A COLORED PETRI NET MODEL FOR CONNECTION MANAGEMENT SERVICES IN MMS

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ABSTRACT – This paper presents a Petri net (PN) model for Connection Management Services (CMS) of the Manufacturing Message Specification (MMS) which describes all possible sequences of service primitives that can occur during establishment, maintenance and termination of the connection and their relation at each service state. The PN model enables a complete and unambiguous specification to be derived for CMS. P-invariants and T-invariants analysis carried out on the connection PN provides useful information on the CMS behavior and specification for its implementation. It is claimed that PNs may offer a hierarchical mathematical description for the entire MMS protocol, a uniform representation for various level of abstraction of MMS protocol and an analytical model for protocol performance evaluation and analysis.

1. INTRODUCTION

The Manufacturing Message Specification (MMS) is an application layer protocol (layer 7 of the ISO Basic Reference model for Open Systems Interconnection [ISO/IS 7498]) designed to facilitate messaging on a manufacturing floor [ISO/DIS 9506]. MMS is the modernized and accepted version of the Manufacturing Message Format Specification (MMFS) of the Manufacturing Automation Protocol (MAP) version 2.1 [McGuffin, 1987]. MMS became a ISO/Draft International Standard in 1987.

The connection management services (CMS) of MMS are used to maintain connections between MMS-users. Initiate, Conclude, Abort, Cancel, and Reject services are included in CMS. In ISO/DIS 9506, connection management services are included in environment and general management services.

In this paper the problem of modeling and analyzing the connection management services with *(colored) Petri nets* has been investigated. Several works have been done in this direction. BURKHARDT et al have established a model for OSI Communication Services and Protocol using *predicate/transition nets* and applied the results successfully to obtain a specification of the OSI-Transport and -Network Service as well as for the specification of the OSI-Transport Protocol classes 2 and 3 [Burkhardt, et al, 1984]. Another extension of the Petri net model, the *numerical Petri net* (NPN), has been employed in BILLINGTON's work for the specification of the OSI-Transport Service (TS) [Billington, 1982, 1983]. The results show that the NPN technique allows all interactions between the TS-provider and its users to be specified for a single connection and provides a very simple description of the queues used to model the interactions between TS-users on a connection. It is claimed that NPNs provide a visually appealing model of signal flow, which facilitates the design process and enables a complete and unambiguous specification to be derived.

It should be pointed out, however, that the application of the conventional Petri net analysis methods in both *predicate/transition net* and *numerical Petri net* models is difficult, if not impossible. For example, there is no obvious way to incorporate memory reference enabling conditions and transition operations in NPN into the Petri net analysis methods, even though they could be modeled by using additional places and transitions in a standard fashion. To apply Petri net analysis methods in our modeling and analyzing, we have developed a rigorous *(colored) Petri net.* model for CMS in MMS. Through P- and T-invariants analysis on the Petri net model, several important properties of CMS have been derived out analytically which are helpful in understanding the behavior of CMS and useful in its implementation.

The specification of CMS may be considered at two levels of abstraction. In the higher level the qualitative aspect of behavior of CMS is specified and only the set of possible sequences of service primitives and their relation at each service state is considered. At the second level the quantitative aspect of behavior of CMS is described and parameters associated with service primitives may be represented together with the procedures involved with connection establishment, maintenance, and termination. In this paper we only address the problem of modeling and analyzing of CMS at the higher level of abstraction. Section 2 introduces the system structure of CMS and gives an overview of the service primitives in CMS. Section 3 reviews briefly ordinary and colored Petri net theory. Section 4 presents a Petri net model for the connection between two MMS-users and the analytical results derived from the model. Section 5 shows the colored Petri net model for the general case, multiple connections, and proves that the behavior of multiple connections is identical to single connection behavior. In the final section we discus how to apply the Petri net models in the implementation of the MMS connection services and performance evaluation.

2. OVERVIEW AND BACKGROUND

The system structure for the connection management is given by the block diagram of figure 1 (where user 1 is the requester and user 2 the responder). The connection management services in MMS contain the **Initiate, Conclude, Abort, Cancel,** and **Reject** services. These services allow the MMS-user to:

a)initiate communication with another MMS-user in the MMS environment, and to establish the requirements and capabilities that support that communication;
b)conclude communication with another MMS-user in the MMS environment in a graceful manner;
c)abort communications with another MMS-user in the MMS environment in an abrupt manner;
d)cancel pending service requests; and
e)receive notification of protocol errors that occur.

The state diagram for entering and leaving the MMS environment defined in MMS is given in the figure 2. The initial state for both calling and called MMS-users is the state " No MMS Environment ". The diagram is depicted from the point of view of an MMS-user. The restrictions on use of MMS services are the follows:

1) The " **No MMS Environment** " State-:while in the state " No MMS Environment ", an MMS-user may only issue the initiate.request service primitive.

2) The " **Establishing MMS Environment (calling)** " State-:while in the state " Establishing MMS Environment (calling)", an MMS- user may only issue the abort.request service primitive.
3) The " **Establishing MMS Environment (called)**" State-:while in the state " Establishing MMS Environment (called)", an MMS-user may only issue the initiate.response or abort.request service primitives.
4) The " **MMS Environment** " State-:this state is divided into a series of substates described in clauses 8-16 of MMS. The only events which cause an exit from the " MMS Environment " state are the issuance of an abort.request, the issurance of a conclude.request service primitive, the receipt of an abort.indication, or the receipt of a conclude.indication service primitive.
5) The " **Relinquishing MMS Environment (Requester)** " State-:while in the state " Relinquishing MMS Environment (Requester)", the only request primitive that the MMS-user may issue is the abort.request service primitive.
6) The " **Relinquishing MMS Environment (Responder)** " State-: while in the state " Relinquishing MMS Environment (Responder)", the MMS-user may only issue the abort.request and the conclude.response service primitives.

In the next three sections, we will construct a mathematical model for MMS connection management using Petri nets based on the above restrictions and perform various analysis to reveal the qualitative behavior of the connection management protocol.

3. BASIC CONCEPTS FOR PETRI NETS AND COLORED PETRI NETS

In this section, we give a brief introduction to (ordinary) Petri net and colored Petri net theory through some definitions and examples. These definitions are quite standard and come mostly from Peterson (1981) and Jensen (1981).

Petri nets (both ordinary and colored) are tools for modeling the dynamic behavior of discrete event systems. They consist mainly of two types of elements: *places* and *transitions*. The set of places represents the system' s *states*, and the transitions represent *events* which change the state of the system. A place can contain a non-negative integer number of tokens. The state of the system modeled by a Petri net is given by its marking, the number of tokens in each of its places. The system evolves by firing its transitions. The rules governing firing will be given below. We will first discuss ordinary Petri nets (or simply, Petri nets) and then extend the results to colored Petri nets.

3.1 Mathematical Structure of Petri Nets

Definition 1: A ***Petri net*** (PN) is a quadruple PN=(P, T, I, O) where:
1)P and T are finite sets of ***places*** and ***transitions***, respectively, such that $P \cap T = \phi$ and $P \cup T \neq \phi$,

2) I: $P \times T \rightarrow N$ is the ***input function***,

3) O: $P \times T \rightarrow N$ is the ***output function***,
where N is the set of natural numbers.

A PN can be represented by a bipartite directed multigraph, the *Petri net graph*. Places are represented by *circles* and transitions by *bars*. There is an arc joining a place p to a transition t iff $I(p,t) \neq 0$. Analogously, there is an arc from a transition t to a place p iff

$O(p,t) \neq 0$. Natural numbers $I(p,t)$ and $O(p,t)$ are called the weights of the arcs. Arcs are labeled with their weights. Labels will be omitted if the arc's weight is equal to one.

