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Composite Adaptive Control of Uncertain Nonlinear Systems Using Immersion and Invariance Method

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Abstract **- The design of a novel composite adaptive control system for a class of uncertain systems using immersion and invariance (I&I) theory is presented. The interest here is to achieve a composite I&I adaptive control in the presence of model parametric uncertainties. Two sources of parameter information are combined for the parameter adaptation, which consists of tracking-error based adaptation law and prediction-error based adaptation law. Particularly, the tracking-error based adaptation law is constructed using I&I theory, which leads to a more flexible and effective design process of adaptation law. Stability analysis is presented using Lyapunov theory. Representative simulations are carried out on the mass-damper-spring system, which illustrate the superiority of the proposed composite I&I control scheme over the standard one.**

Index Terms – Adaptive Control, Composite Adaptive, Immersion and Invariance

I. INTRODUCTION

The problem of control design for dynamic systems with various uncertainties has been widely discussed in the literature [1-2]. Many advanced methods have been proposed to deal with this problem, among which adaptive control is one of the most common methods. In principle, adaptive control is the preferred method to get the satisfactory performance of a system in the presence of uncertainty or unknown variation in plant parameters [3-4].

Intuitively, "adaptive" means to regulate a behavior in response to a change in the system dynamics. The regulation is made by adjusting controller parameters on-line based on the measurable system signals. Various mechanisms are applied to adjusting parameters, leading to various adaptive controller schemes, where model-reference adaptive control and self-tuning regulators are two main types [2]. Some recent related works can be found in [5-7] and the references therein. Besides the continuing progress of adaptive control in theoretical, it is also of increasing interest to employ adaptive techniques in many engineering control systems [2, 8].

Among numerous advances in adaptive control research, one remarkable work is the so-called immersion and invariance (I&I) theory [9]. This theory offers a systematic process for adaptive law design, without requiring a prior Lyapunov function. Moreover, compared to the design of classical adaptive law, it has much more design freedom, which makes the dynamic behavior of the estimation error adjustable to some extent. Based on the significant theoretical advantages, many attempts have been made to adaptive controller design using I&I [10-13]. These applications show great potential of this theory for reducing the effects of plant uncertainty. More details about this theory can be found in [8-9].

It is interesting to note that all the aforementioned results use only tracking errors to update the adaptive laws. However, the tracking error is not the only source of parameter information [2]. The parameter estimation error, namely the prediction error, can also offer useful information for parameter adaptation. When using the combination of the two information sources, a better result in parameter estimation often can be obtained. This method, called composite adaptive control, is one of the focused research area in adaptive control. Some efforts can be found in the literature [5, 14-16]. However, it appears that no attempt has been made to develop composite I&I adaptive control of uncertain nonlinear systems. Therefore, it is of great interest to propose a novel composite adaptive control theory based on the I&I.

Focusing on this issue, an extension of I&I theory, which is called composite I&I adaptive control, is proposed in this paper. The main contribution of this paper lies in the design of a novel I&I-based composite adaptive controller for a class of uncertain systems. The rest of the paper is organized as follows. Section II is the problem formulation and presents the control objective. The I&I-based composite adaptive control design is presented in Section III, along with analysis of the closed-loop system stability. Then Section IV presents simulation results to validate the proposed method, and finally conclusions can be found in Section V.

II. PROBLEM FORMULATION

Consider a class of uncertain systems that can be described as follows

$$\begin{aligned}\dot{x}_1 &= F_1(x_1) + G_1(x_1)x_2 \\ \dot{x}_2 &= F_2(x_1, x_2) + G_2(x_1, x_2, \boldsymbol{\theta})u\end{aligned} \tag{1}$$

where x_1, x_2 are the states, u is the scalar control input, $\boldsymbol{\theta} \in R^n$ is the vector of unknown constant parameters, and

$F_1(x_1)$, $F_2(x_1,x_2)$, $G_1(x_1)$, $G_2(x_1,x_2,\boldsymbol{\theta})$ are nonlinear functions of the states. Consider the stabilization problem for (1) under the assumption that the vector of unknown parameters can be linearly parameterised. Therefore, (1) can be rewritten as

$$\begin{aligned} \dot{x}_1 &= F_1(x_1)+G_1(x_1)x_2 \\ \dot{x}_2 &= F_2(\boldsymbol{x})+\boldsymbol{\varphi}(\boldsymbol{x})^T\boldsymbol{\theta}_1+\boldsymbol{g}(\boldsymbol{x})^T\boldsymbol{\theta}_2 u \end{aligned} \tag{2}$$

where $G_2(x_1,x_2,\boldsymbol{\theta})u=\boldsymbol{\varphi}(\boldsymbol{x})^T\boldsymbol{\theta}_1+\boldsymbol{g}(\boldsymbol{x})^T\boldsymbol{\theta}_2 u$, $\boldsymbol{x}=[x_1,x_2]^T$ and $\boldsymbol{\theta}=[\boldsymbol{\theta}_1^T,\boldsymbol{\theta}_2^T]^T$. Moreover, $\boldsymbol{\varphi}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ are the known "regressors".

