7th International Conference on Advanced Computational Intelligence
Mount Wuyi, Fujian, China; March 27-29, 2015

Continuous-Time Group Consensus Using Distributed Event-Triggered Control

Hongwen Ma, Ding Wang, Derong Liu, and Chao Li

***Abstract*— Group consensus has both positive and negative communication weights, which is an extension to traditional consensus problem. Additionally, distributed event-triggered control has advantages over periodic control considering energy consumption and communication constraints. Thus, it is important to study group consensus using event-triggered control. Moreover, by calculating the maximum and minimum of the corresponding parameters, we can simplify the event-triggered function. The implementation will validate the effectiveness of our distributed control protocol.**

I. Introduction

Multi-agent systems have attracted a lot of attention in recent years. Vicsek et al. [1] proposed a novel type of phase transition in a system of self-driven particles, which is the origin of nearest neighbour rules. Then according to Vicsek's model, Jadbabaie et al. [2] introduced nearest neighbour rules into the multi-agent systems. For more details, please refer to survey papers [3]–[6] and the references cited therein.

Group consensus is one aspect in the extension of consensus problems. In [7], Yu and Wang proposed a new distributed control protocol with group consensus with finite switching topologies and bounded communication delays. Then, Tan et al. [8] relaxed the assumption such that the sums of adjacent weights are identical. However, all the above papers focus on the study of periodic control protocol, which is a serious drawback when we consider energy consumption and communication constraints on wireless platform. Hence, distributed event-triggered control is a suitable choice for solving this problem.

Event-triggered control is designed to improve the efficiency of control. Heemels et al. [9] gave an overview of event-triggered and self-triggered control in recent years. Event-triggered control in multi-agent systems is both conceptually interesting because designing a distributed control protocol based on event-triggered technique requires only relative information from local neighbours [10], and practically interesting because it can solve real-time scheduling problem excluding the infamous Zeno behavior [11]. Nevertheless, the communication weights in multi-agent systems using distributed event-triggered control mentioned above are all positive, while group consensus takes negative weights into consideration. Therefore, it is worth investigating group consensus using distributed event-triggered control.

Motivated by the above discussions, this paper established the conditions for achieving group consensus using distributed event-triggered control. The multi-agent systems are modeled containing two sub-networks. Moreover, the communication weights between the two sub-networks are not simply zeros but with balanced in- and out-degree.

The remainder of this paper is organized as follows. Basic definitions of group consensus and algebraic graph theory are given in Section II. Distributed event-triggered control for group consensus is developed in Section III. Implementations are conducted to validate the effectiveness of the developed criteria in Section IV. Conclusion is given in Section V.

The following notations are utilized throughout this paper: $x = (x_1, x_2, \dots, x_n)^\mathsf{T} \in \mathbb{R}^n$ and $\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$ represents the Euclidian norm of vector x. $A \in \mathbb{R}^{m\times n}$ and $\|A\|$ represents its corresponding Frobenius norm.

II. Backgrounds and Preliminaries

A. Algebraic Graph Theory

A triplet $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ is called a weighted graph if $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ is the set of N nodes, $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is the set of edges, and $\mathcal{A} = (\mathcal{A}_{ij}) \in \mathbb{R}^{N\times N}$ is the $N\times N$ matrix of the weights of $\mathcal{G}$. Here we denote $\mathcal{A}_{ij}$ as the element of the ith row and jth column of matrix $\mathcal{A}$. The ith node in graph $\mathcal{G}$ represents the ith agent, and a directed path from node i to node j is denoted as an ordered pair $(v_i, v_j) \in \mathcal{E}$, which means that agent i can directly transfer its information to agent j. $\mathcal{A}$ is called the adjacency matrix of graph $\mathcal{G}$ and we use the notation $\mathcal{G}(\mathcal{A})$: $\mathcal{A}_{ij} \neq 0 \Leftrightarrow (v_j, v_i) \in \mathcal{E}$ to represent the graph $\mathcal{G}$ corresponding to $\mathcal{A}$. In this article, we assume that $\mathcal{G}$ represents an undirected fixed topology. Note that self-loops will not be considered in this paper, i.e., $\mathcal{A}_{ii} = 0, i = 1, 2, \dots, N$. $\mathcal{G}$ is called connected if there is a path between any two nodes of $\mathcal{G}$. Let

$$\mathcal{D} = \begin{bmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_N \end{bmatrix}$$

be the $N\times N$ diagonal matrix where $d_i = \sum_{v_j\in\mathcal{N}_i} \mathcal{A}_{ij}$ and $\mathcal{N}_i = \{v_j \in \mathcal{V} | (v_j, v_i) \in \mathcal{E}\}$ is the set of neighbour nodes of node $i, i = 1, 2, \dots, N$. Then $\mathcal{D}$ is termed as the indegree matrix of $\mathcal{G}$. The Laplacian matrix is $\mathcal{L} = \mathcal{D} - \mathcal{A}$ corresponding to $\mathcal{G}$.