Definition 2: A ***marking m*** of a PN is a function $m: P \rightarrow N$. It gives the number of tokens contained in each place $p \in P$.

A token can be represented by a dot. The initial state of a system is defined by a net marking called the *initial marking*. Figure 3a shows a PN with its initial marking

$$m_0(p_1) = m_0(p_2) = 1, m_0(p_3) = 0.$$

The marking can be more briefly expressed as a column vector: $m_0 = (1\ 1\ 0)^T$.

Definition 3: A transition t is ***enabled*** wrt a marking m iff: for all $p \in P$, $m(p) \geq I(p,t)$.

In figure 3a, only transition t_1 is enabled and t_2 in figure 3b.

Definition 4 (***execution rule***): Firing an enabled transition consists of removing $I(p,t)$ tokens from each input place p and adding $O(p,t)$ tokens to each output place p.

Figure 3b shows the marking of the PN after firing the enabled transition t_1. The marking reached is $m_1 = (0\ 0\ 1)^T$; in general,

$$\text{for all } p \in P, \ m_1(p) = m_0(p) + O(p,t) - I(p,t).$$

Let m be the marking reached from m_0 by applying the firing sequence s, $m_0 - s \rightarrow m$. If y is the count vector of s (i.e., y represents the number of times each transition has been fired in s), then m can be expressed by the state equation

$$m = m_0 + Ay$$

where $A = O - I = [a(p,t)]$, $a(p,t) = O(p,t) - I(p,t)$, is called the ***incidence matrix*** of the Petri net. In the example of figure 3,

$$A = \begin{bmatrix} -1 & 1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$$

And $s = t_1 t_2 t_1$, i.e., $y = (2\ 1)^T$, leads to the marking $m = m_0 + Ay = (0\ 0\ 1)^T$.

Definition 5 (***reachability set***): The reachability set $R(m)$ for a PN with marking m is the set of all markings of PN which can be reached from m by firing a finite number of transitions of PN.

The reachability set of the PN in figure 3 is $R(m_0) = \{(1\ 1\ 0)^T, (0\ 0\ 1)^T\}$.

3.2 Analysis of Petri Nets

The following are some of the properties and questions that have been studied in the literature about Petri nets [Al-Jaar, 1987]. Later we will use these properties to analyze the behavior of the Petri net model for connection management services.

1) **A deadlock** in a PN occurs when a marking is reached where no transitions in the net can be fired from that point on.
2) A PN is **live** wrt a marking *m* if, for any marking in R(*m*), it is possible to fire any transition in the net. Liveness guarantees the absence of deadlocks.
3) A PN is **reversible** or proper wrt a marking *m* if for every $m' \in R(m)$, $m \in R(m')$. Reversibility guarantees that the system modeled by the PN can re-initialize itself. This is very important for automatic error recovery.

It is easy to show that the PN in figure 3 is live and reversible. Therefore it is deadlock-free.

4) A PN is **bounded** wrt a marking *m* if there exists a finite number k such that for any marking in R(m) the number of tokens in each place of the PN under that marking is less than k. When k=1, the PN is **safe**.
5) A PN is **(structurally) conservative** if, for any initial marking m_0, there exists a weighting vector $w \geq 0 \in N^n$, $n=|P|$ and for all $m \in R(m_0)$,

$$w^T m = w^T m_0$$

i.e., the sum of the tokens weighted by w is constant. When $w^T=(1\ 1\ \ldots\ 1)$, the PN is called strictly conservative.

The weighting vector w is called a ***P-invariant*** or ***S-invariant*** of the PN. It can be proved from the state equation that w is an P-invariant of a PN iff

$$w^T A = 0$$

where A is the incidence matrix of the PN.

6) A PN is **(structurally) consistent** if, for any initial marking *m*, there exists a firing sequence s, called a cyclic firing sequence wrt *m*, such that $m \xrightarrow{s} m$.

The count vector of a cyclic firing sequence of a PN is called a ***T–invariant*** of the net. Similarly, from the state equation, a T–invariant x of a PN must satisfy equation:

$$Ax=0$$

For the PN defined in figure 3, the P–invariants are $(n_1\ n_2\ n_1+n_2)^T$, $n_1, n_2 \geq 0$, $n_1 n_2 \neq 0$. The net is obviously safe and bounded. The P–invariants are $(n\ n)^T$, $n>0$.

3.3 Colored Petri Nets

The major advantage of using colored Petri nets is to short description and analysis of the systems consisting of a number of different processes having a similar structure and behavior. In a colored Petri net, each of its places and transitions is associated with a set of colors. The state of the net is described by the distribution of colors among its places. The conditions for enabling a transition and the firing rule depend on linear functions which label the net's arcs. These functions indicate which colors must mark each place in order to enable a transition wrt a given color as well as which colors must be added to or removed from a place on firing.

Definition 6: A ***colored Petri net*** (CPN) is a 5–tuple CPN=(P, T, C, I, O) where:

1) P and T are finite sets of ***places*** and ***transitions***, respectively, such that $P \cap T = \phi$ and $P \cup T \neq \phi$,
2) C are the sets of ***colors*** associated with the places and the transitions. C(p) and C(t) are finite sets of colors associated with the place p and the transition t.
3) I and O are , respectively, the ***input function*** and the ***output function*** defined on $P \times T$ such that I, O: $C(t) \rightarrow sum[C(p)]$, $(p,t) \in P \times T$, where sum[C(p)] represents the set of non-negative formal sums of elements of C(p).

Definition 7: A ***marking m*** of a CPN is a function defined on P such that m(p): $C(p) \rightarrow N$. m(p) gives the number of tokens of each color in the place p.

Like ordinary Petri nets, a CPN can be represented by a bipartite directed multigraph with I(p,t) and O(p,t) as its arc labels. Each of the nodes (circles and bars) of the CPN graph is marked with its associated set of colors.

The ***incidence matrix*** of a CPN is defined to be $A = O - I = [a(p,t)]$ where $a(p,t) = O(p,t) - I(p,t)$, A is a $|P| \times |T|$ matrix of linear functions.

A transition t is enabled wrt a color $c \in C(t)$ if the current marking *m* is such that $m(p) \geq I(p,t)(c)$ for all $p \in P$. When a transition is enabled wrt the color c, it can fire. This firing is carried out in two steps:

1) Remove I(p,t)(c) colored tokens (i.e., colors) from each input place.
2) Add O(p,t)(c) colored tokens (i.e., colors) from each output place.

Having made these definitions, ***reachability set, deadlock, live, reversible, bounded, conservative***, and ***consistent*** **are** defined exactly as for ordinary Petri nets. The equations for ***P–invariants*** and ***T–invariants*** still hold except that each element of the P–invariant and T–invariant vector is now replaced by a sub-vector weighting the colors associated with the corresponding places and transitions.

4. Petri Net Model for Connection Between Two MMS Users

The objective of this section is to investigate the feasibility of modeling and analyzing the connection management services of MMS using Petri net. We will treat a very simple case, i.e., the connection between two MMS-users, in this section. The results obtained in this section will help us to understand the general results of multi-connection of n MMS-users developed in the next section.

4.1 Connection Petri Net

To model the connection between two MMS-users using a PN, we introduce the following places and transitions based on the description given in the section 2.