The control objective is to find a continuous adaptive control law of the form

$$\begin{aligned} \dot{\hat{\boldsymbol{\theta}}} &= \boldsymbol{\omega}(\boldsymbol{x},\hat{\boldsymbol{\theta}}) \\ u &= u(\boldsymbol{x},\hat{\boldsymbol{\theta}}) \end{aligned} \tag{3}$$

such that all the signals of the closed-loop system (2) with (3) are bounded and $\lim_{t\to\infty} x(t)=0$.

Remark 1. Physical systems that can be modelled by equations similar to (1) is very common in engineering practice. Typical examples in such form include the pendulum model, the mass-damper-spring system, the tunnel-diode circuit system, and so on [1]. Besides, the adaptive control law of higher order systems can be obtained similarly to the one of system (1) through a recursive backstepping procedure. Therefore, system (1) is of significant practical importance.

III. Immersion and Invariance Based Composite Adaptive Control Design

A. Control law design

To begin with, consider a surface defined by

$$p = x_1+\lambda_1\dot{x}_1 \tag{4}$$

where $\lambda_1>0$. Obviously, x_1 will converge to zero if $p=0$.

Differentiating (4) yields

$$\begin{aligned} \dot{p} = \lambda_1\{ &\frac{\partial F_1(x_1)}{\partial x_1}[F_1(x_1)+G_1(x_1)x_2] \\ &+\frac{\partial G_1(x_1)}{\partial x_1}[F_1(x_1)+G_1(x_1)x_2]x_2\} \\ &+\lambda_1 G_1(x_1)[F_2(x_1,x_2)+\boldsymbol{\varphi}(\boldsymbol{x})^T\boldsymbol{\theta}_1+\boldsymbol{g}(\boldsymbol{x})^T\boldsymbol{\theta}_2 u] \\ &+F_1(x_1)+G_1(x_1)x_2 \end{aligned} \tag{5}$$

Rearrange (5) as follows

$$\dot{p} = A(\boldsymbol{x})+B_1(\boldsymbol{x})^T\boldsymbol{\theta}_1+B_2(\boldsymbol{x})^T\boldsymbol{\theta}_2 u \tag{6}$$

where $B_1(\boldsymbol{x})=\lambda_1 G_1(x_1)\boldsymbol{\varphi}(\boldsymbol{x})$, $B_2(\boldsymbol{x})=\lambda_1 G_1(x_1)\boldsymbol{g}(\boldsymbol{x})$, and

$$\begin{aligned} A(\boldsymbol{x}) = \lambda_1 &\frac{\partial F_1(x_1)}{\partial x_1}[F_1(x_1)+G_1(x_1)x_2] \\ &+\lambda_1\frac{\partial G_1(x_1)}{\partial x_1}[F_1(x_1)+G_1(x_1)x_2]x_2+F_1(x_1)+G_1(x_1)x_2 \end{aligned}$$

is employed for notational convenience. Now the control problem is reduced to designing a composite adaptive control law for system (6) such that $p=0$ is a globally stable equilibrium

The stabilizing signal for (6) can be chosen as

$$\begin{aligned} B_2(\boldsymbol{x})^T(\hat{\boldsymbol{\theta}}_{T\_2}+\boldsymbol{\beta}_2+\hat{\boldsymbol{\theta}}_{P\_2})u &= \\ -A(\boldsymbol{x})-B_1(\boldsymbol{x})^T(&\hat{\boldsymbol{\theta}}_{T\_1}+\boldsymbol{\beta}_1+\hat{\boldsymbol{\theta}}_{P\_1})-\lambda_2 p \end{aligned} \tag{7}$$

