

Event-Driven Adaptive Robust Control of Nonlinear Systems With Uncertainties Through NDP Strategy

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Abstract—In this paper, we construct an event-driven adaptive robust control approach for continuous-time uncertain nonlinear systems through a neural dynamic programming (NDP) strategy. Through system transformation and theoretical analysis, the robustness of the original uncertain system can be achieved by designing an event-driven optimal controller with respect to the nominal system under a suitable triggering condition. In addition, it is also observed that the event-driven controller has a certain degree of gain margin. Then, the NDP technique is employed to perform the main controller design task, followed by the uniform ultimate boundedness stability proof with the feedback action of the event-driven adaptive control law. The comparative effect of the present control strategy is also illustrated via two simulation examples. The established method provides a new avenue of combining adaptive dynamic programming-based self-learning control, event-triggered adaptive control, and robust control, to investigate the nonlinear adaptive robust feedback design under uncertain environment.

Index Terms—Adaptive dynamic programming (ADP), adaptive robust control, critic neural network, event-driven control, neural dynamic programming (NDP), uncertain nonlinear systems.

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I. INTRODUCTION

IN MANY areas, such as power, transportation, and aerospace applications, there always exist system uncertainties between mathematical models and practical dynamics, which may bring in degradations of control performance. Under this circumstance, the designed feedback control law should possess a certain degree of robustness with respect to the dynamic uncertainties. As one of the core contents of modern control theory, the optimal regulation problem arises in designing a control law in order to minimize a predefined cost function with respect to a specified plant. The key of utilizing optimal control theory is to establish a method to adjust the design parameters to achieve the desired performance and the stability of controlled plant, and simultaneously, to attain the robustness [1]. Consequently, the robustness and the related optimality of complex systems have been studied by many researchers for a long time [2]–[13]. For instance, Chen *et al.* [3] proposed an advanced robust tracking control method for uncertain multi-input and multioutput nonlinear systems with input saturation. He *et al.* [6], [7] adopted adaptive neural network techniques to handle the system uncertainties and disturbances, so as to control the robotic plant with various nonlinearities, such as input saturation and deadzone. Cheng *et al.* [9] studied the integrated design of the machine body and control algorithm for enhancing the robustness of a closed-chain five-bar machine. Mu and Sun [10] designed a new super-twisting sliding mode observer to achieve the unknown system states of the second order nonlinear plant with bounded uncertainties and disturbances. Significantly, Lin [11] pointed out that the robust control can be designed by solving the corresponding optimal control problem, which provided a novel channel to derive the robust controller and thus led to a great attention of the optimal control design [12], [13].

The adaptive or approximate dynamic programming (ADP) method was originally proposed by Werbos [14] as an effective avenue with adaptive and self-learning ability, in order to conquer the phenomenon of “curse of dimensionality” arising in optimal control problems [15]. It is implemented by solving the Hamilton–Jacobi–Bellman (HJB) equation through the function approximation structures, usually referring to neural networks. Hence, the ADP also can be called neural dynamic programming (NDP) [16], thereby emphasizing the learning property of neural networks. Si and Wang [16] focused on building a thorough framework to develop a generic online learning control system through the fundamental principle

of NDP. By virtue of the fast development of machine learning technique, the combined research of reinforcement learning and ADP has acquired much attention as stated in [17] and [18]. Among that, the novel ideas of ADP have been frequently utilized for designing feedback controller with optimality, such as optimal control of continuous-time systems [19]–[21], constrained-input systems [22]–[24], discrete-time systems [25]–[27], uncertain systems [28]–[30], etc. Among the results, Vamvoudakis and Lewis [19] gave an online adaptive critic algorithm to solve the continuous-time affine nonlinear optimal control problem by integrating the idea of adaptive control. Bian *et al.* [20] proposed a new optimal control strategy for continuous-time nonaffine nonlinear systems with unknown dynamics under the ADP framework. Xu *et al.* [26] developed a novel reinforcement-learning-based neural network output feedback control scheme for single-input and single-output nonlinear systems in the pure-feedback form by using deterministic learning technique. Sokolov *et al.* [27] provided new stability results for action-dependent ADP, by employing a control algorithm that iteratively improved an internal model of the external world in the autonomous system through its continuous interaction with the environment. Wang *et al.* [30] coped with the robust decentralized stabilization of continuous-time nonlinear systems with multicontrol stations and dynamics uncertainties by using an online ADP approach. Recently, Jiang and Jiang [31] developed a new global ADP mechanism for nonlinear adaptive optimal control design, where the difficulty of solving the HJB equation was relaxed to an optimization problem and the neural network approximation was also avoided, thereby bringing in a significant computational improvement. Luo *et al.* [32], [33] proposed the promising off-policy reinforcement learning methods for control design, which was easy to implement and overcome the inefficient exploration problem by evaluating the value function of a target policy with the use of exploratory behavior policies. However, the above results are obtained under the traditional time-driven formulation, which is time-consuming in general.

Unlike the time-triggered control methods, in the event-triggered control mechanism, the sampling instant for updating the feedback controller is determined by a certain triggering condition, rather than relying on a fixed sampling interval. This always results in a significant reduction on computation and communication resources. Recently, the combination of event-triggering mechanism and ADP method has achieved considerable attention [34]–[38]. Note that under the new mechanism, the ADP-based controller is only updated when an event is triggered, and hence, the computational burden of learning and updating can be greatly saved. Vamvoudakis [35] originally proposed an optimal adaptive event-triggered control method for nonlinear continuous-time systems based on the actor-critic framework and neural network approximation. Then, Zhong and He [37] developed an event-triggered ADP control approach for continuous-time affine nonlinear systems with unknown internal states by measuring the input-output data. Zhang *et al.* [38] investigated the H_∞ optimal control problem for a class of continuous-time affine nonlinear

systems by virtue of an event-triggered formulation. However, it is apparent to find that the dynamical uncertainties and the robustness of the controlled plant are not always considered in the existing work of ADP-based event-triggered feedback design, which motivates our research of this paper greatly.

Consequently, in this paper, we investigate the event-driven adaptive robust control for continuous-time affine nonlinear systems with uncertainties using NDP technique. The main idea is inspired by how to embody human brain learning ability and make better use of communication resources when solving the nonlinear adaptive robust control problem, as shown in the aforementioned materials of Section I. Problem statement and basic transformation are presented in Section II. The robust feedback control problem is proven to be related to perform the event-triggered optimal regulation of the nominal system with a certain triggering condition in Section III. Therein, the NDP strategy is employed to facilitate the controller design by constructing a critic neural network. In addition, the uniformly ultimately bounded (UUB) stability of the closed-loop system is also analyzed. The excellent performance of the present control strategy is verified via simulation and comparison studies in Section IV. Concluding remarks with future work discussions and prospects are provided in Section V.

II. PROBLEM STATEMENT AND TRANSFORMATION

Let us study a class of input-affine continuous-time nonlinear systems with the form

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) + Z(x(t)) \quad (1)$$

where $Z(x(t))$ is the uncertain dynamics satisfying

$$Z(x) = G(x)d(\varphi(x)) \quad (2a)$$

$$d^T(\varphi(x))d(\varphi(x)) \leq h^T(\varphi(x))h(\varphi(x)). \quad (2b)$$

In system (1), $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^m$ is the control vector, $f(\cdot)$ and $g(\cdot)$ are differentiable in their arguments with $f(0) = 0$. In (2a) and (2b), the terms $G(\cdot) \in \mathbb{R}^{n \times r}$ with $\|G(x)\| \leq G_{\max}$ and $\varphi(\cdot)$ with $\varphi(0) = 0$ are fixed functions reflecting the structure of uncertainty, $d(\cdot) \in \mathbb{R}^r$ is an uncertain function satisfying $\|d(\varphi(x))\| \leq d_M(x)$ and $d(0) = 0$, and $h(\cdot) \in \mathbb{R}^r$ is a known function satisfying $h(0) = 0$.

In this paper, we will study how to stabilize the uncertain system (1) adaptively. When without considering the uncertainty, the controlled plant turns to its nominal version, which plays an important role in the design process. The nominal system corresponding to (1) is

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t). \quad (3)$$

Similar as the classical literature of nonlinear optimal control, we let $x(0) = x_0$ be the initial state vector and assume that $f + gu$ is Lipschitz continuous on a set Ω in \mathbb{R}^n containing the origin and that the system (3) is controllable.

The following lemma presents the achievement of robustness of uncertain system (1), which can be proved by using the similar framework as [5], [13], and [30].

Lemma 1: Assume that there exist a continuously differentiable and radially unbounded cost function $V(x)$ which satisfies $V(x) > 0$ for all $x \neq 0$ and $V(0) = 0$, a bounded function $\Gamma(x)$ satisfying $\Gamma(x) \geq 0$, as well as a feedback control function $u(x)$, such that

$$(\nabla V(x))^T Z(x) \leq \Gamma(x) \quad (4a)$$

$$U(x, u) + (\nabla V(x))^T (f(x) + g(x)u) + \Gamma(x) = 0 \quad (4b)$$

where $\nabla(\cdot) \triangleq \partial(\cdot)/\partial x$ is employed to denote the gradient operator, $U(x, u) = Q(x) + u^T R u$, and $Q(x) = x^T Q x$ with $Q = Q^T \geq 0$ and $R = R^T > 0$. Under the action of the control input $u(x)$, there exists a neighborhood of the original state, such that system (1) is locally asymptotically stable. In addition, define the cost function of system (3) as

$$J(x_0, u) = \int_0^\infty \{U(x(\tau), u(x(\tau))) + \Gamma(x(\tau))\} d\tau \quad (5)$$

which ensures that $J(x_0, u) = V(x_0)$ holds.

