reducing the order of the closed-loop system and the burden of computation.

Let the observer error be described as $\tilde{\gamma} = \gamma - \hat{\gamma}$ and $\tilde{\alpha} = \alpha - \hat{\alpha}$. Then using (24)-(25) and (27)-(28), the dynamics of the observation errors are given by

$$\dot{\tilde{\gamma}} = g_\gamma \tilde{\alpha} - k_o \tilde{\gamma} + \delta_\gamma, \tag{29}$$

$$\dot{\tilde{\alpha}} = -g_\gamma \tilde{\alpha} + \delta_\alpha, \tag{30}$$

Consider the following Lyapunov candidate

$$W_o = \frac{1}{2}\tilde{\gamma}^2 + \frac{1}{2}\tilde{\alpha}^2. \tag{31}$$

The time time-derivative of W_o along (29)-(30) is

$$\dot{W}_0 = -k_o \tilde{\gamma}^2 - g_\gamma \tilde{\alpha}^2 + g_\gamma \tilde{\alpha}\tilde{\gamma} + \delta_\gamma \tilde{\gamma} + \delta_\alpha \tilde{\alpha} \tag{32}$$

Note that during the flight the coefficients and parameters in $g_\gamma = \bar{q} S C_L^\alpha / mV$ are bounded and satisfy that $\bar{q} > 0$, $S > 0$, $C_L^\alpha > 0$, $m > 0$, $V > 0$. Hence, there exist positive constants $\bar{g}_\gamma$ and $\underline{g}_\gamma$ such that $\bar{g}_\gamma \geq g_\gamma \geq \underline{g}_\gamma > 0$. Then, we have

$$\dot{W}_0 \leq -k_o \tilde{\gamma}^2 - \underline{g}_\gamma \tilde{\alpha}^2 + g_\gamma \tilde{\alpha}\tilde{\gamma} + \delta_\gamma \tilde{\gamma} + \delta_\alpha \tilde{\alpha} \tag{33}$$

By using Young's inequality, we have

$$g_\gamma \tilde{\alpha}\tilde{\gamma} \leq g_\gamma\left(\frac{\tilde{\gamma}^2}{2\varepsilon_1} + \frac{\varepsilon_1 \tilde{\alpha}^2}{2}\right) \leq \bar{g}_\gamma\left(\frac{\tilde{\gamma}^2}{2\varepsilon_1} + \frac{\varepsilon_1 \tilde{\alpha}^2}{2}\right),$$

$$\delta_\gamma \tilde{\gamma} \leq \frac{\delta_\gamma^2}{2} + \frac{\tilde{\gamma}^2}{2}, \quad \delta_\alpha \tilde{\alpha} \leq \frac{\delta_\alpha^2}{2\varepsilon_2} + \frac{\varepsilon_2 \tilde{\alpha}^2}{2},$$

where $\varepsilon_1 > 0, \varepsilon_2 > 0$ are parameters to be determined. Substituting the above inequalities into (33) results in

$$\dot{W}_0 \leq \left(-k_o + \frac{\bar{g}_\gamma}{2\varepsilon_1} + \frac{1}{2}\right)\tilde{\gamma}^2 + \left(-\underline{g}_\gamma + \frac{\bar{g}_\gamma \varepsilon_1}{2} + \frac{\varepsilon_2}{2}\right)\tilde{\alpha}^2 + \frac{\delta_\gamma^2}{2} + \frac{\delta_\alpha^2}{2\varepsilon_2} \tag{34}$$

From (34), it can be seen that by selecting appropriate design parameters, the fuzzy state observer can guarantee the convergence of the observer errors. Hence, in the next section, observer-based adaptive fuzzy output controller is designed to make the resulting closed-loop system stable.

B. Altitude Controller Design

By using the backstepping technique, the altitude controller is designed and the whole procedure is completed in four steps as follow.