with $\lambda_2>0$. $\hat{\boldsymbol{\theta}}_{T\_i}+\boldsymbol{\beta}_i+\hat{\boldsymbol{\theta}}_{P\_i}$ is the estimate of $\boldsymbol{\theta}_i$, where $\hat{\boldsymbol{\theta}}_{T\_i}$ and $\hat{\boldsymbol{\theta}}_{P\_i}$ are tracking-error based estimation and prediction-error based estimation respectively, and $\boldsymbol{\beta}_i(p)$ is a nonlinear vector function to be determined. The advantage offered by this extra term will be shown in the following. With this definition, the estimate error can then be written as

$$\boldsymbol{z} = \hat{\boldsymbol{\theta}}_T+\boldsymbol{\beta}(p)+\hat{\boldsymbol{\theta}}_P-\boldsymbol{\theta} \tag{8}$$

where $\boldsymbol{z}=[\boldsymbol{z}_1^T,\boldsymbol{z}_2^T]^T$. Using (6), (7) and (8), we have

$$\dot{p} = -\phi(\boldsymbol{x})^T\boldsymbol{z}-\lambda_2 p \tag{9}$$

where $\phi(\boldsymbol{x})=\left[B_1(\boldsymbol{x})^T, B_2(\boldsymbol{x})^T u\right]^T$.

B. Composite Adaptive Law Design

In order to get the information of prediction error, both sides of (6) are filtered as follows

$$\frac{\dot{p}}{s+\lambda_f} = \frac{A(\boldsymbol{x})}{s+\lambda_f}+\frac{\phi(\boldsymbol{x})^T\boldsymbol{\theta}}{s+\lambda_f} \tag{10}$$

where $\lambda_f>0$, and s is the Laplace operator. Rearranging (10) yields

$$p = \lambda_f p_f + A_f(\boldsymbol{x})+\phi_f(\boldsymbol{x})^T\boldsymbol{\theta} \tag{11}$$

where $p_f=\dfrac{p}{s+\lambda_f}$, $A_f(\boldsymbol{x})=\dfrac{A(\boldsymbol{x})}{s+\lambda_f}$ and $\phi_f(\boldsymbol{x})^T=\dfrac{\phi(\boldsymbol{x})^T}{s+\lambda_f}$. It can be seen that the derivative of p is removed and (11) is linearly parameterized in terms of $\boldsymbol{\theta}$. Therefore, (11) can be directly used for estimation. Introduce a virtual signal

$$Q = \phi_f(\boldsymbol{x})^T(\hat{\boldsymbol{\theta}}_T + \boldsymbol{\beta} + \hat{\boldsymbol{\theta}}_P) \quad (12)$$

Using (8), (11) and (12), we have

$$\phi_f(\boldsymbol{x})^T \boldsymbol{z} = \phi_f(\boldsymbol{x})^T(\hat{\boldsymbol{\theta}}_T + \boldsymbol{\beta}) - p + \lambda_f p_f + A_f(\boldsymbol{x}) \quad (13)$$

All the signals in the right side of (13) are available. Therefore, $\phi_f(\boldsymbol{x})^T \boldsymbol{z}$, which holds the information of prediction error, can be obtained from (13).

Now consider the design of the composite adaptation law. Differentiating (8) yields the estimate error dynamics as

$$\begin{aligned}\dot{\boldsymbol{z}} &= \dot{\hat{\boldsymbol{\theta}}}_T + \dot{\boldsymbol{\beta}} + \dot{\hat{\boldsymbol{\theta}}}_P - \dot{\boldsymbol{\theta}} \\ &= \dot{\hat{\boldsymbol{\theta}}}_T + \frac{d\boldsymbol{\beta}}{dp}\left[-\phi(\boldsymbol{x})^T \boldsymbol{z} - \lambda_2 p\right] + \dot{\hat{\boldsymbol{\theta}}}_P\end{aligned} \quad (14)$$

Considering (9) and (14), the tracking-error based estimation law $\dot{\hat{\boldsymbol{\theta}}}_T$ can be specified as

$$\dot{\hat{\boldsymbol{\theta}}}_T = \frac{d\boldsymbol{\beta}}{dp}[\lambda_2 p] \quad (15)$$

And the prediction-error based estimation law can be specified as

$$\dot{\hat{\boldsymbol{\theta}}}_P = -r_2 \phi_f(\boldsymbol{x}) \phi_f(\boldsymbol{x})^T \boldsymbol{z} \quad (16)$$

where $r_2 > 0$. Substituting (15) and (16) into (14) yields

$$\dot{\boldsymbol{z}} = \frac{d\boldsymbol{\beta}}{dp}\left[-\phi(\boldsymbol{x})^T \boldsymbol{z}\right] - r_2 \phi_f(\boldsymbol{x}) \phi_f(\boldsymbol{x})^T \boldsymbol{z} \quad (17)$$