Note that the quadratic utility given here is a classical choice for convenience of analysis, as many of the ADP literature have exhibited (see [19], [21], [25], [30], [35], [37], [38]). In addition, the importance of the function $\Gamma(x)$ lies in that it presents an upper bound to the uncertain term $(\nabla V(x))^T Z(x)$, thereby facilitating us to design the robust control of nonlinear system possessing dynamical uncertainty. The following Lemma 2 shows us a specific form of $\Gamma(x)$, which verifies inequality (4a) obviously.

Lemma 2 [30]: For any continuously differentiable function $V(x)$, if we define

$$\Gamma(x) = h^T(\varphi(x))h(\varphi(x)) + \frac{1}{4}(\nabla V(x))^T G(x)G^T(x)\nabla V(x)$$

then, the inequality $(\nabla V(x))^T Z(x) \leq \Gamma(x)$ holds.

Remark 1: According to Lemma 1, the cost function $V(x)$, the bounded function $\Gamma(x)$, and the feedback control $u(x)$ satisfying (4a) and (4b) can guarantee the robust stabilization of system (1). Note that the optimal cost function and optimal control law of system (3) are actually represented as the specific forms of the cost function and feedback control. Hence, in the sequel, we should minimize $J(x_0, u)$ with respect to u , so as to derive the optimal cost function and optimal control law. This serves as the design fundamental of the adaptive robust stabilization of this paper.

Now, we focus on solving the optimal control problem of system (3) with $V(x_0)$ taken as the cost function. The designed optimal feedback control must be admissible, as stated in [19] and [23]. For system (3), it can be observed that

$$V(x_0) = \int_0^T \{U(x, u) + \Gamma(x)\} d\tau + V(x(T)) \quad (6)$$

is equivalent to (4b). Then, (4b) is an infinitesimal version of the cost (6) and is the nonlinear Lyapunov equation.

In what follows, for consistency, we generally take $J(x)$ to denote the cost function, instead of $V(x)$. In light of the classical optimal control theory, we define the Hamiltonian of the transformed problem as

$$H(x, u, \nabla J(x)) = U(x, u) + (\nabla J(x))^T (f + gu) + \Gamma(x).$$

Let Ω be a compact subset of \mathbb{R}^n and $\Psi(\Omega)$ be the set of admissible controls on Ω . The optimal cost function, denoted by $J^*(x_0)$, of the nominal system (3) is defined as

$$J^*(x_0) = \min_{u \in \Psi(\Omega)} J(x_0, u).$$

Note that the optimal cost $J^*(x)$ satisfies the continuous-time HJB equation of the form

$$0 = \min_{u \in \Psi(\Omega)} H(x, u, \nabla J^*(x)). \quad (7)$$

Hence, the optimal control of system (3) is obtained by

$$u^*(x) = -\frac{1}{2}R^{-1}g^T(x)\nabla J^*(x). \quad (8)$$

Using the optimal control $u^*(x)$ and the specific form of the bounded function $\Gamma(x)$, the HJB equation becomes

$$0 = U(x, u^*(x)) + (\nabla J^*(x))^T (f(x) + g(x)u^*(x)) + h^T(\varphi(x))h(\varphi(x)) + \frac{1}{4}(\nabla J^*(x))^T G(x)G^T(x)\nabla J^*(x)$$

with $J^*(0) = 0$. Based on (8), the HJB equation can be also written as

$$0 = Q(x) + (\nabla J^*(x))^T f(x) + h^T(\varphi(x))h(\varphi(x)) - \frac{1}{4}(\nabla J^*(x))^T g(x)R^{-1}g^T(x)\nabla J^*(x) + \frac{1}{4}(\nabla J^*(x))^T G(x)G^T(x)\nabla J^*(x) \quad (9)$$

with $J^*(0) = 0$.

Since it is always difficult to solve the nonlinear optimal control analytically, kinds of ADP-based methods combining the idea of reinforcement learning have been proposed to get the approximate solution [17]–[19]. However, nearly all of the existing methods are implemented predicated on the time-driven formulation, which generally speaking, is time-consuming. In other words, there is a huge room for improvement when considering to make better use of communication resources and reduce the computational burden. Thus, in what follows, we turn to the event-driven adaptive robust controller design through NDP technique.

III. EVENT-DRIVEN ADAPTIVE ROBUST CONTROLLER DESIGN THROUGH NDP TECHNIQUE

In this part, the main controller design method is developed, including the introduction of event-driven formulation, the implementation of neural network learning, and the achievement of robust stabilization.

A. Robust Stabilization With Event-Triggering Mechanism

Let $\mathbb{N} = \{0, 1, 2, \dots\}$ denote the set of all non-negative integers. In order to employ the event-triggering method to control the sampled-data system, we first define a monotonically increasing time sequence of triggering instants as $\{s_j\}_{j=0}^\infty$ with $s_0 = 0$ such that $s_j < s_{j+1}$, $j \in \mathbb{N}$. Assume that the nominal system (3) is sampled at the triggering instants s_j , which results in the sampled state vector $x(s_j) \triangleq \hat{x}_j$ for all $t \in [s_j, s_{j+1})$, $j \in \mathbb{N}$. Then, the event-triggered controller

$\mu(\hat{x}_j)$ is updated based on the sampled state \hat{x}_j rather than the current state $x(t)$. That is to say, the controller $\mu(\hat{x}_j)$ is executed at every triggering instants. By using a zero-order hold, the corresponding control sequence $\{\mu(\hat{x}_j)\}_{j=0}^{\infty}$ becomes a continuous-time input signal $\mu(\hat{x}_j, t)$, that is

$$\mu(\hat{x}_j, t) = u(\hat{x}_j) = u(x(s_j)), \forall t \in [s_j, s_{j+1}), j \in \mathbb{N}. \quad (10)$$

For simplicity, we use the notation $\mu(\hat{x}_j)$ instead of $\mu(\hat{x}_j, t)$. Define the event-triggered error between the sampled state and current state as

$$e_j(t) = \hat{x}_j - x(t), \forall t \in [s_j, s_{j+1}), j \in \mathbb{N}. \quad (11)$$

Thus, the event-triggered optimal control problem can be formulated, where the corresponding sampled-data system is

$$\dot{x}(t) = f(x) + g(x)\mu(x(t) + e_j(t)). \quad (12)$$

Based on (8) and (10), the event-triggered optimal controller can be given by

$$\mu^*(\hat{x}_j) = -\frac{1}{2}R^{-1}g^T(\hat{x}_j)\nabla J^*(\hat{x}_j), \forall t \in [s_j, s_{j+1}) \quad (13)$$

where $\nabla J^*(\hat{x}_j) = (\partial J^*(x)/\partial x)|_{x=\hat{x}_j}$. Under the framework of the event-triggering mechanism, the HJB equation (9) can be rewritten as

$$\begin{aligned} & H(x, \mu^*(\hat{x}_j), \nabla J^*(x)) \\ &= U(x, \mu^*(\hat{x}_j)) + (\nabla J^*(x))^T(f(x) + g(x)\mu^*(\hat{x}_j)) \\ & \quad + h^T(\varphi(x))h(\varphi(x)) + \frac{1}{4}(\nabla J^*(x))^T G(x)G^T(x)\nabla J^*(x) \\ &= Q(x) + (\nabla J^*(x))^T f(x) + h^T(\varphi(x))h(\varphi(x)) \\ & \quad - \frac{1}{2}(\nabla J^*(x))^T g(x)R^{-1}g^T(\hat{x}_j)\nabla J^*(\hat{x}_j) \\ & \quad + \frac{1}{4}(\nabla J^*(\hat{x}_j))^T g(\hat{x}_j)R^{-1}g^T(\hat{x}_j)\nabla J^*(\hat{x}_j) \\ & \quad + \frac{1}{4}(\nabla J^*(x))^T G(x)G^T(x)\nabla J^*(x) \end{aligned} \quad (14)$$

with $J^*(0) = 0$. Note that unlike the time-triggered HJB equation (9), the new formula (14) is in fact the event-triggered HJB equation.

To reveal the relationship between the continuous-time input signals $\mu^*(\hat{x}_j)$ and $u^*(x)$, we recall the following assumption.

Assumption 1 [35]: The control law $\mu(x)$ is Lipschitz continuous with respect to the event-triggered error, i.e., $\|\mu(x(t)) - \mu(\hat{x}_j)\| \leq \mathcal{L}\|e_j(t)\|$, where \mathcal{L} is a positive real constant and $\mu(x) = u(x)$.

Inspired by the excellent work of [35], we present the following lemma that can be derived by conducting a similar proof, where the structure of matrix R should be noticed.

