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Kinematic and Static Analysis of a Cable-driven 3-DOF Delta Parallel Mechanism for Haptic Manipulators

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Abstract: With the rapid development of virtual reality technology, a variety of haptic devices have been designed and applied in the human computer interaction. The parallel mechanism is widely employed due to its high stiffness and resolution. However, its kinematics and dynamics are more complex than its serial counterpart since the parallel links are mechanical coupled. Specially, it is of great significance to make an accuracy gravity compensation for haptic manipulator on which the haptic transparency relies. This paper introduces a cable driven 3-DOF delta parallel mechanism for haptic manipulator and presents the forward and inverse kinematics analyses including the Jacobian matrix and the singularities. According to the principle of virtual work, the static model of the delta parallel manipulator is established and based on this, a practical algorithm for gravity compensation is proposed. The efficiency of the gravity compensation algorithm is validated by the simulation with ADAMS software.

Key Words: Delta, Static Analysis, Gravity Compensation

1 Introduction

Over recent years, haptic device research and development has become a hotspot because it really changes the way of human computer interaction in variety of applications ranging from engineering design and prototyping to medical training. Generally, haptic devices could be divided into three types in structure: serial, parallel and hybrid. Especially, the parallel mechanism is widely used in the haptic device design because of its high stiffness, high resolution and high bearing capacity. And the delta parallel mechanism is a typical representative. In addition, the cable driven design takes advantages of low weight, small backlash and inertia, which is a benefit for the design of control.

The concept of delta parallel robot was first introduced by Clavel in 1988 [1]. In 1995 Tsai invented a novel 3-DOF parallel manipulator which employed only revolute joints and constrained the manipulator output to translational motion [2]. In 1998, Tohoku University developed a compact 6 DOF haptic interface based on the modified delta mechanism as a positioning subsystem [3]. In 2001, the Institute of Systems and Robotics, Ecole Polytechnique Federale de Lausanne (EPFL) developed the delta haptic device based on the state-of-the-art cable driven parallel mechanism [4]. A relatively inexpensive 3-DOF haptic device called falcon which is also based on the modified delta parallel mechanism is made by Novint for game industry. In 2012 Li presented a haptic manipulator using series-parallel mechanism [10]. The hybrid mechanism take advantages of both parallel and serial mechanism and also result in relatively easy kinematic analysis.

In this paper, kinematic and static analyses of the cable driven delta parallel mechanism are presented and a practical algorithm for gravity compensation based on the static model is proposed. This paper is organized as follows. The description of the modified delta parallel mechanism and the inverse and forward kinematics are presented in Section 2. In Section 3, the Jacobian matrix and the singularities of the modified delta parallel mechanism is analyzed. In Section 4, the 3D figures of the workspace are showed. In next section, the static model of the delta parallel manipulator is established and based on this, a practical algorithm for gravity compensation is proposed. The efficiency of the gravity compensation algorithm is validated by the simulation in Section 6. Finally, the conclusion and the future work are given in Section 7.

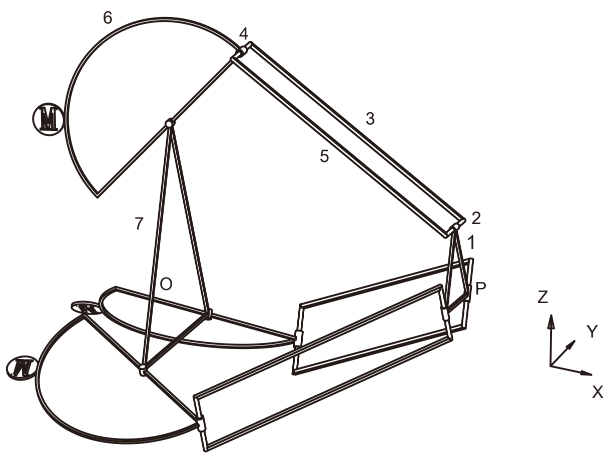

Fig. 1: Schematic of the cable driven delta parallel mechanism

2 Description and Kinematics

2.1 Description

Fig. 1 shows the schematic of the modified delta mechanism, in which the moving platform is labeled 1 and the base platform is labeled 7. It should be noted that the base platform labeled 7 is not a real part but an auxiliary plane formed by connecting three fixed rev center. Three identical parallel limbs separated each other by $2\pi/3$ around the x axis connect the moving platform to the base platform by three revolving joints. Each limb consists of a driving half disk la-

