Interval Type-2 TSK Nominal-fuzzy-model-based Sliding Mode Controller Design for Flexible Air-breathing Hypersonic Vehicles

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***Abstract*—This paper presents a novel interval type-2 TSK nominal-fuzzy-model-based sliding mode controller (IT2-TSK-NFMSMC) for flexible air-breathing hypersonic vehicle (FAHV) in order to stress robustness of the control system in dealing with data-driven based fuzzy modelling deviations, system uncertainty and disturbances. We adopt backstepping structure decomposing FAHV model into 5 control subsystems and design controllers, respectively. More specifically, two subsystems are designed with integral sliding mode model controllers. Another three subsystems which directly coupling with flexible mode disturbances are designed with IT2-TSK-NFMSMCs by the following steps: 1) interval type-2 TSK nominal-fuzzy-models (IT2-TSK-NFM) are generated automatically by using type-2 fuzzy self-organizing methods from experiment datasets; 2) nominal model sliding mode controllers are designed based on the IT2-TSK-NFM, respectively; 3) notch filters are adopted in order to decrease the disturbance effects from the flexible modes; 4) sliding mode compensation controllers are designed through Lyapunov synthesis in order to compensate differences between IT2-TSK-NFM and real models of the FAHV. Several scenarios are studied and the simulation results validate the robustness of the proposed controllers when there exist internal flexible vibration and external system disturbances.**

Keywords—type-2 fuzzy logic system; hypersonic vehicle; nominal model; model-based control; sliding mode; data-driven

I. Introduction

Hypersonic Vehicle has been widely studied since 20 century 60s. Besides generic hypersonic flight vehicle (GHFV), a new branch, which is called air-breathing hypersonic vehicle (AHV), draws dramatic attentions from global researchers for the features of reusability in near space, hyper speed to go any destinations and unnecessary to carry oxidants which can be revolutionary significances in both civilian and military applications [1]. However, the development of hypersonic vehicles are never simple tasks, it relates to areas of hypersonic aerodynamics, air-breathing scramjet engines, high-tech materials, control systems, etc. Though some achievements continue being completed, such as the successful tests of X-43A, X-51A and HiFIRE programs [2, 3]. There still is not any practical applications so far. In addition, due to the fact that the natural frequencies of AHV are lower than normal flight vehicles, the noticeable elastic vibration, system internal and external disturbances and uncertainties will cause devastating impacts on the vehicles operation [4]. Therefore, the research in dealing with aforementioned issues to develop robust and stable controllers for flexible air-breathing hypersonic vehicle (FAHV) model has draw more attentions currently[5].

To date, control system designs in literatures for the simplified FAHV longitudinal model are mainly based on dynamic inversion control and backstepping control schemes, which include PID, sliding mode, neural network, fuzzy logic controllers and so on [6]. However， due to the existing characteristics of strong coupling effects, high nonlinearity and fast time-varying in FAHV real model and vibration from the flexible modes cannot be accurately accessed from sensors in practice, the aforementioned longitudinal model does not include all dynamics at all. Rigorously speaking, the current FAHV longitudinal model can only be treated as a type of nominal model. Nominal-model-based control can ensure the stability and effectiveness of the control system on the basis of robust model-based controller deign for the nominal model by connecting a compensating controller [7]. FLS has been applied to model-based control in many scenarios in both type-1 and type-2 fuzzy-model-based control. Most cases of fuzzy model-based control only expand nonlinear system models into multiple local linearization models which use FLS to make overall integration [8-12]. However, because type-1 and type-2 fuzzy logic systems (T1 & T2 FLS) are universal approximators, they are applicable in data-driven modelling by self-organize approaches [13]. The TSK type FLS, which has linear mathematical expressions consisted with input variables in antecedent part, has better understandable expression for the data-driven model. Moreover, T2 FLS has better ability in describing and handling uncertainties than T1 FLS, which the foot print of uncertainty (FOU) in T2 fuzzy set does. Therefore, it is a feasible attempt that using data-driven methods to construct system models in controller design.

The interval type-2 TSK fuzzy model (IT2-TSK-FM) is usually generated by two ways, e.g., 1) derived from nonlinear models into multiple local linearization models which the fuzzy model only presents as a case operator; 2) based on data-

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driven approach to self-construct the antecedent and consequent parts in the IT2-TSK-FM.

In this paper, we adopt our previous data-driven IT2-TSK approach [14], which the type-1 fuzzy TSK system chosen in this case is ANFIS, to generate the FAHV open loop model as nominal model offline and design an interval type-2 TSK nominal-fuzzy-model-based sliding mode controller (IT2-TSK-NFMSMC) instead of using the current FAHV mathematic models to explore the ability of the method to construct the control system of the FAHV with complex nonlinear model and the ability of resisting uncertainties.