Place P:

p_{i1}: MMS-user i in No MMS Environment;
p_{i2}: MMS-user i in Establishing MMS Environment (calling);
p_{i3}: MMS-user i in Establishing MMS Environment (called);

p_{i4}: MMS-user i in MMS Environment ;
p_{i5}: MMS-user i in Relinquishing MMS Environment (Requester);
p_{i6}: MMS-user i in Relinquishing MMS Environment (Responder);
p_{i7}: MMS-user i received an initiate.indication;
p_{i8}: MMS-user i received an initiate.confirm+;
p_{i9}: MMS-user i received an initiate.confirm-;
p_{i10}: MMS-user i received a conclude.indication;
p_{i11}: MMS-user i received a conclude.confirm+;
p_{i12}: MMS-user i received a conclude.confirm-;

Transition T:

t_{i1}: MMS-user i issues an initiate.request;
t_{i2}: MMS-user i issues an initiate.response+;
t_{i3}: MMS-user i issues an initiate.response-;
t_{i4}: MMS-user i issues a conclude.request;
t_{i5}: MMS-user i issues a conclude.response+;
t_{i6}: MMS-user i issues a conclude.response-;
t_{i7}: MMS-user i receives an initiate.indication;
t_{i8}: MMS-user i receives an initiate.confirm+;
t_{i9}: MMS-user i receives an initiate.confirm-;
t_{i10}: MMS-user i receives a conclude.indication;
t_{i11}: MMS-user i receives a conclude.confirm+;
t_{i12}: MMS-user i receives a conclude.confirm-;
t_{i13}, t_{i14}, t_{i15}, t_{i16}, t_{i17}: MMS-user i issues an abort.request or receives an abort.indication;

where i=1, 2.

The places p_{i1}, p_{i2}, p_{i3}, p_{i4}, p_{i5}, p_{i6}, i=1,2, called ***user states***, are corresponding to the six states defined in the section 2 for each MMS-user. The places p_{i7}, p_{i8}, p_{i9}, p_{i10}, p_{i11}, p_{i12}, i=1,2, called ***connection places***, are introduced to express the interaction between two MMS-users explicitly. The transitions are exactly the same as those defined in ISO/DIS 9506.

Let

$$P=(P_1, P_2), \quad T=(T_1, T_2, T_3)$$
$$P_i=(P_{is}, P_{ic}), \quad T_i=(T_{is}, T_{ir}, T_{ia})$$

where (i=1, 2)

$$P_{is}=(p_{i1}, ..., p_{i6}), \quad P_{ic}=(p_{i7}, ..., p_{i12})$$
$$T_{is}=(t_{i1}, ..., t_{i6}), \quad T_{ir}=(t_{i7}, ..., t_{i12}), \; T_{ia}=(t_{i13}, ..., t_{i17}).$$

With such arrangement of the places and transitions, the incidence matrix of our Petri net model for connection of MMS-user 1 and 2 is

$$A = \begin{bmatrix} A_s & A_r & A_a & 0 & 0 & 0 \\ 0 & A_i & 0 & A_o & 0 & 0 \\ 0 & 0 & 0 & A_s & A_r & A_a \\ A_o & 0 & 0 & 0 & A_i & 0 \end{bmatrix} \tag{1}$$

where

$$A_s = \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

$$A_r = \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad (2)$$

$$A_a = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$A_o = -A_i = I$, and I is the 6×6 identity matrix.

The connection PN with the initial marking is given in Figure 4(a). The number of tokens in places p_{i1} (No MMS Environment) indicates the number of communication channels available to user i. It is clear from Figure 4(a) that two users are two closed subnets of the connection PN.

Two special cases are worth mentioning. The enabled transitions in the initial marking are t_{11}, t_{21} (i.e., both users can issue initiate.request in the initial states). Once the two users issue initiate.request simultaneously, the only enabled transitions in the resulting marking are abort transitions (t_{113}, t_{213}) (see Figure 4(b)). Therefore no MMS Environment can be established in this case and the connection attempt fails. Similarly, when both machines are in the MMS Environment state (Figure 4(c)) and the two users issue the conclude.request simultaneously (transition t_{14}, t_{24}), the only enabled transitions in the resulting marking are again abort transitions (t_{116}, t_{216}) (Figure 4(d)).

Another case is that the connection process may continue indefinitely. In Figure 4(c), if user 1 issues a conclude.request and user 2 responds with a conclude.confirm-, the two users have to return back to MMS Environments (Figure 4(c)). User 1 may re-issue a conclude.request and user 2 may again respond with a conclude.confirm-. This situation could potentially continue forever.

4.2 Invariant Analysis for The Connection PN

To reveal the qualitative behavior of the connection between two MMS-users, we perform the following invariant analysis on the connection PN.

1) **P-invariants:** From section 3 we know that P-invariants satisfy

$$W^T A = 0$$

Let

$$W^T = [W^T_{11}\ W^T_{12}\ \ W^T_{21}\ W^T_{22}] = (w_1, \ldots, w_{24})$$

where W_{i1} and W_{i2} are the weighting vectors corresponding to the user states p_{i1} to p_{i6} and the connection places p_{i7} to p_{i12}, i=1, 2, respectively.

It follows that

$$W^T_{11}A_s + W^T_{22}A_o = 0, \qquad W^T_{12}A_o + W^T_{21}A_s = 0,$$
$$W^T_{11}A_r + W^T_{12}A_i = 0, \quad W^T_{21}A_r + W^T_{22}A_i = 0,$$
$$W^T_{11}A_a = 0, \qquad W^T_{21}A_a = 0,$$

The solution can be found to be

$$W^T_{11} = \alpha(1\ 1\ 1\ 1\ 1\ 1) = an, \qquad W^T_{21} = \beta n \qquad (3)$$
$$W^T_{12} = 0, \qquad W^T_{22} = 0,$$

α and β are two arbitrary positive integers.

The minimal P-invariants are

$$W^T_1 = (n\ 0\ 0\ 0), \qquad W^T_2 = (0\ 0\ n\ 0) \qquad (4)$$

Based on the P-invariants (4), we get the following results,

<u>**Claim 4.1**</u>: *Each subnet of the connection PN corresponding to a user is strictly conservative.*

Proof: Let m' and m" be two arbitrary marking from the reachability set, then, by the definition of P-invariant

$$W^T_1 m' = W^T_1 m'', \qquad W^T_2 m' = W^T_2 m'',$$

that is,

$$m'(p_{11}) + \ldots + m'(p_{16}) = m''(p_{11}) + \ldots + m''(p_{16})$$
$$m'(p_{21}) + \ldots + m'(p_{26}) = m''(p_{21}) + \ldots + m''(p_{26})$$

which proves the claim. #

Claim 4.1 indicates that for the connection PN in Figure 4(a), each user will be in one and only one of six user states. For the user with multiple channels, the claim means that each of user's channels will be in one and only one of the six states. This claim guarantees that in our connection PN no user can be interrupted by other users.

Since each of the places $p_{i1},\ldots, p_{i6}$, $i=1, 2$, can enable an abort transition, the following can be derived from claim 4.1:

Claim 4.2: *The connection PN cannot be deadlock.*

Note that without abort-transitions, the connection PN may be deadlock. Therefore is necessary to introduce abort services. By investigating the reachability set of the connection PN, we get a further result,

Claim 4.3: The connection PN is both live and reversible.

It can be shown, however, that the connection PN is not bounded, which means physically there may be as many as possible messages left in the connection places due to the protocol errors. This can be avoid in the implementation through the proper supervision by the MMS-provider.

2) **T-invariants:** T-invariants satisfy the equation

$$AX=0$$

Let

$$X^T = [X^T_{11}\ X^T_{12}\ X^T_{13}\ X^T_{21}\ X^T_{22}\ X^T_{23}] = (x_1,\ldots,x_{34})$$

where X_{i1}, X_{i2}, and X_{i3} are the count vectors corresponding to the transitions t_{i1} to t_{i6}, t_{i7} to t_{i12}, and the abort transitions t_{i13} to t_{i17}, $i=1, 2$, respectively.

