The Variable Structure PI Speed Controller for AC Servo System

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Abstract: The high performance AC servo system is required to achieve fast response without overshoot. The traditional proportional-integral (PI) controller which utilizes fixed parameters, however, cannot well guarantee the requirement for both response and overshoot at the same time. Thus a variable structure PI (VSPI) controller is proposed in this paper, and the stability conditions of the closed-loop system based on which is given according to Lyapunov stability criterion. When the absolute value of speed error is smaller than the threshold value, the proportional gain and integral gain are regulated adaptively by a linear function; Otherwise, the proportional gain and the integral gain remain the fixed value. Experiment results show that compared with the traditional PI controller, the VSPI controller under a speed step input can achieve fast response with effectively reduced overshoot.

Key Words: AC servo system, overshoot, fast response, the variable structure PI controller

1 Introduction

With the development of bus technology, digital control technology and computer technology, the high performance AC servo system has been widely used in High-end CNC systems and industrial robots[1-4]. High-end CNC systems and industrial robots have a quite high demand for the the speed and robustness of the velocity loop of AC servo system. The traditional PI controller is widely used in the AC servo system due to its simple structure, easy implementation and parameter setting. However, the PI controller will be saturated which can lead to large overshoot even oscillation when the step command is too large[5-8]. In general, AC servo system speed control requires large proportional gain to ensure fast response of the system, and small integral gain to avoid overshoot. However, both large proportional gain and integral gain are needed to ensure the steady-state accuracy. Therefore, the fixed PI parameters are difficult to solve the contradiction between fast response and overshoot.

Scientists have adopted many speed control strategies such as fractional-order speed control and sliding mode speed control[9][10]. Unfortunately, these methods are all complicated and difficult to realize. Some advanced intelligent control strategies such as fuzzy PI control[11] which has been presented and greatly developed in recent years. However, the control rules are highly dependent on expert experience.

A variable structure PI (VSPI) controller is proposed in this paper in order to solve the problem, and the stability conditions of the closed-loop system based on which is given according to Lyapunov stability criterion. When the absolute value of speed error is smaller than the threshold value, the proportional gain and integral gain are regulated adaptively by a linear function; Otherwise, the proportional gain and the integral gain remain the fixed value to avoid overshoot. Experiment results verify the feasibility and effectiveness of the proposed method.

2 The Speed Loop Control Model

Speed closed-loop control is a necessary link in order to ensure the rapidity and stability of the speed response. The motion equation of the AC servo system is:

\[ J \frac{d\omega_r}{dt} + B\omega_r = k_T i_q - T_L \]  

(1)

where \( J \) is the moment of inertia, \( \omega_r \) is the rotor mechanical speed, \( B \) is friction coefficient, \( k_T \) is the torque constant, \( i_q \) is the q-axis current, \( T_L \) is the load torque.

In general, the PI controller is used in the speed control, and its expression is:

\[ u = k_p e(t) + k_i \int_0^t e(t)dt \]  

(2)

where \( u \) is the output of PI controller, \( e(t) \) is the error of speed, \( k_p \) is the proportional gain, \( k_i \) is the integral gain.

The bandwidth of the current loop is far higher than the velocity loop, so that the current loop can be equivalent to the amplification link that the value is 1. The maximum output of the PI speed controller will be limited to the maximum current \( I_{\text{max}} \) due to inverter capacity, motor rated
power and so on. Fig. 1 shows the block diagram of speed controller with integral separation.

\[
\Phi(s) = \frac{k(s + a_0)}{s^2 + ks + c}
\]

where \( k = k_p k_T / J \), \( a_0 = k_i / k_p \), \( c = k_i k_T / J \).

Integral separation algorithm can be implemented to avoid the effect of integrator when the output of PI controller is saturated. With this approach the integral link is added only when the PI controller operates in linear region, otherwise it is kept constant. The algorithm of integral separation is:

\[
i_q^* = \begin{cases} 
  i_{q_{\text{max}}} & u \geq i_{q_{\text{max}}} \\
  u & -i_{q_{\text{max}}} < u < i_{q_{\text{max}}} \\
  -i_{q_{\text{max}}} & u \leq -i_{q_{\text{max}}}
\end{cases}
\]

It can be seen that the proportional link can improve the system response and reduce the steady-state error, but the excessive proportional gain \( k_p \) can cause the system oscillation. The integral link can reduce the steady-state error of the system, but the excessive proportional gain \( k_i \) can lead to the system instability. Therefore, the fixed PI parameters cannot satisfy the requirement of fast dynamic response without overshoot in AC servo system.