Step1. Define altitude tracking error as

$$e_h = h - h_d$$

where h_d is the tracking reference trajectory of the altitude. Noting that γ can be expressed in terms of its estimate as $\gamma = \hat{\gamma} + \tilde{\gamma}$, then the time-derivative of e_h is

$$\dot{e}_h = V\gamma - \dot{h}_d = V\hat{\gamma} - \dot{h}_d + V\tilde{\gamma}$$

By viewing $\hat{\gamma}$ as a virtual control input, the virtual feedback control law can be designed as

$$\hat{\gamma}_d = \frac{1}{V}(-k_h e_h + \dot{h}_d) \tag{35}$$

where k_h is a positive design constant to be specified later.

Step2. Define $e_\gamma = \hat{\gamma} - \hat{\gamma}_d$. Differentiating e_γ, one has

$$\dot{e}_\gamma = (\theta_\gamma^T + \tilde{\theta}_\gamma^T)\varphi_\gamma(x_\gamma) + \eta_\gamma - \delta_\gamma + g_\gamma \hat{\alpha} + k_o(\dot{h}/V - \hat{\gamma}) - \dot{\hat{\gamma}}_d$$

By viewing $\hat{\alpha}$ as a virtual control input, the virtual feedback control law can be designed as

$$\hat{\alpha}_d = \frac{1}{g_\gamma}[-k_\gamma e_\gamma - \theta_\gamma^T \varphi_\gamma(x_\gamma) - k_o(\dot{h}/V - \hat{\gamma}) + \dot{\hat{\gamma}}_d$$
$$-\bar{\eta}_\gamma \tanh(\bar{\eta}_\gamma e_\gamma / \kappa) - \frac{V e_h}{\bar{V}^2}] \tag{36}$$

where $k_\gamma > 0$, $\kappa > 0$ are the design parameters. The adaptive law of θ_γ is designed as

$$\dot{\theta}_\gamma = \beta_\gamma e_\gamma \varphi_\gamma(x_\gamma) - \sigma_\gamma(\theta_\gamma - \theta'_\gamma) \tag{37}$$

where $\beta_\gamma > 0$, $\sigma_\gamma > 0$ and θ'_γ are the design parameters.

Step3. Define $e_\alpha = \hat{\alpha} - \hat{\alpha}_d$. Differentiating e_α, we obtain

$$\dot{e}_\alpha = \theta_\alpha^T \varphi_\alpha(x_\alpha) + \tilde{\theta}_\alpha^T \varphi_\alpha(x_\alpha) + \eta_\alpha - \delta_\alpha - g_\gamma \hat{\alpha} + g_\alpha q - \dot{\hat{\alpha}}_d$$

By viewing q as a virtual control input, the virtual feedback control law can be designed as

$$q_d = \frac{1}{g_\alpha}(-k_\alpha e_\alpha - \theta_\alpha^T \varphi_\alpha(x_\alpha) + g_\gamma \hat{\alpha} + \dot{\hat{\alpha}}_d$$
$$-\bar{\eta}_\alpha \tanh(\bar{\eta}_\alpha e_\alpha / \kappa) - g_\gamma e_\gamma) \tag{38}$$

where $k_\alpha > 0$, $\kappa > 0$ are the design parameters. The adaptive law of θ_α is designed as

$$\dot{\theta}_\alpha = \beta_\alpha e_\alpha \varphi_\alpha(x_\alpha) - \sigma_\alpha(\theta_\alpha - \theta'_\alpha) \tag{39}$$

where $\beta_\alpha > 0$, $\sigma_\alpha > 0$ and θ'_α are the design parameters.