It follows that

$$A_sX_{k1}+ A_rX_{k2} + A_aX_{k3} =0, \qquad k=1,2$$
$$A_iX_{12} + A_oX_{21} =0,$$
$$A_oX_{11} + A_iX_{22} =0.$$

The solution can be found to be

$$X_{21}=X_{12}, \quad X_{22}=X_{11},$$
$$X_{13}=S_1X_{11} + S_2X_{12}, \qquad (5)$$
$$X_{23}=S_2X_{11} + S_1X_{12}.$$

where

$$S_1 = \begin{bmatrix} 1&0&0&0&0&0 \\ 0&-1&-1&0&0&0 \\ 0&1&0&-1&0&1 \\ 0&0&0&1&0&0 \\ 0&0&0&0&-1&-1 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 0&-1&-1&0&0&0 \\ 1&0&0&0&0&0 \\ 0&1&0&-1&0&1 \\ 0&0&0&0&-1&-1 \\ 0&0&0&1&0&0 \end{bmatrix} \qquad (6)$$

and $X_{11}, X_{12} \in N^6$ satisfy the constraints:

$$S_1X_{11} + S_2X_{12} \in N^6, \qquad S_1X_{11} + S_2X_{12} \in N^6,$$

It can be shown that

$$S_1X_{11} + S_2X_{12} \in N^6 \Leftrightarrow S_1X_{11} + S_2X_{12} \in N^6,$$

Hence, since $X^T_{11}=(x_1,\ldots,x_6)$, $X^T_{12}=(x_7,\ldots,x_{12})$, the above constraints can be reduced to

$$x_1 \geq x_8 + x_9, \qquad x_4 \geq x_{11} + x_{12},$$
$$x_7 \geq x_2 + x_3, \qquad x_{10} \geq x_5 + x_6, \qquad (7)$$
$$x_2 + x_6 + x_8 + x_{12} \geq x_4 + x_{10}$$

A ***normal connection cycle*** is any firing sequence which leads the connection PN back to its initial marking (i.e., No MMS Environments) without leaving any messages (tokens) at the connection places. Therefore a normal connection cycle is cyclic firing sequence wrt the initial marking shown in the Figure 4(a). The equations in the first line of the (5) implies that

Claim 4.4: *For any normal connection cycles, the numbers of firings of each of the transitions $t_{11}, \ldots, t_{16}$ ($t_{21}, \ldots, t_{26}$) must be equal to that of firings of each of the corresponding transitions $t_{27}, \ldots, t_{212}$ ($t_{17}, \ldots, t_{112}$), respectively.*

The claim indicates that for a normal connection cycle, the number of firing a request transition by one user must be same as that of firing the corresponding response transition by the other user. This result agrees with our intuition.

It follows from the equations in the second line of (5) that

Claim 4.5: *For any normal connection cycles, the numbers of firings of each of the abort transitions $t_{113}, t_{114}\, t_{115}, t_{116}, t_{117}$ must be equal to that of firings of each of the corresponding abort transitions $t_{214}, t_{213}\, t_{215}, t_{217}, t_{216}$, respectively.*

The claim 4.5 indicates that for a normal connection cycle the number of firings of the abort transition at a calling user state by a calling MMS-user must be same as that of firings of the abort transition at the corresponding called user state by the called MMS-user. In the MMS protocol this is guaranteed by forcing the two abort transitions to fire simultaneously. The claim, however, does not imply that the two abort transitions have to be fired at the same time. It also should be pointed out that the T–invariant condition is just a necessary condition for the normal connection cycles.

Combining the equations (5) and (7), we get a further result

Claim 4.6: *For any normal connection cycles, the numbers of firings of the non-abort transitions entering a user state is no less than that of firings of the non-abort transitions leaving the same user state. And the difference between the two firing numbers is equal to the number of firings of the abort transition at that user state.*

Actually, claim 5 and claim 6 are the consequences of claim 4 and the simple fact that the number of firings of the transitions entering a user state is equal to the number of firings of the transitions leaving the same user state; since at any user state the abort transition always leaves that state.

Even though the results obtained seem trivial, the significance of the above analysis is that instead of specifying those properties as the requirements, we derive them analytically as the necessary conditions for the normal connection cycles.

5. Colored Petri Net for Multiple Connection

The connection PN model obtained in the above section shows that both MMS-users have the identical structure and the similar behavior. The results also indicate that a large number of new places and transitions are required to model the connection among more than two MMS-users using ordinary PN. To avoid dealing with a large connection system consisting of many identical MMS-users with similar behavior, we construct a colored Petri net (CPN) for the multiple connection in this section.

5.1 Colored Connection Petri Net

Figure 5 gives a CPN for multiple connections among n MMS-users which shows an identical structure as that of one MMS-user in the connection PN (figure 4). The 12 places and the 17 transitions have the same functions as those defined in the last section. To associate colors to each of the places and transitions, as well as to specify the input and the output relations, we introduce two color sets

$$M = \{m_1, ..., m_n\}$$
$$CNN=\{(m_i,m_j) \mid i\neq j,\ m_i \text{ and } m_j\in M\}$$

and four linear functions over the color sets

$$\begin{array}{lll} REQ: & CNN\rightarrow M, & REQ((m_i,m_j))=mi \\ RSP: & CNN\rightarrow M, & RSP((m_i,m_j))=mj \\ ID: & CNN\rightarrow CNN, & ID((m_i,m_j))=(m_i,m_j) \\ REV: & CNN\rightarrow CNN, & REV((m_i,m_j))=(m_j,m_i) \end{array}$$

where M is the set of MMS-users, and CNN is the set of connection peers, i.e., (m_i,m_j) means MMS-user m_i is calling MMS-user m_j.

The color function of the CPN is defined as:

$$\begin{array}{lll} C(p_1)=M, & C(p)=CNN & \text{for all } p\in P,\ p\neq p_1. \\ C(t)=CNN, & \text{for all } t\in T. & \end{array}$$

The incidence matrix is defined as

$$A = \begin{bmatrix} A_s & A_r & A_a \\ A_o & A_i & 0 \end{bmatrix} \qquad (8)$$

where

$$A_s = \begin{bmatrix} -REQ & 0 & RSP & 0 & RSP & 0 \\ ID & 0 & 0 & 0 & 0 & 0 \\ 0 & -ID & -ID & 0 & 0 & 0 \\ 0 & REV & 0 & -ID & 0 & REV \\ 0 & 0 & 0 & ID & 0 & 0 \\ 0 & 0 & 0 & 0 & -ID & -ID \end{bmatrix}$$

$$A_r = \begin{bmatrix} -RSP & 0 & REQ & 0 & REQ & 0 \\ 0 & -ID & -ID & 0 & 0 & 0 \\ ID & 0 & 0 & 0 & 0 & 0 \\ 0 & ID & 0 & -REV & 0 & -ID \\ 0 & 0 & 0 & 0 & -ID & -ID \\ 0 & 0 & 0 & ID & 0 & 0 \end{bmatrix} \qquad (9)$$

$$A_a = \begin{bmatrix} REQ & RSP & REQ & RE0 & RSP \\ -ID & 0 & 0 & 0 & 0 \\ 0 & -ID & 0 & 0 & 0 \\ 0 & 0 & -ID & 0 & 0 \\ 0 & 0 & 0 & -ID & 0 \\ 0 & 0 & 0 & 0 & -ID \end{bmatrix}$$

$$A_o = -A_i = \begin{bmatrix} ID & 0 & 0 & 0 & 0 & 0 \\ 0 & ID & 0 & 0 & 0 & 0 \\ 0 & 0 & ID & 0 & 0 & 0 \\ 0 & 0 & 0 & ID & 0 & 0 \\ 0 & 0 & 0 & 0 & ID & 0 \\ 0 & 0 & 0 & 0 & 0 & ID \end{bmatrix}$$

In figure 5 we assume that all unlabeled places and transitions have the color set CNN and all unlabeled arcs have the label ID. The *initial marking* is

$$m_0(p_1)(m_i)=n_i \geq 1, \qquad \text{for all } m_i \in C(p_1)=M,$$

or

$$m_0(p_1) = \sum n_i m_i$$

$$m_0(p)=0, \qquad \text{for all } p \in P,\ p \neq p_1.$$

where n_i represents the number of channels available at MMS-user m_i.

