

Consensus of Heterogeneous Multi-agent Systems With Switching Topologies Using Input-output Feedback Linearization

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Abstract: In this paper, the leader-following consensus problem of multi-agent systems with nonlinear heterogeneous dynamics and switching topologies is considered. All agent outputs are required to track a leader output which is assumed as a desired value. By input-output feedback linearization, the nonlinear heterogeneous system is decomposed into equivalent linear systems and internal dynamics. Then a distributed auxiliary controller for every agent using local information is designed and a sufficient consensus condition is given under switching topologies with every possible topology containing a directed spanning tree. Furthermore, it is proved that the consensus can be achieved with the same condition for multi-agent systems under switching topologies frequently containing a directed spanning tree. Simulation results verify the effectiveness of the control protocol.

Key Words: Multi-agent Systems, Switching Topologies, Nonlinear Heterogeneous Systems, Communication Failures, Input-output Feedback

1 Introduction

In recent years, the coordination control of multi-agent systems is a hot topic and has received spreading attention from various fields of science, including control engineering, mathematics, physics, robots and artificial intelligence [1-3]. As a basic and important problem, the distributed consensus problem of multi-agent systems is mainly categorized into leaderless consensus problem and leader-following consensus problem. Leaderless consensus means all agents synchronize to a common value that cannot generally be controlled. Leader-following consensus problem requires that all agents follow the desired trajectory of a leader whose motion is independent of the other agents.

In the distributed consensus problem, all agents can only share information with their neighbors locally. Many existing works on disturbed consensus of multi-agent systems with linear dynamics were studied in [4-6]. Distributed consensus of multi-agent systems with nonlinear dynamics was investigated in [7,8]. Wen et al. in [9] studied the leader-following consensus for a class of multi-agent systems with first-order Lipschitz-type nonlinear dynamics by a delayed-input approach. Considering the communicational delay, a consensus algorithm for the second-order nonlinear multi-agent systems with a leader was proposed in [10]. The tracking consensus problems were solved for first-, second- and high-order nonlinear multi-agent systems without requiring the knowledge of system dynamics in [11-13], respectively. To our knowledge, only a few literatures studied the consensus problem for heterogeneous multi-agent systems. A novel method was proposed in [14] to solve the output synchronization problem for heterogeneous networks of non-introspective agents. Iterative learning based method to solve the finite-time consensus tracking problem of het-

erogeneous linear multi-agent systems was proved useful in [15]. Input-output feedback linearization method was adopted to design the secondary voltage controller for microgrids in [16], then this method was extended to heterogeneous cooperative systems with fixed topology in [17].

As is well known that network connectivity of multi-agent systems is an important factor in achieving consensus. In the previous literatures, it is required that the communication topology is fixed. However, the communication connectivity between agents with their neighbors may be broken or changed due to the external disturbances or technological limitations in reality likely power systems, networking and wireless communication systems, etc. Therefore, the leader-following consensus problem for multi-agent systems with switching topologies is significant. Wen et al in [18] used a directed spanning tree instead of strongly connected topologies to solve the tracking consensus problem under switching topologies. In [19], robust consensus schemes were given for leaderless and leader-following multi-agent systems under fixed and switching topologies with a joint connectedness assumption. In the existing literatures on the consensus problem of multi-agent systems with switching topologies, it is commonly assumed that the communication topology always meet some connection conditions. However, as the multi-agent systems evolves with time, the communication between the agents with their neighbors may be intermittent due to sensor failures or actuator failures, i.e., switching topologies frequently contain a directed spanning tree [20,21]. In other words, all agents can not share information with their neighbors in case of communication failures.

In this paper, we consider the leader-following consensus problem for nonlinear heterogeneous multi-agent systems, where the dynamics systems are non-identical for different agents. Compared with [17], the cases of switching directed topologies and communication failures are considered. It is assumed that the disabled communication in case of communication failures can be repairable by a restoration mechanism. All agent outputs are required to track the leader out-

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put which is assumed as a reference value. By introducing an auxiliary control, the input-output feedback linearization method is adopted to transform the nonlinear heterogeneous dynamics to linear dynamics. Furthermore, under switching directed topologies with every possible topology containing a directed spanning tree, distributed control protocol is derived and a sufficient consensus condition related with the control gain and dwell time for switching is given based on Lyapunov technique and switching theory. Then it is proved that the consensus can be achieved under switching directed topologies and communication failures with the same condition. To the best of our knowledge, it is the first time to give the consensus analysis for heterogeneous multi-agent systems under switching topologies by using input-output feedback linearization.