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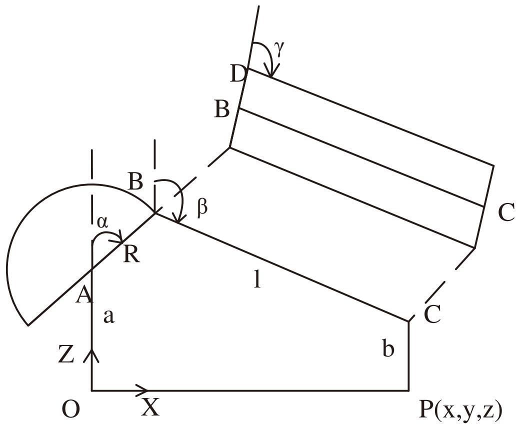

Fig. 2: the ith $(i = 1, 2, 3)$ limb for kinematic analysis

beled 6 and a planar four-bar parallelogram labeled 2, 3, 4, 5. And the four corner revolute axes of the planar four-bar parallelogram are perpendicular to the axes connecting the moving platform and the base platform. The actuate motors are tangent to the half disks through cables.

Apart from the inherent mechanical features of parallel mechanism, the cable driven delta mechanism takes advantages of low weight, small backlash and inertia which is benefit for the design of control. They also tend not to intrude in the visual space of the user since only thin cables and a small structure which supports them are required [5].

2.2 Forward Kinematics

The forward kinematics describes the transformation from the joint space to the end Cartesian space. Compared with the serial manipulator, it is more difficult to solve the forward kinematic problem for the parallel manipulator. For the modified delta manipulator, we should solve the position of the end-effector given the controlled joint angles. A direct solution for the forward kinematics is presented utilizing the geometrical relationship of this manipulator. The solution can be reduced to a quadratic equation in a single unknown using method of elimination. Obviously the polynomial equations result in not only a solution and the real solution need to be selected on the basis of configuration constraint considering the position of moving platform is always positive in X direction.

Due to the triple-symmetry of this manipulator, each limb can be treated separately. Take the ith $(i = 1, 2, 3)$ limb as an example and the description is showed in Fig. 2. A reference frame (XYZ) is attached to the point O, which is the center of the triangle formed by connecting the revolute center point of three identical half disks. The X-direction is defined perpendicular to the base platform, the Y-direction is defined as horizontal and the Z-direction is upright. The position of end-effector is described by the triple $P(x, y, z)$ in the XYZ coordinate frame. The radius of the half disk is R and the length of the parallel bar is l. The angle α, β, γ is respective the control angle measured from $\overrightarrow{OA}$ to $\overrightarrow{AB}$, the pitch angle measured from vertical direction to $\overrightarrow{BC}$, the yaw angle measured from $\overrightarrow{BD}$ to $\overrightarrow{BC}$. The length of the vector $\overrightarrow{OA}$, $\overrightarrow{CP}$ is respective a, b.

The reference frame of the ith $(i = 1, 2, 3)$ limb is located at the point O but rotated along the X axis by an angle $\phi_i = 0°, 120°, 240°$. The transformation matrix is given by

$$R(\phi_i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi_i) & -\sin(\phi_i) \\ 0 & \sin(\phi_i) & \cos(\phi_i) \end{bmatrix} \tag{1}$$

According to the geometrical relationship and the geometrical parameters defined in Fig.2, the kinematic equations are obtained:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R(\phi_i) \begin{bmatrix} R\sin(\alpha_i) + l\sin(\beta_i)\sin(\gamma_i) \\ l\cos(\gamma_i) \\ a - b + a\cos(\alpha_i) + b\cos(\beta_i)\sin(\gamma_i)) \end{bmatrix} \tag{2}$$

This results in a set of 9 equations with 9 unknowns $(x, y, z, \beta_i, \gamma_i)$ for $i = 1, 2, 3$. As motioned above, utilizing method of elimination and considering the $x > 0$ the only solution (x, y, z) will be solved.

2.3 Inverse Kinematics

The inverse kinematics describes the transformation from the end Cartesian space to the joint space. For the modified delta manipulator, we solve the controlled joint angles given the position of the end-effector. Similar to the forward kinematics, consider equation(2) and eliminate unknowns β_i and γ_i, then utilizing a half-angle tangent formula of α_i resulting in a quadratic equation. However, solving these quadratic equations can result in four solutions for each limb for a given position of the end-effector and produce at most two unique physical postures. In the practical applications, we could remain the unique actual solution by identifying the angle range of the special configuration[2].