Under the initial marking m_0, transition t_1 is the only enabled one (wrt any of its colors). When t_1 is firing wrt the color (m_i,m_j), a token with color m_i will be removed from place p_1, and a token with color (m_i,m_j) will be added to place p_2 and place p_7, respectively. It is very obvious from the CPN that for multiple channels, concurrency exists everywhere.

Next we present P-invariant and T-invariant analysis for the connection CPN. We will see that the results here are the direct extension of the results obtained for two MMS-users.

5.2 Invariant Analysis for The Connection CPN

1) P-invariants: P-invariants satisfy

$$W^T A = 0$$

where we consider every linear function in the incidence matrix as a matrix with appropriate dimensions. This is equivalent to consider the formal sum of colors as a linear space with every color as a base element. We assume that the same color has the same index in all the color sets associated with the places and transitions.

Let

$$W^T = [W^T{}_1 \ W^T{}_2]$$

and

$$W^T{}_1 = [W^T{}_{11} \ W^T{}_{12} \ W^T{}_{13} \ W^T{}_{14} \ W^T{}_{15} \ W^T{}_{16}]$$

W_1 and W_2 are the weighting vector about places p_1 to p_6 and p_7 to p_{12}, respectively. And W_{1i} is the weighting vector about the place p_i, $i=1,\ldots,6$. Therefore W_{11} is a $n\times 1$ vector, W_{1i}, $i=2,\ldots, 6$, are $n(n-1)\times 1$ vectors.

Expanding the P-invariant equation, we get

$$\begin{aligned} &W^T{}_1 A_s + W^T{}_2 A_o = 0 \\ &W^T{}_1 A_r + W^T{}_2 A_i = 0 \\ &W^T{}_1 A_a = 0 \end{aligned}$$

It is easy to see that $X\times ID = X$, so

$$\begin{aligned} W^T{}_{12} &= W^T{}_{11}\, REQ, \\ W^T{}_{13} &= W^T{}_{11}\, RSP, \\ W^T{}_{14} &= W^T{}_{11}\, REQ, \\ W^T{}_{15} &= W^T{}_{11}\, REQ, \\ W^T{}_{16} &= W^T{}_{11}\, RSP, \end{aligned} \qquad (9)$$

$$\begin{aligned} &W^T{}_1 A_s + W^T{}_2 = 0 \\ &W^T{}_1 A_r - W^T{}_2 = 0 \end{aligned} \qquad (10)$$

Since

$$RSP\times ID(m_i,m_j)=m_j, \quad REQ\times REV(m_i,m_j)=m_j, \text{ for all } (m_i,m_j)\in CNN$$

then

$$RSP\times ID - REQ\times REV = 0$$

We find that W_{1i} in (9) satisfy $W^T{}_1A_s=W^T{}_1A_r=0$, therefore the general form of P-invariants is

$$W^T{}_{12} = W^T{}_{11}\, REQ, \quad W^T{}_{13} = W^T{}_{11}\, RSP, \quad W^T{}_{14} = W^T{}_{11}\, REQ, \quad W^T{}_{15} = W^T{}_{11}\, REQ, \quad W^T{}_{16} = W^T{}_{11}\, RSP, \qquad W^T{}_2 = 0, \qquad (11)$$

where $W_{11} \in R^n$ is an arbitrary positive vector.

Claim 5.1: *For every MMS-user m_i and marking m reached from the initial marking, the sum of the number of m_i tokens in p_1, the number of (m_i,m_k),tokens $k=1, \ldots, n$, in p_2, p_4, p_5, and the number of (m_i,m_k) tokens , $k=1, \ldots, n$, in p_3, p_6 is a constant, n_i.*

Proof: Let m be an arbitrary marking from the reachability set $R(m_0)$, then, by the definition of P-invariant

$$W^Tm = W^T m_0 = W^T{}_{11}m_0(p_1),$$

where W is shown in (11), it follows that

$$W^Tm = W^T{}_{11}m(p_1)+W^T{}_{12}m(p_2)+W^T{}_{13}m(p_3)+W^T{}_{14}m(p_4)+W^T{}_{15}m(p_5)+W^T{}_{16}m(p_6)$$
$$= W^T{}_{11}\{m(p_1)+REQ[m(p_2)+m(p_4)+m(p_5)]+RSP[m(p_3)+m(p_6)]\}$$

Since $W^T{}_{11}$ is an arbitrary positive vector, we get

$$m(p_1)+REQ[m(p_2)+m(p_4)+m(p_5)]+RSP[m(p_3)+m(p_6)]=m_0(p_1)$$

Note that

REQ$[m(p_2)+m(p_4)+m(p_5)]$=(the number of (m_1,m_k) tokens, $k=1, \ldots, n$, in (p_2,p_4,p_5), ..., the number of (m_n,m_k) tokens, $k=1, \ldots, n$, in $(p_2,p_4,p_5))^T$
RSP$[m(p_3)+m(p_6)]$=(the number of (m_k,m_1) tokens, $k=1, \ldots, n$, in (p_3, p_6), ..., the number of (m_k,m_n) tokens, $k=1, \ldots, n$, in $(p_3, p_6))^T$

Therefore , for any reachable marking m,

the number of m_i tokens in p_1+ the number of (m_i,m_k) tokens, $k=1, \ldots, n$, in p_2, p_4, p_5 + the number of (m_i,m_k) tokens, $k=1, \ldots, n$, in p_3, p_6 $=n_i$. #

Claim 5.1 is the extension of claim 4.1 for multiple connection CPN, which simply says that each channel of a MMS-user can only be in one of the six states $p_1, \ldots, p_6$.

Similar to the situation in the ordinary connection PN, we can prove easily the following results:

Claim 5.2: *The connection CPN cannot be deadlock wrt any colors.*

Claim 5.3: *The connection CPN is live and reversible wrt any colors.*

2) **T-invariants:** T-invariants satisfy the equation

$$AX=0$$

Let

$$X^T = [X^T_1 \ X^T_2 \ X^T_3]$$

and

$$X^T_1 = [X^T_{11} \ X^T_{12} \ X^T_{13} \ X^T_{14} \ X^T_{15} \ X^T_{16}]$$
$$X^T_3 = [X^T_{31} \ X^T_{32} \ X^T_{33} \ X^T_{34} \ X^T_{35}]$$

where X_{i1}, X_{i2}, and X_{i3} are the count vectors corresponding to the transitions t_{i1} to t_{i6}, t_{i7} to t_{i12}, and the abort transitions t_{i13} to t_{i17}, i=1, 2, respectively; X_{i1} and X_{3i} are n(n-1)¥1 count vectors of transitions t_i and $t_{(12+i)}$, respectively.