Lemma 3: Suppose that the Assumption 1 holds and let $R = r^T r$. Then, we have

$$\begin{aligned} & H(x, \mu^*(\hat{x}_j), \nabla J^*(x)) - H(x, u^*(x), \nabla J^*(x)) \\ &= (u^*(x) - \mu^*(\hat{x}_j))^T R(u^*(x) - \mu^*(\hat{x}_j)) \\ &\leq \mathcal{L}^2 \|r\|^2 \|e_j(t)\|^2. \end{aligned} \quad (15)$$

For the event-triggered control problem, a triggering condition should be designed to determine the event-triggering

instants and guarantee the stability of the closed-loop system. Now, we need to derive an appropriate triggering condition for the robustness of the original uncertain system (1) with the event-triggered optimal controller (13), which can guarantee the asymptotic stability of system (1).

Theorem 1: For the nominal system (3), suppose that $J^*(x)$ is the solution of the HJB equation (7) and $\mu^*(\hat{x}_j)$ is the event-triggered optimal controller. For all $t \in [s_j, s_{j+1}), j \in \mathbb{N}$, if the triggering condition is defined as

$$\begin{aligned} \|e_j(t)\|^2 &\leq \frac{(1 - \eta_1^2)\lambda_{\min}(Q)\|x\|^2 + \|r\mu^*(\hat{x}_j)\|^2}{\mathcal{L}^2 \|r\|^2} \\ &\triangleq \|e_T\|^2 \end{aligned} \quad (16)$$

where e_T denotes the threshold, $\lambda_{\min}(Q)$ is the minimal eigenvalue of Q , and $\eta_1 \in (0, 1)$ is a designed sample frequency parameter, then the closed-loop form of system (1) with the event-triggered controller (13) is asymptotically stable.

Proof: Choose $L_1(t) = J^*(x(t))$ as the Lyapunov function. The derivative of $L_1(t)$ along the system trajectory (1) with the controller (13) can be formulated as

$$\dot{L}_1(t) = (\nabla J^*(x))^T(f(x) + g(x)\mu^*(\hat{x}_j) + Z(x)). \quad (17)$$

From (8), we can get

$$g^T(x)\nabla J^*(x) = -2Ru^*(x). \quad (18)$$

Since $J^*(x)$ satisfies (4a) and (4b), we have

$$(\nabla J^*(x))^T Z(x) \leq \Gamma(x) \quad (19a)$$

$$(\nabla J^*(x))^T f(x) = -Q(x) + u^{*T}(x)Ru^*(x) - \Gamma(x). \quad (19b)$$

Through making a combination of (18), (19a), and (19b), the derivative (17) becomes

$$\begin{aligned} \dot{L}_1(t) &\leq (\nabla J^*(x))^T f(x) + (\nabla J^*(x))^T g(x)\mu^*(\hat{x}_j) + \Gamma(x) \\ &\leq -Q(x) + u^{*T}(x)Ru^*(x) - 2u^{*T}(x)R\mu^*(\hat{x}_j). \end{aligned} \quad (20)$$

Adding and subtracting a quadratic term $\mu^{*T}(\hat{x}_j)R\mu^*(\hat{x}_j)$, it follows from (20) that

$$\begin{aligned} \dot{L}_1(t) &\leq -Q(x) - \mu^{*T}(\hat{x}_j)R\mu^*(\hat{x}_j) \\ &\quad + (u^*(x) - \mu^*(\hat{x}_j))^T R(u^*(x) - \mu^*(\hat{x}_j)). \end{aligned} \quad (21)$$

According to Lemma 3, (21) can be deduced to

$$\begin{aligned} \dot{L}_1(t) &\leq -x^T Qx + \mathcal{L}^2 \|r\|^2 \|e_j(t)\|^2 - \|r\mu^*(\hat{x}_j)\|^2 \\ &\leq -\eta_1^2 \lambda_{\min}(Q)\|x\|^2 + (\eta_1^2 - 1)\lambda_{\min}(Q)\|x\|^2 \\ &\quad + \mathcal{L}^2 \|r\|^2 \|e_j(t)\|^2 - \|r\mu^*(\hat{x}_j)\|^2. \end{aligned}$$

Hence, if the triggering condition (16) is satisfied, we obtain $\dot{L}_1(t) \leq -\eta_1^2 \lambda_{\min}(Q)\|x\|^2 < 0$ for any $x(t) \neq 0, t \in [s_j, s_{j+1})$. This proves that the triggering condition (16) can ensure the asymptotic stability of the uncertain system (1). ■

Remark 2: In the triggering condition (16), the sample frequency parameter η_1 should be selected to ensure that the term $\|e_T\|^2$ is positive. Actually, it is easy to attain the goal because η_1 is constrained to be located in an interval $(0, 1)$.

For the continuous-time nonlinear system with the event-triggered control input, the minimal intersample time $s_{\min} = \min_{j \in \mathbb{N}} \{s_{j+1} - s_j\}$ might be zero and the accumulations of event times may occur (i.e., the Zeno behavior). To exclude the Zeno behavior, we present the following lemma.

Lemma 4: Considering the uncertain system (1) with the event-triggered control law (13), the minimal intersample time s_{\min} determined by (16) is lower bounded by a nonzero positive constant such that

$$s_{\min} \geq \frac{1}{\mathcal{K}} \ln(1 + S_{j,\min}) > 0$$

which illustrates the exclusion of the Zeno behavior, where

$$S_{j,\min} = \min_{j \in \mathbb{N}} \left\{ \frac{\|e_j(s_{j+1})\|}{\|\hat{x}_j\| + \pi} \right\} > 0$$

and $e_j(s_{j+1}) = \hat{x}_j - x(s_{j+1})$, \mathcal{K} is a positive constant, and π is a small positive constant satisfying $\|f(x) + g(x)u + Z(x)\| \leq \mathcal{K}\|x\| + \mathcal{K}\pi$.

The proof of Lemma 4 is similar with [38] and thus it is omitted here. Note that the constants \mathcal{K} and π are existing since $f + gu$ is Lipschitz continuous and the term $Z(x)$ is upper bounded as $\|Z(x)\| \leq G_{\max} d_M(x)$.

Incidentally, it is important to note that the event-triggered controller $\mu^*(\hat{x}_j)$ has a certain degree of gain margin, as shown in the following corollary. This result can be obtained easily as Theorem 1 and hence the proof is omitted here.

Corollary 1: For system (3) with $J^*(x)$ and $\mu^*(\hat{x}_j)$, if the triggering condition is given by

$$\begin{aligned} \|e_j(t)\|^2 &\leq \frac{(1 - \hat{\eta}_1^2) \lambda_{\min}(Q) \|x\|^2 + \xi \|r \mu^*(\hat{x}_j)\|^2}{\xi \mathcal{L}^2 \|r\|^2} \\ &\triangleq \|\dot{e}_T\|^2, \forall t \in [s_j, s_{j+1}), j \in \mathbb{N} \end{aligned} \quad (22)$$

where ξ is a constant satisfying $\xi \geq 1$, \dot{e}_T is the threshold, and $\hat{\eta}_1 \in (0, 1)$ is the sample frequency parameter, then the closed-loop system (1) with event-triggered controller $\xi \mu^*(\hat{x}_j)$ is asymptotically stable.

Next, the idea of NDP is employed to approximate the optimal cost function $J^*(x)$ and the optimal control law $\mu^*(\hat{x}_j)$ with event-driven formulation, which is named as the event-driven NDP algorithm.

B. Event-Driven NDP Design With Implementation

In the event-based NDP algorithm, only a single critic neural network with three-layer structure is required to approximate the cost function, which actually, reduces the implementation complexity of building both the critic network and the action network [19], [24], [35]. According to the universal approximation property, $J(x)$ can be reconstructed by a neural network on a compact set Ω as

$$J(x) = \omega_c^\top \sigma_c(x) + \varepsilon_c(x)$$

where $\omega_c \in \mathbb{R}^l$ is the ideal weight vector, $\sigma_c(x) \in \mathbb{R}^l$ is the activation function, l is the number of neurons in the hidden layer, and $\varepsilon_c(x)$ is the approximation error of the neural network. Then

$$\nabla J(x) = (\nabla \sigma_c(x))^\top \omega_c + \nabla \varepsilon_c(x). \quad (23)$$

Under the framework of ADP, since the ideal weight vector is unavailable, a critic neural network needs to be constructed with respect to the estimated weight elements as the form

$$\hat{J}(x) = \hat{\omega}_c^\top \sigma_c(x)$$

to approximate the cost function. Then, we have

$$\nabla \hat{J}(x) = (\nabla \sigma_c(x))^\top \hat{\omega}_c. \quad (24)$$

According to (13) and (23), we describe the event-triggered optimal control law as follows:

$$\mu(\hat{x}_j) = -\frac{1}{2} R^{-1} g^\top(\hat{x}_j) \left((\nabla \sigma_c(\hat{x}_j))^\top \omega_c + \nabla \varepsilon_c(\hat{x}_j) \right).$$

By combining (13) with (24), the event-triggered approximate optimal control law can be formulated as

$$\hat{\mu}(\hat{x}_j) = -\frac{1}{2} R^{-1} g^\top(\hat{x}_j) (\nabla \sigma_c(\hat{x}_j))^\top \hat{\omega}_c. \quad (25)$$