Now the extra term $\boldsymbol{\beta}(p)$ in I&I theory is to be designed. By a proper selection of $\boldsymbol{\beta}(p)$, the estimate error dynamics (17) can be rendered stable. Moreover, it is also observable that different selections of $\boldsymbol{\beta}(p)$ will lead to different dynamic behaviour of (17), which implies the dynamic behaviour of the estimation error can be regulated to some extent. Here one choice for $\boldsymbol{\beta}(p)$ is

$$\frac{d\boldsymbol{\beta}}{dp} = r_1 \phi(\boldsymbol{x}) \quad (18)$$

with $r_1 > 0$. Substituting (18) into (17), we have

$$\dot{\boldsymbol{z}} = -r_1 \phi(\boldsymbol{x}) \phi(\boldsymbol{x})^T \boldsymbol{z} - r_2 \phi_f(\boldsymbol{x}) \phi_f(\boldsymbol{x})^T \boldsymbol{z} \quad (19)$$

Remark 2. With $\boldsymbol{\beta}$, $\dot{\hat{\boldsymbol{\theta}}}_P$ and $\dot{\hat{\boldsymbol{\theta}}}_T$ derived from (18), (16) and (15) sequentially, the stabilizing control signal can be obtained from (7).

C. Stability Analysis

Using (9) and (19), rewrite the dynamics of the closed-loop system

$$\begin{cases}\dot{p} = -\phi(\boldsymbol{x})^T \boldsymbol{z} - \lambda_2 p \\ \dot{\boldsymbol{z}} = -r_1 \phi(\boldsymbol{x}) \phi(\boldsymbol{x})^T \boldsymbol{z} - r_2 \phi_f(\boldsymbol{x}) \phi_f(\boldsymbol{x})^T \boldsymbol{z}\end{cases} \quad (20)$$

To examine the stability of the whole closed-loop system, consider the following Lyapunov candidate function

$$V = \frac{1}{2}p^2 + \frac{1}{2}\lambda_2^{-1} \boldsymbol{z}^T r_1^{-1} \boldsymbol{z} \quad (21)$$

Differentiating (21) and using (20), we have

$$\begin{aligned}\dot{V} = &-\lambda_2 p^2 - \phi(\boldsymbol{x})^T \boldsymbol{z} p - \lambda_2^{-1}\left(\phi(\boldsymbol{x})^T \boldsymbol{z}\right)^2 \\ &- r_1^{-1}\lambda_2^{-1} r_2 \left(\phi_f(\boldsymbol{x})^T \boldsymbol{z}\right)^2\end{aligned} \quad (22)$$

By Young's inequality, it can be concluded that

$$\begin{aligned}\dot{V} &\le -\frac{1}{2}\lambda_2 p^2 - \frac{1}{2}\lambda_2^{-1}\left(\phi(\boldsymbol{x})^T \boldsymbol{z}\right)^2 \\ &\quad - r_1^{-1}\lambda_2^{-1} r_2 \left(\phi_f(\boldsymbol{x})^T \boldsymbol{z}\right)^2 \\ &\le 0\end{aligned} \quad (23)$$

Therefore, $(p, \boldsymbol{z}) \in \mathcal{L}_\infty$ and $(p, \phi(\boldsymbol{x})^T \boldsymbol{z}, \phi_f(\boldsymbol{x})^T \boldsymbol{z}) \in \mathcal{L}_2$. Since the terms $(\phi(\boldsymbol{x})^T \boldsymbol{z}, \phi_f(\boldsymbol{x})^T \boldsymbol{z})$ and their time derivates are bounded, it follows that $\lim_{t\to\infty}(p, \phi(\boldsymbol{x})^T \boldsymbol{z}, \phi_f(\boldsymbol{x})^T \boldsymbol{z}) = \boldsymbol{0}$ by Barbalat's lemma. Finally, we can guarantee $\lim_{t\to\infty} x_1(t) = 0$ from (4), which completes the proof.

Remark 3. Note that the Lyapunov function mentioned above is used only for stability analysis. For conventional adaptive schemes, parameter update laws are usually constructed by careful cancelation of terms in the time-derivative of Lyapunov functions. However, it can be observed from the above composite adaptive design process that there is no need of a Lyapunov function priorly at parameter adaptive law design level.