Expanding the T-invariant equation, we get

$$A_sX_1 + A_rX_2 + A_aX_3 = 0,$$
$$A_oX_1 + A_iX_2 = 0.$$

which implies that

$$X_2 = X_1,$$
$$(A_s + A_r)X_1 + A_aX_3 = 0,$$

It follows further that

$$(RSP+REQ)(X_{11}-X_{13} -X_{15})=REQ(X_{31}+X_{33}+X_{34})+RSP(X_{32}+X_{35})$$

The first equation can be reduced to

$$[REQ + RSP - REQ(ID + REV)](X_{12} - X_{14} + X_{16})=0$$

but

$$REQ(ID + REV)=REQ + RSP$$

The general form of the solution is

$$\begin{aligned} &X_2= X_1 \\ &X_{31}= X_{32}= X_{11} - X_{12} - X_{13} \\ &X_{33}=(ID + REV)(X_{12} - X_{14} + X_{16}) \\ &X_{34}= X_{35}= X_{14} - X_{15} - X_{16} \end{aligned} \qquad (12)$$

with the constraints:

$$X_{11} \geq X_{12} + X_{13}, \quad X_{12}+ X_{16} \geq X_{14}, \quad X_{14} \geq X_{15} + X_{16}$$

From the first equation of (12), we get

__Claim 5.4:__ *For any normal connection cycles, the numbers of firings of each color of the transitions t_1, ..., t_6 must be equal to that of firings of the same color of the corresponding transitions t_7, ..., t_{12} , respectively.*

The claim indicates that for a normal connection cycle, the number of firings of a request transition wrt a given color must be same as that of firings of the corresponding response transition wrt the same color.

The rest of the equations in (12) imply that

__Claim 5.5:__ *For any normal connection cycles, the numbers of firings of each color of the abort transitions t_{13}, t_{16} must be equal to that of firings of the same color of* *the corresponding abort transitions t_{14}, t_{17}, respectively; and the number of firings of (m_i,m_j) the abort transitions t_{15} must be equal to that of firings of (m_j,m_i) of t_{15}.*

The claim indicates that for a normal connection cycle the number of firings of an abort transition at a calling user state wrt a given color must be same as that of firings of the abort transition at the corresponding called user state wrt the same color . This is guaranteed by MMS.

Combing the equations (12) and (13), we get a further result

Claim 5.6: *For any normal connection cycles, the numbers of firings of the non-abort transitions entering a user state wrt a given color is no less than that of firings of the non-abort transitions leaving the same user state wrt the same color . Also he difference between the two firing numbers is equal to the number of firings of the abort transition at that user state wrt the same color .*

Note that in our connection CPN model for multiple connections, we already assumed that every MMS-user can talk with each other (as assumed in MMS), if there are limitations on the connectivity among some MMS-users, an additional place has to be added to the connection CPN to indicate the connectivity of MMS-users.

6. CONCLUSIONS

As can be seen from figures 4 and 5 the Petri net models provide a complete, unambiguous and compact description of connection management services of MMS at the higher level of abstraction of specification and their graphical nature helps to visualize the system structure and dynamics. Petri net models describe all of the possible sequences of service primitives that can occur during establishment, maintenance and termination of the connection and their relationships at each service state. By P-invariants analysis we have proved that the connection PN models the mutual exclusion in the states of communication channel, a property that has to be followed for any valid model. The P-invariants analysis also indicates that automatic error recovery procedures can be incorporated into our PN model later since it is deadlock-free and reversible. T-invariants analysis gives the necessary conditions for normal connection cycles on the possible sequences of service primitives. These conditions provide useful information about what procedures we have to take in the implementation of CMS to guarantee the establishment of a correct connection.

Other important facts about the PN model which also indicate the advantages of using Petri nets include:

A hierarchical mathematical description for the entire MMS protocol can be developed using Petri nets. PN methods allow various phases of CMS to be constructed separately. In the connection PN, the connection maintenance phase is represented by a single place "MMS Environment" (P_{14}, P_{24} in figure 4 and P_4 figure 5) which actually consists of a number of service activities. For each of the service activities, e.g., file management services, a sub-PN model can be constructed. The PN model for MMS Environment may be built upon those sub-PN models by merging the common places and transitions, or introducing additional places and transitions if necessary. Once the MMS Environment place is invoked, one of the sub-PNs will be executed accordingly. Similar approaches can be taken for each of the services of MMS. Therefore a hierarchical mathematical description for the entire MMS protocol can be established.

A uniform representation for various level of abstraction of the MMS protocol can be provided by using Petri nets. Physically, tokens in the connection PN represent messages and service states in CMS. Therefore the specification of CMS at different levels of abstraction can be achieved easily by associating tokens with attributes containing information needed at the corresponding abstraction level. Decisions can be made about what to do next at each of places by referring to this information and actions can be taken during every transition firing by fetching parameters from the attributes for a specific service. Since attributes corresponding to different abstraction levels may be incorporated into tokens separately, this approach leads to a stepwise refinement technique for both specification and implementation of CMS, thereby increasing ease of understanding and programming. Clearly, this method for specification at different levels of abstraction can be applied to other MMS services.

An analytical model for protocol performance evaluation and analysis can be constructed from Petri net models. Recent studies on stochastic Petri nets (SPNs) show that SPN is a very promising tool for system performance evaluation and analysis [**Molloy** 1982]. A SPN can be constructed from a PN by assigning to transitions the (random) time used to complete their firings or to places the time during which the conditions represented by them hold. Time consideration allows a performance analysis to be carried out on the system modeled by the PN through calculating production time, resource utilization, throughput rate, etc. Equivalence between (semi-) Markov processes and (generalized) SPNs provide various analytical and convenient methods for performance index calculation. For CMS, the average connection cycle time, the machine utilization and the user waiting time offer important measures for optimization of system design and implementation. More important, the sensitivity of these performance indices on variations of CMS system structure and parameters can be determined using SPN, which provides very useful information for the improvement of system design and implementation and the selection of service parameters (the optimal operating conditions). Similar performance analysis can be carried out on other MMS services.

Describing all these aspects of the MMS protocol with the single mathematical structure, the Petri nets, is presently being explored.

It is claimed the Petri nets are a powerful and generic tool for the modeling and analyzing of CMS and other MMS services. More research is needed to verify this claim.

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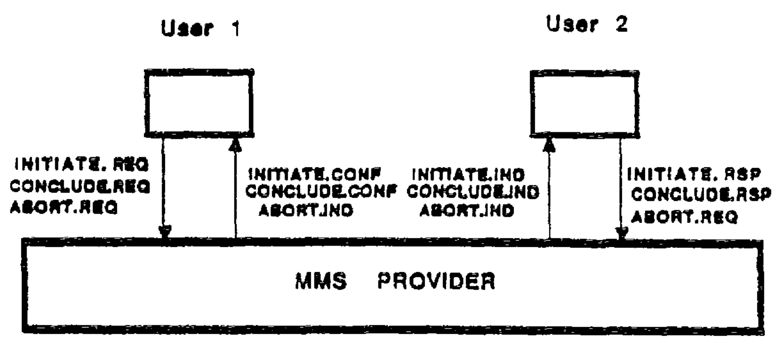
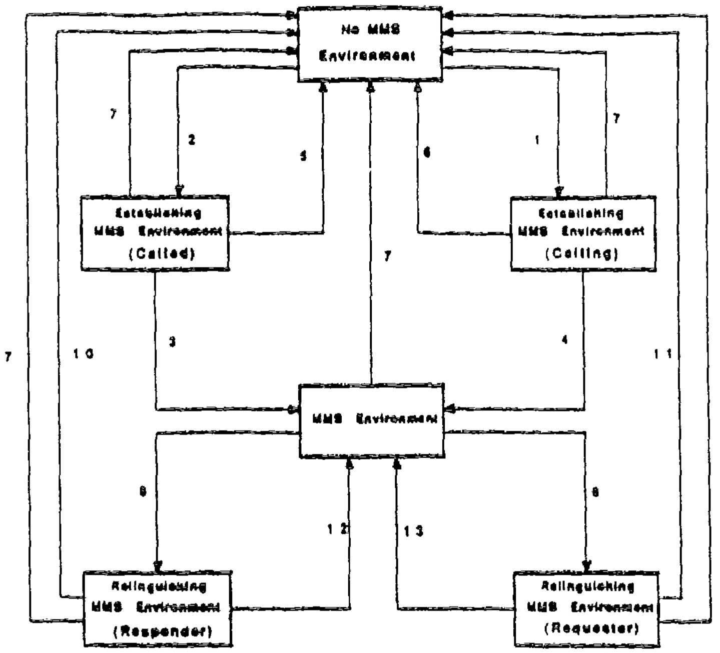