3 Jacobian and Singularity Analysis

The Jacobian is a powerful tool in the design and performance measurement of a manipulator. It is often used for the control of manipulator since for a given desired end-effector velocity, it is possible to map the velocity back to the joint space. Similarly, the Jacobian matrix provides the static force transformation from the Cartesian space to the controlled joint space. In addition, Jacobian analysis is also used to determine the singular positions of a manipulator, since when a manipulator is at a singular position, the Jacobian matrix is also singular [6]. The definition of the Jacobian matrix is the transformation from the joint velocities to the end-effector velocity as follow(3):

$$\dot{x} = J\dot{q} \tag{3}$$

Where $\dot{x}$ is an n-dimensional output velocity vector of the end-effector, $\dot{q}$ is an m-dimensional vector that represents a set of controlled joint vector and J is the $n * m$ Jacobian matrix.

In the case of the delta mechanism, the vector operation can simplify the Jacobian analysis. We give a simplified version of the deduction established by Codourey [8]. Take the ith $(i = 1, 2, 3)$ limb as an example and the description is

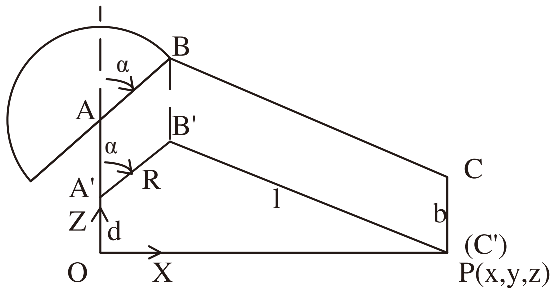

Fig. 3: the ith $(i = 1, 2, 3)$ limb for Jacobian analysis

showed in Fig. 3. As the moving platform can only be translated, the moving platform will always keep the same orientation. So, point A, B, C can be moved to $A^{'}, B^{'}, C^{'}(P)$ with a distance b. This simplification will be easier for the derivation of Jacobian matrix without affecting the results. Fist, a loop closure vector equation can be chosen as:

$$\|\overrightarrow{B_i^{'}P}\|^2 = l^2 \;\; i = 1, 2, 3 \tag{4}$$

Let $\mathbf{v}_i$ be the vector $\overrightarrow{B_i^{'}P}$. According to the geometrical relationship showed in Fig. 3, $\mathbf{v}_i$ can be written as:

$$\mathbf{v}_i = \overrightarrow{O_iP} - \overrightarrow{O_iA_i^{'}} - \overrightarrow{A_i^{'}B_i^{'}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} - R(\phi_i) \begin{bmatrix} R\sin(\alpha_i) \\ 0 \\ -R\cos(\alpha_i) \end{bmatrix} \tag{5}$$

The time derivative of equation

$$\mathbf{v}_i^T \cdot \mathbf{v}_i = l^2 \tag{6}$$

leads to:

$$\mathbf{v}_i^T \dot{\mathbf{v}}_i + \dot{\mathbf{v}}_i{}^T \mathbf{v}_i = 0 \tag{7}$$

Due to the commutativity property of the product, the above equation is simplified as follow:

$$\mathbf{v}_i^T \dot{\mathbf{v}}_i = 0 \tag{8}$$

According to equation (5), the time derivative of $\mathbf{v}_i$ is given by:

$$\dot{\mathbf{v}}_i = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} - R(\phi_i) \begin{bmatrix} R\cos(\alpha_i) \\ 0 \\ -R\sin(\alpha_i) \end{bmatrix} \dot{\alpha}_i \tag{9}$$

Let

$$\dot{\alpha} = \begin{bmatrix} \dot{\alpha}_1 & \dot{\alpha}_2 & \dot{\alpha}_3 \end{bmatrix}^T \tag{10}$$

$$\dot{\mathbf{X}} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T \tag{11}$$

$$\mathbf{w}_i = R(\phi_i) \begin{bmatrix} R\cos(\alpha_i) \\ 0 \\ -R\sin(\alpha_i) \end{bmatrix} \tag{12}$$

Rearranging equation (8) into a vector form, the following is obtained:

$$\begin{bmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \\ \mathbf{v}_3^T \end{bmatrix} \dot{\mathbf{X}} + \begin{bmatrix} \mathbf{v}_1^T\mathbf{w}_1 & 0 & 0 \\ 0 & \mathbf{v}_2^T\mathbf{w}_2 & 0 \\ 0 & 0 & \mathbf{v}_3^T\mathbf{w}_3 \end{bmatrix} \dot{\alpha} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

So the Jacobian matrix J can be written as:

$$J = -\begin{bmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \\ \mathbf{v}_3^T \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{v}_1^T\mathbf{w}_1 & 0 & 0 \\ 0 & \mathbf{v}_2^T\mathbf{w}_2 & 0 \\ 0 & 0 & \mathbf{v}_3^T\mathbf{w}_3 \end{bmatrix} \tag{14}$$

It should however be noted that, the Jacobian matrix is not only a function of joint angles α_i as is usually in the case for serial robots, but also a function of the end effector position X.