IV. NUMERICAL SIMULATION

To verify the proposed composite adaptive control scheme, an academy example, the mass-damper-spring system modelled as [17], is used here. Consider a mass m sliding on a horizontal surface. The mass is subjected to a control force u, a resistive force $k_v\dot{x}$ due to friction, a restoring force of the spring $k_p x$, and an unknown constant disturbance d . x is the displacement from the origin, whose dynamics can be described as

$$m\ddot{x} = u + d - k_p x - k_v \dot{x} \quad (24)$$

where m , k_p and k_v are unknown positive constant. The control objective is to design an adaptive state feedback control law making x asymptotically track the reference command x_r .

To begin with, rewrite the model into the form of (1)

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \boldsymbol{\varphi}(\boldsymbol{x})^T \boldsymbol{\theta}_1 + \theta_2 u \end{aligned} \quad (25)$$

where $x_1 = x$, $\boldsymbol{\varphi}(\boldsymbol{x}) = [-x_1, -x_2, 1]^T$, $\boldsymbol{\theta}_1 = [k_p/m, k_v/m, d/m]^T$ and $\theta_2 = 1/m$.

Then define the surface p as

$$p = (x - x_r) + \lambda_1 (x - x_r)' \quad (26)$$

According to Section III, the stabilizing signal for (25) and (26) can be chosen as

$$\begin{aligned} \lambda_1(\hat{\boldsymbol{\theta}}_{T\_2} + \boldsymbol{\beta}_2 + \hat{\boldsymbol{\theta}}_{P\_2})u = -(x_2 - \dot{x}_r - \lambda_1 \ddot{x}_r) \\ -\lambda_1 \boldsymbol{\varphi}(\boldsymbol{x})^T (\hat{\boldsymbol{\theta}}_{T\_1} + \boldsymbol{\beta}_1 + \hat{\boldsymbol{\theta}}_{P\_1}) - \lambda_2 p \end{aligned} \quad (27)$$

And the tracking-error based estimation of I&I $\hat{\boldsymbol{\theta}}_T$ and $\boldsymbol{\beta}$ can be specified as

$$\begin{aligned} \frac{d\boldsymbol{\beta}}{dp} &= r_1 [\lambda_1 \boldsymbol{\varphi}(\boldsymbol{x})^T, \lambda_1 u]^T \\ \dot{\hat{\boldsymbol{\theta}}}_T &= r_1 \lambda_2 p [\lambda_1 \boldsymbol{\varphi}(\boldsymbol{x})^T, \lambda_1 u]^T \end{aligned} \quad (28)$$

And the prediction-error based estimation law can be specified as

$$\dot{\hat{\boldsymbol{\theta}}}_P = -r_2 \phi_f(\boldsymbol{x}) \phi_f(\boldsymbol{x})^T \boldsymbol{z} \quad (29)$$

where $\phi_f(\boldsymbol{x})^T \boldsymbol{z}$ can be calculated from (13).

The initial states of the mass-damper-spring system are set as $(x_1, x_2) = (0, 0)$, and the parameters are chosen as $(m, k_p, k_v, d) = (10, 1, 1, 1)$. A relatively aggressive reference command is considered here, which is a sinusoidal signal with an amplitude of 10 and a frequency of $1\,rad/s$. To illustrate the effectiveness of the proposed composite adaptive control scheme, the results conducted on the standard I&I-based adaptive control of the mass-damper-spring system are presented simultaneously, which are used as the baseline for comparison. Moreover, during simulations all the parameters of both the two control systems are the same except that there is an extra parameter estimation part of (29) for the composite adaptive control system.

The initial values for the estimates of unknown parameters are firstly set as $\hat{\boldsymbol{\theta}}(0) = [0.1, 0.1, 0.1, 0.1]^T$, which are around the actual values. The simulation results are shown in Figs. 1-4. Good tracking performances of x_1 for the sinusoidal reference command can be seen in Fig. 1 and Fig. 2 for both composite and standard I&I adaptive control. Moreover, it can be observed from Fig. 3 and Fig. 4 that the time histories of surface p and $\phi(\boldsymbol{x})^T \boldsymbol{z}$ asymptotically converge to zero. According to (4) and (9), this further implies the convergence of x_1 to the reference command. Overall, the state behaviour of both the two control systems are almost the same, except that surface p of composite I&I adaptive control oscillates slightly.