The rest of this paper is organized as follows: Section 2 introduces some basic knowledge from graph theory and problem formulation. In Section 3, some lemmas and assumptions are stated, and based on the direct dependency of the agent outputs on the control inputs, main consensus analysis results are given. Simulation results are presented in Section 4. The paper ends with a conclusion in Section 5.

2 Preliminaries

2.1 Basic Graph Theory

A digraph described the information exchange between agents is usually expressed as $G = (V, E)$, where $V = \{1, 2, \dots, N\}$ is a nonempty set of nodes representing N agents and $E \subseteq v \times v$ is the set of edges. A weighted matrix $A = [a_{ij}] \in R^{N \times N}$ ($i \neq j$) represents the topology of a graph. A directed edge of G is denoted by (v_j, v_i) , representing that node i receives the information from node j . In this case, node i is called as a neighbor of node j . If $(v_j, v_i) \in E$, $a_{ij} > 0$, otherwise $a_{ij} = 0$. The neighbors of the i th node is denoted by the set $N_i = \{j \mid (v_j, v_i) \in E\}$. The in-degree of node i is defined as $d_i = \sum_{j=1}^N a_{ij}$. The Laplacian matrix of graph G is defined as $L = D - A$ with $D = \text{diag}(d_1, d_2, \dots, d_N)$. A directed graph is called strongly connected, if there is a directed path from any node i to node j . A spanning tree is a directed graph (digraph), which has a root such that there is a directed edge from the root to every other node in the graph.

Note that the interaction topology is dynamically changing. Let $\hat{g} = \{g^1, \dots, g^s\}$, $s \geq 1$ denotes the set of all possible directed interaction graphs. $g^{\sigma(t)} \in \hat{g}$ denotes the interaction graph at time t , and $\sigma(t) : [0 : \infty] \rightarrow \{1, \dots, s\}$ is a switching signal.

2.2 Problem Formulation

Consider nonlinear and heterogeneous systems with N ($N \geq 2$) agents. The dynamics of the i th agent is described as

$$\begin{cases} \dot{x}_i = f_i(x_i) + g_i(x_i)u_i \\ y_i = h_i(x_i) + d_i u_i \end{cases} \quad (1)$$

where $x_i \in R^{n_i}$ is the state vector of the agent i , $u_i \in R$ is the control input, $y_i \in R$ is the output of i th agent, and the smooth functions f_i, g_i and h_i are nonlinear dynamics of the i th agent. It is assumed that $f_i(\cdot) : R^{n_i} \rightarrow R^{n_i}$ is locally Lipschitz in R^{n_i} with $f_i(0) = 0$. It is assumed that the directed feed term is a nonzero constant number, i.e., $d_i \neq 0$.

The agent state dynamics f_i, g_i and h_i and state dimensions n_i may be distinct.

For the consensus tracking problem, the goal is to design distributed control input u_i to ensure that the output y_i of all agents to track the leader output. The leader, generated the desired trajectory, has nonlinear dynamics as

$$\begin{cases} \dot{x}_0 = f_0(x_0) \\ y_0 = h_0(x_0) \end{cases} \quad (2)$$

where f_0, g_0 are assumed to be smooth nonlinear functions of class C^∞ , and y_0 is the output of the leader. It is assumed that all of the agent states are measurable. In this paper, the tracking consensus problem of the MISO systems (1) is considered. The conclusion can be extended to MIMO systems.

Definition 1. The multi-agent systems (1) is said to achieve consensus with reference dynamics (2), if their outputs reach agreement, i.e.,

$$\lim_{t \rightarrow \infty} (y_i(t) - y_0(t)) = 0, \quad i = 1, \dots, N \quad (3)$$

3 Feedback Linearization and Consensus Analysis

Note that the control input u_i is differentiable. Differentiate the output of each agent in (1) to obtain

$$\dot{y}_i(t) = \frac{\partial h_i}{\partial x_i} f_i(x_i) + \frac{\partial h_i}{\partial x_i} g_i(x_i) u_i + d_i \dot{u}_i \quad (4)$$

