Optimal Traffic Sensor Location for Origin–Destination Estimation Using a Compressed Sensing Framework

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***Abstract*— A series of flow estimation problems, especially origin–destination estimation, involves optimally locating sensors on a transportation network to measure traffic counts. As compressed sensing (CS) provides a new method to solve the estimation problem, its sensor location strategy needs to be researched in order to facilitate the reconstruction. This paper first points out that the accurate flow recovery is difficult by introducing a necessary condition, and then categorizes the location determination into two cases: sensor number with restriction and without restriction. For both cases, we elucidate their theoretical foundations of locating methods and propose an algorithm based on column coherence minimization, which optimally facilitates the reconstruction for CS framework. Numerical experiments indicate that the selected sensor locations fit the flow recovery and the proposed algorithm, compared with other methods, can lead to a slightly better result under certain observations.**

***Index Terms*— Traffic sensor location, traffic flow estimation, compressed sensing.**

NOTATIONS

$\lvert\cdot\rvert$	absolute value if $\cdot$ is a real number; cardinality if $\cdot$ is a vector or set.
$\lVert\cdot\rVert_p$	ℓ_p norm.
$Rank(\cdot)$	rank of a matrix.
$\mu(\cdot)$	column coherence of a matrix.
$<\cdot,\cdot>$	inner product of two vectors.
$G(N_0, A, A_c)$	abstract road network, N_0: node set, A: link set, A_c: observable link set.
X	sensor located link set ($X \subseteq A_c$) or the traffic flow measurement vector.
x_i	traffic counts from link i.
V	original traffic flow vector.
v_j	the j-th entry of the vector V.
N	dimension of V.
P	assignment matrix, also measurement matrix.
p_{ij}	cell in P, $p_{ij} \in [0, 1]$.
$p_{i\cdot}$	the i-th row of P, $p_{i\cdot} = [p_{i1} \cdots p_{iN}]$.
m	number of sensors.
L	$N \times N$ transformation basis for V.
W	coordinates of V under the basis L.
P_c	assignment matrix for A_c.
M	number of rows for P_c.
k	sparse degree of V.
Φ	CS matrix for P, $\Phi = P \cdot L$.
$\tilde{\varphi_{\cdot j}}$	the j-th column in Φ.
Φ_c	CS matrix for P_c, $\Phi_c = P_c \cdot L$.
Φ_{ccom}	$\lvert P_c\rvert \times \binom{N}{2}$ temporary matrix.
N_{com}	number of columns of Φ_{ccom}, $N_{com} = \binom{N}{2}$.
Φ_m	temporary matrix constructed by selecting m rows from Φ_{ccom}.
φ_{ij}	cell in Φ_m or Φ_{ccom}.
$\mathbb{C}^{m\times N}$	$m \times N$ complex matrix.
$\mathbb{R}^{m\times m}$	$m \times m$ real matrix.

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I. INTRODUCTION

FLOW estimation (such as Origin-Destination (OD), link, route flow) is crucial for transportation planning and traffic management. This type of problems consists of inferring the whole flow vector by given available traffic count measurements from a subset of links. Mathematically, these three kinds of flow estimations are similarly formulated as an under determined linear system according to the flow conservation constraints, among which OD estimation is the most representative [1]–[3]. It is solved by various models like Maximum Likelihood, Generalized Least Squares, Bayesian Inference, Bi-level Programming, etc [4]–[7]. Recently, Compressed Sensing (CS) has been employed into this issue [8], [9]. It seeks an appropriate transformation basis to convert the original flow vector into a sparse one approximately and reconstructs it by ℓ_1-minimization with a proper measurement matrix.

In the estimation process, a sub-problem—the optimal sensor deployment—cannot be avoided and has been researched for decades. The problem arises because the transportation agencies, in practice, face budget constraints in implementing comprehensive sensor deployment plans, and assuming the full observability of link flows is unreasonable. Meanwhile, due to the linear dependence of some link traffic counts, full observation is extravagant and unnecessary. In general, optimal sensor location issues can be classified into two categories: those focusing on the algebraic and topological properties of the network structure, represented by [10]–[12], and those associating with observed traffic states (traffic flow is mainly focused in this paper) as well as the estimation techniques [13], [14]. This paper mainly concentrates on the latter scope, regarding ℓ_1-minimization as the reconstruction approach. The reconstruction is severely influenced by the sensor location strategy, which motivates us to conduct a detailed research. The main contributions of this paper can be listed as follows. Firstly, theoretical analysis of accurate flow reconstruction in CS framework is given, which indicates the exact recovery is much difficult unless extremely strict conditions are satisfied. Optimization is an appropriate way to improve the probability of accurate reconstruction. Secondly, sensor location determination is categorized into two cases and the location strategies for each are presented. Since the problem is NP-hard, an approximate algorithm based on column coherence minimization is designed. Compared with our previous work, column coherence minimization leads to a much more complex case but a more accurate reconstruction [8]. Thirdly, detailed numerical experiments and a comparative analysis are given.

The remainder of the paper is organized as follows: In the next section, an investigation of the sensor deployment rules and optimization models is shown. Section III gives necessary and sufficient conditions of flow estimation using CS, and points out that the accurate reconstruction is difficult. Section IV demonstrates the analytical process and general principles of sensor location determination for sparse recovery. The algorithm based on column coherence, which can promote the probability of reconstruction rather than row coherence, is also designed in this section. To illustrate the algorithm, Section V shows the numerical experiments with the interpretation of the results. Comparative analysis with other determination rules is conducted as well. Section VI concludes this paper and puts several additional discussions.