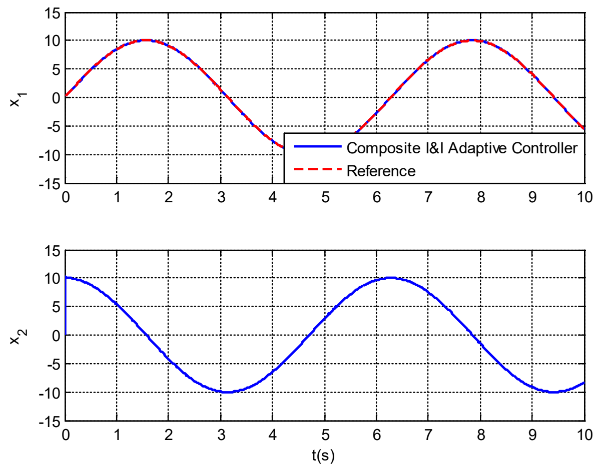

Fig. 1 Trajectories of States (Composite I&I Adaptive Controller)

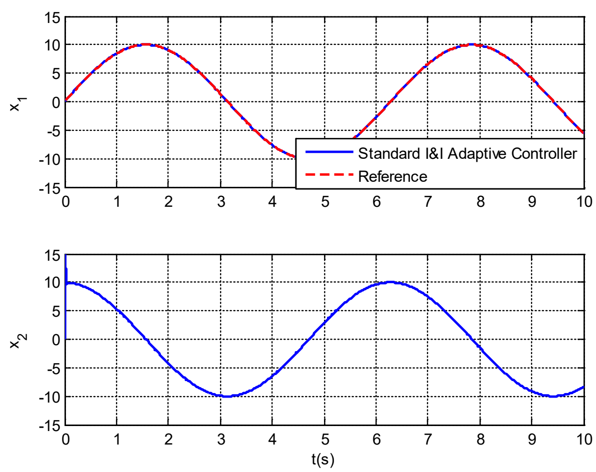

Fig. 2 Trajectories of States (Standard I&I Adaptive Controller)

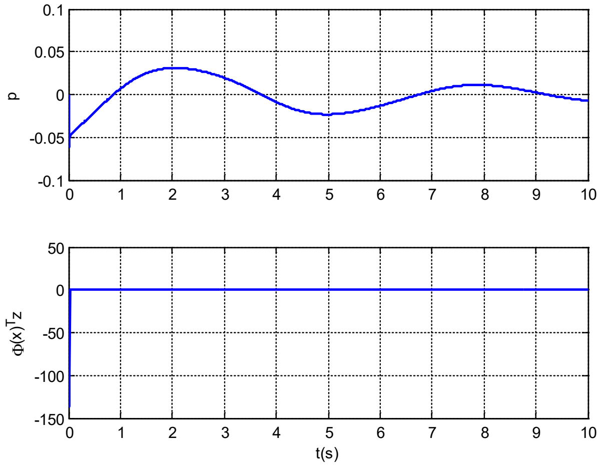

Fig. 3 p and $\phi(\boldsymbol{x})^T z$ (Composite I&I Adaptive Controller)

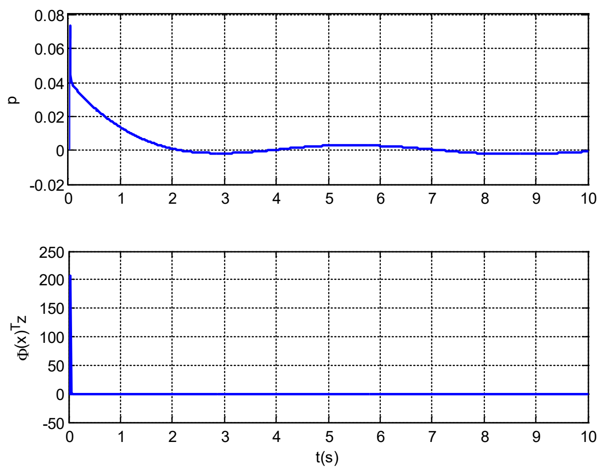

Fig. 4 p and $\phi(\boldsymbol{x})^T z$ (Standard I&I Adaptive Controller)