Define the auxiliary control

$$\omega_i = \frac{\partial h_i}{\partial x_i} f_i(x_i) + \frac{\partial h_i}{\partial x_i} g_i(x_i) u_i + d_i \dot{u}_i \quad (5)$$

It is clear that the system (1) is input-output linearizable, since the state feedback control

$$d_i \dot{u}_i = \omega_i - \frac{\partial h_i}{\partial x_i} f_i(x_i) - \frac{\partial h_i}{\partial x_i} g_i(x_i) u_i \quad (6)$$

reduces the input-output map to

$$\dot{y}_i(t) = \omega_i \quad (7)$$

the control input u_i can be computed directly by (6). By the input-output feedback linearization, the dynamics of each agent is decomposed into the first order dynamical system in (7) and a set of internal dynamics

$$\dot{\mu}_i = W_i(y_i, \mu_i), \quad \forall i \quad (8)$$

where $\mu_i \in R^{n_i}$ represents the internal dynamics.

Remark 1. The nonlinear nature of the dynamics in system (1) can be overcome by the input-output feedback linearization. By choosing appropriate auxiliary control ω_i , a distributed controller u_i derived by (6) can guarantee the tracking consensus.

3.1 Consensus Analysis With Every Possible Topology Containing A Directed Spanning Tree

Note that the tracking synchronization problem of system (1) with switching topologies is considered in this paper. In this subsection, consensus tracking is considered for multi-agent systems (1) with each possible topology containing a directed spanning tree. The set of all switching directed

graphs is $\hat{g} = \{g^1, \dots, g^s\}$. It is assumed that there exists an infinite sequence of uniformly bounded non-overlapping time interval $[t_k, t_{k+1})$, $k \in \mathbb{N}$. Let $t_1 = 0$, $t_{k+1} - t_k < T$ and $T > 0$. The positive constant T is called the dwell time. The graph keeps unchanged in the interval $t \in [t_k, t_{k+1})$ and switches at a series of fixed time instants $\{t_1, t_2, \dots\}$.

It is assumed that each agent communicate with others through switching directed graphs $\hat{g} = \{g^1, \dots, g^s\}$. The neighborhood tracking error of extended outputs can be denoted as

$$e_{yi} = \sum_{j \in N_i} a_{ij}^{\sigma(t)} (y_i - y_j) + b_i^{\sigma(t)} (y_i - y_0) \quad (9)$$

The agents for which $b_i^{\sigma(t)} \neq 0$ are called controlled agents.

The neighborhood tracking error vector for the graph \hat{g} is written as

$$e_y = (L^{\sigma(t)} + B^{\sigma(t)}) (y - \underline{y}_0) \equiv H^{\sigma(t)} \delta_y \quad (10)$$

where $y = [y_1, y_2, \dots, y_N]^T \in \mathbb{R}^N$, $H^{\sigma(t)} = L^{\sigma(t)} + B^{\sigma(t)}$, $\delta_y = y - \underline{y}_0$, $e_y = [e_{y1}, e_{y2}, \dots, e_{yN}]^T \in \mathbb{R}^N$ and $\underline{y}_0 = 1_N \otimes y_0 \in \mathbb{R}^N$ with 1_N the vector of ones with the length of N .

Assumption 1. The graph $g^{\sigma(t)} \in \hat{g}$, $\forall \sigma(t) \in \{1, 2, \dots, s\}$ has a spanning tree and $b_i^{\sigma(t)} \neq 0$ for at least one root node.

Assumption 2. The vector $\underline{y}_0 = 0$.

Remark 2. The Assumption 1 is very mild comparing with the strongly connected diagraphs. A strongly connected directed graph must have a spanning tree, but not vice versa. For Assumption 2, the output of the leader is expected to be a desired value for some multi-agent systems such that the output voltage and frequency of the leader are constant references in [8]. Comparing with the uncontrolled common value in leaderless consensus problem, the introduction of a leader can guarantee all agents outputs converge to a desired common value.

Lemma 1. Suppose that Assumption 1 holds. Then

$$\|\delta_y\| \leq \|e_y\| / \underline{\sigma}(H^{\sigma(t)}) \quad (11)$$

where $\underline{\sigma}(H^{\sigma(t)})$ is the minimum singular value of matrix $H^{\sigma(t)}$, $\forall \sigma(t) \in \{1, 2, \dots, s\}$.