3 Design of Variable Structure PI Controller

The variable structure PI controller is proposed in order to solve the contradiction between fast response and overshoot. The expression is as follows:

\[
k_p = \begin{cases} 
  \frac{k_{p_2} - k_{p_1}}{b} e + k_{p_1} & 0 \leq e \leq b \\
  \frac{k_{p_2} - k_{p_1}}{-b} e + k_{p_1} & -b \leq e \leq 0 \\
  k_{p_2} & |e| > b
\end{cases}
\]

\[
k_i = \begin{cases} 
  \frac{k_{i_2} - k_{i_1}}{b} e + k_{i_1} & 0 \leq e \leq b \\
  \frac{k_{i_2} - k_{i_1}}{-b} e + k_{i_1} & -b \leq e \leq 0 \\
  k_{i_2} & |e| > b
\end{cases}
\]

where \( b \) is the limitation of speed error, \( b \geq 0 \), \( k_{p_1} \) is the maximum proportional gain, \( k_{p_2} \) is the minimum proportional gain, \( k_{i_1} \) is the maximum integral gain, \( k_{i_2} \) is the minimum integral gain.

The output of the proportional term \( U_p \) and the incremental integral term \( \Delta U_i \) are as follows:

\[
U_p = k_p e
\]

\[
\Delta U_i = k_i U_p
\]

The curve of proportional gain \( k_p \) is shown in Fig. 2. The proportional gain \( k_p \) should be reduced when PI controller saturates, but \( k_p \) cannot be always reduced that it would be kept as the minimum proportional gain \( k_{p_2} \). The proportional gain \( k_p \) is regulated adaptively by a linear function in order to ensure the fast response of servo system when PI controller is out of saturation. The equation (9) ensures that \( U_p \) is monotonic with speed error \( e \).
\[
\dot{U}_p = \begin{cases} 
\frac{k_p - k_n}{b} e + k_p > 0 & 0 \leq e \leq b \\
\frac{k_p - k_n}{b} e + k_n > 0 & -b \leq e \leq 0 \\
k_p > 0 & |e| > b
\end{cases}
\] (9)

By the equation above, the maximum proportional gain \(k_p\) and the minimum proportional gain \(k_{p2}\) are presented as follows:
\[
\begin{align*}
  k_p & \geq 2k_p, \\
  k_{p2} & > 0
\end{align*}
\] (10)

The curve of integral gain \(k_i\) is shown in Fig.3. The integral gain \(k_i\) should be reduced when PI controller saturates in order to prevent overshoot and oscillation of servo system. The integral gain \(k_i\) is regulated adaptively by a linear function when PI controller is out of saturation.

![Fig.3 The curve of proportional gain \(k_i\)](image)

4 Stability Analysis of Variable Structure PI Controller

Combining equation (1) with the speed error \(e = \omega^* - \omega\), the differential equation of speed error can be expressed as:
\[
\dot{e} = \frac{B}{J} \left( \omega^* - e \right) - \frac{k_x i_q^* - T_L}{J}
\] (11)

Through equation (5) and (6), \(i_q^*\) can be got:
\[
i_q^* = \begin{cases} 
a_1 e^2 + k_p e + a_2 e \theta + k_\theta \theta & 0 \leq e \leq b \\
-a_1 e^2 + k_p e - a_2 e \theta + k_\theta \theta & -b \leq e \leq 0 \\
k_p e + k_\theta \theta & |e| > b
\end{cases}
\] (12)

where \(\theta = \int_0^t e(t) dt\), \(a_1 = \frac{k_p - k_n}{b}\), \(a_2 = \frac{k_{p2} - k_n}{b}\).

The Lyapunov candidate function is chosen as:
\[
V = \frac{1}{2} Je^2 + \frac{k_x k_\theta}{2} (\theta - \theta_s)^2
\] (13)

where \(\theta_s\) is the steady state of the integral regulator. The speed error \(e\) is 0 and \(i_q^* = k_i \theta\) in steady state. Replacing them in (11), \(\theta\) can be expressed as:
\[
\dot{\theta} = \frac{B \omega^* + T_L}{k_x k_\theta}
\] (14)

According to Lyapunov stability criterion, system stability condition is as follow:
\[
\dot{V} = Je \dot{e} + k_i \dot{e} (\theta - \theta_s) \leq 0
\] (15)

Three cases can be described as follows according to the variation range of speed error.