Step4. Define $e_q = q - q_d$. Differentiating e_q, we have

$$\dot{e}_q = \theta_q^T \varphi_q(x_q) + \tilde{\theta}_q^T \varphi_q(x_q) + \eta_q + g_q \delta_e - \dot{q}_d$$

Finally, the feedback control law δ_e can be designed as

$$\delta_e = \frac{1}{g_q}(-k_q e_q - \theta_q^T \varphi_q(x_q) + \dot{q}_d - \overline{\eta}_q \tanh(\overline{\eta}_q e_q / \kappa) - g_\alpha e_\alpha) \quad (40)$$

where $k_q > 0$, $\kappa > 0$ are the design parameters. The adaptive law of θ_q is designed as

$$\dot{\theta}_q = \beta_q e_q \varphi_q(x_q) - \sigma_q(\theta_q - \theta_q') \quad (41)$$

where $\beta_q > 0$, $\sigma_q > 0$ and θ_q' are the design parameters.

C. Velocity Controller Design

The velocity controller is developed. Define velocity tracking error as

$$e_V = V - V_d$$

The time-derivative of e_V is

$$\dot{e}_V = \theta_V^T \varphi_V(x_V) + \tilde{\theta}_V^T \varphi_V(x_V) + \eta_V + g_V(\alpha)\phi - \dot{V}_d$$

Then the velocity controller is designed as

$$\phi = \frac{1}{g_V(\hat{\alpha})}(-k_V e_V - \theta_V^T \varphi_V(x_V) + \dot{V}_d - \overline{\eta}_V \tanh(\overline{\eta}_V e_V / \kappa)) \quad (42)$$

where $k_V > 0$, $\kappa > 0$ are the design parameters. The adaptive law of θ_V is designed as

$$\dot{\theta}_V = \beta_V e_V \varphi_V(x_V) - \sigma_V(\theta_V - \theta_V') \quad (43)$$

where $\beta_V > 0$, $\sigma_V > 0$ and θ_V' are the design parameters.

VI. Stability Analysis

Theorem 1: Consider the closed-loop system consisting of the model (12)-(16) with control laws (35), (36) , (38) , (40), (42) and adaptive laws (37), (39), (41), (43). Then all the signals involved are bounded and the output tracking error converges to a small neighborhood of the origin.

Proof: Consider the following Lyapunov function:

$$W = W_o + W_h + W_\gamma + W_\alpha + W_q + W_V \quad (44)$$

where

$$W_o = \frac{1}{2}\tilde{\gamma}^2 + \frac{1}{2}\tilde{\alpha}^2, W_h = \frac{e_h{}^2}{2\overline{V}^2}, W_\gamma = \frac{1}{2}e_\gamma^2 + \frac{1}{2\beta_\gamma}\tilde{\theta}_\gamma^T \tilde{\theta}_\gamma,$$

$$W_\alpha = \frac{1}{2}e_\alpha^2 + \frac{1}{2\beta_\alpha}\tilde{\theta}_\alpha^T \tilde{\theta}_\alpha, W_q = \frac{1}{2}e_q^2 + \frac{1}{2\beta_q}\tilde{\theta}_q^T \tilde{\theta}_q,$$

$$W_V = \frac{1}{2}e_V^2 + \frac{1}{2\beta_V}\tilde{\theta}_V^T \tilde{\theta}_V.$$

Differentiating $W_i\ (i = h, \gamma, \alpha, q, V)$ and combining (35)-(43), we obtain

$$\dot{W}_h = \frac{1}{\overline{V}^2}(-k_h e_h^2 + V e_h \tilde{\gamma} + V e_h e_\gamma) \quad (45)$$

$$\begin{aligned}\dot{W}_\gamma &= -k_\gamma e_\gamma^2 + e_\gamma \eta_\gamma - e_\gamma \overline{\eta}_\gamma \tanh(\overline{\eta}_\gamma e_\gamma / \kappa) - e_\gamma \delta_\gamma \\ &+ g_\gamma e_\gamma e_\alpha - \frac{V e_h e_\gamma}{\overline{V}^2} + \frac{\sigma_\gamma}{\beta_\gamma}\tilde{\theta}_\gamma^T(\theta_\gamma - \theta_\gamma')\end{aligned} \quad (46)$$