Proof. Since the diagraph $g^{\sigma(t)}$ have a spanning tree and $b_i^{\sigma(t)} \neq 0$ for at least one node, $H^{\sigma(t)}$ is nonsingular. Combining (10), we have $\delta_y = (H^{\sigma(t)})^{-1} e_y$, therefore $\|\delta_y\| = \|(H^{\sigma(t)})^{-1} e_y\| \leq \|e_y\| / \underline{\sigma}(H^{\sigma(t)})$.

Remark 3. Note that the vector δ_y cannot be computed locally at each node. In this subsection, the neighborhood tracking error is used to solve the tracking problem. Lemma 1 implies that if $e_y(t)$ is asymptotically stable, the vector δ_y is also asymptotically stable, i.e., consensus problem is sloved.

Lemma 2. [22] Suppose that Assumption 1 holds, then there exists a positive vector $\xi^{\sigma(t)} = (\xi_1^{\sigma(t)}, \dots, \xi_N^{\sigma(t)})^T \in \mathbb{R}^N$, such that $H^{\sigma(t)} \xi^{\sigma(t)} = \mathbf{1}_n$, and $\Xi^{\sigma(t)} H^{\sigma(t)} + (H^{\sigma(t)})^T \Xi^{\sigma(t)} > 0$, where $\Xi^{\sigma(t)} = \text{diag}\{1/\xi_1^{\sigma(t)}, \dots, 1/\xi_N^{\sigma(t)}\}$, for all $t \geq 0$.

Theorem 1. Suppose Assumptions 1 and 2 are satisfied. Assume that the zero dynamics of each node $\dot{\mu}_i =$

$W_i(0, \mu_i)$, $\forall i$, is asymptotically stable and $d_i \neq 0$ in system (1). Let the auxiliary control ω_i in (5) be chosen as

$$\omega_i = -c e_{yi} \quad (12)$$

if the positive constants c and T satisfy the following inequality

$$-\frac{c}{2} \xi_0 \gamma T + \ln r_0 < 0 \quad (13)$$

where c is the control gain and T is the dwell time, $\underline{\sigma}(\cdot)$ and $\bar{\sigma}(\cdot)$ are the minimum and maximum singular value of a matrix, respectively. $Q^{\sigma(t)} = \Xi^{\sigma(t)} H^{\sigma(t)} + (H^{\sigma(t)})^T \Xi^{\sigma(t)}$, $\gamma = \min\{\underline{\sigma}(Q^{\sigma(t)}) | \sigma(t) \in \{1, \dots, s\}\}$, $r_0 = \xi_1 / \xi_0$, $\xi_0 = \min_{i,j} \xi_j^i$, $\xi_1 = \max_{i,j} \xi_j^i$, $i \in \{1, \dots, s\}$, $j = 1, 2, \dots, N$. Then the outputs $y_i(t)$ of system (1) are synchronous to the leader output $y_0(t)$.

Proof. From (12), the global input vector ω is

$$\omega = [\omega_1, \omega_2, \dots, \omega_N]^T = -c e_y \quad (14)$$

Consider the following Lyapunov function candidate:

$$V(t) = \frac{1}{2} e_y^T(t) \Xi^{\sigma(t)} e_y(t) \quad (15)$$

where $\Xi^{\sigma(t)}$ is a positive definite matrix and $\Xi^{\sigma(t)} = \text{diag}\{1/\xi_i^{\sigma(t)}\}$.

The time derivative of the Lyapunov function (15) is

$$\dot{V}(t) = e_y^T(t) \Xi^{\sigma(t)} H^{\sigma(t)} (\omega - \dot{\underline{y}}_0) \quad (16)$$

Based on (14) and Assumption 2, we have

$$\dot{V}(t) = -\frac{c}{2} e_y^T(t) \left(\Xi^{\sigma(t)} H^{\sigma(t)} + (H^{\sigma(t)})^T \Xi^{\sigma(t)} \right) e_y(t) \quad (17)$$

According to [23], $Q^{\sigma(t)} = \Xi^{\sigma(t)} H^{\sigma(t)} + (H^{\sigma(t)})^T \Xi^{\sigma(t)}$ is positive definite. Then

$$\begin{aligned} \dot{V}(t) &= -\frac{c}{2} e_y^T(t) Q^{\sigma(t)} e_y(t) \\ &\leq -\frac{c}{2} \sigma(Q^{\sigma(t)}) \|e_y\|^2 \leq -\frac{c}{2} \gamma \|e_y\|^2 \end{aligned} \quad (18)$$

where $\gamma = \min\{\underline{\sigma}(Q^{\sigma(t)}) | \sigma(t) \in \{1, \dots, s\}\} > 0$ and $\underline{\sigma}(Q^{\sigma(t)})$ is the minimum singular value of matrix $Q^{\sigma(t)}$. $\underline{\sigma}(Q^{\sigma(t)}) > 0$ because $Q^{\sigma(t)}$ is positive definite.