When speed error \(0 \leq e \leq b\), replacing (12) in (11):
\[
\dot{e} = \frac{B}{J} \left( \omega^* - e \right) - \frac{k_x (a_1 e^2 + k_p e + a_2 e \theta + k_\theta \theta) - T_L}{J}
\] (16)

Combining equation (15) with (14), (16):
\[
\dot{V} = -k_x a_1 e^3 - (B + k_x k_p + a_2 \theta) e^2
\] (17)
\[
-((B \omega^* + T_L) - k_x k_\theta \theta) e
\]

Replacing \(a_1\), \(a_2\) and \(\theta \leq i_{q_{\text{max}}}\) in (17), thus the stability condition of closed-loop system is:
\[
B + k_x k_p \theta \leq \frac{(k_n - k_\theta)}{b} i_{q_{\text{max}}}
\] (18)

When speed error \(-b \leq e \leq 0\), replacing (12) in (11):
\[
\dot{e} = \frac{B}{J} \left( \omega^* - e \right) - \frac{k_x (-a_1 e^2 + k_p e - a_2 e \theta + k_\theta \theta) - T_L}{J}
\] (19)

Combining equation (15) with (14), (19):
\[
\dot{V} = k_x a_1 e^3 - (B + k_x k_p - a_2 \theta) e^2
\] (20)
\[
-((B \omega^* + T_L) - k_x k_\theta \theta) e
\]

Replacing \(a_1\), \(a_2\) and \(\theta \leq i_{q_{\text{max}}}\) in (20), thus the stability condition of closed-loop system is expressed as (18).

When speed error \(|e| > b\), replacing (12) in (11):
\[
\dot{e} = \frac{B}{J} \left( \omega^* - e \right) - \frac{k_x (a_1 e^2 + k_p e + a_2 e \theta + k_\theta \theta) - T_L}{J}
\] (21)

Combining equation (15) with (14), (21):
\[
\dot{V} = -(B + k_x k_p) e^2 -((B \omega^* + T_L) - k_x k_\theta \theta) e
\] (22)
As the equation (22) always holds, replacing \(|\theta_t| \leq i_{q_{max}}\) in (14), stability condition of closed-loop system can be obtained as:
\[
B|\omega^*| + |T_L| \leq k_r k_i i_{q_{max}} \tag{23}
\]
In a word, stability condition of closed-loop system is:
\[
B|\omega^*| + |T_L| \leq k_r k_i i_{q_{max}} \tag{24}
\]

5 Parameter tuning of Variable Structure PI Controller

According to the block diagram of speed controller with integral separation in Fig.1, the transfer function of the output \(U(s)\) and the speed input \(R(s)\) is obtained as follows:
\[
\frac{U(s)}{R(s)} = k_p H(s) \tag{25}
\]
where
\[
H(s) = \frac{s^2 + (\frac{B}{J} + \frac{k}{J}k_p s^2 + \frac{Bk_i}{J} k_p)}{s^2 + (\frac{B}{J} + \frac{k}{J}k_p s^2 + \frac{Bk_i}{J} k_p)} \tag{26}
\]
The output of the PI controller is less than \(i_{q_{max}}\), so it can be obtained that:
\[
k_p |\omega^*| \leq k_p |\omega| \leq i_{q_{max}} \tag{27}
\]
Combining equation (25) with (27):
\[
|U(s)| = |k_p R(s) H(s)| \leq i_{q_{max}} |H(s)| \leq i_{q_{max}} \tag{28}
\]
So the transfer function \(H(s)\) should satisfy the following equation:
\[
|H(j\omega)| \leq 1 \quad \forall j\omega \tag{29}
\]