The rest of this paper is organized as below: Section 2 gives the model descriptions of the FAHV and a brief introduction of the IT2-TSK rule structure; Section 3 provides the detail procedures and key components in designing the IT2-TSK-NFMC based on backstepping structure. This section continues by further describing the data-driven IT2-TSK fuzzy modelling which is adopted in this study; Section 4 gives stability analysis of the IT2-TSK-NFMC in each subsystems; Section 5 gives simulations and comparisons in several scenarios; Section 6 draws conclusions.

II. Preliminaries

A. Flexible Air-breathing Hypersonic Vehicle Model

This paper studies the simplified longitudinal dynamics of flexible air-breathing hypersonic vehicle (FAHV) models [4], which the flexible modes are reflected inside of forces and moments. Moreover, this model will be used as the open-loop subsystem data generator.

The nonlinear longitudinal dynamic motion equations of the FAHV are given as:

$$\dot{V} = \left(T\cos\alpha - D\right)/m - g\sin\gamma \tag{1}$$

$$\dot{\gamma} = \left(L + T\sin\alpha\right)/(mV) - g\cos\gamma / V \tag{2}$$

$$\dot{h} = V\sin\gamma \tag{3}$$

$$\dot{\alpha} = q - \dot{\gamma} \tag{4}$$

$$\dot{q} = M_{yy} / I_{yy} \tag{5}$$

$$\ddot{\eta}_i = -2\xi_i\omega_i\dot{\eta}_i - \omega_i^2\eta_i + N_i, \qquad i = 1,2,3 \tag{6}$$

Eleven flight states are included in this model, where $[V,\gamma,h,\alpha,q]$ represents rigid-body states: the vehicle velocity, flight path angle (FPA), altitude, angle of attack (AA) and pitch rate (PR) separately, and $\boldsymbol{\eta} = [\eta_i, \dot{\eta}_i]_{i=1,2,3}$ stands for the first three flexible modes. The resonance frequencies of the flexible modes are set as $\omega_1 = 21.17 rad/s$, $\omega_2 = 53.92 rad/s$ and $\omega_3 = 109.1 rad/s$ with the damping ratio constant ξ_i =0.02 based on experimental data. The canard deflection δ_c and elevator deflection δ_e are presented through the canard deflection gain k_{ec} as: $\delta_c = k_{ec}\delta_e$, where $k_{ec} = -C_L^{\delta_e} / C_L^{\delta_c}$. Coefficients of the thrust *T*, drag *D*, lift *L*, pitching moment *M* and generalized forces N_i are given below:

$$\begin{cases} C_{T,\phi}(\alpha) = C_T^{\phi\alpha^3}\alpha^3 + C_T^{\phi\alpha^2}\alpha^2 + C_T^{\phi\alpha}\alpha + C_T^{\phi} \\ C_T(\alpha) = C_T^3\alpha^3 + C_T^2\alpha^2 + C_T^1\alpha + C_T^0 \\ C_L(\alpha,\boldsymbol{\delta},\boldsymbol{\eta}) = C_L^{\alpha}\alpha + C_L^{\delta_e}\delta_e + C_L^{\delta_c}\delta_c + C_L^0 + \boldsymbol{C}_L^{\eta}\boldsymbol{\eta} \\ C_D(\alpha,\boldsymbol{\delta},\boldsymbol{\eta}) = C_D^{\alpha^2}\alpha^2 + C_D^{\alpha}\alpha + C_D^{\delta_e^2}\delta_e^2 + C_D^{\delta_e}\delta_e + C_D^{\delta_c^2}\delta_c^2 \\ \qquad + C_D^{\delta_c}\delta_c + C_D^0 + \boldsymbol{C}_D^{\eta}\boldsymbol{\eta} \\ C_M(\alpha,\boldsymbol{\delta},\boldsymbol{\eta}) = C_M^{\alpha^2}\alpha^2 + C_M^{\alpha}\alpha + C_M^{\delta_e}\delta_e + C_M^{\delta_c}\delta_c + C_M^0 + \boldsymbol{C}_M^{\eta}\boldsymbol{\eta} \\ \boldsymbol{C}_j^{\eta} = \left[C_j^{\eta_1} \;\; 0 \;\; C_j^{\eta_2} \;\; 0 \;\; C_j^{\eta_3} \;\; 0\right], \quad j = T,L,D,M \\ \boldsymbol{N}_i^{\eta} = \left[N_i^{\eta_1} \;\; 0 \;\; N_i^{\eta_2} \;\; 0 \;\; N_i^{\eta_3} \;\; 0\right], \quad i = 1,2,3 \end{cases} \tag{7}$$