II. Literature Review

A. Traffic Sensor Location

Traditionally, the traffic sensor location problem has been mostly addressed as a sub-problem of the broader OD demand estimation, rather than as an independent problem in the context of link-based applications. Lam and Lo proposed the traffic flow volume coverage principles for determining the priority of link observation in 1990 [15]. Yang *et al.* [16] analyzed the reliability of OD estimation based on link flows. They put forward the maximum possible relative error (MPRE) as the evaluation indicator between estimated and actual flows. If a particular OD pair is not covered by the observations, the resulting MPRE will be infinite as well. This leads to the proposition of covering rule which tries to guarantee that a portion of each demand flow is observable. Bianco et al. gave an iterative two-stage procedure and several priority-based greedy heuristics to determine the sensor locations [17]. For the screen line-based traffic-counting location problem, Yang *et al.* [18] further extended the MPRE concept and considered how to decide the minimum number of counting stations with their optimal locations. Bierlaire [19] presented a similar "total demand scale (TDS)" metric to calculate the difference between the maximum and minimum possible total demand estimates in a polyhedron constrained by traffic measurements. Chen *et al.* [20] adopted the TDS metric to evaluate the quality of estimated OD demand and to calculate the possible traffic counting station locations. Yang and Zhou [21] further defined four location rules in terms of geographical connectivity. Yim and Lam [22] tested a number of rules in several large traffic networks. Based on the entropy measure proposed by Van Zuylen and Willumsen [23], Chung [24] introduced an optimum sampling framework to take into account the information content of the prior OD estimate. Furthermore, Ehlert *et al.* [25] proposed a second-best location procedure to select informative links in a traffic network with partial detector coverage. Eisenman *et al.* [26] proposed a Kalman filtering-based conceptual framework to characterize the error propagation dynamics in estimation, and they developed a simulation-based approach to numerically assess the value of point sensors for real-time network traffic estimations and predictions in a large scale network. In general, principles of sensor location deployment are summarized as follows [27].

- R1: (OD covering rule [16]) Sensors should be located on links so that some positive fraction of trips between any OD pair is observed. In words, each OD pair is covered by the observation set.
- R2: (Maximal flow fraction rule [21]) Sensors should be located on links so that, for each covered OD pair, the flow fraction between this OD pair out of flows on these links is as large as possible.
- R3: (Route covering rule [18], [28]) Sensors should be located on links so that, for each OD pair, all the routes connecting this OD should be covered.
- R4: (Maximal flow intercepting rule [21]) Sensors should be located on links so that the observed flow is as much as possible.
- R5: (Maximal OD demand fraction rule [29]) Sensors should be located on links that maximize the sum of intercepted OD demand fraction. This rule is similar to R2 but the demand fraction is computed as the ratio between the route flow on an observed link of an OD pair and the number of trips of the OD pair itself, rather than against the flow on the observed link.
- R6: (Maximal net route flow captured rule [21]) Sensors should be located on links so that, for a given number of observations, the best location set is the one that captures the largest net OD route flows.
- R7: (Maximal net OD flow captured rule [22]) Sensors should be located on links so that, for a given number of observations, the best location set is the one that captures the largest number of net OD trips.
- R8: (Link independence rule [21]) Sensors should be located on links so that the resultant traffic flows on the chosen links should be linearly independent.
- R9: (Minimal observation rule [30]) Sensors should be as few as possible on the premise that the feasibility and accuracy of OD estimation are satisfied.

It should be noted that these rules may not be obeyed simultaneously for a specific network. Some of them are even contradictory in certain cases. In practice, which rules to be satisfied depends on the focus of the flow estimation model. Yang et al. calculated the minimum observation set that covered all the OD pairs and captured the maximal net route flow, considering R1 and R6 [16], [21]. Similarly, Yang and Ma established the constraints for R3 and selected the observed links covering all routes [18], [28]. When the number of available sensors is not enough to cover all the OD pairs, the objective could be the maximization of the total number of OD pairs that can be covered, as discussed by Elhert *et al.* [25]. Yim and Lam [22] maximized the total net flow captured (R7) by locating k counting sensors on the links. Larsson *et al.* [31] adopted a constrained optimization model to locate at most k counting sensors on the links so that the total intercepted flow is maximized (R4). In their research, however, no attention was paid to link flow dependencies (R8) and to double counting. For R3, Yang *et al.* [32] further incorporated time into the objectives and proposed a multi-level optimization model as well as its genetic solution algorithm. Another scholar considered the time dimension is Fei, who focused on the distinction of travel patterns and defined the optimal locations in this case [33]. Wang *et al.* [13] directly introduced OD estimation error rather than MPRE into location determination explicitly. In his model, a confidence level of the priori route flow was designated at first and the traffic counting links were determined by reducing the uncertainty of the priori link information.

While significant progress has been made in formulating and solving the sensor location problem, several challenging theoretical and practical issues remain to be addressed. CS has brought an innovative framework for traditional flow estimation. It develops to recover the sparse flow vector from a group of lower dimensional measurements. This property stimulates scholars to study its applications on traffic flow estimation.

B. Compressed Sensing

Originally derived from signal processing, Compressed Sensing (or Compressive Sensing) was first proposed by Donoho [34]. Its basic thought is to recover original signal vector directly from an under sampled measurement. Similar problems are encountered in other fields, and the use of ℓ_1-minimization and related methods was greatly popularized with the work of Tibshirani on the so-called LASSO (Least Absolute Shrinkage and Selection Operator) [35]. In CS, two crucial problems, reconstruction and measurement, have been received most concentrations. The following two paragraphs will give a brief summary of them.

Let $x \in \mathbb{C}^N$ be an original signal vector and $y \in \mathbb{C}^m$ be an under determined measurement, that is $y = Ax$ with A called compressed sensing matrix or measurement matrix ($m \ll N$). If x is k-sparse, then it can be calculated through $\min_x \parallel x \parallel_0,\ s.t.\ Ax = y$. Unfortunately, it is NP-hard in general [36], [37]. Thus two convex relaxations—ℓ_1-minimization (also called basis pursuit) and greedy algorithms (such as various matching pursuits)—have been proposed [39] and [41]. Quite surprisingly for both types of approaches, various recovery results are available, which provide conditions on the matrix A and on the sparsity $\parallel x \parallel_0$ such that the recovered solution coincides with original x. These approaches partially give a solution of the reconstruction.

The two convex relaxations above are not contradictory to the NP-hardness of ℓ_0-minimization, since they are only effective to a subclass of matrices A. This leads to another central problem: the design of measurement matrix. Candès and Tao [41] pointed out that for a real-valued case, a sufficient condition of sparse recovery is A satisfies Restricted Isometry Property (RIP). In signal processing, a particular type of random matrices, such as the Gassian matrix, Bernoulli Matrix or Partial Fourier Matrix, is often used due to its RIP satisfaction with high probability [42]. Yet how to acquire a deterministic and explicit measurement matrix seems more desirable and complicated.

When CS applied to traffic flow estimation, the measurement matrix cannot be generated randomly. It depends on the deployment of traffic counting sensors. Moreover, the reconstruction requires the counting link set to meet some particular conditions. Consequently, starting from accurate flow reconstruction, how to determine the optimal traffic sensor locations is studied in the rest of this paper.