As for the Hamiltonian, when taking the neural network expression (23) into account, it becomes

$$\begin{aligned} H(x, \mu(\hat{x}_j), \omega_c) &= U(x, \mu(\hat{x}_j)) + h^\top(\varphi(x))h(\varphi(x)) \\ &\quad + \omega_c^\top \nabla \sigma_c(x) (f(x) + g(x)\mu(\hat{x}_j)) \\ &\quad + \frac{1}{4} \omega_c^\top \nabla \sigma_c(x) G(x) G^\top(x) (\nabla \sigma_c(x))^\top \omega_c \\ &\triangleq e_{cH} \end{aligned} \quad (26)$$

where

$$\begin{aligned} e_{cH} &= -(\nabla \varepsilon_c(x))^\top (f(x) + g(x)\mu(\hat{x}_j)) \\ &\quad - \frac{1}{2} \omega_c^\top \nabla \sigma_c(x) G(x) G^\top(x) \nabla \varepsilon_c(x) \\ &\quad - \frac{1}{4} (\nabla \varepsilon_c(x))^\top G(x) G^\top(x) \nabla \varepsilon_c(x) \end{aligned}$$

represents the residual error due to the neural network approximation. Using (25), the approximate Hamiltonian can be obtained by

$$\begin{aligned} \hat{H}(x, \mu(\hat{x}_j), \hat{\omega}_c) &= U(x, \mu(\hat{x}_j)) + h^\top(\varphi(x))h(\varphi(x)) \\ &\quad + \hat{\omega}_c^\top \nabla \sigma_c(x) (f(x) + g(x)\mu(\hat{x}_j)) \\ &\quad + \frac{1}{4} \hat{\omega}_c^\top \nabla \sigma_c(x) G(x) G^\top(x) (\nabla \sigma_c(x))^\top \hat{\omega}_c \\ &\triangleq e_c. \end{aligned} \quad (27)$$

By letting the error of estimating the critic network weight be $\tilde{\omega}_c = \omega_c - \hat{\omega}_c$ and combining (26) with (27), we find that

$$\begin{aligned} e_c &= -\tilde{\omega}_c^\top \nabla \sigma_c(x) (f(x) + g(x)\mu(\hat{x}_j)) \\ &\quad + \frac{1}{4} \tilde{\omega}_c^\top \nabla \sigma_c(x) G(x) G^\top(x) (\nabla \sigma_c(x))^\top \tilde{\omega}_c \\ &\quad - \frac{1}{2} \omega_c^\top \nabla \sigma_c(x) G(x) G^\top(x) (\nabla \sigma_c(x))^\top \tilde{\omega}_c + e_{cH} \end{aligned} \quad (28)$$

which shows the relationship between the terms e_c and e_{cH} .

For the purpose of critic learning, it is desired to train $\hat{\omega}_c$ to minimize the objective function $E_c = 0.5 e_c^\top e_c$. Note that the approximated control law (25) is often used for conducting the learning stage because of the unavailability of the

$$\begin{cases} \dot{z} = \begin{bmatrix} f(x) + g(x)\hat{\mu}(\hat{x}_j) \\ 0 \\ -\alpha_c \left(\phi^\top \tilde{\omega}_c - \frac{1}{4} \tilde{\omega}_c^\top B \tilde{\omega}_c + \frac{1}{2} \omega_c^\top B \tilde{\omega}_c - e_{cH} \right) \left(\phi + \frac{1}{2} B \omega_c - \frac{1}{2} B \tilde{\omega}_c \right) \end{bmatrix}, & t \in [s_j, s_{j+1}) \\ z(t) = z(t^-) + \begin{bmatrix} 0 \\ x - \hat{x}_j \\ 0 \end{bmatrix}, & t = s_{j+1} \end{cases} \quad (31)$$

$$\|\tilde{\omega}_c\| > \sqrt{\frac{2\|R^{-1}\|^2 g_{\max}^2 \nabla \sigma_{c\max}^2 + \lambda_4 + \sqrt{(2\|R^{-1}\|^2 g_{\max}^2 \nabla \sigma_{c\max}^2 + \lambda_4)^2 + 2\lambda_3(4\|R^{-1}\|^2 g_{\max}^2 \nabla \varepsilon_{c\max}^2 + \alpha_c^2 e_{cH\max}^2)}}{2\lambda_3}} \quad (33)$$

optimal control law $\mu(\hat{x}_j)$. At present, we adopt the standard steepest descent algorithm to adjust the weight vector as $\dot{\hat{\omega}}_c = -\alpha_c(\partial E_c / \partial \hat{\omega}_c)$, which, based on (27), is in fact

$$\begin{aligned} \dot{\hat{\omega}}_c &= -\alpha_c e_c \left(\frac{\partial e_c}{\partial \hat{\omega}_c} \right) \\ &= -\alpha_c \left(U(x, \hat{\mu}(\hat{x}_j)) + h^\top(\varphi(x))h(\varphi(x)) + \phi^\top \hat{\omega}_c \right. \\ &\quad \left. + \frac{1}{4} \hat{\omega}_c^\top \nabla \sigma_c(x) G(x) G^\top(x) (\nabla \sigma_c(x))^\top \hat{\omega}_c \right) \\ &\quad \times \left(\phi + \frac{1}{2} \nabla \sigma_c(x) G(x) G^\top(x) (\nabla \sigma_c(x))^\top \hat{\omega}_c \right) \end{aligned} \quad (29)$$

where $\phi = \nabla \sigma_c(x)(f(x) + g(x)\hat{\mu}(\hat{x}_j))$ and $\alpha_c > 0$ is the designed learning rate of the critic network. Then, recalling $\dot{\tilde{\omega}}_c = -\dot{\hat{\omega}}_c$ and (28), we can further derive that the error dynamical equation of approximating the cost function by the critic network is

$$\begin{aligned} \dot{\tilde{\omega}}_c &= \alpha_c e_c \left(\frac{\partial e_c}{\partial \hat{\omega}_c} \right) \\ &= -\alpha_c \left(\phi^\top \tilde{\omega}_c - \frac{1}{4} \tilde{\omega}_c^\top \nabla \sigma_c(x) G(x) G^\top(x) (\nabla \sigma_c(x))^\top \tilde{\omega}_c \right. \\ &\quad \left. + \frac{1}{2} \omega_c^\top \nabla \sigma_c(x) G(x) G^\top(x) (\nabla \sigma_c(x))^\top \tilde{\omega}_c - e_{cH} \right) \\ &\quad \times \left(\phi + \frac{1}{2} \nabla \sigma_c(x) G(x) G^\top(x) (\nabla \sigma_c(x))^\top \omega_c \right. \\ &\quad \left. - \frac{1}{2} \nabla \sigma_c(x) G(x) G^\top(x) (\nabla \sigma_c(x))^\top \tilde{\omega}_c \right). \end{aligned} \quad (30)$$

Actually, we can observe that the closed-loop sampled-data system is an impulsive dynamical system with flow dynamics for all $t \in [s_j, s_{j+1})$ and jump dynamics for all $t = s_{j+1}, j \in \mathbb{N}$. When defining an augmented state vector as $z = [x^\top, \hat{x}_j^\top, \tilde{\omega}_c^\top]^\top$ and based on (11), (12), and (30), the dynamics of the impulsive system can be described by (31), shown at the top of the page, where $B \triangleq \nabla \sigma_c(x) G(x) G^\top(x) (\nabla \sigma_c(x))^\top$, $z(t^-) = \lim_{\varrho \rightarrow 0} z(t - \varrho)$, and the term “0” represents a null vector with appropriate dimension.

C. Closed-Loop Stability With Event-Triggered Control

In this part, we turn to the stability issue of the closed-loop system. At present, we focus on the UUB stability, as most literature of the ADP field have done

(see [19], [22], [24], [28]–[30], [37], [38]). Before proceeding, the following assumptions are needed, as often used in ADP like [19], [24], [34], and [38].

Assumption 2: For the system dynamics $g(x)$, we have the following two assumptions.

- 1) The dynamics $g(x)$ is Lipschitz continuous such that $\|g(x) - g(\hat{x}_j)\| \leq A\|e_j(t)\|$, where A is a positive constant.
- 2) The dynamics $g(x)$ is upper bounded such that $\|g(x)\| \leq g_{\max}$, where g_{\max} is a positive constant.

Assumption 3: Assume that the following bounded conditions hold on a compact set Ω .

- 1) The ideal weight vector, i.e., ω_c is upper bounded such that $\|\omega_c\| \leq \omega_{c\max}$.
- 2) The derivative of the activation function, i.e., $\nabla \sigma_c(x)$ is Lipschitz continuous such that $\|\nabla \sigma_c(x) - \nabla \sigma_c(\hat{x}_j)\| \leq B\|e_j(t)\|$, where B is a positive constant.
- 3) The derivative term $\nabla \sigma_c(x)$ is upper bounded such that $\|\nabla \sigma_c(x)\| \leq \nabla \sigma_{c\max}$, where $\nabla \sigma_{c\max}$ is a positive constant.
- 4) The derivative of the approximation error, i.e., $\nabla \varepsilon_c(x)$ is upper bounded such that $\|\nabla \varepsilon_c(x)\| \leq \nabla \varepsilon_{c\max}$, where $\nabla \varepsilon_{c\max}$ is a positive constant.
- 5) The residual error term, i.e., e_{cH} is upper bounded by a positive constant $e_{cH\max}$.