From (15), we can obtain

$$V(t) \leq \bar{\sigma}(\Xi^{\sigma(t)}) \|e_y\| \quad (19)$$

Inserting (18) to (19), we can get

$$\dot{V}(t) \leq -\frac{c}{2} \frac{\gamma}{\bar{\sigma}(\Xi^{\sigma(t)})} V(t) = -\frac{c}{2} \xi_0 \gamma V(t) \quad (20)$$

where $\xi_0 = \min_{i,j} \xi_j^i$, $j = 1, 2, \dots, N$, $i \in \{1, \dots, s\}$. Therefore, the Lyapunov function is exponential decreasing during each time interval $t \in [t_k, t_{k+1})$.

Note that the topology of the multi-agent system switches at $t = t_{k+1}$. It follows from (20) for $k = 1$ that

$$V(t_2^-) < V(t_1) e^{-\frac{c}{2} \xi_0 \gamma (t_2 - t_1)} < e^{-\frac{c}{2} \xi_0 \gamma T} V(t_0) \quad (21)$$

Combining (15), we obtain

$$V(t_2) \leq r_0 V(t_2^-) < r_0 e^{-\frac{\xi}{2} \xi_0 \gamma T} V(0) = e^{(-\frac{\xi}{2} \xi_0 \gamma T + \ln r_0)} V(0) \quad (22)$$

One can get the following inequality by recursion for $t = t_k, k \geq 2$ that

$$V(t_k) < e^{(k-1)(-\frac{\xi}{2} \xi_0 \gamma T + \ln r_0)} V(0) \quad (23)$$

Therefore, at arbitrary switching instant $t = t_k, \forall k \in \mathbb{N}$, the Lyapunov function is decreasing.

Then for $t \in (t_k, t_{k+1}), \forall k \in \mathbb{N}$, the following inequality holds.

$$\begin{aligned} V(t) &< e^{-\frac{\xi}{2} \xi_0 \gamma (t-t_k)} V(t_k) \\ &< e^{-\frac{\xi}{2} \xi_0 \gamma (t-t_k) + (k-1)(-\frac{\xi}{2} \xi_0 \gamma T + \ln r_0)} V(0) \\ &< e^{(k-1)(-\frac{\xi}{2} \xi_0 \gamma T + \ln r_0)} V(0) \\ &< e^{(\frac{k-1}{k+1})(-\frac{\xi}{2} \xi_0 \gamma T + \ln r_0)} V(0) \end{aligned} \quad (24)$$

From (23) and (24), we have that $V(t)$ will decay to zero as $t \rightarrow \infty$. Then the neighborhood tracking error will also goes to zero. By Lemma 1, the global disagreement vector will converge to zero as $t \rightarrow \infty$. Therefore the synchronization between the outputs $y_i(t)$ of system (1) and the leader output $y_0(t)$ can be achieved. The proof is thus completed.

Remark 4. In condition (13), the parameters ξ_0, γ, r_0 is determined by the communication topologies. Theorem 1 implies that the control gain c is related with the switching topologies and dwell time. The synchronization speed and the dwell time can be adjusted by c .

3.2 Consensus Analysis With Switching Topologies and Communication Failures

In this subsection, the case of communication failures is considered, which means that the set of all possible directed graphs contains a completely disconnected graph. To possibly achieve the desired consensus, it is assumed that the disabled communication graph could be recovered for a short period of time. The set of all switching directed graphs is $\hat{g} = \{g^0, g^1, \dots, g^s\}$, g^0 denotes the completely disconnected graph. Suppose that there exists an infinite sequence of uniformly bounded non-overlapping time intervals $[t_k, t_{k+1})$ with $t_1 = 0, k \in \mathbb{N}$ and communication graph switches in every bounded time interval with finite switching times h_k . Let $t_k = t_k^1 < t_k^2 < \dots < t_k^{h_k-1} < t_k^{h_k} = t_{k+1}$, the dwell time $T < t_k^i - t_k^{i-1}, i \in \{1, \dots, h_k - 1\}$. Assume that the communication graph $g^{\sigma(t)}$ contain a spanning tree for $\sigma(t) \in \{1, 2, \dots, s\}$ in the interval $t \in [t_k, t_k^{h_k-1})$ and the graph g^0 is completely disconnected in the interval $t \in [t_k^{h_k-1}, t_{k+1})$ as shown in Fig. 1.