III. Conditions of Accurate Flow Reconstruction Based on Compressed Sensing

Consider a road network $G(N_0, A, A_c)$. The OD flow estimation from traffic counts is usually formulated as a group of under-determined measurements in CS framework:

$$X = P \cdot V = P \cdot L \cdot W = \Phi \cdot W \tag{1}$$

where Φ is called compressed sensing matrix and L is a full rank transformation basis. When W is sparse, it is believed that we can reconstruct the original solution via ℓ_1-minimization. However, accurate reconstruction requires Φ to meet particular conditions. The proposition below shows a necessary condition of this.

Proposition 1: Let W be a k-sparse vector, and $X = \Phi \cdot W$ is a lower dimensional linear measurement. Φ is an $m \times N$ measurement matrix. If the original W can be accurately reconstructed from the measurement, then any $2k$ columns from Φ are linearly independent.

Proof: Suppose not. There are $2k$ linear correlated columns in Φ. Denote $\Phi = (\tilde{\varphi}_{\cdot 1}, \tilde{\varphi}_{\cdot 2}, \cdots, \tilde{\varphi}_{\cdot N})$. Without loss of generality, assume the first $2k$ columns are correlated. Then $\exists \alpha_1, \alpha_2, \cdots, \alpha_{2k} \in \mathbb{R}$ satisfy

$$\sum_{i=1}^{2k} \alpha_i \cdot \tilde{\varphi}_{\cdot i} = 0 \tag{2}$$

where $\alpha_1, \alpha_2, \cdots, \alpha_{2k}$ are constants and do not all equal to zero. Consider two particular vectors:

$$W^{(1)} = [\alpha_1, \cdots, \alpha_k, 0, \cdots, 0]^T$$
$$W^{(2)} = [0, \cdots, 0, -\alpha_{k+1}, \cdots, -\alpha_{2k}]^T$$

Thus Eq. (2) can be written as

$$\Phi[\alpha_1, \cdots, \alpha_k, 0, \cdots, 0]^T = \Phi[0, \cdots, 0, -\alpha_{k+1}, \cdots, -\alpha_{2k}]^T$$
$$\Rightarrow \Phi \cdot W^{(1)} = \Phi \cdot W^{(2)}$$

Since α_i are not all zeros, we have $W^{(1)} \neq W^{(2)}$. This indicates two different k-sparse vectors are mapped into an identical measurement through Φ. Thus accurate reconstruction is impossible, which brings a contradiction. ■

Prop. 1 shows that if we want to recover an original k-sparse flow vector exactly, the number of independent observations should be at least $2k$. In general, this condition is usually not satisfied because compared with the original flow entries, the independent observable links are much fewer. More extremely, in case that the original flow vector is not sparse, we need to search for a transformation basis to complete a "compress" process. Indeed, in order to achieve this goal, one may simply store only the k largest coordinates under the transformation (called the best k-term approximation), while reconstructing V from its compressed version. The non-stored coordinates are simply set to zero. It is obvious that more reconstruction of entries requires more independent observations.

The sufficient condition of accurate recovery is called RIP, which concerns about the restricted isometry constant defined as below [41].

Definition 1: For an $m \times N$ measurement matrix Φ, the k-restricted isometry constant δ_k of Φ is the smallest quantity such that

$$(1 - \delta_k)\|W\|_2^2 \leq \|\Phi W\|_2^2 \leq (1 + \delta_k)\|W\|_2^2 \tag{3}$$

holds for all k-sparse vectors W.

If $\delta_k \in (0, 1)$, then we call Φ satisfies the RIP. A matrix having a small restricted isometry constant is beneficial for the reconstruction. This means every subset of k or fewer columns is approximately an orthonormal system. Note that the RIP cannot guarantee the unique recovery of original sparse vector by ℓ_1-minimization unless the restricted isometry constant is small enough. One particular sufficient condition is [43]:

$$\delta_k + \delta_{2k} + \delta_{3k} < 1 \tag{4}$$

Unfortunately, this condition is also an extremely strict requirement for Φ, which can hardly be achieved in the transportation application. Nevertheless, it is still the basic principle of sensor location deployment. Minimizing the restricted isometry constant can lead to an optimal set of observation that facilitates the sparse recovery.

IV. Traffic Sensor Location Determination

The major problem of sensor location determination is to search a particular $X \subset A_c$ as the observed link set, so that the sparse recovery by ℓ_1-minimization from Eq. (1) becomes optimal. As mentioned before, limited by the financial budget practically, distributing large quantities of detectors may not be possible. The numbers of sensors may vary in different situations. So we categorize this problem into two cases below.

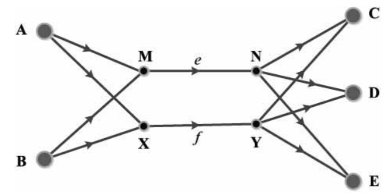

Fig. 1. A simple network.

A. Sensor Number Without Restriction

For each link in G, there exists a measurement row vector. For example, as the simple network shown in Fig. 1, the linear measurements can be written as

$$\begin{pmatrix} x_e \\ x_f \end{pmatrix} = \begin{pmatrix} \alpha_{AC} & \alpha_{AD} & \alpha_{AE} & \beta_{BC} & \beta_{BD} & \beta_{BE} \\ 1-\alpha_{AC} & 1-\alpha_{AD} & 1-\alpha_{AE} & 1-\beta_{BC} & 1-\beta_{BD} & 1-\beta_{BE} \end{pmatrix} V'$$

where $V' = [u_{AC}, u_{AD}, u_{AE}, u_{BC}, u_{BD}, u_{BE}]^T$ is the OD vector. The corresponding measurement vector of link e is $[\alpha_{AC}, \alpha_{AD}, \alpha_{AE}, \beta_{BC}, \beta_{BD}, \beta_{BE}]$, which represents the flow proportion assigned to link e from each OD pair. It is clear that for each link of the road network, a measurement row vector can be uniquely identified in this way. Generally, let $p_{i\cdot} = [p_{i1}, p_{i2}, \cdots, p_{iN}]$ be the measurement vector of link i. And the measurement vector sets associated with A_c and X can be constructed respectively. We denote the two sets as P_c and P. When there is no restriction imposed on sensor number, it means all of the rows in P_c can be selected.

From traffic measurement, we always hope to obtain maximum original flow information from X. In order to avoid redundant information, the sensor number should be $Rank(P_c)$. Because for each $P \subset P_c$ that satisfies $Rank(P) = Rank(P_c)$, the remaining measurement vectors in P_c are all in the space spanned by P and thus can be linearly represented. These observations are linear combinations of X. Actually, the rule R8 previously mentioned in Section II describes this principle. The following two propositions manifest that each X compatible with $Rank(P) = Rank(P_c)$ is equivalent in this sense, and the maximal sensor number without redundant information is $Rank(P_c)$.