$$\begin{cases} L \approx 0.5\rho V^2 s C_L(\alpha,\boldsymbol{\delta},\boldsymbol{\eta}), \;\; D \approx 0.5\rho V^2 s C_D(\alpha,\boldsymbol{\delta},\boldsymbol{\eta}) \\ T \approx 0.5\rho V^2 s\left[C_{T,\phi}(\alpha)\phi + C_T(\alpha) + \boldsymbol{C}_T^{\eta}\boldsymbol{\eta}\right] \\ M_{yy} \approx z_T T + 0.5\rho V^2 s\bar{c} C_M(\alpha,\boldsymbol{\delta},\boldsymbol{\eta}) \\ N_i \approx 0.5\rho V^2 s\left[N_i^{\alpha^2}\alpha^2 + N_i^{\alpha}\alpha + N_i^{\delta_e}\delta_e + N_i^{\delta_c}\delta_c + N_i^0 + \boldsymbol{N}_i^{\eta}\boldsymbol{\eta}\right] \end{cases} \tag{8}$$

where $\boldsymbol{\delta} = [\delta_c, \delta_e]^T$, the air density ρ is defined as $\rho = \rho_0 \exp(-h/h_0)$ with $\rho_0 = 6.7429\times10^{-5} Slug/ft^3$ and $h_0 = 24000 ft$. A second-order engine model is introduced as :

$$\ddot{\phi} = -2\xi_n\omega_n\dot{\phi} - \omega_n^2\phi + \omega_n^2\phi_c \tag{9}$$

where ζ_n is the engine damping ratio, ω_n is the nominal engine frequency. The actuator-limitations are set as:

$$-20^\circ \le \delta_e, \delta_c \le 20^\circ,\; 0.05 \le \phi \le 1.5 \tag{10}$$

The output vector is chosen as $\boldsymbol{y} = [V, h]^T$.

B. Interval Type-2 TSK Fuzzy Model Generating

The source data is generated from the FAHV longitudinal model in section 2A. The structure of interval typ-2 TSK fuzzy models (IT2-TSK-FMs) are mainly based on interval type-2 TSK fuzzy system (IT2-TSK) whose antecedent part is consist of interval type-2 fuzzy sets whereas consequent part is crisp linear parameters (A2-C0). Moreover, IT2-TSK system has already been proved as universal approximator [15]. Without losing generality, the i^{th} rule of A2-C0 IT2-TSK system is given in MIMO form:

Rule i: If x_1 is $\tilde{P}_1^i$ and … and x_{in} is $\tilde{P}_{in}^i$, then (11)

$$y_i^o = W_{0,i}^o + \sum_{m=1}^{in} W_{m,i}^o x_m \quad i = 1,\dots,r;\; m = 1,\dots,in;\; o = 1,\dots,n$$

where $\tilde{P}_m^i$ denotes Gaussian interval type-2 fuzzy set (IT2-FS) which corresponds to m^{th} input variable x_m. Moreover, in the function of o^{th} output y_i^o, $W_{m,i}^o$ denotes crisp consequent parameter of the m^{th} input variable. r is the number of rules, in is the number of input variables, n is the number of outputs.

However, the IT2-TSK fuzzy system in (11) does not fit the control-ready form in the coming section 3B. It should be transferred into IT2 fuzzy linear model to represent the FAHV model as shown in (12).

Assumption 1: Supposing the input variables $\boldsymbol{U} = x_1, \ldots, x_l$ are control variables and $\boldsymbol{L} = x_{l+1}, \ldots, x_{in}$ are state variables, then (11) can be transferred as:

$$\text{Rule i: If } x_1 \text{ is } \tilde{P}_1^i, \ldots, x_l \text{ is } \tilde{P}_l^i, \text{ and } x_{l+1} \text{ is } \tilde{P}_{l+1}^i, \ldots, x_{in} \text{ is } \tilde{P}_{in}^i, \quad (12)$$
$$\text{then } y_i^o = W_{0,i}^o + \sum_{m_1=1}^{l} \bar{W}_{m_1,i}^o \overline{U^{m_1}} + \sum_{m_2=1}^{in-l} \underline{W}_{m_2,i}^o \overline{L^{m_2}}$$

where $m_1 = 1, \ldots, l$, $m_2 = l+1, \ldots, in$, $\overline{U^{m_1}}$, $\overline{L^{m_2}}$ stands for m_1^{th} and m_2^{th} variables in $\boldsymbol{U}$ and $\boldsymbol{L}$ respectively, $\bar{W}_{m_1,i}^o = W_{m_1,i}^o$, $\underline{W}_{m_2,i}^o = W_{m_2+l,i}^o$, other definitions are the same as (11). With singleton fuzzification, product inference, BMM type reduction and center-average defuzzification, the overall IT2-TSK fuzzy model is inferred as:

$$y^o = \mathrm{A}^o \boldsymbol{L}^T + \mathrm{B}^o \boldsymbol{U}^T + \boldsymbol{wb}^o \tag{13}$$

where

$$\mathrm{A}^o = \frac{1}{2}\left(\frac{\sum_{i=1}^{r} \bar{f}_i(\boldsymbol{x}) \bar{\boldsymbol{W}}_i^o}{\sum_{i=1}^{r} \bar{f}_i(\boldsymbol{x})} + \frac{\sum_{i=1}^{r} \underline{f}_i(\boldsymbol{x}) \bar{\boldsymbol{W}}_i^o}{\sum_{i=1}^{r} \underline{f}_i(\boldsymbol{x})} \right) \in 1 \times (in - l),$$

$$\mathrm{B}^o = \frac{1}{2}\left(\frac{\sum_{i=1}^{r} \bar{f}_i(\boldsymbol{x}) \underline{\boldsymbol{W}}_i^o}{\sum_{i=1}^{r} \bar{f}_i(\boldsymbol{x})} + \frac{\sum_{i=1}^{r} \underline{f}_i(\boldsymbol{x}) \underline{\boldsymbol{W}}_i^o}{\sum_{i=1}^{r} \underline{f}_i(\boldsymbol{x})} \right) \in 1 \times l,$$

$$wb^o = \frac{1}{2}\left(\frac{\sum_{i=1}^{r} \bar{f}_i(\boldsymbol{x}) w_{0,i}^o}{\sum_{i=1}^{r} \bar{f}_i(\boldsymbol{x})} + \frac{\sum_{i=1}^{r} \underline{f}_i(\boldsymbol{x}) w_{0,i}^o}{\sum_{i=1}^{r} \underline{f}_i(\boldsymbol{x})} \right) \in 1 \times 1,$$

$\bar{f}_i(\boldsymbol{x}) = \prod_{m=1}^{in} \bar{\mu}_m^i(x_m)$, $\underline{f}_i(\boldsymbol{x}) = \prod_{m=1}^{in} \underline{\mu}_m^i(x_m)$ are firing values, $\bar{\mu}_m^i(x_m)$, $\underline{\mu}_m^i(x_m)$ are upper membership function (UMF) and lower membership function (LMF) values of uncertainty means IT2 fuzzy sets with input variable value x_m ,

III. Interval Type-2 TSK Nominal-fuzzy-model-based Controller Design

The detail design of the interval type-2 TSK nominal-fuzzy-model-based sliding mode controller (IT2-TSK-NFMSMC) for the flexible air-breathing hypersonic vehicle (FAHV) will be given in this section. The controller is developed based on backstepping control scheme. It consists of five subsystem controllers (SC) corresponding to five rigid-body dynamics in (1)-(5), velocity, flight path angel (FPA) and pitch rate (PR) subsystems are developed with the IT2-TSK-NFMSMC for their complex flexible mode coupling effects and the other two are developed with integral sliding mode controllers (ISMC). Moreover, the key components in designing the IT2-TSK-NFM-SMC consists of interval type-2 TSK nominal-fuzzy-mode-based control, sliding mode control, compensate control, tracking differentiator and notch filter which the first three are used to construct the controller itself and the last two are used to generate virtual commands and to active suppress the flexible mode excitation, respectively.

The overall control scheme and the IT2-TSK-NFMSMC are shown in Fig. 1 and Fig.2 separately.

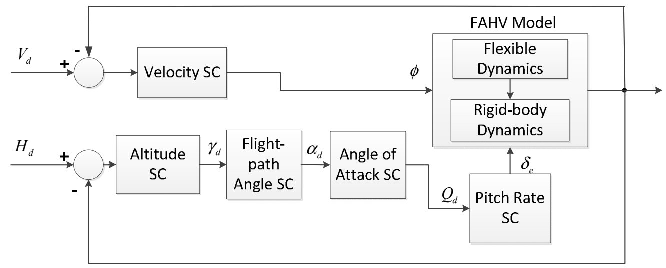

Fig. 1. Overall Control Scheme

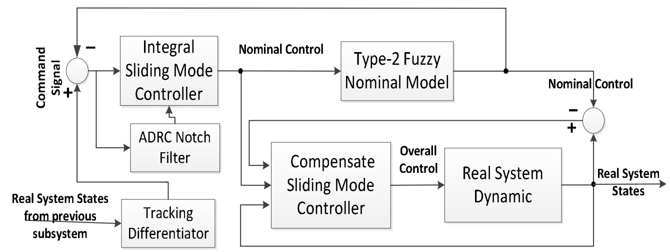

Fig. 2. IT2-TSK-NFMSMC Subsystem Controller

A. Overall Backstepping Control Scheme of FAHV

Defining the tracking errors of five rigid-body state variables of V, h, γ, α, q *as (13):*