$$\begin{aligned}\dot{W}_\alpha &= -k_\alpha e_\alpha^2 + e_\alpha \eta_\alpha - e_\alpha \overline{\eta}_\alpha \tanh(\overline{\eta}_\alpha e_\alpha / \kappa) - e_\alpha \delta_\alpha \\ &+ g_\alpha e_\alpha e_q - g_\gamma e_\gamma e_\alpha + \frac{\sigma_\alpha}{\beta_\alpha}\tilde{\theta}_\alpha^T(\theta_\alpha - \theta_\alpha')\end{aligned} \quad (47)$$

$$\begin{aligned}\dot{W}_q &= -k_q e_q^2 + e_q \eta_q - e_q \overline{\eta}_q \tanh(\overline{\eta}_q e_q / \kappa) \\ &- g_\alpha e_\alpha e_q + \frac{\sigma_q}{\beta_q}\tilde{\theta}_q^T(\theta_q - \theta_q')\end{aligned} \quad (48)$$

$$\begin{aligned}\dot{W}_V &= -k_V e_V^2 + e_V \eta_V - e_V \overline{\eta}_V \tanh(\overline{\eta}_V e_V / \kappa) \\ &+ \frac{\sigma_V}{\beta_V}\tilde{\theta}_V^T(\theta_V - \theta_V') + A\tilde{\alpha}\end{aligned} \quad (49)$$

where $A = \frac{\overline{q}S\phi}{m}[C_T^{\phi\alpha^3}(\alpha^2 + \alpha\hat{\alpha} + \hat{\alpha}^2) + C_T^{\phi\alpha^2}(\alpha + \hat{\alpha}) + C_T^{\phi}]$.

Here, assume A is bounded during the flight, i.e. $|A| < \overline{A}$.

Note the following inequality holds:

$$\frac{\sigma_k}{\beta_k}\tilde{\theta}_k^T(\theta_k - \theta_k') \le -\frac{\sigma_k}{2\beta_k}\tilde{\theta}_k^T\tilde{\theta}_k + \frac{\sigma_k}{2\beta_k}\left\|\theta_k^* - \theta_k'\right\|^2,\ k = V, \gamma, \alpha, q$$

$$e_k \eta_k - e_k \overline{\eta}_k \tanh(\overline{\eta}_k e_k / \kappa) \le 0.2785\kappa = \kappa',\ k = V, \gamma, \alpha, q$$

$$-e_k \delta_k \le \frac{e_k^2}{2} + \frac{\delta_k^2}{2},\ k = \gamma, \alpha, \quad A\tilde{\alpha} \le \frac{\overline{A}^2}{2\varepsilon_3} + \frac{\varepsilon_3 \tilde{\alpha}^2}{2},$$

where $\varepsilon_3 > 0$ is a design parameter.