Another method adopted by Stamper to compute the Jacobian matrix is to consider and differentiate a set of loop closure constraint equations to obtain the velocity relationships between the operational space variables and the joint space variables [6]. And Stamper also finds several singular positions by examining the conditions that cause the Jacobian matrices to be singular. The singular positions happen under the following conditions:

$$\alpha_i - \beta_i = 0 \; or \; \pi \tag{15}$$

or

$$\beta_i = 0 \; or \; \pi \tag{16}$$

or

$$\gamma_i = 0 \; or \; \pi \tag{17}$$

These singular positions happen at the limit of the manipulator workspace. And in the real configuration, there is none singular position within its workspace, which is a significant advantage of this mechanism.

4 Workspace Analysis

Although parallel manipulators provide several advantages compared with serial manipulators, the limited workspace is the most common criticism. From a practical perspective, the desired workspace is not the larger the better, its quality is also important. Stamper optimizes the total volume of well-conditioned workspace by maximizing a global condition index [7]. However, for most practical applications, it is an essential task to determine the variable parameters depending on the design requirements. While the accurate analytical solutions for the problem is quite difficult and complex. And for most cases, we do not need the accurate solutions while we need an appropriate estimation. Furthermore, along with rapid progress of computers computation performance, the workspace of a manipulator can be efficiently by a simple numerical discretization method utilizing the forward kinematics of the mechanism. Here we give the example of the determination of the delta manipulator with the following parameters: R=120, a=140, l=255, b=65. And the unit is millimeter. Fig. 4 shows MATLAB results of the 3D workspace of the manipulator. The results display a symmetry about the X axis in $120°$ sectors, which conforms to the symmetry structure of the delta mechanism.

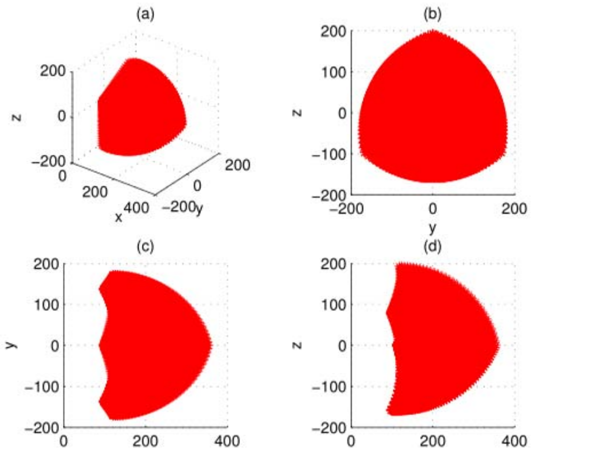

Fig. 4: The 3D workspace:(a)3D workspace (b)workspace front view (c)workspace top view (d)workspace side view Units:mm

5 Static Analysis

In most applications, the haptic interface is used as a desktop device and provides a not-so-big force feedback to the human operator. Considering the use of comfort, gravity compensation is indispensable. Otherwise the human operators will suffer from fatigue when they use the device for a long time. And the gravity of the device absolutely affects the sensation of the operators. Usually, Lagrange-Euler formulation and Newton-Euler formulation are two main analytical approaches of the dynamics of robots. However, it is a difficult problem to solve the dynamics of the parallel manipulator and it is also time consuming which maybe causes a serious time delay. Given that the velocity of the end-effector is slow, we adopt the static model to analyze the modified delta mechanism.

In most practical project applications, a complete model taking into account the precise force analysis is not necessary for which leads to very complicated solutions. Thus, for analytical purposes, we develop the static model with the following simplifying hypotheses:

1) the masses of four-bar parallelogram are optimally separated into two portions and placed at their two extremities, $2/3$ at its upper extremity and $1/3$ at its lower extremity[8].
2) friction effects and elasticity are neglected.