Theorem 2: Suppose that Assumptions 2 and 3 hold. The tuning law for the critic network is given by (29). Then, the closed-loop system (12) is asymptotically stable and the critic weight estimation error is guaranteed to be UUB if the adaptive triggering condition

$$\begin{aligned} \|e_j(t)\|^2 &\leq \frac{(1 - \eta_2^2) \lambda_{\min}(Q) \|x\|^2 + \|r\hat{\mu}(\hat{x}_j)\|^2}{2\ell^2 \|\hat{\omega}_c\|^2 \|R^{-1}\|} \\ &\triangleq \|\hat{e}_T\|^2 \end{aligned} \quad (32)$$

where $\eta_2 \in (0, 1)$ is the parameter to be designed reflecting the sample frequency and $\ell^2 = A^2 \nabla \sigma_{c\max}^2 + B^2 g_{\max}^2$ and the inequality (33), shown at the top of the page, where λ_3 and λ_4 are given in (41a) and (41b), respectively, are satisfied for the critic network.

Proof: Considering the impulsive dynamical system (31), we choose a Lyapunov function candidate composed of three terms as follows:

$$L_2(t) = L_{21}(t) + L_{22}(t) + L_{23}(t) \quad (34)$$

where

$$\begin{aligned} L_{21} &= J^*(x(t)) \\ L_{22} &= J^*(\hat{x}_j) \\ L_{23} &= \frac{1}{2} \tilde{\omega}_c^\top(t) \tilde{\omega}_c(t). \end{aligned}$$

Note that the proof should be divided into two parts, focusing on the continuous and the jump dynamics, respectively. Hence, two cases are analyzed in the sequel, including events are not triggered and events are triggered.

1) *For the Case That Events Are Not Triggered, i.e., $\forall t \in [s_j, s_{j+1})$:* Taking the time derivative of the Lyapunov function along the trajectory of system (31) yields

$$\dot{L}_2(t) = \dot{L}_{21}(t) + \dot{L}_{22}(t) + \dot{L}_{23}(t).$$

Observing $\dot{L}_2(t)$, the second term is $\dot{L}_{22} = 0$ while the first and third terms are

$$\dot{L}_{21} = (\nabla J^*(x))^\top (f(x) + g(x)\hat{\mu}(\hat{x}_j))$$

and

$$\begin{aligned} \dot{L}_{23} &= -\alpha_c \tilde{\omega}_c^\top \left(\phi + \frac{1}{2} B \omega_c - \frac{1}{2} B \tilde{\omega}_c \right) \\ &\quad \times \left(\phi^\top \tilde{\omega}_c - \frac{1}{4} \tilde{\omega}_c^\top B \tilde{\omega}_c + \frac{1}{2} \omega_c^\top B \tilde{\omega}_c - e_{cH} \right) \end{aligned}$$

respectively.

For the first term \dot{L}_{21} , based on (18) and (19b), we have

$$\begin{aligned} \dot{L}_{21} &= (\nabla J^*(x))^\top f(x) + (\nabla J^*(x))^\top g(x)\hat{\mu}(\hat{x}_j) \\ &= -Q(x) + u^{*\top}(x) R u^*(x) - \Gamma(x) - 2u^{*\top}(x) R \hat{\mu}(\hat{x}_j) \\ &= -x^\top Q x - \Gamma(x) - \|r\hat{\mu}(\hat{x}_j)\|^2 + \|r\|^2 \|u^*(x) - \hat{\mu}(\hat{x}_j)\|^2. \end{aligned} \quad (35)$$

Note that the following inequality holds:

$$\begin{aligned} &\|r\|^2 \|u^*(x) - \hat{\mu}(\hat{x}_j)\|^2 \\ &= \|r\|^2 \left\| \frac{1}{2} R^{-1} g^\top(\hat{x}_j) (\nabla \sigma_c(\hat{x}_j))^\top \tilde{\omega}_c \right. \\ &\quad \left. - \frac{1}{2} R^{-1} g^\top(x) (\nabla \sigma_c(x))^\top \tilde{\omega}_c \right. \\ &\quad \left. - \frac{1}{2} R^{-1} g^\top(x) \left((\nabla \sigma_c(x))^\top \tilde{\omega}_c + \nabla \varepsilon_c(x) \right) \right\|^2 \\ &\leq \|r\|^2 \left\| R^{-1} \left(g^\top(\hat{x}_j) (\nabla \sigma_c(\hat{x}_j))^\top - g^\top(x) (\nabla \sigma_c(x))^\top \right) \tilde{\omega}_c \right\|^2 \\ &\quad + \left\| R^{-1} g^\top(x) \left((\nabla \sigma_c(x))^\top \tilde{\omega}_c + \nabla \varepsilon_c(x) \right) \right\|^2. \end{aligned} \quad (36)$$

According to Assumptions 2 and 3, we have

$$\begin{aligned} &\left\| g^\top(\hat{x}_j) (\nabla \sigma_c(\hat{x}_j))^\top - g^\top(x) (\nabla \sigma_c(x))^\top \right\|^2 \\ &= \left\| \nabla \sigma_c(\hat{x}_j) g(\hat{x}_j) - \nabla \sigma_c(x) g(x) \right\|^2 \\ &= \left\| (\nabla \sigma_c(\hat{x}_j) - \nabla \sigma_c(x)) g(\hat{x}_j) + \nabla \sigma_c(x) (g(\hat{x}_j) - g(x)) \right\|^2 \\ &\leq 2 \left\| (\nabla \sigma_c(\hat{x}_j) - \nabla \sigma_c(x)) g(\hat{x}_j) \right\|^2 \\ &\quad + 2 \left\| \nabla \sigma_c(x) (g(\hat{x}_j) - g(x)) \right\|^2 \\ &\leq 2 \left(A^2 \nabla \sigma_{c\max}^2 + B^2 g_{\max}^2 \right) \|e_j(t)\|^2. \end{aligned} \quad (37)$$

Based on (36) and (37), (35) can be rewritten as

$$\begin{aligned} \dot{L}_{21} &\leq -x^\top Q x - \|r\hat{\mu}(\hat{x}_j)\|^2 \\ &\quad + 2 \left\| R^{-1} \right\| \left\| \tilde{\omega}_c \right\|^2 \left(A^2 \nabla \sigma_{c\max}^2 + B^2 g_{\max}^2 \right) \|e_j(t)\|^2 \\ &\quad + 2 \left\| R^{-1} \right\|^2 g_{\max}^2 \nabla \sigma_{c\max}^2 \|\tilde{\omega}_c\|^2 \\ &\quad + 2 \left\| R^{-1} \right\|^2 g_{\max}^2 \nabla \varepsilon_{c\max}^2. \end{aligned} \quad (38)$$

When expanding the third term \dot{L}_{23} , we find that

$$\begin{aligned} \dot{L}_{23} &= -\frac{1}{8} \alpha_c \left(\tilde{\omega}_c^\top B \tilde{\omega}_c \right)^2 - \alpha_c \tilde{\omega}_c^\top \phi \phi^\top \tilde{\omega}_c + \alpha_c \tilde{\omega}_c^\top \phi e_{cH} \\ &\quad - \alpha_c \left(\tilde{\omega}_c^\top B \omega_c \right) \left(\phi^\top \tilde{\omega}_c \right) + \frac{3}{4} \alpha_c \left(\tilde{\omega}_c^\top B \tilde{\omega}_c \right) \left(\phi^\top \tilde{\omega}_c \right) \\ &\quad + \frac{3}{8} \alpha_c \left(\tilde{\omega}_c^\top B \omega_c \right) \left(\tilde{\omega}_c^\top B \tilde{\omega}_c \right) - \frac{1}{4} \alpha_c \left(\tilde{\omega}_c^\top B \omega_c \right)^2 \\ &\quad + \frac{1}{2} \alpha_c \left(\tilde{\omega}_c^\top B \omega_c \right) e_{cH} - \frac{1}{2} \alpha_c \left(\tilde{\omega}_c^\top B \tilde{\omega}_c \right) e_{cH}. \end{aligned} \quad (39)$$

By applying the Young's inequality to the terms $\alpha_c \tilde{\omega}_c^\top \phi e_{cH}$ and $(1/2) \alpha_c (\tilde{\omega}_c^\top B \omega_c) e_{cH}$, letting $\lambda_{0m} > 0$ and $\lambda_{0M} > 0$ be the lower and upper bounds of the norm of matrix B , and taking account of the bounded conditions $\|B \omega_c\| \leq \nabla \sigma_{c\max}^2 G_{\max}^2 \omega_{c\max} \triangleq \lambda_1$ and $\|\phi\| \leq \lambda_2$, we can obtain that (39) can be derived as

$$\dot{L}_{23} \leq -\lambda_3 \|\tilde{\omega}_c\|^4 + \lambda_4 \|\tilde{\omega}_c\|^2 + \frac{1}{2} \alpha_c^2 e_{cH\max}^2 \quad (40)$$

where

$$\lambda_3 = \frac{1}{8} \alpha_c \lambda_{0m}^2 - \frac{3}{8 \vartheta_1^2} \alpha_c \lambda_{0M}^2 - \frac{3}{16 \vartheta_2^2} \alpha_c \lambda_{0M}^2 \quad (41a)$$

$$\begin{aligned} \lambda_4 &= \alpha_c \lambda_1 \lambda_2 + \frac{3 \vartheta_1^2}{8} \alpha_c \lambda_2^2 + \frac{3 \vartheta_2^2}{16} \alpha_c \lambda_1^2 + \frac{1}{4} \lambda_1^2 \\ &\quad + \frac{1}{2} \alpha_c \lambda_{0M} e_{cH\max} - (\alpha_c - 1) \lambda_{\min}(\phi \phi^\top) \end{aligned} \quad (41b)$$

and ϑ_1 and ϑ_2 are nonzero constants selected during the design process. Though there exist both positive and negative components in λ_3 and λ_4 , we can guarantee that they are eventually positive values by choosing appropriate parameters such as ϑ_1 and ϑ_2 . For (41b), note that if the persistence of excitation like condition is satisfied, we have $\lambda_{\min}(\phi \phi^\top) > 0$ [19].