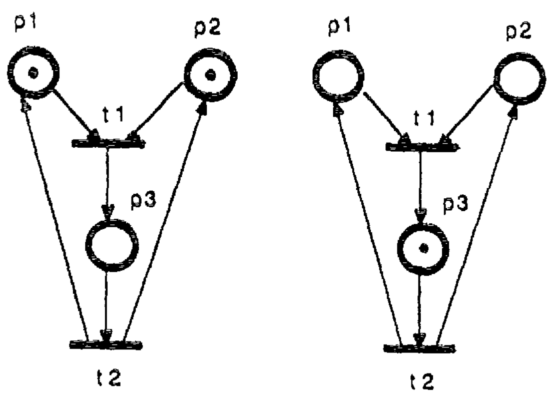
FIGURE 1. THE SYSTEM STRUCTURE OF MMS CONNECTION MANAGEMENT



Transitions:

1- initiate.request
2- initiate.indication
3- initiate.response+
4- initiate.confirm+
5- initiate.response-
6- initiate.confirm-
7- abort.req or abort.indication
8- conclude.request
9- conclude.indication
10- conclude.response+
11- conclude.confirm+
12- conclude.response-
13- conclude.confirm-

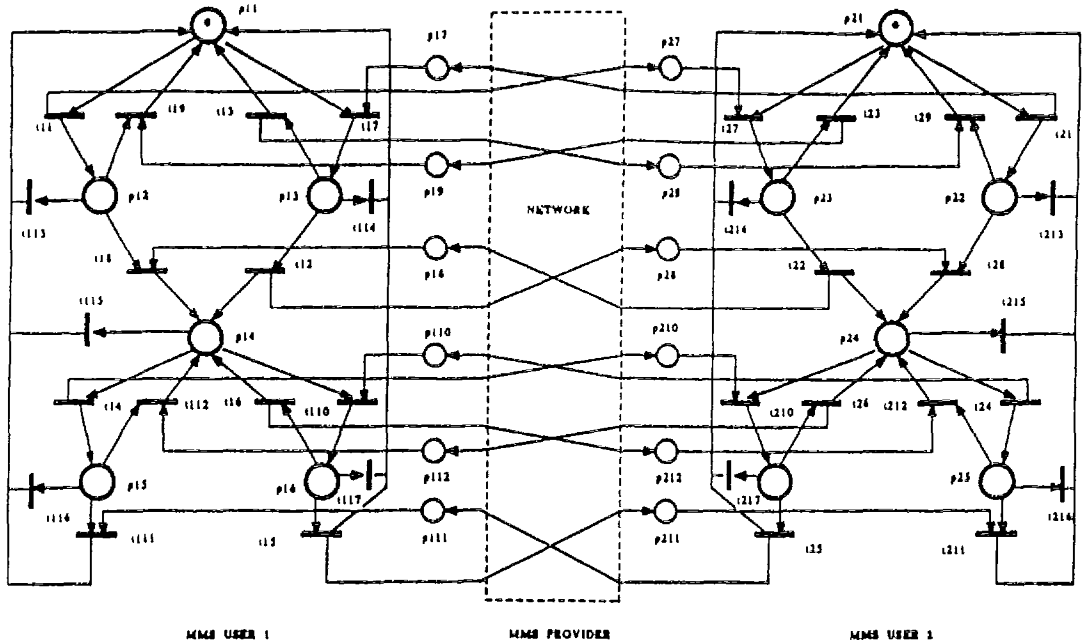
FIGURE 2. CONNECTION MANAGEMENT STATE DIAGRAM



3(a) 3(b)

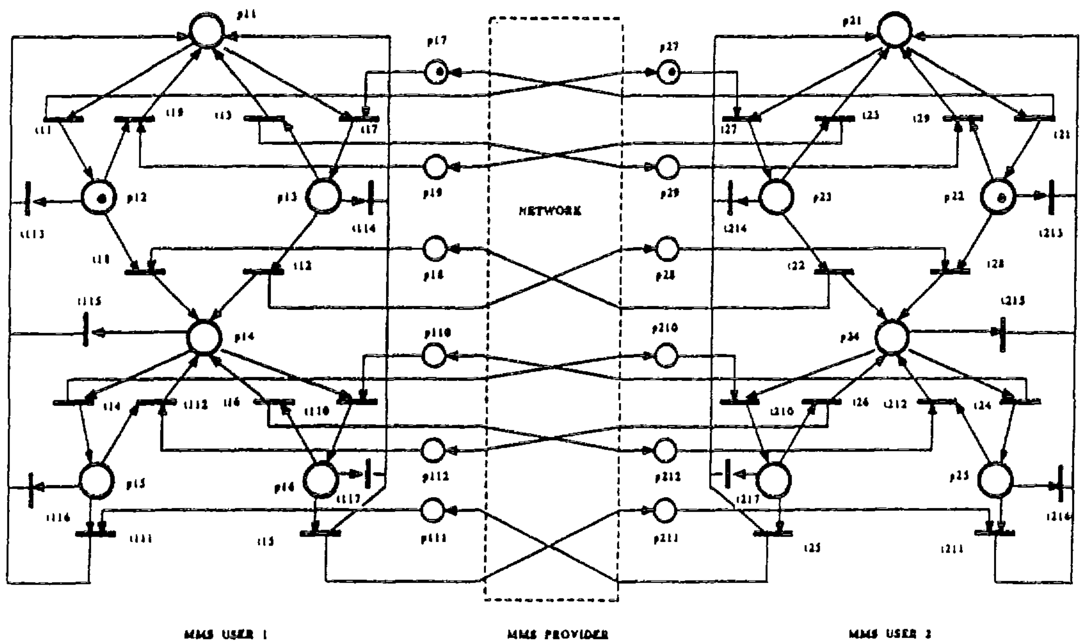
$$I = \begin{vmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{vmatrix} \qquad O = \begin{vmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{vmatrix}$$

FIGURE 3. A MARKED PN (A) BEFORE (B) AFTER FIRING TRANSITION T1.



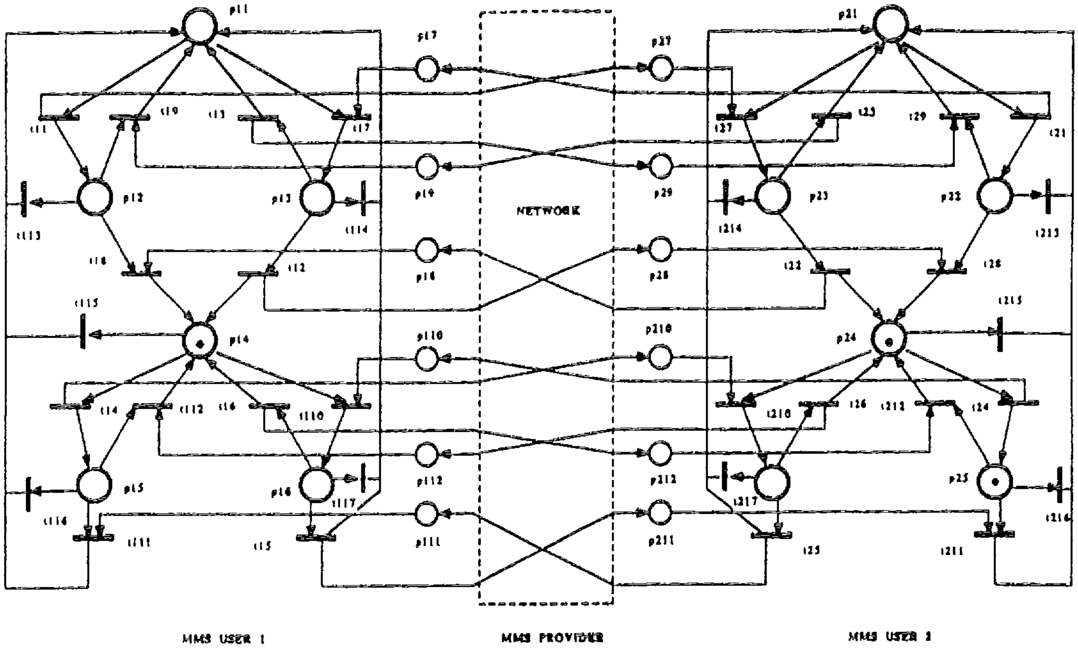
4(a) The connection PN with initial marking