$$e_Z = Z - Z_d \tag{14}$$

where Z stands for V, h, γ, α, q , the subscript d stands for reference command. Differentiating (14) and substitute (1)-(5), we can get: $\dot{e}_V = f_v + g_v \phi - \dot{V}_d$ (15); $\dot{e}_h = V \tilde{u}_h - \dot{h}_d$ (16); $\dot{e}_\gamma = f_\gamma + g_\gamma \tilde{u}_\gamma - \dot{\gamma}_d$ (17); $\dot{e}_\alpha = f_\alpha + g_\alpha \tilde{u}_\alpha - \dot{\alpha}_d$ (18); $\dot{e}_q = f_q + g_q \delta_e - \dot{q}_d$ (19) where ϕ, δ_e are actual control outputs and $\tilde{u}_h, \tilde{u}_\gamma, \tilde{u}_\alpha$ are virtual control outputs which refers to γ, α, q in the corresponding subsystems respectively.

However, in backstepping control scheme design for the FAHV, (15)-(19) are only derived from rigid-body dynamics. There still are strong coupling effects in *states of velocity, FPA and PR* from flexible modes directly which can be seen in (7)-(8). To a certain extend, (15)-(19) can be only treated as nominal models of the FAHV. Therefore, we adopt nominal control strategy and the IT2-TSK-FM to design the IT2-TSK-NFMSMC for the (15), (17), (19) three state subsystems and *to design traditional robust ISMCs in altitude and AA subsystems because they are influenced by flexible mode effects indirectly*. The next two subsections will give detailed IT2-TSK-NFMSMC and ISMC design process with one case of each group respectively.

B. IT2-TSK Nominal-fuzzy-mode- Based Sliding Mode Controller Design

1) Sliding Mode Controller Design for IT2-TSK Nominal Fuzzy Model

Without loss of generality, we define the nominal model and real model as the following form respectively:

$$\dot{Y}_n = f_n^Y + g_n^Y \upsilon_n^Y \tag{20}$$

$$\dot{Y}_o = f_o^Y + g_o^Y u_o^Y + d_o^Y \tag{21}$$

where n stands for nominal, Y stands for V, γ, q , o stands for real, d_o^Y stands for bounded disturbances. (20) is derived from IT2-TSK-FM in the form of (13), where $f_n^Y = \mathrm{A}^Y \boldsymbol{L}^T + wb^Y$, $g_n^Y = \mathrm{B}^Y$ and $\upsilon_n^Y = \left(\boldsymbol{U}^Y\right)^T$. The modelling error between nominal model and real one can be defined as (22):

$$e_{n,o}^Y = Y_o - Y_n \tag{22}$$

Defining the sliding surfaces in the each IT2-TSK-NFM subsystem as:

$$s_n^Y = e_{n,o} + \lambda_n \int e_{n,o} \tag{23}$$

where λ_n=-f_n^Y / Y_n .

Substituting (20) back into (15), (17) and (19), we can get:

$$\dot{e}_{n,o}^Y = f_n^Y + g_n^Y \upsilon_n^Y - \dot{Y}_d \tag{24}$$

$$\upsilon_n^Y = \left(\tilde{\upsilon}_n^Y - f_n^Y + \dot{Y}_d\right) / g_n^Y \tag{25}$$

where $\tilde{\upsilon}_n^Y$ is virtual control of the corresponding subsystems,

Defining $\tilde{\upsilon}_n^Y$ with the ISMC form as:

$$\tilde{\upsilon}_n^Y = -k_n^Y s_n^Y \tag{26}$$

where k_n^Y is positive constant values.

2) Compensate Sliding Mode Controller Design

After the ISMC design of the IT2-TSK-NFM for the FAHV, we design a compensate sliding mode controller to compensate the tracking errors and to ensure the velocity, FPA and PR subsystems' stability by the following two theorems and the stability analysis will be given in next section.