By combining (38) with (40), we can obtain

$$\begin{aligned} \dot{L}_2(t) &\leq -x^\top Q x - \|r\hat{\mu}(\hat{x}_j)\|^2 \\ &\quad + 2 \left\| R^{-1} \right\| \left\| \tilde{\omega}_c \right\|^2 \left(A^2 \nabla \sigma_{c\max}^2 + B^2 g_{\max}^2 \right) \|e_j(t)\|^2 \\ &\quad - \lambda_3 \|\tilde{\omega}_c\|^4 + \left(2 \left\| R^{-1} \right\|^2 g_{\max}^2 \nabla \sigma_{c\max}^2 + \lambda_4 \right) \|\tilde{\omega}_c\|^2 \\ &\quad + 2 \left\| R^{-1} \right\|^2 g_{\max}^2 \nabla \varepsilon_{c\max}^2 + \frac{1}{2} \alpha_c^2 e_{cH\max}^2. \end{aligned} \quad (42)$$

Introducing η_2 and considering the fact that

$$-x^\top Q x \leq -\eta_2^2 \lambda_{\min}(Q) \|x\|^2 + (\eta_2^2 - 1) \lambda_{\min}(Q) \|x\|^2$$

if the triggering condition (32) and the inequality (33) are satisfied, we can conclude that the time derivative inequality (42) becomes $\dot{L}_2(t) \leq -\eta_2^2 \lambda_{\min}(Q) \|x\|^2 < 0$. In other words, the derivative of the Lyapunov function candidate is negative during the flow for all $t \in [s_j, s_{j+1})$.

2) For the Case That Events Are Triggered, i.e., $\forall t = s_{j+1}$: Considering the difference of the Lyapunov function candidate (34) yields

$$\Delta L_2(t) = L_2(\hat{x}_{j+1}) - L_2(x(s_{j+1}^-)) = \Delta L_{21} + \Delta L_{22} + \Delta L_{23}.$$

According to (32), (33), and (42), we know that $\dot{L}_2(t) < 0$ for all $t \in [s_j, s_{j+1})$. Since the system state and cost function are continuous, we can obtain the following two inequalities:

$$\Delta L_{21} = J^*(\hat{x}_{j+1}) - J^*(x(s_{j+1}^-)) \leq 0$$

and

$$\Delta L_{23} = \frac{1}{2} \tilde{\omega}_c^\top(\hat{x}_{j+1}) \tilde{\omega}_c(\hat{x}_{j+1}) - \frac{1}{2} \tilde{\omega}_c^\top(x(s_{j+1}^-)) \tilde{\omega}_c(x(s_{j+1}^-)) \leq 0.$$

Hence, we can further find that

$$\Delta L_2(t) \leq \Delta L_{22} = J^*(\hat{x}_{j+1}) - J^*(\hat{x}_j) \leq -v(\|e_{j+1}(s_j)\|)$$

where $v(\cdot)$ is a class- κ function [39] and $e_{j+1}(s_j) = \hat{x}_{j+1} - \hat{x}_j$. This implies that the Lyapunov function candidate (34) is also decreasing at the triggering instants $\forall t = s_{j+1}$.

By combining the above two cases, if the triggering condition (32) and the inequality (33) hold, we can derive the conclusion that the closed-loop impulsive system is asymptotically stable and the weight estimation error of the critic network is UUB. The proof is completed. ■

Remark 3: Note that the error in $\hat{H}(x, \mu(\hat{x}_j), \hat{\omega}_c)$ is introduced by the neural network approximation and the time-triggered/event-triggered transformation from (8) to (13). Actually, we can acquire the nearly optimal performance for the nominal system with the event-triggered approximate control law (25) by adjusting the parameter η_2 in (32). In fact, the choice of the parameter η_2 affects the value of $\|\hat{e}_T\|^2$ and further affects the triggering condition. When the triggering condition is changed, the sampling frequency will exhibit different property. Thus, it is one of the parameters that determine how frequent that the system states are sampled. In other words, there exists a tradeoff between the approximation accuracy and computation reduction that is determined by the sampling frequency.

IV. SIMULATION STUDIES

We conduct two simulation experiments to demonstrate the effectiveness of the nonlinear adaptive robust control method.

Example 1: In this experiment, we consider an input-affine continuous-time nonlinear system including an uncertain term

$$\dot{x} = \begin{bmatrix} -x_1 - 2x_2 \\ x_1 - x_2 - \cos x_1 \sin x_2^2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(x) + \begin{bmatrix} p_1 x_1 \sin x_2^2 \\ 0 \end{bmatrix} \quad (43)$$

where $x = [x_1, x_2]^\top \in \mathbb{R}^2$ and $u(x) \in \mathbb{R}$ are the state and control vectors, while $Z(x) = [p_1 x_1 \sin x_2^2, 0]^\top$ (with $p_1 \in [-2, 2]$) represents the uncertainty. Letting $\varphi(x) = x$ and considering the uncertain structure, we can select $G(x) = [1, 0]^\top$, $d(\varphi(x)) = p_1 x_1 \sin x_2^2$, and $h(\varphi(x)) = 2x_1 \sin x_2^2$. Let $Q(x) = 2x^\top x$ and $R = I$ (I denotes an identity matrix

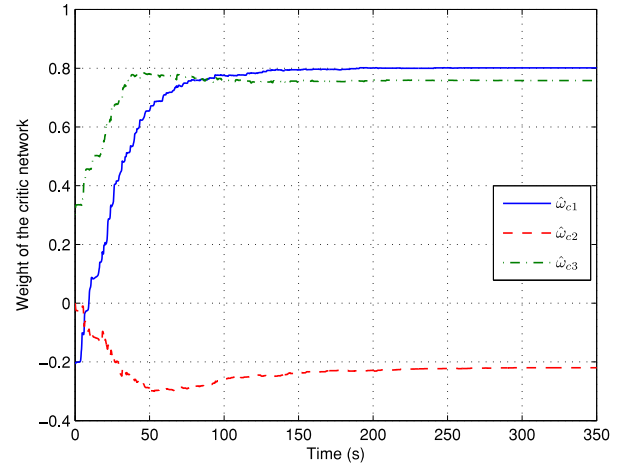


Fig. 1. Convergence process of weight vector of the critic network.

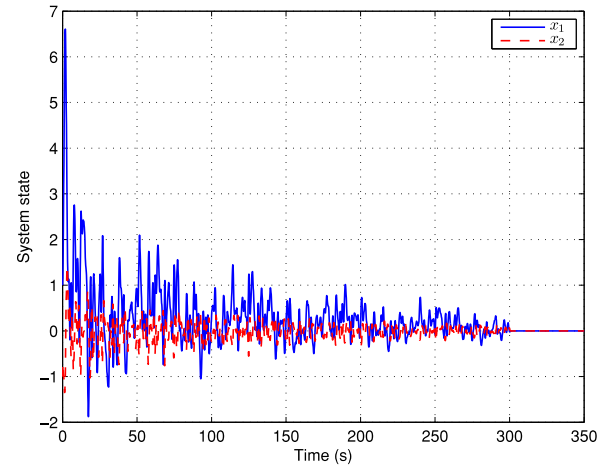


Fig. 2. State trajectory during the learning phase.

with suitable dimension). We employ the event-driven NDP method to solve the transformed optimal control problem of the nominal system

$$\dot{x} = \begin{bmatrix} -x_1 - 2x_2 \\ x_1 - x_2 - \cos x_1 \sin x_2^2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(x). \quad (44)$$

Denote the weight vector of the critic network as $\hat{\omega}_c = [\hat{\omega}_{c1}, \hat{\omega}_{c2}, \hat{\omega}_{c3}]^\top$. The activation function of the critic network is selected as $\sigma_c(x) = [x_1^2, x_1 x_2, x_2^2]^\top$. This setting fashion reflects an experimental choice after considering a tradeoff between control accuracy and calculation complexity. Besides, we let the learning rate of the critic network be $\alpha_c = 0.1$ and the initial state of system (44) be $x_0 = [1, -1]^\top$.