The PI parameter satisfying equation (29) should be expressed as \(12-13\):
\[
\begin{align*}
    k_p & \leq \frac{B}{k_r} \\
    k_i & \leq \frac{k_p^2}{Jk_r} (\sqrt{2} - 1)
\end{align*}
\]
Combining with equation (10), the parameter of the variable PI controller can be obtained as:
\[
\begin{align*}
    k_p & \geq \frac{B}{k_r} \\
    k_i & \geq 2k_p \\
    k_i & \leq \frac{k_p^2}{Jk_r} (\sqrt{2} - 1) \\
    k_i & \leq \frac{k_p^2}{Jk_r^2} (\sqrt{2} - 1)
\end{align*}
\]

6 Experiment Results

In order to verify the efficiency of the proposed controller, the closed-loop control system of PMSM is built on the platform of digital controller TMS320LF2812 and EPM1270T144 CPLD. The block diagram of speed control is shown in Fig.4 and the experiment platform is shown in Fig.5. Parameters of PMSM used in the experiment are given as follows: armature resistance \(R=1.6\, \Omega\), armature inductance of d axis \(L_d=16.03\, mH\), armature inductance of q axis \(L_q=17.15\, mH\), rotor flux linkage \(\phi_f=0.16\, Wb\), rotor inertia \(J=1.1*10^{-3}\, kgm^2\), number of poles \(P=3\), rated speed \(n=2000\, rpm\), rated current 6.5A, rated torque 4.5Nm. The position photoelectric encoder are 2500 lines.

![Fig.5 Block diagram of speed control](image-url)
The PWM carrier frequency is 16KHZ and the period is 62.5μs. The parameters of PI current controller are as follows: the proportional gain $k_p = 1.3$, the integral gain $k_i = 0.05$. The velocity loop frequency is 1KHZ and the period is 1ms. The limit output of speed loop is rated current 6.5A.

The parameters of traditional PI velocity controller are as follows: the proportional gain $k_p = 15$, the integral gain $k_i = 0.3$. The parameters of variable structure PI velocity controller are as follows: the maximum proportional gain $k_{p1} = 15$, the minimum proportional gain $k_{p2} = 15$, the maximum integral gain $k_{i1} = 0.3$, the minimum integral gain $k_{i2} = 0.15$.

Fig. 6 shows the response curves of the variable structure PI controller and the traditional PI controller when the speed step command is 1200r/min with no load. It can be seen that the settling time of traditional PI controller is 121ms and the overshoot is 4%; The settling time of variable structure PI controller is 109ms without overshoot. The parameters of variable structure PI speed controller are the minimum proportional gain and the minimum integral gain in the starting phase in order to prevent overshoot. It is regulated adaptively by a linear function near the desired speed until it reached the maximum proportional gain and maximum integral gain. However, it is easy to produce overshoot because the traditional PI controller parameters remain unchanged.

Fig. 7 shows the response curves of the variable structure PI controller and the traditional PI controller when the speed step command is 1200r/min with constant load 2Nm. It can be seen that the settling time of traditional PI controller is 148ms and the overshoot is 2%; The settling time of variable structure PI controller is 130ms without overshoot. Comparing with the experimental results with no load, it can be concluded that control time of two controllers increase because the moment of inertia increases with load torque. However, the traditional PI controller becomes worse due to fixed parameters.

Fig. 8 shows the response curves of the variable structure PI controller and the traditional PI controller when the speed step command is 1200~500r/min with no load. The traditional PI controller achieves the steady state at 1068ms and the overshoot is 5.2%; The variable structure PI controller achieves the steady state at 1052ms without overshoot. It is easy to produce overshoot because the traditional PI controller parameters remain unchanged.

Fig. 9 shows the response curves of the variable structure PI controller and the traditional PI controller when the speed step command is 1200~500r/min with constant load 2Nm. The traditional PI controller achieves the steady state at 1082ms and the overshoot is 6%; The variable structure PI controller achieves the steady state at 1060ms without overshoot.
overshoot. From Fig. 5, Fig. 6, Fig. 7 and Fig. 8, it can be demonstrated that the variable structure PI controller can improve the dynamic performance and the steady state accuracy compared with the traditional PI controller.

7 Conclusion

In order to solve the problem that the traditional PI controller is often unable to satisfy the fast response without overshoot for speed control of AC servo system, the variable structure PI (V SPI) controller is proposed in this paper. The stability conditions of the closed-loop system are given according to Lyapunov stability criterion. The experiment results show that the variable structure PI controller can solve the contradiction between the overshoot and the fast response.

REFERENCES