FIGURE 4. THE PETRI NET FOR THE CONNECTION BETWEEN TWO MMS USERS



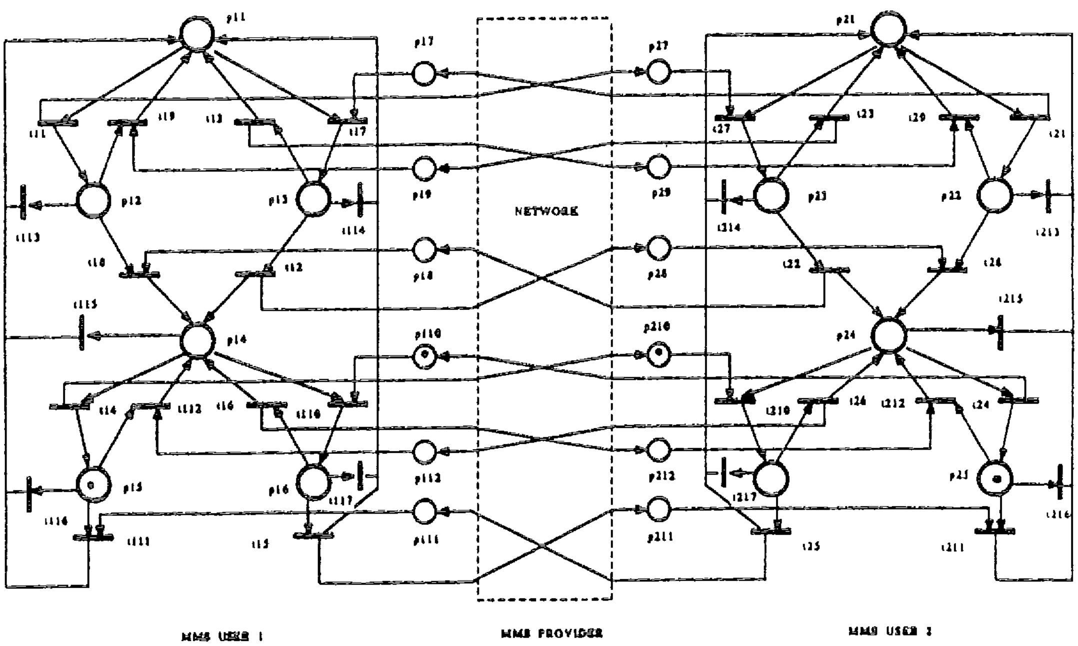
4(b) The connection PN after firing t11 and t21

FIGURE 4. THE PETRI NET FOR THE CONNECTION BETWEEN TWO MMS USERS



4(c) The connection PN in MMS Environment

FIGURE 4. THE PETRI NET FOR THE CONNECTION BETWEEN TWO MMS USERS



4(d) The connection PN after firing t14 and t24 from MMS Environment

FIGURE 4. THE PETRI NET FOR THE CONNECTION BETWEEN TWO MMS USERS

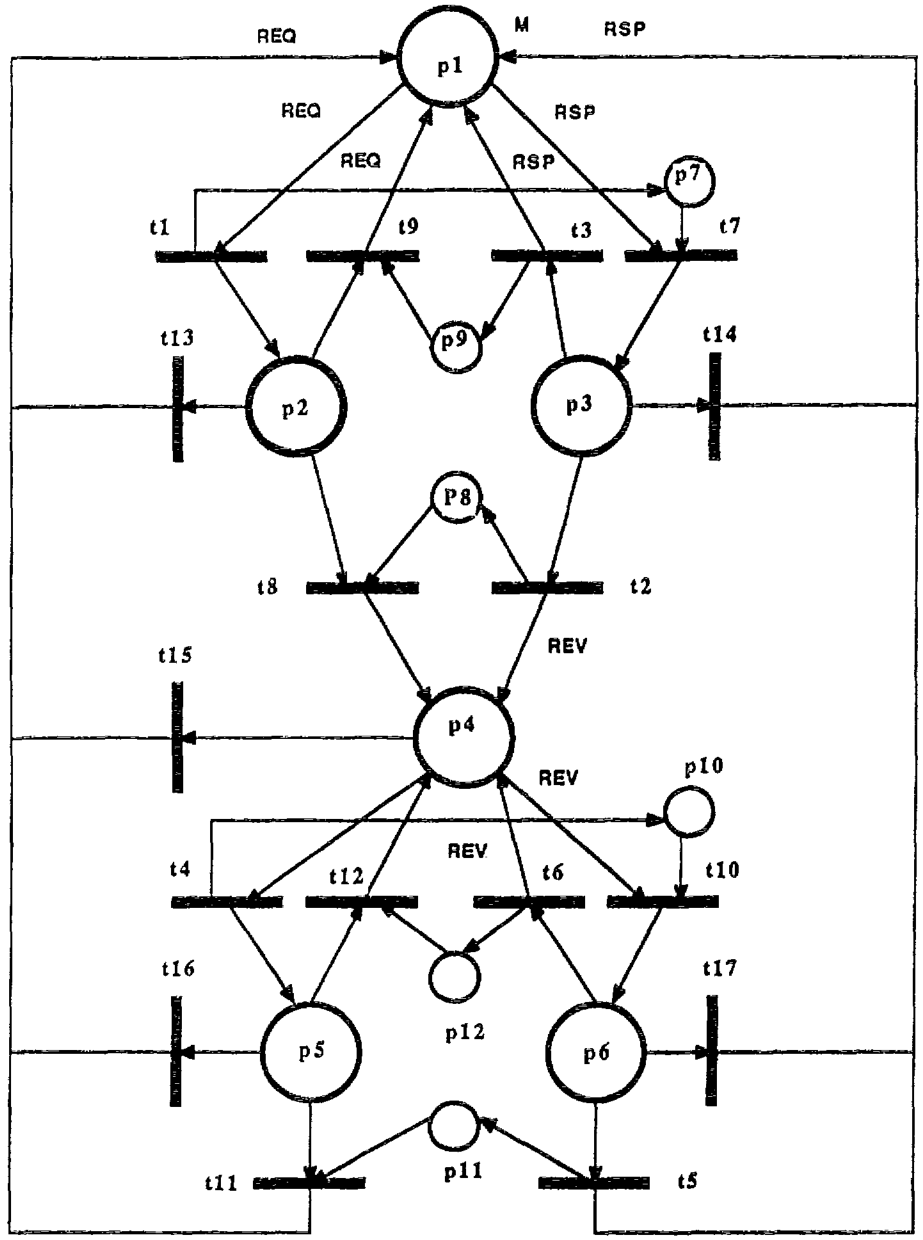


FIGURE 5. THE COLORED PETRI NET FOR MULTIPLE CONNECTION