Proposition 2: Let V be a k-sparse $N \times 1$ column vector. $(X_1 \subseteq A_c, P_1 \subseteq P_c), (X_2 \subseteq A_c, P_2 \subseteq P_c)$ are two groups of observations: $X_1 = P_1 \cdot V$, $X_2 = P_2 \cdot V$. The dimensions of X_1, X_2 and P_1, P_2 are $m \times 1$ and $m \times N$ $(m < N)$. Assume $Rank(P_1) = Rank(P_2) = Rank(P_c) = m$. Then, the sparse solutions V_1 and V_2 under ℓ_1-reconstruction are equivalent $(V_1 = V_2)$.

Proof: It is only required to prove that the reconstructions from the two groups of observations have identical solution spaces. From $m = Rank(P_1) = Rank(P_c)$, we know that there is at least one group of m linearly independent row vectors in P_1. And these vectors also belong to P_c according to $P_1 \subseteq P_c$, thus construct a basis of P_1 and P_c both. Therefore, all the row vectors in P_c can be linearly represented by P_1 (otherwise, $Rank(P_c) > m$). Particularly, P_2 can be represented by P_1 as well for $P_2 \subseteq P_c$. Thus $\exists T \in \mathbb{R}^{m\times m}$, $Rank(T) = m$, that satisfies $P_1 = T \cdot P_2$. Based on this, we have

$$X_1 = P_1 \cdot V = T \cdot P_2 \cdot V = T \cdot X_2$$

Hence,

$$\begin{aligned} V_1 &= \underset{X_1=P_1\cdot V}{\arg\min} \left\| V \right\|_1 = \underset{T^{-1}X_1=T^{-1}P_1\cdot V}{\arg\min} \left\| V \right\|_1 \\ &= \underset{X_2=P_2\cdot V}{\arg\min} \left\| V \right\|_1 = V_2 \end{aligned}$$

■

Proposition 3: Let V be a k-sparse $N \times 1$ column vector. $(X_1 \subseteq A_c, P_1 \subseteq P_c), (X_2 \subseteq A_c, P_2 \subseteq P_c)$ are two groups of observations: $X_1 = P_1\cdot V$, $X_2 = P_2\cdot V$. The dimensions of X_1, X_2 and P_1, P_2 are $m \times 1$, $n \times 1$, $m \times N$, $n \times N (m < n < N)$, respectively. Assume $Rank(P_1) = Rank(P_2) = Rank(P_c) = m$. Then, the sparse solutions V_1 and V_2 under ℓ_1-reconstruction are equivalent ($V_1 = V_2$).

Proof: Without loss of generality, the sequence of the equations in the second observation can be adjusted and expressed in partitioned blocks as

$$\begin{pmatrix} X_m \\ X_{n-m} \end{pmatrix} = \begin{pmatrix} P_2^m \\ P_2^{n-m} \end{pmatrix} \cdot V \tag{5}$$

Where $Rank(P_2^m) = m$. Therefore, we can eliminate the redundant equations via Gaussian elimination and get

$$\begin{pmatrix} X_m \\ \mathbf{0}_{n-m} \end{pmatrix} = \begin{pmatrix} P_2^m \\ \mathbf{0}_{n-m} \end{pmatrix} \cdot V \tag{6}$$

Obviously, the solution spaces of Eq. (5) and Eq. (6) are identical. According to Prop. 2, Eq. (6) is equivalent to $X_1 = P_1 \cdot V$. Hence,

$$V_1 = \underset{X_1=P_1\cdot V}{\arg\min} \left\| V \right\|_1 = \underset{X_2=P_2\cdot V}{\arg\min} \left\| V \right\|_1 = V_2$$

■

Prop. 2 and Prop. 3 indicate that any link set with full rank can extract equivalent information when the sensor number is not with any restriction. Yet when the sensor number limited within a range below $Rank(P_c)$, it will lead to a quite different situation.

B. Sensor Number With Restriction

As mentioned before, the maximal sensor number that can be deployed in the network is usually limited due to the budget constraints. Suppose the maximal number of traffic sensors is m. If $m \geq Rank(P_c)$, it is equivalent to the case without restriction. When $m < Rank(P_c)$, which means that the maximal non-redundant observations provided by P_c are not available completely, maximum linearly independent information is still preferable. Thus a particular set of m linearly independent links should be selected.

In Section III, minimizing the restricted isometry constant is set to be the basic principle of sensor location deployment. Direct study of this constant, however, is relatively complex. Instead, scholars focus on the coherence which is easier to handle.

Definition 2 (Coherence [44]): Suppose $\Phi \in \mathbb{C}^{m\times N}$, $\Phi = (\tilde{\varphi}_{\cdot 1}, \cdots, \tilde{\varphi}_{\cdot N})$, $m \leq N$ and $\|\tilde{\varphi}_{\cdot i}\|_2 = 1 (i = 1, 2, \cdots, N)$. Then the column coherence of Φ is defined as

$$\mu(\Phi) = \max_{i\neq j} \left| < \tilde{\varphi}_{\cdot i}, \tilde{\varphi}_{\cdot j} > \right|$$

The relationship between restricted isometry constant and coherence is shown by Bourgain *et al.* [45]:

Theorem 1: Suppose the coherence of $\Phi = (\tilde{\varphi}_{\cdot 1}, \cdots, \tilde{\varphi}_{\cdot N}) \in \mathbb{C}^{m\times N} (m \leq N)$ is μ. Then, Φ satisfies the RIP of order k with the restricted isometry constant $\delta_k \leq (k-1)\mu$.

Obviously, the restricted isometry constant will decrease when μ becomes smaller, thus facilitates the recovery of W in Eq. (1). According to this fundamental theoretical basis, the optimal measurement matrix P should have the smallest $\mu(\Phi)$ and satisfy $Rank(P) = m$ simultaneously. This problem can be mathematically written as

$$P^* = \arg\min_P \mu(P \cdot L), \quad s.t.\ P \subset P_c$$

The following algorithm gives an approximate searching strategy.

Input: abstract road network $G(N_0, A)$; observable link set A_c; sensor number m; sparse transformation basis L.

Output: sensor deployed link set X.

Step 1: calculate the measurement row vector set P_c for A_c according to the traffic assignment proportions.

Step 2: if a particular row $P_{i\cdot} = \mathbf{0}$ in P_c, the i-th link cannot cover any original flow pair and should be deleted from P_c.