As indicated in [12], [19], [22], [24], and [28] the system state of the controlled plant should be persistently excited long enough, to make sure that the critic network can approximate the optimal cost function as accurate as possible. As a result, we add a probing noise to guarantee the persistency of excitation condition. We experimentally choose $\eta_2 = 0.6$ and $\ell = 12$. In addition, the sampling time is chosen as 0.1 s. During simulation, we observe that the weight vector of the critic network converges to $[0.8013, -0.2200, 0.7583]^\top$ as shown in Fig. 1. In fact, we can observe that the convergence

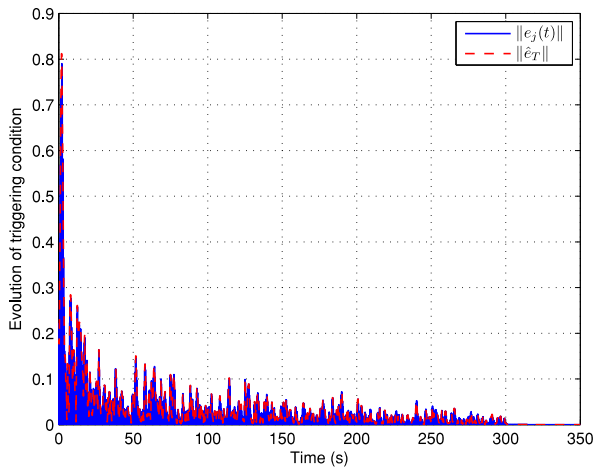
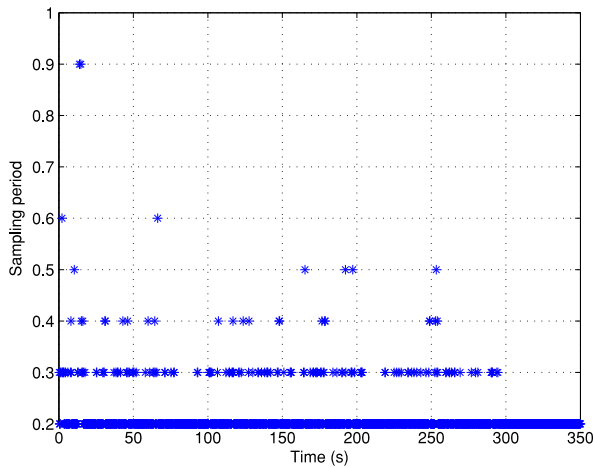
Fig. 3. Evolution of the triggering condition with $\|e_j(t)\|$ and $\|\hat{e}_T\|$.

Fig. 4. Sampling period during the learning process of the control input.

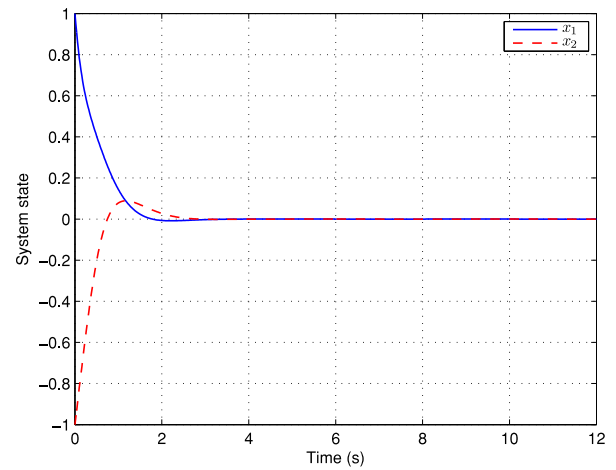


Fig. 5. State trajectory reflecting the robust stabilization.

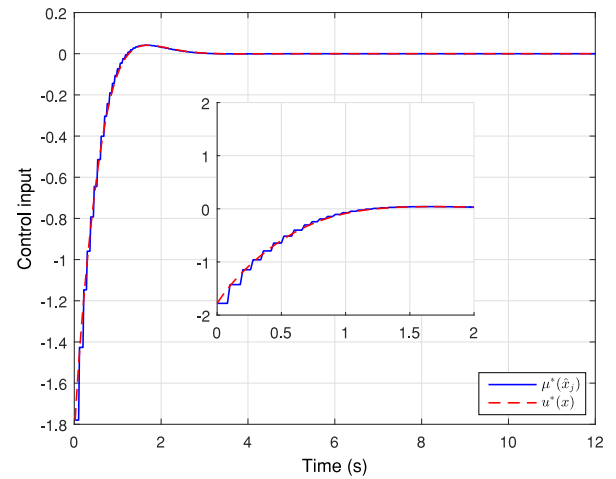


Fig. 6. Event- and time-triggered control inputs.

trend of the weight vector has appeared after 300 s. Then, the probing signal is removed. The evolution of the state trajectory during the learning phase is presented in Fig. 2. We see that the state vector converges to zero after the probing noise is turned off. In addition, the evolution of the triggering condition is shown in Fig. 3, from which we can find that the event-triggered error $e_j(t)$ and the threshold \hat{e}_T converge to zero as the state vector approaches zero. In addition, the event-triggered error is forced to zero when the triggering condition is not satisfied, which implies that the system states are sampled at the triggering instants. The sampling period during the event-triggered learning process of the control law is depicted in Fig. 4. We find that the event-triggered controller only needs 1640 samples of the state while the time-triggered controller uses 3500 samples, which means fewer transmissions are required between the plant and the controller under the event-triggering framework.

Next, we choose $p_1 = -2$ to evaluate the robust control performance with the obtained control law $\mu^*(\hat{x}_j)$ and the triggering condition (16). Let $\mathcal{L} = 12$ and $\eta_1 = 0.5$. The sampling time is chosen as 0.02 s for the uncertain system (43). From Fig. 5, we can observe the state trajectory of (43) can converge to the equilibrium point under the near-optimal control law

TABLE I
INCREASE OF SAMPLE NUMBERS

Symbol	Case 1	Case 2	Case 3	Case 4	Case 5
η_1	0.1	0.3	0.5	0.7	0.9
N_s	162	165	169	176	200

and the triggering condition (16). Particularly, Fig. 6 compares the performance of control inputs obtained under the event-triggered and the time-triggered frameworks. The event-driven controller is found to approach the time-driven controller gradually. Additionally, Fig. 7 displays the evolution of triggering condition during the robust control implementation.

Moreover, it also should be mentioned that the sample frequency of sampled-data system can be adjusted by the parameter η_1 . When η_1 is closer to 1, the system states are sampled more frequently and the same for the controller updating, which also brings in an increase of the computational burden. As for this simulation, the number of samples, N_s , is largening with the increase of the parameter η_1 , which can be illustrated by several case studies in Table I.

At last, we select $\xi = 3$ and conduct the simulation again. Figs. 8 and 9 display the state response of (43) and the evolution of triggering condition during the adaptive robust

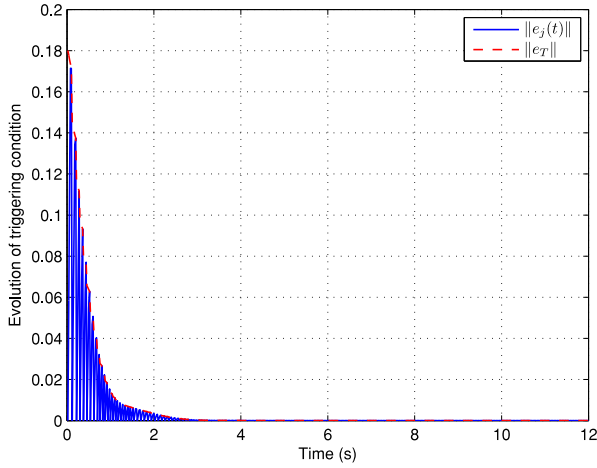
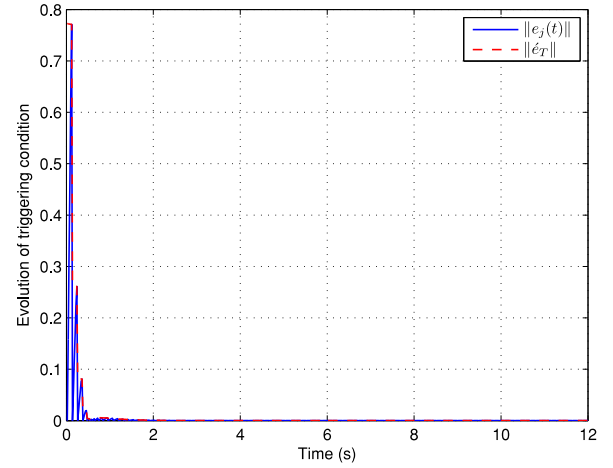
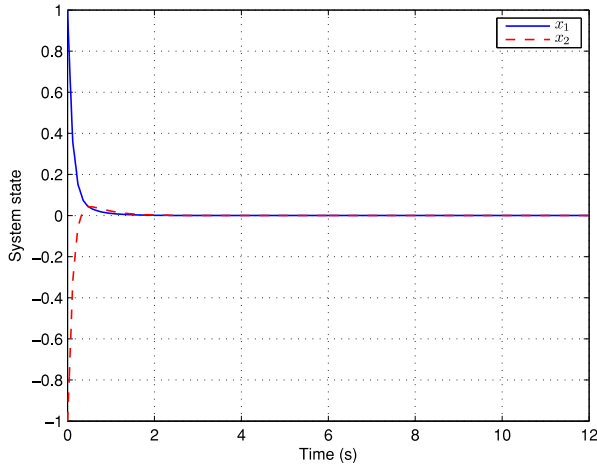
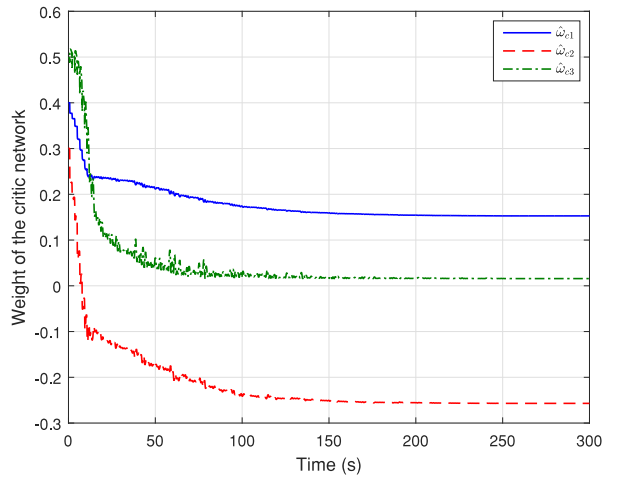
Fig. 7. Evolution of the triggering condition with $\|e_j(t)\|$ and $\|e_T\|$.Fig. 9. Evolution of the triggering condition with $\|e_j(t)\|$ and $\|e_T\|$ in case that $\xi = 3$.Fig. 8. State trajectory reflecting the robust stabilization in case that $\xi = 3$.