Theorem 1: the compensate control law between nominal model and real model is defined as:

$$u_o^Y = -k_o^Y s_n^Y + \frac{1}{g_{oa}^Y}\left(g_n^Y \upsilon_n^Y + \frac{f_n^Y Y_o}{Y_n}\right) - \frac{f_n^Y Y_o}{g_{oa}^Y Y_n} - h\left(Y_o, t\right) \operatorname{sgn}\left(s_n^Y\right) \tag{27}$$

Theorem 2: the $h\left(x_o, t\right)$ is defined as:

$$\begin{aligned} h\left(Y_o, t\right) = d_{\max}^Y &+ \frac{1}{2}\left|\left(\frac{1}{g_{o\min}^Y} - \frac{1}{g_{o\max}^Y}\right)\left(g_n^Y \upsilon_n^Y + \frac{f_n^Y Y_o}{Y_n}\right)\right| \\ &+ \frac{1}{2}\left|-\frac{f_{o\max}^Y}{g_{o\min}^Y} + \frac{f_{o\min}^Y}{g_{o\max}^Y}\right| + \frac{1}{g_{o\min}^Y}\eta^Y \left|s_n^Y(0)\right| \exp\left(-\eta^Y t\right) \end{aligned} \tag{28}$$

where $f_{\mathrm{o}\min}^Y \le f_{\mathrm{o}}^Y \le f_{\mathrm{o}\max}^Y$, $g_{\mathrm{o}\min}^Y \le g_{\mathrm{o}}^Y \le g_{\mathrm{o}\max}^Y$, $d^Y \le \left|d_{\max}^Y\right|$, $f_{oa}^Y = \frac{1}{2}\left(f_{o\min}^Y + f_{o\max}^Y\right)$, $g_{oa}^Y = \frac{1}{2}\left(g_{o\min}^Y + g_{o\max}^Y\right)$

C. Non IT2-TSK Nominal-fuzzy-model Based Controller Design

The compensate SMCs design process in altitude and the AA subsystems are the same as the first part of subsection B. However, the only difference between these two part is non IT2-TSK Nominal-fuzzy-model based compensate SMCs use the real model dynamics.

D. Tackiing Differentiator Design

Because each subsystem in (15)-(19) is a first-order subsystem, then tracking differentiator need to generate first derivate information other than the first-order arranged signal itself. The discrete TD algorithms are implemented as follows [16]:

Each channel:

$$\begin{cases} \overline{SC}_d(t+1) = \overline{SC}_d(t) + \tau * \partial \overline{SC}_d(t) / \partial t \\ \partial \overline{SC}_d(t+1) / \partial t = \partial \overline{SC}_d(t) / \partial t + \tau * fs_1(t) \\ fs_1(t) = -\lambda(\lambda(\overline{SC}_d(t) - \overline{SC}_r) + 3\partial \overline{SC}_d(t) / \partial t) \end{cases} \tag{29}$$

where $\overline{SC}_d$ and $\overline{SC}_r$ are arranged tracking references and final tracking target respectively. λ and τ are factors which can decide the speed of the arranged process and the length of calculation step size respectively. t is the number of iteration.

E. Notch Filter Subsystems' Design

Notch filters are integrated in the velocity and altitude dynamics to suppress specific high-frequency compensation signals to avoid exciting the flexible modes. In this paper, we adopt linear notch filter [1] with the form of:

$$N\left(f\right) = \frac{f^2 + \varpi_{jk}^2}{f^2 + \varpi_{jk} f / K_{jk} + + \varpi_{jk}^2} \tag{30}$$

where ϖ_{jk}, K_{jk} , $j = \gamma, h$, $k = 1, 2, 3$ denote central frequencies and quality factors of the notch filter respectively.

The central frequencies are chosen as previously stated in section 2 A. The quality factors are chosen as K_{jk}=1.1 .

IV. Subsystem Stability Analysis

In this section, we will give stability analysis of the subsystems in which the IT2-TSK-NFMSMC implemented.

Differentiating (23), we can get the first order derivative of the sliding surface:

$$\dot{s}_n^Y = \dot{e}_{n,o}^Y + \lambda_n^Y e_{n,o}^Y = \dot{Y}_o - \dot{Y}_n + \lambda_n^Y\left(Y_o - Y_n\right) \tag{31}$$

Dividing (31) with g_o^Y , yields:

$$\frac{1}{g_o^Y}\dot{s}_n^Y = u_o^Y + d_o^Y - \frac{g_n^Y \tilde{\upsilon}_n^Y}{g_o^Y} + \left(\frac{f_o^Y}{g_o^Y} - \frac{f_n^Y}{g_o^Y}\frac{Y_o}{Y_n}\right) \tag{32}$$

Substituting (27) back into (32):

$$\begin{aligned} \frac{1}{g_o^Y}\dot{s}_n^Y = &-k_o^Y s_n^Y + \frac{1}{g_{oa}^Y}\left(g_n^Y \upsilon_n^Y + \frac{f_n^Y Y_o}{Y_n}\right) - \frac{f_n^Y Y_o}{g_{oa}^Y Y_n} - h\left(Y_o, t\right)\operatorname{sgn}\left(s_n^Y\right) \\ &- \frac{g_n^Y \upsilon_n^Y}{g_o^Y} + \left(\frac{f_o^Y}{g_o^Y} - \frac{f_n^Y}{g_o^Y}\frac{Y_o}{Y_n}\right) + d_o^Y \end{aligned} \tag{33}$$