Substituting the above inequalities into (45)-(49) and considering (34), one has

$$\begin{aligned}\dot{W} &\le (-k_o + \frac{\overline{g}_\gamma}{2\varepsilon_1} + 1)\tilde{\gamma}^2 + (-\underline{g}_\gamma + \frac{\overline{g}_\gamma \varepsilon_1}{2} + \frac{\varepsilon_2 + \varepsilon_3}{2})\tilde{\alpha}^2 \\ &+ \frac{1}{\overline{V}^2}(-k_h + \frac{1}{2})e_h^2 + (-k_\gamma + \frac{1}{2})e_\gamma^2 + (-k_\alpha + \frac{1}{2})e_\alpha^2 - k_q e_q^2 - k_V e_V^2 \\ &- \frac{\sigma_\gamma}{2\beta_\gamma}\tilde{\theta}_\gamma^T\tilde{\theta}_\gamma - \frac{\sigma_\alpha}{2\beta_\alpha}\tilde{\theta}_\alpha^T\tilde{\theta}_\alpha - \frac{\sigma_q}{2\beta_q}\tilde{\theta}_q^T\tilde{\theta}_q - \frac{\sigma_V}{2\beta_V}\tilde{\theta}_V^T\tilde{\theta}_V \\ &+ \frac{\sigma_\gamma}{2\beta_\gamma}\left\|\theta_\gamma^* - \theta_\gamma'\right\|^2 + \frac{\sigma_\alpha}{2\beta_\alpha}\left\|\theta_\alpha^* - \theta_\alpha'\right\|^2 + \frac{\sigma_q}{2\beta_q}\left\|\theta_q^* - \theta_q'\right\|^2 \\ &+ \frac{\sigma_V}{2\beta_V}\left\|\theta_V^* - \theta_V'\right\|^2 + 4\kappa' + \frac{(\varepsilon_2 + 1)\delta_\alpha^2}{2\varepsilon_2} + \delta_\gamma^2 + \frac{\overline{A}^2}{2\varepsilon_3}.\end{aligned} \quad (50)$$

Choose the appropriate design parameters k_o, ε_1, ε_2, ε_3, $k_i (i = h, \gamma, \alpha, q, V)$, $\sigma_k (k = \gamma, \alpha, q, V)$, $\beta_k (k = \gamma, \alpha, q, V)$ such that $(-k_o + \frac{\overline{g}_\gamma}{2\varepsilon_1} + 1) < 0$, $(-\underline{g}_\gamma + \frac{\overline{g}_\gamma \varepsilon_1}{2} + \frac{\varepsilon_2 + \varepsilon_3}{2}) < 0$, $k_i > 0\ (i = q, V)$, $k_i > 1/2\ (i = h, \gamma, \alpha)$, $\sigma_k > 0\ (k = \gamma, \alpha, q, V)$ and $\beta_k > 0\ (k = \gamma, \alpha, q, V)$. Then (50) becomes

$$\dot{W} \le -cW + \lambda \quad (51)$$

where

$$\begin{aligned}c &= 2 \times \max\{(-k_o + \frac{\overline{g}_\gamma}{2\varepsilon_1} + 1), (-\underline{g}_\gamma + \frac{\overline{g}_\gamma \varepsilon_1}{2} + \frac{\varepsilon_2}{2}), \\ &-k_i + \frac{1}{2}(i = h, \gamma, \alpha), -k_i(i = q, V), -\frac{\sigma_k}{2\beta_k}(k = \gamma, \alpha, q, V)\}\end{aligned} \quad (52)$$

$$\begin{aligned}\lambda &= \frac{\sigma_\gamma}{2\beta_\gamma}\left\|\theta_\gamma^* - \theta_\gamma'\right\|^2 + \frac{\sigma_\alpha}{2\beta_\alpha}\left\|\theta_\alpha^* - \theta_\alpha'\right\|^2 + \frac{\sigma_q}{2\beta_q}\left\|\theta_q^* - \theta_q'\right\|^2 \\ &+ \frac{\sigma_V}{2\beta_V}\left\|\theta_V^* - \theta_V'\right\|^2 + 4\kappa' + \frac{(\varepsilon_2 + 1)\delta_\alpha^2}{2\varepsilon_2} + \delta_\gamma^2 + \frac{\overline{A}^2}{2\varepsilon_3}\end{aligned} \quad (53)$$

By assumption 3, δ_α and δ_γ are bounded which imply λ in (53) is bounded. Hence, from (51), all the signals involved in the closed-loop system are bounded and the output tracking error converges to a small neighborhood of the origin.

VII. SIMULATION

In this simulation study, the adaptive type-2 fuzzy output feedback control schemes are applied to the tracking control of the FAHVs.