Fig. 10. Convergence process of weight vector of the critic network.

control implementation in case that $\xi = 3$. It can be apparently observed an improvement of the closed-loop stability when compared with the performance of the norm case (i.e., $\xi = 1$ reflected in Fig. 5). Obviously, the above results demonstrate the effectiveness and superiority of the event-driven adaptive robust control method by virtue of the NDP strategy.

Example 2: A robotic arm is a kind of mechanical component possessing similar abilities to a human arm. It may be the sum total of the mechanism or may be a part of a complex robot. In this experiment, we consider a single link robot arm chosen in [36] with the dynamics

$$\ddot{\theta}(t) = -\frac{M\bar{g}\bar{H}}{\bar{G}} \sin(\theta(t)) - \frac{D}{\bar{G}}\dot{\theta}(t) + \frac{1}{\bar{G}}u(t) \quad (45)$$

where $\theta(t)$ is the angle position of the robot arm and $u(t)$ is the control input. Other parameters of the robot arm are given in Table II. If we define $x = [x_1, x_2]^T$, where $x_1 = \theta$ and $x_2 = \dot{\theta}$, then the dynamics (45) can be rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -4.905 \sin x_1 - 0.2x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u. \quad (46)$$

We make a modification to the plant (46) by introducing an uncertain term $Z(x) = [-1, 1.5]^T p_2 x_1 x_2 \sin x_1 \cos x_2$

with $p_2 \in [-1, 1]$. Then, we can select $G(x) = [-1, 1.5]^T$, $d(\varphi(x)) = p_2 x_1 x_2 \sin x_1 \cos x_2$, and $h(\varphi(x)) = x_1 x_2 \sin x_1 \cos x_2$. Other parameters are initialized the same as example 1.

Next, we utilize the NDP strategy to solve the event-driven optimal control problem of the nominal system and then derive the adaptive robust control law of the uncertain system. Through the learning phase, the weight vector of the critic network converges to $[0.1528, -0.2570, 0.0158]^T$, as illustrated in Fig. 10. Via simulation, it is observed that the time-driven controller uses 3000 samples of the state while the event-driven controller only needs 1314 samples, thereby giving rise to a great reduction of the data transmission.

TABLE II
PARAMETERS OF THE ROBOT ARM

Symbol	Meaning	Value
M	The mass of the payload	10
\bar{g}	The acceleration of gravity	9.81
\bar{H}	The length of the arm	0.5
\bar{G}	The moment of inertia	10
D	The viscous friction	2

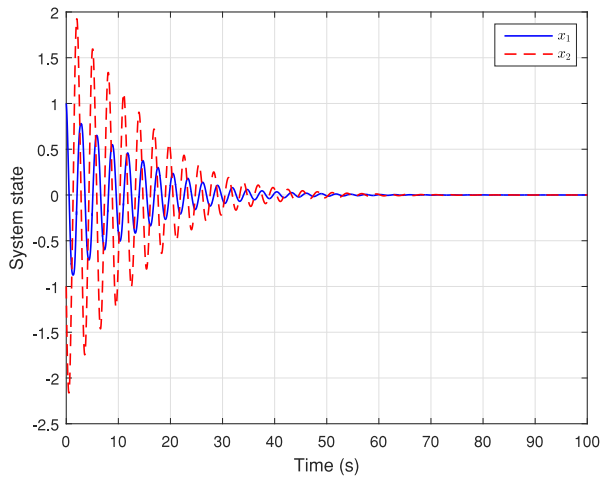


Fig. 11. State trajectory reflecting the robust stabilization.

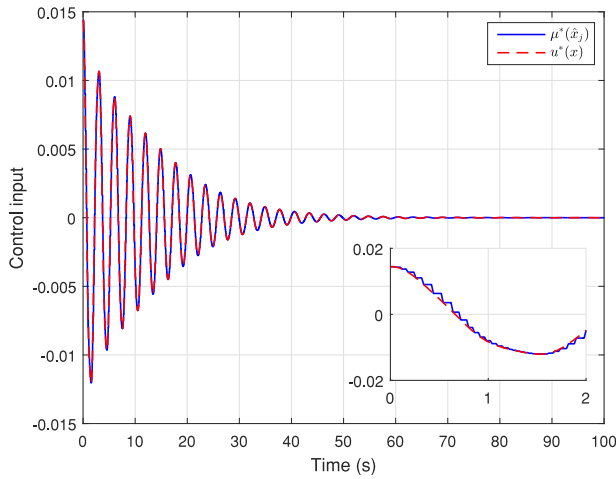


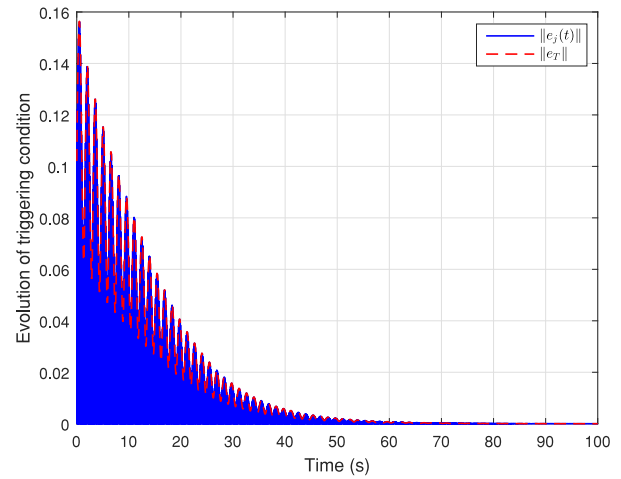
Fig. 12. Event- and time-triggered control inputs.

At last, the robust stabilization performance is investigated by choosing $p_2 = 1$ and applying the obtained event-driven control law to the uncertain plant for 100 s. Then, the system response is depicted in Fig. 11, while the comparison of control inputs between the event-driven and time-driven cases is presented in Fig. 12. Besides, the evolution of triggering condition is displayed in Fig. 13 during the adaptive robust stabilization. It is clear to find that the above results verify the effectiveness of the event-driven nonlinear adaptive robust control method based on the NDP technique.

V. CONCLUSION

A novel event-driven formulation is developed to design the adaptive robust control for a class of continuous-time uncertain nonlinear systems with a suitable triggering condition. An artificial neural network is constructed for implementing the NDP technique and establishing the event-driven approximate optimal control law with closed-loop stability analysis as well as simulation experiments.

It is worth mentioning that the controlled plant we studied in this paper represents a class of nonlinear systems with known affine nominal dynamics and unknown unmatched uncertainty.

Fig. 13. Evolution of the triggering condition with $\|e_j(t)\|$ and $\|e_T\|$.

The present method provides a channel for solving the adaptive robust stabilization problem for this kind of uncertain systems. Note that this formulation emphasizes the robustness of the NDP-based control approach. In other words, unlike the basic regulation design with optimality, this paper provides an adaptive robust control strategy for uncertain nonlinear systems by virtue of the problem transformation and the NDP technique.

The future work contains extending the present approach to adaptive robust feedback design of more complex nonlinear systems. Additionally, it is noticed that although the system uncertainty has been taken into consideration in this paper, a meaningful cost function with respect to the original uncertain system is not defined and discussed. From this point of view, the future work also includes investigating the optimality of the adaptive robust control law under event-driven formulation.

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