Choosing the Lyapunov function candidate as (34) for analyzing the subsystems of the IT2-TSK-NFMSMC:

$$V = \frac{1}{2g_o^Y}\left(s_n^Y\right)^2 \tag{34}$$

Taking the first order derivative of (34), yields:

$$\begin{aligned}\dot{V} &= \frac{1}{g_o^Y} s_n^Y \dot{s}_n^Y \qquad (35)\\ &\le -k_o^Y\left(s_n^Y\right)^2 + \left|s_n^Y\right|\left\{\left|\frac{1}{g_{oa}^Y} - \frac{1}{g_o^Y}\right|\left|g_n^Y \upsilon_n^Y + \frac{f_n^Y Y_o}{Y_n}\right| + \left|\frac{f_o^Y}{g_o^Y} - \frac{f_n^Y Y_o}{g_{oa}^Y Y_n}\right| \right.\\ &\qquad \left. + d_o^Y - h\left(Y_o, t\right)\right\}\\ &\le -k_o^Y\left(s_n^Y\right)^2 + \left|s_n^Y\right|\left\{\left|\frac{1}{g_{oa}^Y} - \frac{1}{g_o^Y}\right|\left|g_n^Y \upsilon_n^Y + \frac{f_n^Y Y_o}{Y_n}\right| + \left|\frac{f_o^Y}{g_o^Y} - \frac{f_n^Y Y_o}{g_{oa}^Y Y_n}\right| \right.\\ &\qquad \left. + \left|d_o^Y\right| - h\left(Y_o, t\right)\right\}\end{aligned}$$

Utilizing (27) and (35), we can obtain:

$$\dot{V} \le -k_o^Y\left(s_n^Y\right)^2 \tag{36}$$

thus from (34) and (36) gives:

$$s_n^Y \le \left|s_n^Y(0)\right| \exp\left(-\eta^Y t\right) \tag{37}$$

which indicates the sliding surface s_n^Y will converge to zero exponentially and the tracking error between IT2-TSK-NFM and the real model will converge to zero.

V. Simulation

In this section, we will give a specific way to generate the training data and to learn the interval type-2 TSK fuzzy model (IT2-TSK-FM) in the velocity, FPA and PR subsystems respectively. Moreover, in order to verify the effectiveness and robustness of the proposed interval type-2 TSK nominal fuzzy model-based sliding mode controller (IT2-TSK-NFMSMC), two scenarios, which includes and excludes velocity channel output noise, are studied with comparisons of type-1 ones (T1-TSK-NFMSMC) and the original backstepping model-based sliding mode controller (BSMC).

A. Training Data and Tracking Commands Analysis

The training data is generated in the flexible air-breathing hypersonic vehicle (FAHV) open loop model with 22 scenarios and generate 44,022 data pairs in total which are shown in TABLE I. The training data sets are collected to cover as much as possible of the FAHV flight situations. Moreover, the training variables chosen in different subsystems are put in the TABLE II. All the input variables are normalized into [-1,1] before the learning process whereas the output variable is not scaled. Furthermore, we choose 3 evenly distributed interval type-2 Gaussian fuzzy sets with uncertainty means in each input variable's universe of discourse to construct the antecedent part of the IT2-TSK-NFM.

TABLE I. IT2-TSK-NFM Training Data Generating Detail

Excitation states	Trim value	Excitation Form
δ_e	$\delta_{e0} = 0$	$\delta_{e0} + 0.2 * \sin(30t)$
δ_e	$\delta_{e0} = 0$	$\delta_{e0} + 0.2 * \sin(30t)$
ϕ	$\phi_0 = 0.12$	$\phi_0 + \left[0.05 \cdot (1 \text{ to } 11)\right] \cdot \sin(6/30t)$

TABLE II. IT2-TSK-NFM Learning Input-Output Variables Setting

Subsystems	Input-Output Variables Setting	
	Input Variables	***Output Variables***
Velocity	$V, h, \gamma, \alpha, \delta_e, \phi$	$\dot{V}$
FPA	$V, h, \gamma, \alpha, \delta_e, \phi$	$\dot{\gamma}$
PR	$V, h, \alpha, \delta_e, q$	$\dot{q}$

Moreover, the open-loop responses of the FAHV longitudinal model are descending flight action which means the IT2-TSK-FM can just model the flight states' dynamics below the velocity and altitude initial states. On the basis of constant dynamic pressure, the tracking differentiators (TD) are used to generate reference tracking commands which are set as +48.6 feet/s and -10,000 feet in velocity and altitude channels respectively .