H. W. Ma, D. Wang, D. R. Liu, and C. Li are with the State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China (email: {mahongwen2012, ding.wang, derong.liu, lichao2012}@ia.ac.cn).

This work was supported in part by the National Natural Science Foundation of China under Grants 61034002, 61233001, 61273140, 61304086, and 61374105, in part by Beijing Natural Science Foundation under Grant 4132078, and in part by the Early Career Development Award of SKLMCCS.

B. Group Consensus

Given the network with N agents where $x_i \in \mathbb{R}^n$ represents the state of the ith agent. In physical implementations, the state of a node can represent the voltage or current of smart grid [12], temperature of rooms [13], and attitude of unmanned aerial vehicles [14], etc.

In this article, we assume that each agent has the dynamics as follows:

$$\dot{x}_i(t) = u_i(t),\ i = 1, 2, \ldots, N, \tag{1}$$

where $x_i \in \mathbb{R}$. We investigate the case that agents in a network can reach two consistent states asymptotically with event-triggered distributed control.

Before proceeding, we introduce the concepts of **group consensus** proposed in [7]. Suppose that the complex network $\mathcal{G}$ contains $N_1 + N_2$ $(N_1, N_2 > 0)$ agents consisting of two sub-networks $\mathcal{G}_1 = \{\mathcal{V}_1, \mathcal{E}_1, \mathcal{A}_1\}$ and $\mathcal{G}_2 = \{\mathcal{V}_2, \mathcal{E}_2, \mathcal{A}_2\}$, where $x^1 = (x_1, x_2, \ldots, x_{N_1})^\top$ and $x^2 = (x_{N_1+1}, x_{N_1+2}, \ldots, x_{N_1+N_2})^\top$ represent the states of $\mathcal{G}_1$ and $\mathcal{G}_2$, respectively. Thus, all the agents are divided into two groups with communication between the two groups. Furthermore, the whole graph is $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and the corresponding state is $x = (x_1, x_2, \ldots, x_{N_1+N_2})^\top$. Consequently, denote the index sets of sub-networks by $\mathcal{I}_1 = \{1, 2, \ldots, N_1\}$ and $\mathcal{I}_2 = \{N_1+1, N_1+2, \ldots, N_1+N_2\}$, and denote the node sets by $\mathcal{V}_1 = \{v_1, v_2, \ldots, v_{N_1}\}$ and $\mathcal{V}_2 = (v_{N_1+1}, v_{N_1+2}, \ldots, v_{N_1+N_2})$, where $\mathcal{I} = \mathcal{I}_1 \bigcup \mathcal{I}_2$ and $\mathcal{V} = \mathcal{V}_1 \bigcup \mathcal{V}_2$. More specifically, the neighbour sets of the corresponding sub-networks are $\mathcal{N}_{1i} = \{v_j \in \mathcal{V}_1 | (v_j, v_i) \in \mathcal{E}\}$ and $\mathcal{N}_{2i} = \{v_j \in \mathcal{V}_2 | (v_j, v_i) \in \mathcal{E}\}$, where $\mathcal{N}_i = \{v_j \in \mathcal{V} | (v_j, v_i) \in \mathcal{E}\} = \mathcal{N}_{1i} \bigcup \mathcal{N}_{2i}, \forall i \in \mathcal{I}$.

A new distributed control protocol is proposed as follows:

$$u_i(t) = \begin{cases} - \sum\limits_{v_j \in \mathcal{N}_{1i}} \mathcal{A}_{ij}(x_i(t) - x_j(t)) \\ \quad - \sum\limits_{v_j \in \mathcal{N}_{2i}} \mathcal{A}_{ij} x_j(t), \quad \forall i \in \mathcal{I}_1; \\ - \sum\limits_{v_j \in \mathcal{N}_{2i}} \mathcal{A}_{ij}(x_i(t) - x_j(t)) \\ \quad - \sum\limits_{v_j \in \mathcal{N}_{1i}} \mathcal{A}_{ij} x_j(t), \quad \forall i \in \mathcal{I}_2, \end{cases} \tag{2}$$

where $\mathcal{A}_{ij} \geq 0, \forall i, j \in \mathcal{I}_1$ and $\forall i, j \in \mathcal{I}_2$; $\mathcal{A}_{ij} \in \mathbb{R}, \forall (i, j) \in \Xi = \{(i, j) | i \in \mathcal{I}_1, j \in \mathcal{I}_2\} \bigcup \{(j, i) | i \in \mathcal{I}_1, j \in \mathcal{I}_2\}$.