Step 3: if $m \geq Rank(P_c)$, let $m = Rank(P_c)$.

Step 4: search the link set with minimal $\mu(\Phi)$. Since it is an NP-hard problem, we attempt to acquire a near-optimal solution in order to avoid the global searching.

Step 4.1: preprocessing. Calculate $\Phi_c = P_c\cdot L$. Establish a new temporary matrix Φ_{ccom}, whose columns are the multiplication in each entry of any two columns from Φ_c. That is

$$\Phi_c = \begin{pmatrix} \varphi_{c11} & \cdots & \varphi_{c1N} \\ \vdots & \ddots & \vdots \\ \varphi_{cM1} & \cdots & \varphi_{cMN} \end{pmatrix}$$

$$\Phi_{ccom} = \begin{pmatrix} \varphi_{c11}\cdot\varphi_{c12} & \cdots & \varphi_{c12}\cdot\varphi_{c13} & \cdots & \varphi_{c1(N-1)}\cdot\varphi_{c1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \varphi_{cM1}\cdot\varphi_{cM2} & \cdots & \varphi_{cM2}\cdot\varphi_{cM3} & \cdots & \varphi_{cM(N-1)}\cdot\varphi_{cMN} \end{pmatrix}$$

where M is the number of rows in P_c. Clearly, Φ_{ccom} has M rows as well, and its number of columns is $N_{com} = \binom{N}{2}$ (N is the dimension of L). Here we define another new temporary matrix Φ_m. This matrix is constructed by selecting m rows from Φ_{ccom}. For example, if we choose the first m rows of Φ_{ccom}, the Φ_m will be

$$\Phi_m = \begin{pmatrix} \varphi_{c11}\cdot\varphi_{c12} & \cdots & \varphi_{c1(N-1)}\cdot\varphi_{c1N} \\ \vdots & \ddots & \vdots \\ \varphi_{cm1}\cdot\varphi_{cm2} & \cdots & \varphi_{cm(N-1)}\cdot\varphi_{cmN} \end{pmatrix} \tag{7}$$

Through the row ordinals $(1, 2, \cdots, m$ in current example), this Φ_m determines a measurement matrix and

$$P = \Phi \cdot L^{-1} = \begin{pmatrix} \varphi_{c11} & \cdots & \varphi_{c1N} \\ \vdots & \ddots & \vdots \\ \varphi_{cm1} & \cdots & \varphi_{cmN} \end{pmatrix} \cdot L^{-1} \tag{8}$$

And the related sensor deployed links are link 1 to m. According to Def. 2, the column coherence of Φ can be denoted as

$$\mu(\Phi) = \max_j \left| \sum_i \varphi_{ij} \right|, \varphi_{ij} \in \Phi_m \tag{9}$$

Since the rows of Φ_m all come from Φ_{ccom}, each cell φ_{ij} in Φ_m also equals to the corresponding element in Φ_{ccom}. Note that our objective is to minimize $\mu(\Phi)$. Thus in a general case, the problem is converted into seeking the optimal Φ_m that satisfies

$$\Phi_m^* = \underset{\Phi_m \subset \Phi_{ccom}}{\arg\min}\ \mu(\Phi_m)$$

Unfortunately, it is NP-hard. The following two steps are an iterative algorithm to solve this optimization approximately.

Step 4.2: initial solution. For convenience, we denote

$$\varphi_{i\cdot}^{\max} = \max_j |\varphi_{ij}|$$

which means $\varphi_{i\cdot}^{\max}$ is the maximum cell in the i-th row of Φ_m or Φ_{ccom}. The initial solution Φ_m is determined by selecting the m rows which have minimal $\varphi_{i\cdot}^{\max}$ from Φ_{ccom} and the Φ determined by this Φ_{ccom} is linearly independent. For example, without loss of generality, suppose the rows of Φ_{ccom} satisfy $\varphi_{1\cdot}^{\max} \le \varphi_{2\cdot}^{\max} \le \cdots \le \varphi_{M\cdot}^{\max}$. The initial Φ_m will start with its first m rows as Eq. (7) shows. In order to guarantee the linear independence, the compressed sensing matrix Φ determined by the initial Φ_m has to be checked. If it is not full ranked, we should replace one selected row by increase the total sum of $\varphi_{i\cdot}^{\max}$ in Φ_m most slightly. That is replacing the m-th row with the $(m+1)$-th row as

$$\Phi_m = \begin{pmatrix} \varphi_{c11} \cdot \varphi_{c12} & \cdots & \varphi_{c1(N-1)} \cdot \varphi_{c1N} \\ \vdots & \ddots & \vdots \\ \varphi_{c(m-1)1} \cdot \varphi_{c(m-1)2} & \cdots & \varphi_{c(m-1)(N-1)} \cdot \varphi_{c(m-1)N} \\ \varphi_{c(m+1)1} \cdot \varphi_{c(m+1)2} & \cdots & \varphi_{c(m+1)(N-1)} \cdot \varphi_{c(m+1)N} \end{pmatrix} \tag{10}$$

If the Φ determined by the above Φ_m is still not full ranked, then repeat the replacement. Since $m \le Rank(P_c)$, we will always get a Φ_m with a full ranked Φ. It is our initial solution. After the initial solution acquired, if Φ_{ccom} has some rows whose $\varphi_{i\cdot}^{\max}$ is smaller than the maximal $\varphi_{i\cdot}^{\max}$ in Φ_m, then delete these rows with their correspondence in P_c. This is because the Φ containing these rows are proved to be linearly dependent during our selection process. For convenience, the new Φ_{ccom} and P_c after deletions are still denoted by the original symbol.

Step 4.3: update. On the basis of Eq (9). We further assume

$$\mu(\Phi) = \max_j \left| \sum_i \varphi_{ij} \right| = \left| \sum_i \varphi_{ij'} \right|$$

which means the coherence is determined by the j'-th column. Our central task is to update the rows in Φ_m, so that $\mu(\Phi)$ decreases during the update process. It can be divided into three cases:

1° $\sum_i \varphi_{ij'} = 0$. It immediately results in $\mu(\Phi) = 0$ and the computation terminates;

2° $\sum_i \varphi_{ij'} > 0$. Swap the row with

$$\max\nolimits_{i,\varphi_{ij'}>0} \varphi_{ij'} \quad s.t.\ \ \varphi_{ij'} \in \Phi_m$$

from Φ_m and the row with

$$\min\nolimits_{i,\varphi_{ij'}>0} \varphi_{ij'} \quad s.t.\ \ \varphi_{ij'} \in \Phi_{ccom}, \varphi_{ij'} \notin \Phi_m$$

from Φ_{ccom}. In other words, the former formula means the row in Φ_m that contains the maximal positive cell in the j'-th column. While the latter one represents the row in Φ_{ccom} but not in Φ_m which contains the minimal positive cell in the j'-th column.