B. Without Subsystem Output Noise

In this experiment scenario, we adopt the backstepping nominal sliding mode controller (NMSMC) which uses the FAHV longitudinal mathematic model and the compensator in section 3 B, IT2-TSK-NFMSMC (shown as IT2NFMSMC for short) which adopt interval type-2 TSK fuzzy model respectively to make comparison simulations. As shown in Fig. 3, IT2-TSK-NFMSMC behaves less than the NMSMC due to the modeling flying envelope is different from the reference signals, however, both the two controllers can control FAHV in a reasonable command tracking errors. However, there exist some differences in subfigure of accelerator ϕ. The IT2-TSK-FNMSMC have smoother than the NMSMC.

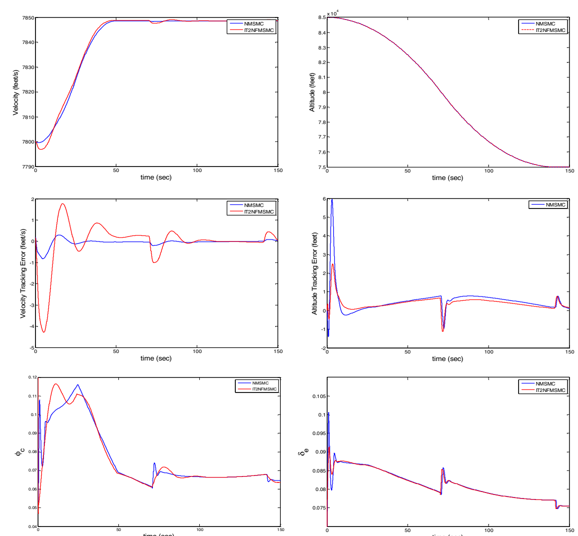

Fig. 3. Simulation Comparisions without Noise in Subsystem

C. *With Subsystem Output Noise*

In this experiment scenario, we adopt the same two controllers as stated in the last subsection. Furthermore, a Gaussian white noise with variance of 1% of α, δ_e, ϕ and mean value 0 is added into the respective signal channels to simulate the actuators output noise. As shown in Fig. 4, under

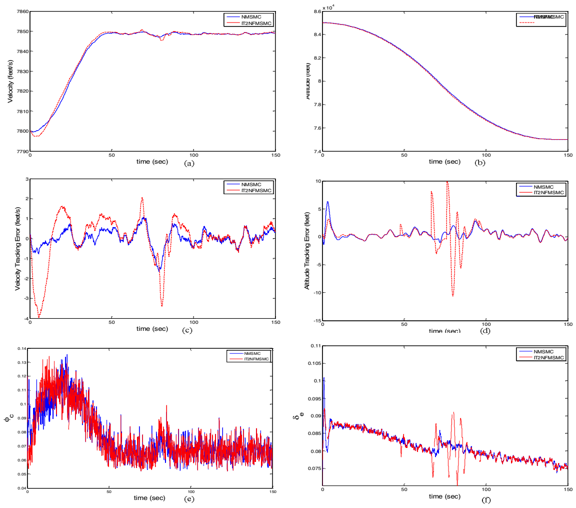

Fig. 4. Simulation Comparisons with Measuring Noise in Velocity

the strong structural uncertainties, however, the IT2-TSK-NFMSMC performs worse than the NMSMC which is constructed with the FAHV rigid-body dynamics, but the proposed controller can still handle the situation with reasonable performance.

VI. CONCLUSION

In this paper a novel interval type-2 TSK nominal-fuzzy-model-based sliding mode controller (IT2-TSK-NFMSMC) for flexible air-breathing hypersonic vehicle (FAHV) is presented to give a trial on stressing the robustness of the data-driven based fuzzy modelling control system. The backstepping control scheme is adopted and the proposed IT2-TSK-NFMSMC is embedded in the velocity, flight path angle and pitch rate subsystems whereas traditional integral sliding mode controller is embedded in altitude and angle of attack subsystems. More specifically, the IT2-TSK-NFMSMC is designed by the following steps: 1) using our previous interval type-2 fuzzy self-organizing approach and training data sets of the FAHV longitudinal model dynamics to construct interval type-2 TSK nominal-fuzzy-models (IT2-TSK-NFM); 2) designing sliding mode controllers based on the IT2-TSK-NFM; 3) adopting notch filters to active decreasing disturbance effects from the flexible modes; 4) designing sliding mode compensate controllers through Lyapunov synthesis to compensate difference between IT2-TSK-NFM and real model of the FAHV. The simulation comparisons show strong robustness and effectiveness of the proposed IT2-TSK-NFMSMC compared to traditional mathematic model-based control in dealing with the flexible mode uncertainties and the actuator output noises which means the data-driven based interval type-2 TSK nominal-fuzzy-model-based control is applicable but need further study in practice applications.

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