Definition 1: If the states of the agents in $\mathcal{G}$ satisfy the following two conditions:

$$\lim_{t \to \infty} \|x_i(t) - x_j(t)\| = 0, \quad \forall i, j \in \mathcal{I}_1; \tag{3}$$

$$\lim_{t \to \infty} \|x_i(t) - x_j(t)\| = 0, \quad \forall i, j \in \mathcal{I}_2, \tag{4}$$

then the multi-agent systems (1) are said to reach a **group consensus** asymptotically.

Note that given the set

$$\Upsilon = \{x_1 = x_2 = \ldots = x_{N_1}, \\ x_{N_1+1} = x_{N_1+2} = \ldots = x_{N_1+N_2}\},$$

then Υ is a globally attractive and invariant manifold if the group consensus can be reached. In what follows, we will discuss the group consensus using distributed event-triggered control protocol.

Definition 2 (cf. [7]): The communication topology $\mathcal{G} = \{\mathcal{G}_1, \mathcal{G}_2\}$ of the multi-agent systems is consisted of $N_1 + N_2$ nodes. Given any $i \in \mathcal{I}_1$, the out-degree and in-degree of node v_i in $\mathcal{G}_1$ to $\mathcal{G}_2$ are defined as follows:

$$d_{\text{out}}(v_i, \mathcal{G}_2) = \sum_{j=N_1+1}^{N_1+N_2} \mathcal{A}_{ji},\ d_{\text{in}}(v_i, \mathcal{G}_2) = \sum_{j=N_1+1}^{N_1+N_2} \mathcal{A}_{ij}.$$

Given $i \in \mathcal{I}_1$, if $d_{\text{in}}(v_i, \mathcal{G}_2) = 0$ and $d_{\text{out}}(v_i, \mathcal{G}_2) = 0$, then we say $v_i \in \mathcal{V}_1$ is **in-degree balanced** and **out-degree balanced** to $\mathcal{G}_2$, respectively. Similarly, given $i \in \mathcal{I}_2$, if $d_{\text{in}}(v_i, \mathcal{G}_1) = 0$ and $d_{\text{out}}(v_i, \mathcal{G}_1) = 0$, then we say $v_i \in \mathcal{V}_2$ is in-degree balanced and out-degree balanced to $\mathcal{G}_1$, respectively. Furthermore, if all nodes in $\mathcal{G}_1(\mathcal{G}_2)$ are out(in)-degree balanced to $\mathcal{G}_2(\mathcal{G}_1)$, we say that $\mathcal{G}_1(\mathcal{G}_2)$ is out(in)-degree balanced to $\mathcal{G}_2(\mathcal{G}_1)$, and vice versa.

We use $\mathcal{L}$ to represent the Laplacian matrix of the communication topology $\mathcal{G}$, where $\mathcal{L} = (l_{ij}) \in \mathbb{R}^{(N_1+N_2) \times (N_1+N_2)}$ is defined as follows:

$$l_{ij} = \begin{cases} -\mathcal{A}_{ij}, & j \neq i; \\ \sum\limits_{k=1, k \neq i}^{N_1+N_2} \mathcal{A}_{ik}, & j = i. \end{cases}$$

Suppose $\mathcal{L}$ has a block form

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_{11} & \mathcal{L}_{12} \\ \mathcal{L}_{21} & \mathcal{L}_{22} \end{bmatrix}, \tag{5}$$

where $\mathcal{L}_{11} \in \mathbb{R}^{N_1 \times N_1}$ and $\mathcal{L}_{22} \in \mathbb{R}^{N_2 \times N_2}$, then the multi-agent systems (1) with u_i given in (2) is equivalent to the following form:

$$\begin{aligned} \dot{x}^1(t) &= -\mathcal{L}_{11} x^1 - \mathcal{L}_{12} x^2; \\ \dot{x}^2(t) &= -\mathcal{L}_{21} x^1 - \mathcal{L}_{22} x^2, \end{aligned} \tag{6}$$

where $\mathcal{L}_{12} = \mathcal{L}_{21}$. Before we establish our main theorem, we introduce Assumption 1 and Lemma 1 that will be utilized in the subsequent section.

Assumption 1: Considering the balance of the two sub-networks $\mathcal{G}_1$ and $\mathcal{G}_2$ mentioned in Definition 2, we propose three assumptions for the convenience of later proofs as follows:

(A1) $\sum\limits_{j=N_1+1}^{N_1+N_2} \mathcal{A}_{ij} = 0, \forall i \in \mathcal{I}_1$;

(A2) $\sum\limits_{j=1}^{N_1} \mathcal{A}_{ij} = 0, \forall i \in \mathcal{I}_2$;

(A3) $(x^1)^\top \mathcal{L}_{12} x^2$ is in the form of $(x_{i_1} - x_{j_1})(x_{i_2} - x_{j_2})$, *where* $(i_1, j_1) \in \mathcal{E}_1$ and $(i_2, j_2) \in \mathcal{E}_2$.