As a result of the communication failures, the agents in system (1) can only exchange information with their neighbors for $t \in [t_k, t_k^{h_k-1})$. In this case, the neighborhood tracking error of extended outputs can be specified as

$$e_y = \begin{cases} (L^{\sigma(t)} + B^{\sigma(t)})(y - y_0) \equiv H^{\sigma(t)} \delta_y, & t \in [t_k, t_k^{h_k-1}) \\ 0, & t \in [t_k^{h_k-1}, t_{k+1}), k \in \mathbb{N}, \end{cases} \quad (25)$$

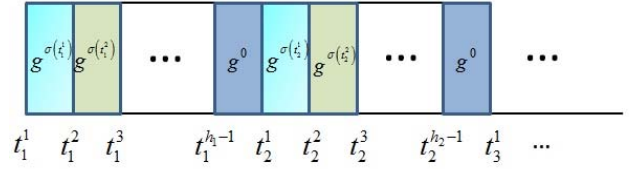


Fig. 1: The switching diagrams with communication failures

Theorem 2. Suppose that the communication graph $g^{\sigma(t)}$ contain a spanning tree for $\sigma(t) \in \{1, 2, \dots, s-1\}$ and g^0 is completely disconnected. Assume that the zero dynamics of each node $\dot{\mu}_i = W_i(0, \mu_i), \forall i$, are asymptotically stable and $d_i \neq 0$ in system (1), if the conditions (12) and (13) in Theorem 1 are still satisfied, then consensus of multi-agent systems(1) can be achieved with the control input u_i in (6).

proof. Consider the following Lyapunov function candidate:

$$V(t) = \begin{cases} \frac{1}{2} \delta_y^T(t) \Xi^{\hat{\sigma}(t)} \delta_y(t), & t \in [t_k, t_k^{h_k-1}) \\ \frac{1}{2} \delta_y^T(t) \delta_y(t), & t \in [t_k^{h_k-1}, t_{k+1}) \end{cases} \quad (26)$$

where $\delta_y = y - y_0, \Xi^{\hat{\sigma}(t)} = \text{diag}\{1/\xi_1^{\hat{\sigma}(t)}, \dots, 1/\xi_N^{\hat{\sigma}(t)}\}, k \in \mathbb{N}$.

For $t \in [t_k, t_k^{h_k-1})$, the derivative of the Lyapunov function (26) is

$$\begin{aligned} \dot{V}(t) &= \delta_y^T(t) \Xi^{\hat{\sigma}(t)} \dot{\delta}_y(t) \\ &= \delta_y^T(t) \Xi^{\hat{\sigma}(t)} (\omega - \dot{y}_0) \\ &= -c \delta_y^T(t) \Xi^{\hat{\sigma}(t)} e_y \\ &= -\frac{c}{2} \delta_y^T(t) \left(\Xi^{\hat{\sigma}(t)} H^{\hat{\sigma}(t)} + (H^{\hat{\sigma}(t)})^T \Xi^{\hat{\sigma}(t)} \right) \delta_y(t) \end{aligned} \quad (27)$$

Obviously, it is similar with (17). If the condition (13) is satisfied, the Lyapunov function is decreasing for $t \in [t_1, t_1^{h_1-1})$ based on Theorem 1. Then one can get the following inequality by recursion

$$V\left(\left(t_1^{h_1-1}\right)^-\right) < e^{(h_1-2)(-\frac{\xi}{2} \xi_0 \gamma T) + (h_1-3)(\ln r_0)} V(0) \quad (28)$$

Combining (26), one has

$$\begin{aligned} V\left(t_1^{h_1-1}\right) &\leq \xi_1 V\left(\left(t_1^{h_1-1}\right)^-\right) \\ &< \xi_1 e^{(h_1-2)(-\frac{\xi}{2} \xi_0 \gamma T) + (h_1-3)(\ln r_0)} V(0) \end{aligned} \quad (29)$$