Simulations are conducted on the model of the FAHVs with flexible states described as (1)-(6). The initial flying states are chosen as $[V,h,\gamma,\alpha,Q]^T=[7846.4\ ft/s, 85000\ ft, 0rad, 0.0219rad, 0rad/s]^T$, $\eta=[0.594,0,-0.0976,0,-0.0335,0]^T$ and the initial control inputs are chosen as $[\phi,\delta_e]^T=[0.12,0.12rad]^T$.

The control objective is to control the states V,h to track their reference trajectories. The reference trajectory is generated by filtering the given 1000 ft/s step in velocity channel and a 10000 ft change in altitude channel through second-order prefilters.

The premise variables of IT2-FLS are selected as $x_V=[V,\hat{a}]^T$, $x_\gamma=[V,\hat{a}]^T$, $x_\alpha=[V,\hat{a}]^T$, $x_\gamma=[V,\hat{a},\hat{\gamma}]^T$. The consequent IT2-FSs are chosen to be T1-FSs, their values are initially set as random numbers between -1 and 1. Specifying the design parameters $\beta_k=1\,(k=V,\gamma,\alpha,q)$, $\sigma_k=0.1\,(k=\gamma,\alpha,q)$ and $\sigma_V=0.01$, hence the adaptive laws in (37), (39), (41) and (43) are constructed.

The rest parameters of the proposed controllers are chosen as $k_o=10$, $k_i(i=h,\gamma,\alpha,q,V)$, $k_V=2$, $k_h=2.5$, $k_\gamma=4$, $k_\alpha=1$, $k_q=40$ and $\kappa=1$. In usual backstepping control design, the analytic computation of the virtual control signal derivatives is tedious. Here in simulation, we use command filters to obtain the derivatives of the virtual control signals as well as reference command signals.

To illustrate the performance of the proposed output feedback control system, the control laws are conducted on FAHVs model with 0% and 30% uncertainties added in all aerodynamic coefficients in (7)-(10). The trajectories of the velocity and altitude and the corresponding tracking errors are showed in Fig.3. It can be seen that the velocity V and altitude h can track their desired trajectories. Other flight states remain bounded as shown in Fig.4. Hence, the proposed adaptive type-2 fuzzy output feedback controller does provide good tracking performance, despite the presence of incomplete state measurement and the model uncertainties.

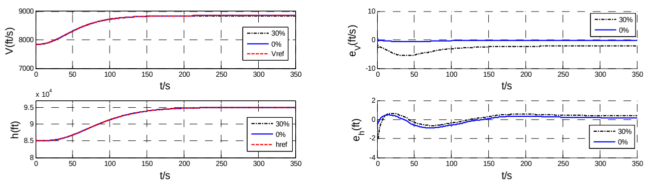

Figure 3. Tracking performance of altitude and velocity.

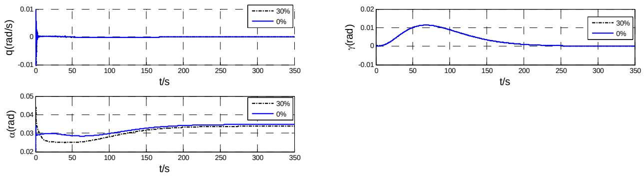

Figure 4. Tracking performance of other flight states.

VIII. CONCLUSION

In this paper we propose a new adaptive fuzzy output feedback control scheme for the tracking control of the flexible air-breathing vehicles. This control scheme involves IT2-FLS, which contributes to approximating the unknown dynamics of FAHVs. Based on these results, the approach of designing reduced-order fuzzy states observer is presented, which gives the estimates of the unmeasured states. Under the framework of the backstepping control method, an observer-based adaptive type-2 fuzzy controller is constructed to deal with the tracking problem for FAHVs. Furthermore, it is proven that the proposed controller guarantees that all signals in the closed-loop systems are bounded and the tracking error converges to a small neighborhood of the origin. Simulation results of the tracking control have been shown to demonstrate the effectiveness of the proposed results.

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