With the above mentioned simplifying hypothesis, the delta robot can be reduced to 4 bodies only: the moving platform and 3 half disks. Take the i^{th} $(i = 1, 2, 3)$ limb as an example and the static analysis is showed in Fig. 5. The centroid of the half disk locates at point E from which the distance is r away to the center point A. According to the virtual work principle, we obtain:

$$\mathbf{T} + \mathbf{T}_d + \mathbf{T}_p + J^T\mathbf{G}_m = J^T\mathbf{F} \tag{18}$$

Then, the torque of the joint space is given by:

$$\mathbf{T} = -\mathbf{T}_d - \mathbf{T}_p - J^T\mathbf{G}_m + J^T\mathbf{F} \tag{19}$$

Where: $\mathbf{T}$ is the vector of torques that has to be applied to the joint space;

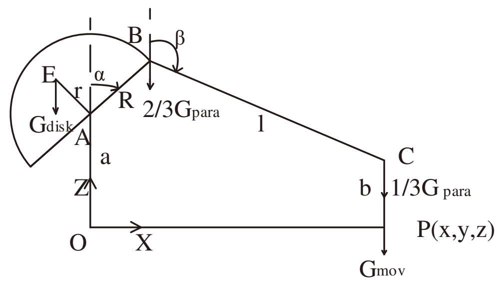

Fig. 5: the ith $(i = 1, 2, 3)$ limb for static analysis

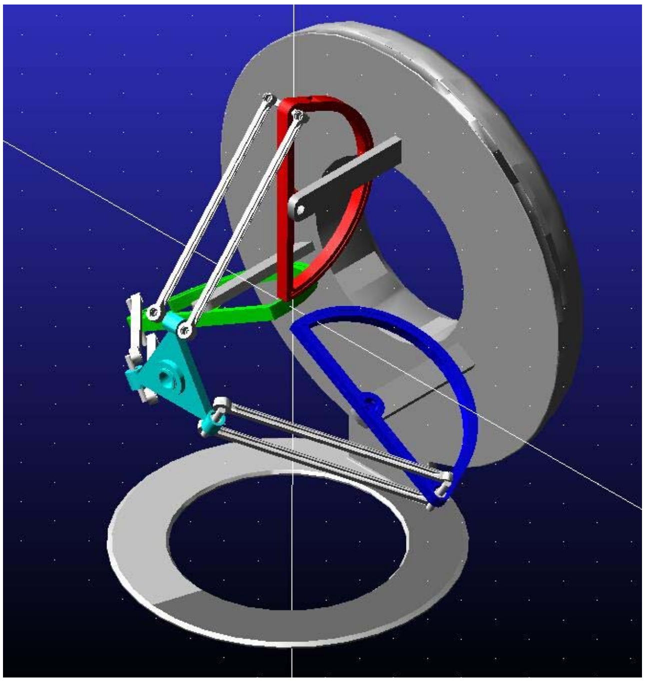

Fig. 6: ADAMS model of the modified delta mechanism

$\mathbf{T}_d$ is the torque vector produced by the gravitational force of the half disks and is given by:

$$\mathbf{T}_d = m_{disk}gr\,[-\cos(\alpha_1)\ 1/2\cos(\alpha_2)\ 1/2\cos(\alpha_3)]^T \tag{20}$$

$\mathbf{T}_p$ is the torque produced by the gravitational force of the parallel bars and is given by:

$$\mathbf{T}_p = 2/3m_{para}gR\,[\sin(\alpha_1)\ \ -1/2\sin(\alpha_2)\ \ -1/2\sin(\alpha_3)]^T \tag{21}$$

$\mathbf{G}_m$ is the torque produced by the gravitational force of the moving platform and the parallel bars and is given by:

$$\mathbf{G}_m = (m_{para} + m_{mov})g\,[0\ 0\ \ -1]^T \tag{22}$$

$\mathbf{F}$ is the desired force of the end-effector.

6 Simulation

In order to demonstrate the validity of the algorithms, we construct the virtual prototype in ADAMS software as