For $t \in [t_k^{h_k-1}, t_{k+1})$, we have $\dot{y} = \omega = -ce_y = 0$. Then the control inputs u_i computed by (6) will make the outputs y_i be constant. Then one has

$$V(t) = V\left(t_1^{h_1-1}\right), t \in [t_1^{h_1-1}, t_2) \quad (30)$$

Combining (26), one gets that

$$V(t_2) \leq \frac{1}{\xi_0} V\left(t_1^{h_1-1}\right) < e^{(h_1-2)(-\frac{\xi}{2} \xi_0 \gamma T + \ln r_0)} V(0) \quad (31)$$

From Theorem 1, we can get that the Lyapunov function (26) is decreasing for $t \in [t_k, t_k^{h_k-1})$ by recursion. According to (30), the Lyapunov function (26) remain the same for finite time $t \in [t_k^{h_k-1}, t_{k+1})$. Then we have that $V(t)$ will decay to zero as $t \rightarrow \infty$. Therefore, the multi-agent systems (1) is said to achieve consensus with reference dynamics (2). The proof is completed.

Remark 5. Comparing Theorem 1 with Theorem 2, we can obtain that the introduction of communication failures does not affect the sufficient condition (13), which is the different from the works in [20,21]. The reason is that the control inputs u_i derived by (6) can use agents own dynamics information to keep their outputs constant under communication failures. Once the disabled communication graph is recovered, the agent outputs will continue to track the output of the leader. Therefore, the control protocol is robust.

Remark 6. In Theorem 1 and 2, the local asymptotically stability of zero dynamics in (8) is required. Note that the local asymptotically stability of internal dynamics is proved by the local asymptotically stability of zero dynamics $\dot{\mu}_i = W_i(0, \mu_i)$, $\forall i$, in [24].

4 Simulation

In this section, a simulation example is given to verify the validity of the results.

Each agent is assumed to be a single-link manipulator actuated by a field-controlled DC motor [25] with a output which can be modeled as

$$\begin{aligned}\dot{x}_1 &= -ax_1 + u \\ \dot{x}_2 &= -bx_2 + k - dx_1x_3 \\ \dot{x}_3 &= \theta x_1x_2 \\ y &= x_3 + u\end{aligned}\quad (32)$$

where x_1, x_2 and x_3 are the field current, armature current and angular velocity, respectively, and a, b, d, k and θ are positive constants. For angular velocity control, the output is chosen as $y = x_3 + u$. The leader output is known as the reference angular velocity $y_0 = 5$. We choose the model parameters $a = 1, b = 2, d = 1.8, k = 3.5, \theta = 0.2$ for agent 1 and 2, and $a = 0.5, b = 1.3, d = 2.8, k = 4.5, \theta = 1.2$ for agent 3 and 4. The initial values of agents are $x_1 = [2, 3, 5], x_2 = [6, 8, 2], x_3 = [4, 10, 8], x_4 = [1, 3, 10]$. According to [25], the zero dynamics of system (32) are asymptotically stable.

4.1 Case 1

In this case, every possible topology contains a directed spanning tree. According to [19], we take multi-agent systems consisting of five nodes with directed switching topologies g^1, g^2 as shown in Fig. 2. It can be observed that g^1, g^2 both contain a directed spanning tree with the leader as root node. In the digraphs, only agent 1 and 4 are connected to the leader.

The associated adjacency matrices of the digraphs Fig. 2 are

$$A^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 3 & 0.5 & 0 & 2 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.5 \\ 2.5 & 0 & 0 & 1.5 & 0 \end{bmatrix}$$