Lemma 1 (cf. [15]): With Assumptions (A1), (A2) and distributed control protocol (2), the multi-agent systems (1) can reach the group consensus asymptotically if and only if
(i) $\mathcal{L}$ has only two simple zero eigenvalues while the others have positive real parts;
(ii) $\mathcal{L}_1$ and $\mathcal{L}_2$ are in-degree and out-degree balanced to each other.

III. Distributed Event-Triggered Control for Group Consensus

Each agent will update its own control input $u_i(t)$ at event times decided by information from itself and from its neighbours. We denote these event times by $t_0^i, t_1^i, \ldots, t_k^i, \ldots, \forall i \in \mathcal{I}$. Defining the error measurement function for agent i as

$$\varepsilon_i(t) = x_i(t_k^i) - x_i(t), \quad t \in [t_k^i, t_{k+1}^i), k = 0, 1, \ldots. \tag{7}$$

The distributed control protocol (2) can be written in the form of

$$u_i(t) = \begin{cases} -\sum\limits_{v_j \in \mathcal{N}_{1i}} \mathcal{A}_{ij}(x_i(t_k^i) - x_j(t_{\tilde{k}(t)}^j)) \\ \quad -\sum\limits_{v_j \in \mathcal{N}_{2i}} \mathcal{A}_{ij} x_j(t_{\tilde{k}(t)}^j), \quad \forall i \in \mathcal{I}_1; \\ -\sum\limits_{v_j \in \mathcal{N}_{2i}} \mathcal{A}_{ij}(x_i(t_k^i) - x_j(t_{\tilde{k}(t)}^j)) \\ \quad -\sum\limits_{v_j \in \mathcal{N}_{1i}} \mathcal{A}_{ij} x_j(t_{\tilde{k}(t)}^j), \quad \forall i \in \mathcal{I}_2, \end{cases} \tag{8}$$

where $\tilde{k}(t) \triangleq \arg \min\limits_{h \in \mathbb{N}:\, t \geq t_h^j} \{t - t_h^j\}$ and $t_{\tilde{k}(t)}^j$ is the latest event time of agent j within $t \in [t_k^i, t_{k+1}^i)$. Suppose $\mathcal{L}x \triangleq q = (q_1, q_2, \ldots, q_{N_1+N_2})^\top$. Then, $q_i(t) = \sum\limits_{v_j \in \mathcal{N}_i} \mathcal{A}_{ij}(x_i(t) - x_j(t))$, $\forall i \in \mathcal{I}$. We introduce a notation $|N_i^{\mathcal{A}}| = \sum\limits_{v_j \in \mathcal{N}_i} \mathcal{A}_{ij}$ to simplify the expression of the following proof.

Theorem 1: If $\mathcal{G}$ is undirected connected graph with Laplacian matrix $\mathcal{L}$ which is satisfied with the condition (i) in Lemma 1, then with the Assumption 1, given the multi-agent systems (1) with the distributed control protocol (8) and the distributed event-triggered mechanism

$$\varepsilon_i^2(t) = \alpha \frac{\beta_{\min}}{\gamma_{\max}} q_i^2(t), \tag{9}$$

the multi-agent systems (1) can asymptotically reach group consensus. In addition, $\alpha \in (0,1), \beta_{\min} = \min\limits_{i \in \mathcal{I}}\{1 - c|N_i^{\mathcal{A}}|\}, \gamma_{\max} = \max\limits_{i \in \mathcal{I}}\{|N_i^{\mathcal{A}}|/c\}$ and $c \in \bigcap\limits_{i \in \mathcal{I}} (0, 1/|N_i^{\mathcal{A}}|)$.