3° $\sum_i \varphi_{ij'} < 0$. Swap the row with

$$\min\nolimits_{i,\varphi_{ij'}<0} \varphi_{ij'} \quad s.t.\ \ \varphi_{ij'} \in \Phi_m$$

from Φ_m and the row with

$$\max\nolimits_{i,\varphi_{ij'}<0} \varphi_{ij'} \quad s.t.\ \ \varphi_{ij'} \in \Phi_{ccom}, \varphi_{ij'} \notin \Phi_m$$

from Φ_{ccom}. Similarly, the former formula means the row in Φ_m that contains the minimal negative cell in the j'-th column. While the latter one represents the row in Φ_{ccom} but not in Φ_m which contains the maximal negative cell in the j'-th column. Linear independence of Φ should also be examined. The updated matrix is denoted as Φ_m'. If $\mu(\Phi) > \mu(\Phi')$, then let $\Phi_m = \Phi_m'$ and go to the next round of iteration; else select the m rows from P_c which are determined by Φ_m as the output and the computation terminates.

In Step 4.2, the initial solution is based on the following theoretical estimation. On the one hand, there is

$$\begin{aligned} \mu(\Phi_m) &= \max_{1\le j\le N_{com}} \left| \sum_{i=1}^{m} \varphi_{ij} \right| \ge \frac{1}{N_{com}} \sum_{j=1}^{N_{com}} \left| \sum_{i=1}^{m} \varphi_{ij} \right| \\ &\ge \frac{1}{N_{com}} \left| \sum_{j=1}^{N_{com}} \sum_{i=1}^{m} \varphi_{ij} \right|, (\varphi_{ij} \in \Phi_m) \end{aligned}$$

On the other hand, we have

$$\begin{aligned} \mu(\Phi_m) &= \max_{1\le j\le N_{com}} \left| \sum_{i=1}^{m} \varphi_{ij} \right| \le \max_{1\le j\le N_{com}} \sum_{i=1}^{m} |\varphi_{ij}| \\ &\le \sum_{i=1}^{m} \max_{1\le j\le N_{com}} |\varphi_{ij}| = \sum_{i=1}^{m} \varphi_{i\cdot}^{\max}, (\varphi_{ij} \in \Phi_m) \end{aligned}$$

As we have selected the m rows with the smallest $\varphi_{i\cdot}^{\max}$, $\mu(\Phi)$ determined by the initial solution actually possess the minimal upper bound.

Since the searching is NP-hard, the update process may not achieve a global optimal solution. However, limited by its upper bound, the initial $\mu(\Phi)$ is relatively small. In the swap operation, the $\max_i \varphi_{ij'}$ $(\varphi_{ij'} > 0)$ or $\min_i \varphi_{ij'}$ $(\varphi_{ij'} < 0)$ may appear in any row of Φ_m. For each case, the row swap will be conducted $(|A_c| - m)$ times at most. Thus the maximal number

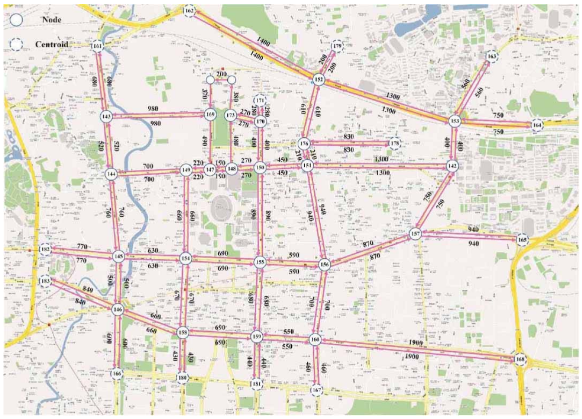

Fig. 2. Real road network.

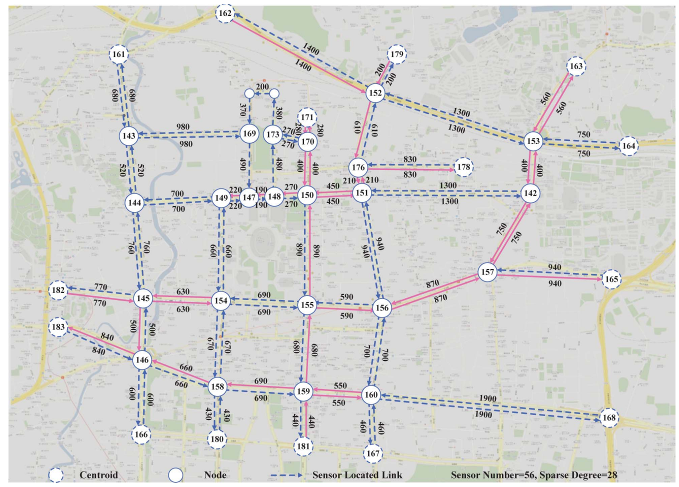

Fig. 3. Optimal sensor locations (sensor number=56, sparse degree=28).

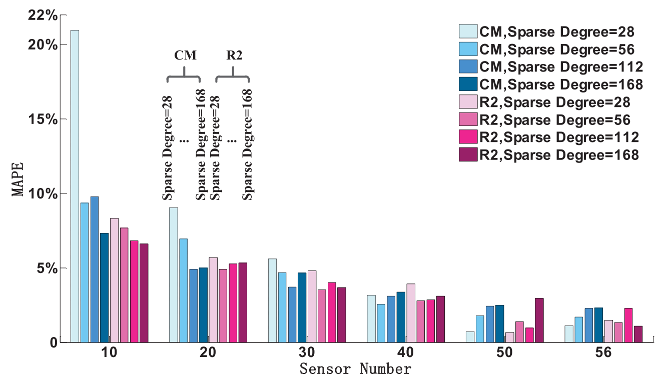

Fig. 4. MAPE of non-sparse entries from cm and R2.

of swap is $m(|A_c| - m)$. Therefore, for each $1 \leq j' \leq N_{com}$, we have $m \cdot N_{com} \cdot (|A_c| - m)$ operations at most. Obviously, the algorithm possesses polynomial complexity.