Now the initial values for the estimates of unknown parameters are set as $\hat{\boldsymbol{\theta}}(0) = [0.5, 0.5, 0.5, 0.1]^T$, which are far away from the actual values. The simulation results are shown in Figs. 5-8. Significant differences between the results of composite and standard I&I adaptive control can be observed obviously this time. Fig. 5 displays that the composite I&I adaptive controller can still provide rapid and accurate tracking of the reference command, while the standard I&I adaptive controller is severely degraded as shown in Fig. 6. Similar results can be found for p and $\phi(\boldsymbol{x})^T z$. Fig. 8 shows that adaptation error $\phi(\boldsymbol{x})^T z$ of the standard I&I adaptive control oscillates severely, while Fig. 7 shows a stable and damped behavior of $\phi(\boldsymbol{x})^T z$ for the composite I&I adaptive control. Actually, a faster adaptation without getting the oscillatory behavior is the key feature of composite adaptive control. With these simulation results, it clearly demonstrates the superiority of the proposed composite I&I adaptive control system.

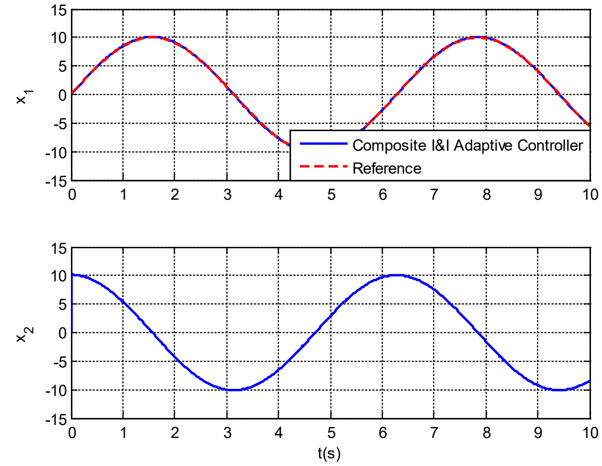

Fig. 5 Trajectories of States (Composite I&I Adaptive Controller)

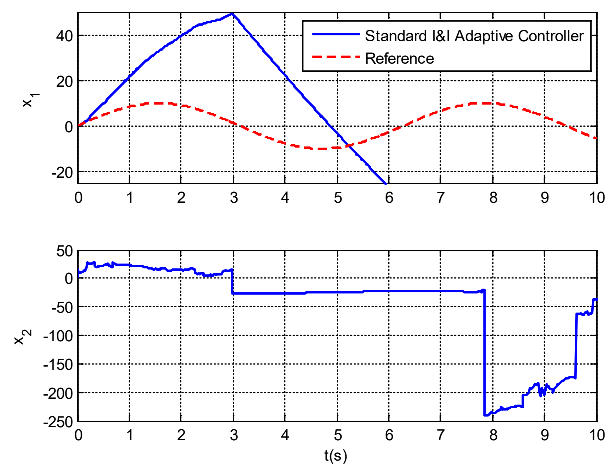

Fig. 6 Trajectories of States (Standard I&I Adaptive Controller)

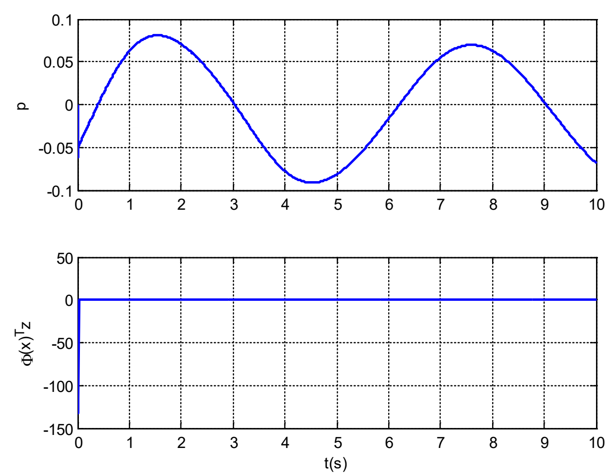

Fig. 7 p and $\phi(\boldsymbol{x})^T z$ (Composite I&I Adaptive Controller)

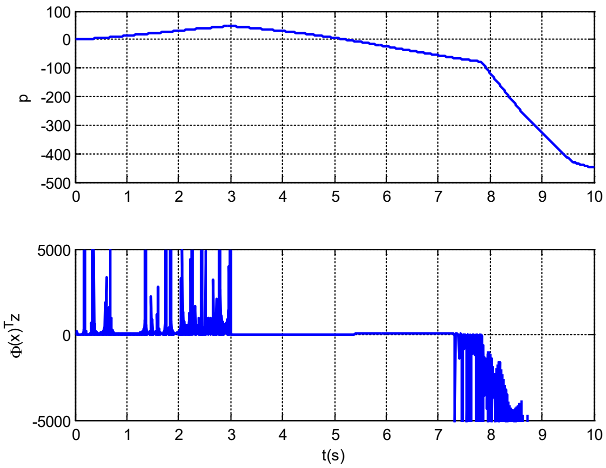

Fig. 8 p and $\phi(x)^T z$ (Standard I&I Adaptive Controller)