Proof: From Assumption (A1), we know that

$$\sum_{v_j \in N_{2i}} \mathcal{A}_{ij} = 0, \forall i \in \mathcal{I}_1,$$

then the first part of control protocol (8) can be rewritten as

$$\begin{aligned} u_i(t) =& -\sum_{v_j \in \mathcal{N}_{1i}} \mathcal{A}_{ij}(x_i(t_k) - x_j(t_{\tilde{k}(t)}^j)) \\ & -\sum_{v_j \in \mathcal{N}_{2i}} \mathcal{A}_{ij} x_j(t_{\tilde{k}(t)}^j) \\ =& -\sum_{v_j \in \mathcal{N}_{1i}} \mathcal{A}_{ij}(x_i(t_k) - x_j(t_{\tilde{k}(t)}^j)) \\ & -\sum_{v_j \in \mathcal{N}_{2i}} \mathcal{A}_{ij}(x_i(t_k) - x_j(t_{\tilde{k}(t)}^j)) \\ =& -\sum_{v_j \in \mathcal{N}_{i}} \mathcal{A}_{ij}(x_i(t_k) - x_j(t_{\tilde{k}(t)}^j)), \end{aligned}$$

$\forall i \in \mathcal{I}_1$. Similarly, the second part of control protocol (8) can be rewritten in the same form. Thus,

$$\begin{aligned} \dot{x}_i(t) &= u_i(t) \\ &= -\sum_{v_j \in \mathcal{N}_i} \mathcal{A}_{ij}(x_i(t_k^i) - x_j(t_{\tilde{k}(t)}^j)) \\ &= -\sum_{v_j \in \mathcal{N}_i} \mathcal{A}_{ij}(x_i(t) - x_j(t)) \\ &\quad -\sum_{v_j \in \mathcal{N}_i} \mathcal{A}_{ij}(\varepsilon_i(t) - \varepsilon_j(t)), \ \forall i \in \mathcal{I}. \end{aligned} \tag{10}$$

Furthermore, we rewrite (10) in a compact vector form as

$$\dot{x}(t) = -\mathcal{L}x(t) - \mathcal{L}\varepsilon(t). \tag{11}$$

We choose a Lyapunov function for the closed-loop system as follows:

$$V = \frac{1}{2} x^\top \mathcal{L} x.$$

Then,

$$\dot{V} = x^\top \mathcal{L} \dot{x} = -x^\top \mathcal{L}\mathcal{L}(x + \varepsilon) = -q^\top q - q^\top \mathcal{L}\varepsilon.$$

Before proceeding, we introduce a basic inequality

$$|rs| \leq \frac{c}{2} r^2 + \frac{1}{2c} s^2, \quad r \in \mathbb{R}, s \in \mathbb{R}, \forall c > 0 \tag{12}$$

to better demonstrate our following proof.

$$\begin{aligned} \dot{V} =& -\sum_{i \in \mathcal{I}} q_i^2 - \sum_{i \in \mathcal{I}} \sum_{v_j \in \mathcal{N}_i} \mathcal{A}_{ij} q_i (\varepsilon_i - \varepsilon_j) \\ =& -\sum_{i \in \mathcal{I}} q_i^2 - \sum_{i \in \mathcal{I}} \sum_{v_j \in \mathcal{N}_i} \mathcal{A}_{ij} q_i \varepsilon_i + \sum_{i \in \mathcal{I}} \sum_{v_j \in \mathcal{N}_i} \mathcal{A}_{ij} q_i \varepsilon_j \\ \leq& -\sum_{i \in \mathcal{I}} q_i^2 + \sum_{i \in \mathcal{I}} \sum_{v_j \in \mathcal{N}_i} \mathcal{A}_{ij} \left(\frac{c}{2} q_i^2 + \frac{1}{2c} \varepsilon_i^2\right) \\ & + \sum_{i \in \mathcal{I}} \sum_{v_j \in \mathcal{N}_i} \mathcal{A}_{ij} \left(\frac{c}{2} q_i^2 + \frac{1}{2c} \varepsilon_j^2\right) \qquad \text{(c.f. (12))} \\ =& -\sum_{i \in \mathcal{I}} q_i^2 + c \sum_{i \in \mathcal{I}} |N_i^{\mathcal{A}}| q_i^2 + \frac{1}{2c} \sum_{i \in \mathcal{I}} |N_i^{\mathcal{A}}| \varepsilon_i^2 \\ & + \frac{1}{2c} \sum_{i \in \mathcal{I}} \sum_{v_j \in \mathcal{N}_i} \mathcal{A}_{ij} \varepsilon_j^2. \end{aligned}$$

Due to the symmetry of $\mathcal{L}$, the last term above can be rewritten as

$$\frac{1}{2c} \sum_{i \in \mathcal{I}} \sum_{v_j \in \mathcal{N}_i} \mathcal{A}_{ij} \varepsilon_j^2 = \frac{1}{2c} \sum_{i \in \mathcal{I}} \sum_{v_j \in \mathcal{N}_i} \mathcal{A}_{ij} \varepsilon_i^2 = \frac{1}{2c} \sum_{i \in \mathcal{I}} |N_i^{\mathcal{A}}| \varepsilon_i^2.$$