V. Numerical Experiments in a Medium Real Network

As a sub-problem, the sensor location strategy needs to be evaluated through the flow estimation. In this paper, we choose OD estimation scenario to complete this evaluation. A simplified real road network extracted from the area of Tianhe Sports Center in the city of Guangzhou in China (Fig. 2) is used to conduct our numerical tests. This area covers about 18.7 km^2, containing 93 directional links and 38 nodes. The length of each link measured from Google map is marked as well. 15 nodes on the edge are defined as centroid nodes and we have 210 OD pairs.

Several other experimental parameters should be clearly specified before the introduction of our results. Firstly, User Equilibrium (UE) traffic assignment strategy is adopted. This will simulate more realistic transportation situations, despite its measurement matrix is much more complicated in contrast with all-or-nothing assignment whose measurement is simply a 0-1 matrix. Secondly, four types of OD demands with different sparse degrees are set to investigate the effects of sensor location determinations and estimation results. This mainly aims to test when the original OD is not sparse enough, how "accurate" can the reconstruction reach through ℓ_1-minimization. As the rank of total measurements is 56, we set the sparse degrees to be 28, 56, 112 and 168 respectively, and the integer non-sparse entries all come from a $[200, 10000]$ uniform distribution. In actual transportation networks, each pair of OD flow rarely equals to zero especially when the time interval investigated is relatively long. Thus rather than simply treating the sparse entries as zero, they are set to be uniformly distributed integers from 0 to 25% of the non-sparse entries average. Thirdly, in order to simulate the randomness, perturbations that obey Gassian distribution are added into each entry of the original sparse OD. The perturbations, however, have to be limited to a small range that will not change the feature of original OD. In our experiment, the maximal amplitude of perturbations does not exceed 20% of the entries, which means $perturb \sim Norm(0, 0.2 * EntryValue)$.

One of optimal sensor location sets generated by our algorithm is depicted in Fig. 3. Due to space limitation, full results are not presented here. The comparisons between the coherence minimization and the sensor location rules summarized in Section II are also carried out. Since our test scenario is the OD estimation, we exclude the rules for the path flows. That is, the rules R3, R5 and R6 are not considered here. In addition, the OD covering (R1), Link independence (R8) and Minimal observation (R9) rules are deemed as the basic rules which are obeyed in all of our experiments. For the rest, the R4 and R7 can be equivalently interpreted as selecting the links with its related $\| x \|_1$ maximized mathematically. This is because transit traffic flow is not employed and all of travel demands come from the centroid nodes we set. Hence we need to investigate three sensor location principles: coherence minimization (CM), R2 and R4/R7.

Fig. 4 gives the mean absolute percent errors (MAPE) of the non-sparse entries computed by CM and R2 principles, which is defined as

$$MAPE = \frac{1}{k} \sum_{v_j \neq 0} \frac{|\hat{v}_j - v_j|}{v_j} \times 100\%$$

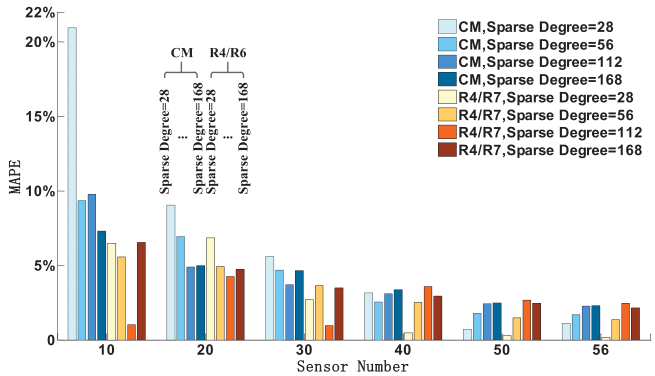

Fig. 5. MAPE of non-sparse entries from cm and R4/R7.

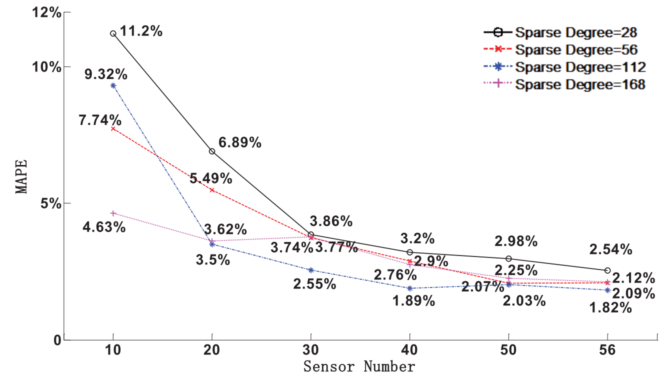

Fig. 6. MAPE of sparse entries from CM.

where $\hat{\upsilon}_j$ and υ_j are estimated and original OD demand, respectively. As one can see, the errors of tests are all below 22%. The general trend is the MAPE decreases with the sensor number grown. This phenomenon is easy to explain, because more sensors will no doubt, lead to relatively more observed information that can be used in the OD recovery. When the sensor number is too small such as 10 or 20, both algorithms (especially CM) will bring much higher reconstruction errors. It indicates that to maintain a certain number of sensors is essential for reliable OD estimation. In addition, when the sparse degree becomes larger under similar conditions, the accuracy gets lower with higher observations. Yet this pattern seems not applicable to the cases with fewer observations, which might be caused by the worse performance of reconstruction algorithms with this type of situations. At higher observation levels, CM result is particularly better than R2 for the small sparse degrees. This manifests the advantage of CM locations lies on the sparse demand operation.

The comparisons between CM and R4/R7 rules are shown in Fig. 5. The total error trend is similar with Fig. 4. And within the same group, it is more clearly that the MAPE increases during sparse degree growing. Moreover, CM locations on average outperform R4/R7 in the high observation groups. In order to evaluate whether the principal components of the demands are overwhelmed by the recovery noise, we need to investigate reconstruction results correspond to sparse entries. However, it is essential to slightly modify the MAPE evaluation criterion, due to the potential zero denominators. The MAPE for sparse entries is computed as

$$MAPE = \frac{\frac{1}{N-k}\sum_{\upsilon_j \approx 0}|\hat{\upsilon}_j - \upsilon_j|}{\frac{1}{k}\sum_{\upsilon_j \neq 0}\upsilon_j} = \frac{k\sum_{\upsilon_j \approx 0}|\hat{\upsilon}_j|}{(N-k)\sum_{\upsilon_j \neq 0}\upsilon_j} \times 100\%$$

In general, because of perturbations, the sparse entries are not strictly equivalent to 0. Thus we use $\upsilon_j \approx 0$ to represent them. The results of sparse entries are shown in Fig. 6. The indicators for the four sparse degrees stay at a low level and are all below 12%, which means the principal demands can be effectively separated. The MAPE gradually gets lower in the rise of observations under each sparse degree level. Another obvious conclusion from Fig. 6 is that, however, with the sparse degrees increase, the reconstruction becomes less robust, as the MAPE in 28 and 56 sparse degrees are stable while it fluctuates a little more in 112 and 168.