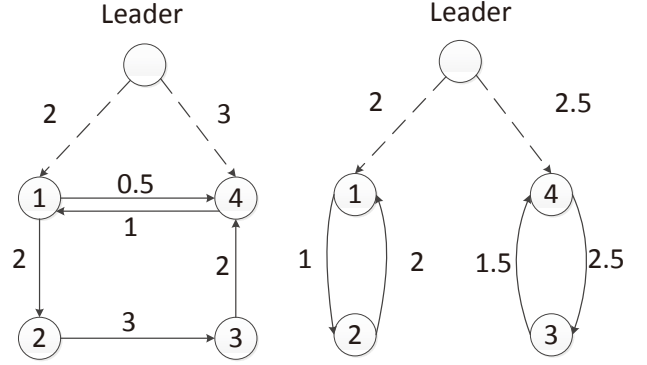


Fig. 2: Topologies of communication graph g^1, g^2

Based on (13), the dwell time T should be larger than 0.38s with $c = 4.5, r_0 = 2.45, \xi_0 = 0.58$ and $\xi_1 = 1.42$. Suppose that the communication topologies switch between g^1, g^2 in every 0.4s. The output trajectories of the uncontrolled multi-agent systems (1) are shown in Fig. 3. The output trajectories of system (1) under auxiliary control (12) are shown in Fig. 4. Simulation results show that tracking consensus is indeed achieved under control input derived by (6).

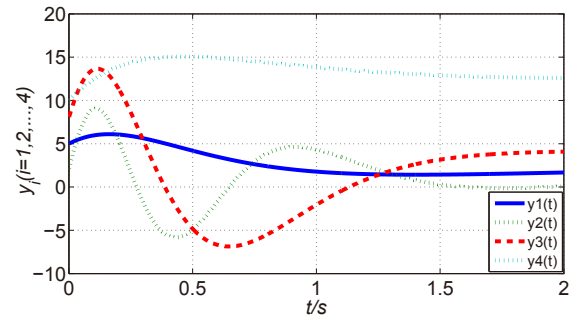


Fig. 3: Output trajectories $y_i(t)$ of uncontrolled systems

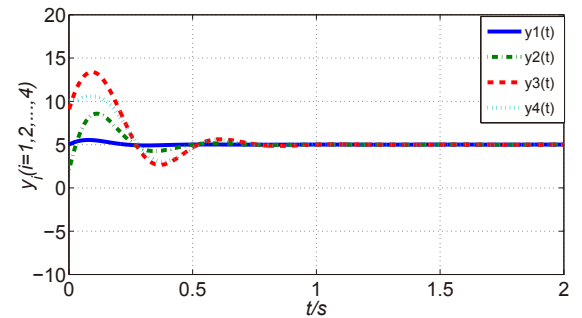


Fig. 4: Output trajectories $y_i(t)$ of controlled systems in Case 1

4.2 Case 2

In this case, we use a completely disconnected graph g^0 to denote the disabled communication graph. The associated adjacency matrix of the digraph g^0 is $A^0 = 0$. The other switching topologies is the same with Case 1. Suppose that the communication topologies switch sequently between g^1, g^0, g^2 in every 0.4s with $c = 4.5$ which is satisfied the condition (13). The output trajectories of the controlled multi-agent systems under switching topologies g^1, g^2 and disabled communication graph g^0 are shown in Fig. 5. Although agents can not share information with their neighbors locally in the case of communication failures for $t = [0.4, 0.8)$, the control input derived by (6) can be adjusted by their own dynamics information to keep their outputs constant. Once the communication graph is recovered for $t = [0.8, 1.2)$ with topologies g^2 , the agent outputs continue to track the leader output $y_0 = 5$ using their own and neighbors' information. The result shows that tracking consensus is indeed achieved.

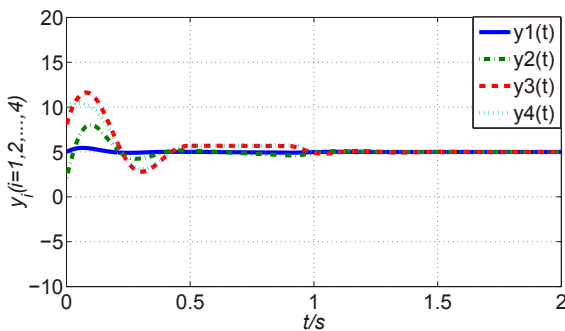


Fig. 5: Output trajectories $y_i(t)$ of controlled systems in Case 2

5 Conclusion

In brief, we have investigated the leader-following consensus problem of nonlinear heterogeneous multi-agent systems with switching topologies. The output of the leader is assumed as a desired value. By input-output feedback linearization, identical linear dynamics and non-identical internal dynamics are available. Under the switching topologies, disturbed control protocol for every agent is designed and a sufficient condition is derived. Furthermore, we prove that the consensus problem can be solved for multi-agent systems with switching topologies and repairable communication failures under the sufficient condition. Some numerical simulations are provided to demonstrate the effectiveness of our proposed consensus algorithm.

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