Therefore,

$$\dot{V} \leq -\sum_{i \in \mathcal{I}} (1 - c|N_i^{\mathcal{A}}|) q_i^2 + \sum_{i \in \mathcal{I}} \frac{|N_i^{\mathcal{A}}|}{c} \varepsilon_i^2.$$

Consequently, in order to enforce $\dot{V} < 0$, we suppose that $\alpha \in (0,1), \beta_{\min} = \min\limits_{i \in \mathcal{I}}\{1 - c|N_i^{\mathcal{A}}|\}, \gamma_{\max} = \max\limits_{i \in \mathcal{I}}\{|N_i^{\mathcal{A}}|/c\}$ and $c \in \bigcap\limits_{i \in \mathcal{I}} (0, 1/|N_i^{\mathcal{A}}|)$. Then if

$$\varepsilon_i^2(t) \leq \alpha \frac{\beta_{\min}}{\gamma_{\max}} q_i^2(t), \ \forall i \in \mathcal{I}, \tag{13}$$

we can obtain that

$$\begin{aligned}\dot{V} &\le -\sum_{i\in\mathcal{I}}(1-c|N_i^{\mathcal{A}}|)q_i^2+\sum_{i\in\mathcal{I}}\frac{|N_i^{\mathcal{A}}|}{c}\varepsilon_i^2\\ &\le -\sum_{i\in\mathcal{I}}\beta_{\min}q_i^2+\sum_{i\in\mathcal{I}}\frac{|N_i^{\mathcal{A}}|}{c}\alpha\frac{\beta_{\min}}{\gamma_{\max}}q_i^2\\ &\le -\sum_{i\in\mathcal{I}}\beta_{\min}q_i^2+\sum_{i\in\mathcal{I}}\gamma_{\max}\alpha\frac{\beta_{\min}}{\gamma_{\max}}q_i^2\\ &=(\alpha-1)\sum_{i\in\mathcal{I}}\beta_{\min}q_i^2\\ &<0.\end{aligned}$$

Moreover,

$$\begin{aligned}V&=\frac{1}{2}x^{\top}\mathcal{L}x\\ &=\frac{1}{2}\left[(x^1)^{\top},(x^2)^{\top}\right]\begin{bmatrix}\mathcal{L}_{11} & \mathcal{L}_{12}\\ \mathcal{L}_{21} & \mathcal{L}_{22}\end{bmatrix}\begin{bmatrix}x^1\\ x^2\end{bmatrix}\\ &=\frac{1}{2}\left((x^1)^{\top}\mathcal{L}_{11}x^1+(x^2)^{\top}\mathcal{L}_{22}x^2+2(x^1)^{\top}\mathcal{L}_{12}x^2\right).\end{aligned}\tag{14}$$

Owing to the fact that $\mathcal{L}$ has two zero eigenvalues and the rest are with positive real numbers, without loss of generality we denote the spectrum of $\mathcal{L}$ by $\lambda(\mathcal{L})=\{\lambda_1,\lambda_2,\ldots,\lambda_{N_1+N_2}\}$, where $\lambda_1=0,\lambda_2=0$. In addition, $\mathcal{G}$ is undirected. Thus, $\lambda_3,\lambda_4,\ldots,\lambda_{N_1+N_2}$ are all positive real numbers. Therefore, $\mathcal{L}$ can be diagonalized with a matrix $U\in\mathbb{R}^{(N_1+N_2)\times(N_1+N_2)}$, i.e.,

$$\mathcal{L}=U^{\top}\begin{bmatrix}\lambda_1 & 0 & \cdots & 0\\ 0 & \lambda_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \lambda_{N_1+N_2}\end{bmatrix}U=U^{\top}DU.$$

Thus, V can be rewritten as

$$V=\frac{1}{2}(Ux)^{\top}D(Ux)=\frac{1}{2}\tilde{x}^{\top}D\tilde{x}=\frac{1}{2}\sum_{i=1}^{N_1+N_2}\lambda_i\tilde{x}_i^2\ge 0.$$

Considering the form of (14),

$$(x^1)^{\top}\mathcal{L}_{11}x^1=\frac{1}{2}\sum_{(i,j)\in\mathcal{E}_1}\mathcal{A}_{ij}(x_i-x_j)^2\tag{15}$$

and

$$(x^2)^{\top}\mathcal{L}_{22}x^2=\frac{1}{2}\sum_{(i,j)\in\mathcal{E}_2}\mathcal{A}_{ij}(x_i-x_j)^2\tag{16}$$

are both in quadratic form. In addition, with (15), (16), (A3) and condition (i) in Lemma 1, $2(x^1)^{\top}\mathcal{L}_{12}x^2$ is in the form of $(x_{i_1}-x_{j_1})(x_{i_2}-x_{j_2})$, which can be transformed into the form of $[(x_{i_1}-x_{j_1})+(x_{i_2}-x_{j_2})]^2$, where $(i_1,j_1)\in\mathcal{E}_1$ and $(i_2,j_2)\in\mathcal{E}_2$. Therefore, with $\dot{V}(t)<0$, $V(t)$ is in the quadratic form subject to $\lim_{t\to\infty}x(t)\in\Upsilon$ defined in Section II. Furthermore, Υ is a globally attractive and invariant manifold. Therefore, group consensus can be asymptotically reached with distributed control protocol (8) and decentralized event-triggered mechanism (9). ■