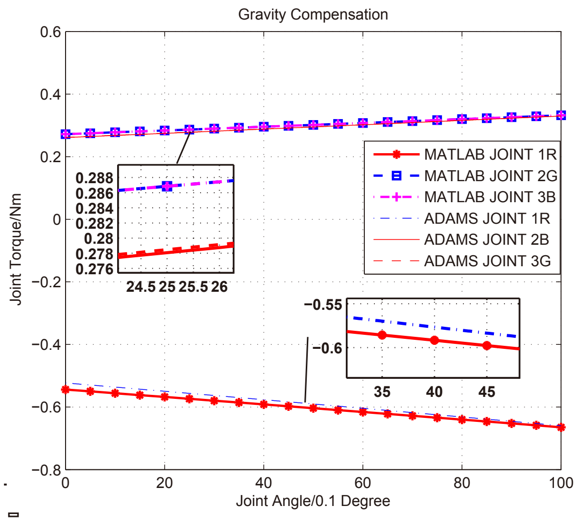

Fig. 7: Torque for gravity compensation 1

Table 1: The Conditions of Experiments

Angles and Force	α_1,α_2,α_3	F
Fig.7	0d-10d	0N
Fig.8	0d-10d	3N
Fig.9	0d-10d	5N

showed in Fig. 6. First, we calculate the torques according to the algorithms presented above in MATLAB software and the results are given in Fig. 7-9. Then with the help of ADAMS, we can get the torque of joint space wherever the end-effector is in the working space. The torque of joints space is also plotted in Fig. 7-9. There are six different curves in every figure and the first three present the results from MATLAB while other three present the results from ADAMS. Three rotating joints of the half disks are respectively labeled JOINT 1R, JOINT 2G, JOINT 3B as showed in Fig. 6. These three sets of experiments are performed under different conditions given by Tables 1 and the mass of each part is give by Tables 2. For convenience, take $\alpha_1 = \alpha_2 = \alpha_3$ and take the direction of desired force in the Z direction considering that there is a great influence on gravity compensation in the Z direction. Due to the symmetry of JOINT 2G and JOINT 3B, this two torque curves overlap both in MATLAB and in ADAMS. Though comparison, it is obviously to find that the difference between two sets of results is less than 0.01Nm. The difference can be read from the enlarged partial view as showed in Fig. 7-9. Therefor, the simulation results prove the validity of the algorithm proposed above.

Table 2: The Mass of Each Part

Part	Mass
Half Disk	0.255kg
Long Link	0.0338kg
Short Link	0.0085kg
Moving Platform	0.7688kg

7 Conclusion and Future Work

In this paper, the modified delta parallel mechanism used in the haptic interface design is introduced. Compared with

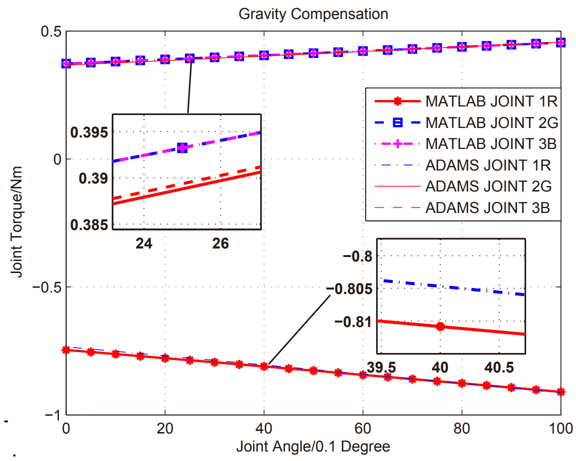

Fig. 8: Torque for gravity compensation 2

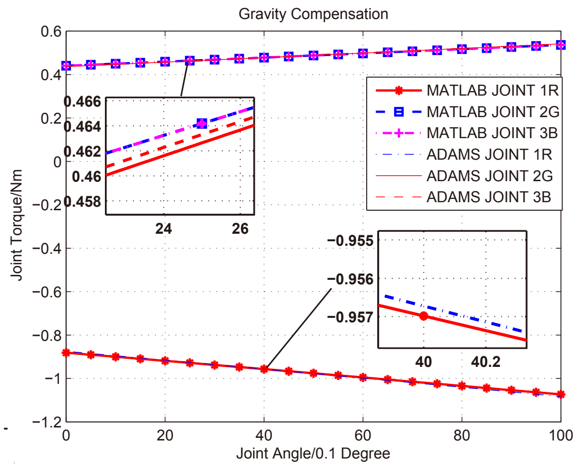

Fig. 9: Torque for gravity compensation 3

serial mechanisms, the parallel ones have higher precision and greater structural stiffness. The forward and inverse kinematics are presented and based on this we give the analyses of Jacobian and the singularities. We adopt the numerical method give the determination of the workspace. According to the virtual work principle, we propose the gravity compensation algorithm on the basis of the static model.The efficiency of the gravity compensation algorithm is validated by the simulation. Future work includes developing the controller of the motors for the modified delta parallel mechanism and conducting the gravity compensation experiments. Based on this, we will study the performance of force feedback.

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