TABLE I

STANDARD DEVIATIONS FOR THREE LOCATION RULES

Spar. Deg.	Alg.	56	50	40	30	20	10
		Sensor Number					
28	CM	0.009	0.006	0.025	0.031	0.042	0.080
	R2	0.019	0.005	0.023	0.029	0.032	0.039
	R4/R7	0.001	0.002	0.003	0.018	0.038	0.033
56	CM	0.019	0.018	0.022	0.034	0.047	0.056
	R2	0.013	0.014	0.025	0.027	0.033	0.046
	R4/R7	0.014	0.013	0.019	0.022	0.030	0.033
112	CM	0.020	0.021	0.026	0.027	0.037	0.070
	R2	0.018	0.007	0.024	0.031	0.039	0.049
	R4/R7	0.019	0.019	0.026	0.007	0.033	0.008
168	CM	0.019	0.022	0.026	0.037	0.040	0.054
	R2	0.009	0.022	0.025	0.030	0.045	0.048
	R4/R7	0.017	0.020	0.023	0.030	0.039	0.051

The standard deviations of each numerical test are listed in Table I. With sensor number decreases, the standard deviations of the whole three methods all get larger. It also conforms to the fact that fewer sensors leads to less accurate recovery. At each sparse degree level, three groups of deviation values are generally closed to each other except 10 observations.

In our previous work, row coherence minimization (RM) is adopted to be the objective to design the selection algorithm [8]. In theory, this condition is weaker than the column coherence minimization (CM) and cannot strictly give the upper limit of the restricted isometry constant. In this sense, column coherence minimization is deemed more suitable for the sparse reconstruction. For these two methods, we conduct a group of test as well. As shown in Fig. 7, when the sensor number is small, the recovery results of CM are closer to RM. Yet when it gets more observations, the reconstruction accuracies of the two methods decrease totally and CM is evidently better than RM. The reduced margins of MAPE from CM compared to RM are illustrated in Table II. As can be seen, the average MAPE reduces 0.2%, 1.7% and 1.1% with 56, 50, 40 observations, whereas it slightly increases at other sensor number levels.

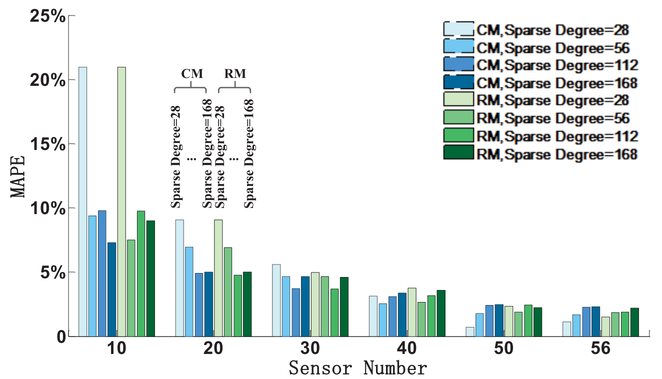

Fig. 7. MAPE of non-sparse entries for row and column coherence minimization.

TABLE II

MAPE REDUCTION BETWEEN CM AND RM (%)

	Sensor Number					
Spar. Deg.	56	50	40	30	20	10
28	0.4	1.7	0.6	-0.6	0	0
56	0.2	0.1	0.1	0	0	-1.8
112	-0.3	0.1	0.1	0	-0.1	0
168	-0.1	-0.2	0.3	0	0	1.7
Aver.	0.2	1.7	1.1	-0.6	-0.1	-0.1

VI. CONCLUSIONS AND DISCUSSIONS

Traffic sensor location is a basic problem in the OD demand or other traffic flow estimation. Compressed sensing provides us a new framework for this type of issues. Oriented to the sparse reconstruction from this framework, this paper gives theoretical foundations on traffic count measurements and provides a new algorithm based on column coherence minimization. The new algorithm can slightly promote the reconstruction accuracy compared with our previous work. Numerical tests show the proposed method has a better result than the current location rules in case that the sparse degree is somewhat high.

As mentioned in Section IV, $m \geq Rank(P_c)$ means any group of m measurements contains redundant equations. In theory, these redundant equations do not bring useful additional information that contributes to the sparse reconstruction. However, in actual transportation networks, a lot of other factors will impact the traffic count measurements. For example, the malfunctioned loop detector may provide wrong traffic count data. The recognition of vehicles by cameras will be affected seriously in rainy, foggy day or at night, which may cause a part of vehicles missing from the sensor detection. All of these potential anomalies cannot be completely avoided and are bound to bring much noise to our measurements. So the under-determined system formulated by Eq. (1) is contaminated and can hardly match the "true" theoretical model exactly. As a supplement, additional observations may provide extra information to adjust the coefficients of the equations. With the under-determined system calibrated in advance, the flow vector reconstructed from compressed sensing is expected more accurate. In this sense, more observation is beneficial.

Objectively, the hypothesis underlying CS based OD estimation is that the true OD has sparsity to some extent. This point, however, is still in dispute among scholars currently. To our knowledge, two approaches can be used to acquire a certain sparse OD demand. One is to shorten the investigated time interval appropriately. As new technological detectors have much faster sampling frequencies as well as computational performances, transportation dynamics seem slower relatively. For example, Hofleitner *et al.* [46] adopted probe vehicles data with 1 minute detection cycle to estimate the primary link travel time of San Francisco via ℓ_1 penalty method, and achieved a good result. The other is to consider time dependent traffic pattern so as to generate sparse OD matrix when the network is automated zoning. For example, it is expected there to be a few popular demands between office locations, parking lots, and so on during morning rush hours. Similarly, we may also expect most origins of flow in a business district to come from the suburbs and residential areas. Recent research from Menon *et al.* [47] has presented some achievements about this issue. And he has further assessed the viability of sparsity assumption in his work concisely. In summary, the CS based flow estimation as well as its sensor location needs more analysis and actual application validation.

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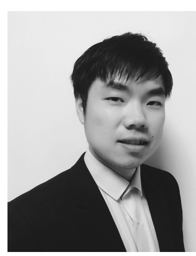

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