TABLE I

ALGORITHM FOR DECENTRALIZED EVENT-TRIGGERED CONTROL

Step 1:	Given the initial conditions with $x(t_0)=x_0$, $t_0=0$ and the communication topology $\mathcal{G}$, where x_0 is the initial state of the multi-agent systems.
Step 2:	Choose the appropriate parameters $c,\alpha,\beta_{\min}$ and $\gamma_{\max}$ according to the given initial conditions.
Step 3:	**While** $\lvert\varepsilon_i(t)\rvert\ge\epsilon$ for any $i\in\mathcal{I}$ where $\epsilon\in\mathbb{R}$ is the given bounded error, **goto Step 4**. Else **goto Step 6**.
Step 4:	If $\varepsilon_i^2(t)<\alpha\frac{\beta_{\min}}{\gamma_{\max}}q_i^2(t)$, $\forall i\in\mathcal{I}$, then $u_i(t)=-\sum_{v_j\in\mathcal{N}_i}\mathcal{A}_{ij}(x_i(t_k^i)-x_j(t_{k(t)}^j))$ and **goto Step 3**. Else **goto Step 5**.
Step 5:	Suppose at $t=t_{k+1}^i$ for any $i\in\mathcal{I}$, $\varepsilon_i^2(t_{k+1}^i)=\alpha\frac{\beta_{\min}}{\gamma_{\max}}q_i^2(t_{k+1}^i)$. Then $t_k^i=t_{k+1}^i$ where $\varepsilon_i(t_{k+1}^i)=0$, and all the agents $j\in\mathcal{N}_i\cup i$ update their control protocols $u_j(t)$. **Goto Step 3**.
Step 6:	Terminate the algorithm.

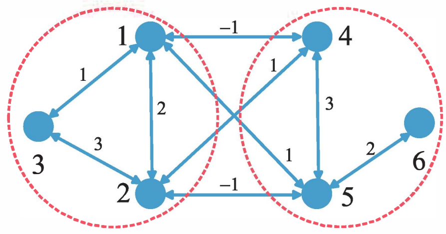

Fig. 1. Topology of six agents

IV. EXAMPLES AND PERFORMANCE ANALYSIS

Given the multi-agent systems with six agents and the communication topology in Fig. 1. Agents 1, 2 and 3 are in one group, while agents 4, 5 and 6 are in another group. Then

$$\mathcal{L}_1=\begin{bmatrix}3 & -2 & -1 & 1 & -1 & 0\\ -2 & 5 & -3 & -1 & 1 & 0\\ -1 & -3 & 4 & 0 & 0 & 0\\ 1 & -1 & 0 & 3 & -3 & 0\\ -1 & 1 & 0 & -3 & 5 & -2\\ 0 & 0 & 0 & 0 & -2 & 2\end{bmatrix}.$$

The eigenvalues of $\mathcal{L}_1$ are $\lambda_1=\lambda_2=0,\lambda_3=2.15,\lambda_4=3.92,\lambda_5=6.70$ and $\lambda_6=9.23$, which are all positive real numbers. $x_0=(-8,9,-10,6,-2,11)^{\top},c=0.1,\alpha=0.5,\beta_{\min}=0.5$ and $\gamma_{\max}=50$. In Fig. 3, it can be seen that the distributed event-triggered control requires fewer control updates than the periodic control in Fig. 2. Furthermore, each agent updates on its own triggering time in Fig. 3. In Fig. 4, $|\varepsilon_5(t)|_{\max}=\sqrt{\alpha\beta_{\min}/\gamma_{\max}}|q_5(t)|$, it illustrates that $|\varepsilon_5(t)|$ will not exceed the boundary of $|\varepsilon_5(t)|_{\max}$ in dot line.

V. CONCLUSIONS

This paper establishes a distributed event-triggered control protocol for group consensus. A distributed event-triggered function is developed to activate the control input. By calculating the maximum and minimum of the corresponding parameters, the event-triggered function can be simplified to reduce memory allocation for control system. In future work, we will focus on the situations with switching topology and time delays, which are more appropriate to the physical implementations.