V. Conclusion

A novel immersion and invariance (I&I) based composite adaptive control is proposed in this paper. The design of a composite adaptive controller for a class of uncertain systems is presented. The main feature of this method lies in the construction of the estimator, which consists of tracking-error based estimation and prediction-error based estimation. I&I technology is used to design the tracking-error based estimation, with an extra nonlinear term that makes the estimation error dynamics adjustable to some extent. Stability analysis of the whole closed-loop system is demonstrated using Lyapunov theory. Two sets of simulations are performed on the mass-damper-spring system. Improved performance of the proposed composite I&I control has been illustrated via these simulations.

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References

[1] K. Hassan, *Nonlinear Systems, 3rd ed.*, Prentice Hall, 2001.
[2] J. Slotine, and W. Li, *Applied Nonlinear Control*, Prentice Hall, Englewood Cliffs, 1991.
[3] P. Ioannou and J. Sun, *Robust Adaptive Control*, New York: Dover Publication, 1996.
[4] K. Astrom and B. Wittenmark, *Adaptive Control, 2nd ed.*, New York: Dover Publication, 2008.
[5] S. Jana and M. Bhat, "Composite adaptive control using output feedback and application to micro air vehicle," *2017 SICE International Symposium on Control Systems*, Okayama, Japan, March 6-9, 2017, pp. 99-106.
[6] S.Kersting and M. Buss, "Direct and indirect model reference adaptive control for multivariable piecewise affine systems," *IEEE Transactions on Automatic. Control*, 2017, in press.
[7] L. Xing, C. Wen, Z. Liu, H. Su, and J. Cai, "Event-Triggered adaptive control for a class of uncertain nonlinear systems," *IEEE Transactions on Automatic. Control*, vol. 62, no. 4, pp. 2071-2076, July 2017.
[8] A. Astolfi, D. Karagiannis, and R. Ortega, *Nonlinear and Adaptive Control with Applications*, London: Springer, 2008, Chapter 9.
[9] A. Astolfi and R. Ortega, "Immersion and invariance: a new tool for stabilization and adaptive control of nonlinear systems," *IEEE Transactions on Automatic. Control*, vol. 48, no. 4, pp. 590-606, Apr. 2003.
[10] J. Zhang, Q. Li, N. Cheng, N. Chen, and B. Liang, "Adaptive dynamic surface control for unmanned aerial vehicles based on attractive manifolds," *Journal of Guidance, Control, and Dynamics,* vol. 36, no. 6, pp.1776–1783, Nov.-Dec. 2013.
[11] Y. Kobayashi and M. Takahashi, "Design of nonlinear adaptive flight control system based on immersion and invariance," In: *Proceedings of AIAA Guidance, Navigation, and Control Conference*, Chicago, USA, August 10-13, 2009.
[12] K. Lee and S. Singh, "Immersion- and invariance-based adaptive missile control using filtered signals," *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering,* vol. 226, no. 6, pp.646–663, 2012.
[13] J. Hu and H. Zhang, "Immersion and invariance based command-filtered adaptive backstepping control of VTOL vehicles," *Automatica*; vol. 49, no. 7, pp. 2160–2167, July 2013.
[14] P. Patre, W. Mackunis, M. Johnson and W. Dixon, "Composite adaptive control for Euler_Lagrange systems with additive disturbances," *Automatica*; vol. 46, no. 1, pp. 140–147, Jan. 2010.
[15] A. Dando, "Spacecraft attitude maneuvers using composite adaptive control with invariant sliding manifold," *Proceedings of the Conference on Decision and Control,* Shanghai, China, Dec. 16-18, 2009, pp. 4535-4550.
[16] J. Na, et al, "Robust adaptive finite-time parameter estimation and control of nonlinear systems," *2011 IEEE International Symposium on Intelligent Control,* Denver, USA, Sep. 28-30, 2011, pp.1014-1019.
[17] R. Marino and P. Tomei, *Nonlinear Control Design: Geometric, Adaptive and Robust,* Prentice Hall, 1996.