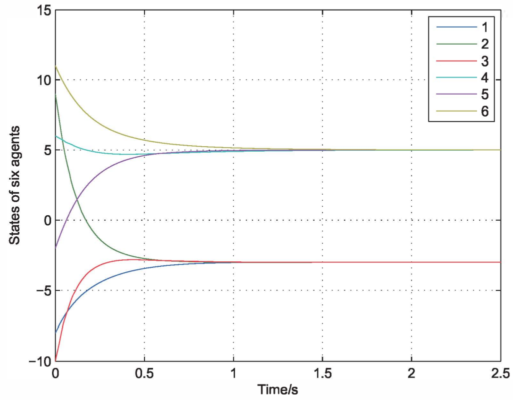

Fig. 2. Group consensus of six agents using periodic control

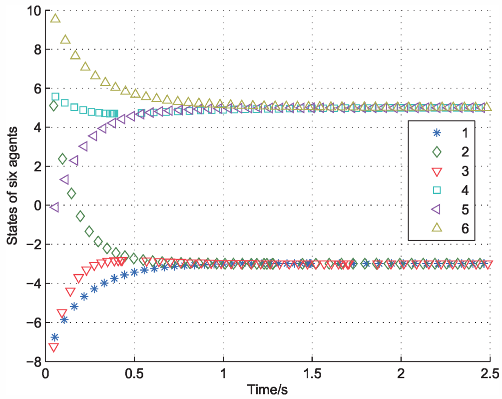

Fig. 3. Group consensus of six agents using distributed event-triggered control

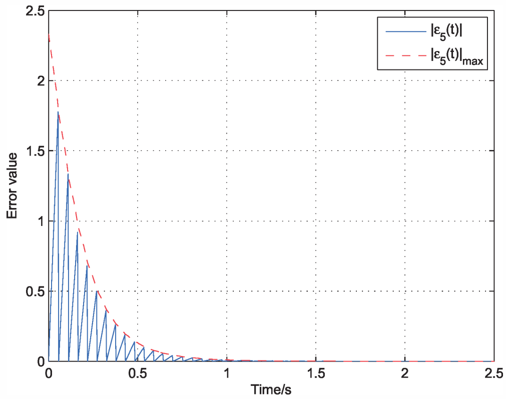

Fig. 4. Error trajectories of agent 5

REFERENCES

[1] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, "Novel type of phase transition in a system of self-driven particles," *Physical Review Letters*, vol. 75, no. 6, pp. 1226–1229, 1995.

[2] A. Jadbabaie, J. Lin, and A. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 988–1001, 2003.

[3] R. Olfati-Saber and R. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.

[4] W. Ren, R. Beard, and E. Atkins, "A survey of consensus problems in multi-agent coordination," in *Proceedings of the American Control Conference*, Portland, OR, USA, 2005, pp. 1859–1864.

[5] R. Olfati-Saber, J. Fax, and R. Murray, "Consensus and cooperation in networked multi-agent systems," in *Proceedings of the IEEE*, vol. 95, no. 1, 2007, pp. 215–233.

[6] W. Ren and R. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.

[7] J. Yu and L. Wang, "Group consensus in multi-agent systems with switching topologies and communication delays," *Systems & Control Letters*, vol. 59, no. 6, pp. 340–348, 2010.

[8] C. Tan, G.-P. Liu, and G.-R. Duan, "Group consensus of networked multi-agent systems with directed topology," in *Proceedings of the IFAC*, Milano, Italy, 2011, pp. 8878–8893.

[9] W. Heemels, K. Johansson, and P. Tabuada, "An introduction to event-triggered and self-triggered control," in *Proceedings of the Conference on Decision and Control*, Maui, Hawaii, USA, 2012, pp. 3270–3285.

[10] D. Dimarogonas, E. Frazzoli, and K. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 5, pp. 1291–1297, 2012.

[11] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1680–1685, 2007.

[12] M. Pipattanasomporn, H. Feroze, and S. Rahman, "Multi-agent systems in a distributed smart grid: Design and implementation," in *Proceedings of Power Systems Conference and Exposition*, Seattle, 2009, pp. 1–8.

[13] P. Davidsson and M. Boman, "A multi-agent system for controlling intelligent buildings," in *Proceedings of the Multi Agent Systems Conference*, Boston, MA, USA, 2000, pp. 377–378.

[14] W. Ren and R. Beard, "Trajectory tracking for unmanned air vehicles with velocity and heading rate constraints," *IEEE Transactions on Control Systems Technology*, vol. 12, no. 5, pp. 706–716, 2004.

[15] J. Yu and L. Wang, "Group consensus of multi-agent systems with directed information exchange," *International Journal of Systems Science*, vol. 43, no. 2, pp. 334–348, 2012.