Adaptive Immersion and Invariance Continuous Finite-time Control of Hypersonic Vehicles*

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Abstract— A novel nonlinear multi-input/multi-output adaptive continuous finite-time control system for air-breathing hypersonic vehicles with parametric uncertainty is proposed in this paper. The control system is based on the frame of backstepping design and time-scale separation principle to decouple the high-order vehicle model into two first-order and one second-order subsystems. For the first-order subsystems, controllers combining fast terminal sliding mode control (TSMC) with adaptive immersion and invariance (I&I) are designed. For the second-order subsystem, the controller is a combination of non-singular TSMC and adaptive I&I. The finite-time stability of each subsystem is analyzed by Lyapunov theory. Simulation experiments are conducted to demonstrate that the control system has the feature of fast and accurate tracking to attitude and velocity commands.

I. INTRODUCTION

Hypersonic flight makes many things possible, such as economically reusable space shuttle, faster global travel and prompt global strike. Since the 1950's, breakthroughs have been made in scramjets which obtain oxygen from atmosphere [1]. Therefore, air-breathing hypersonic vehicles (AHVs) have gained sustained attention and extensive research, because AHVs, powered by scramjet, enable hypersonic flight with much more load compared to rocket-powered aerocraft [2]. However, it's a great challenge to design control systems for AHVs due to their special configuration and complex dynamics [3]. To make those incredible civilian and military applications realities, numbers of research work have been done about AHVs [4].

AHVs are fast time-variant, strong coupling and high nonlinear system with significant uncertainty and flexibility. These properties make conventional linear flight control methods performance degraded. Thus nonlinear control methods have become the research focus. In [5], a MIMO adaptive sliding mode controller is designed based on feedback linearization framework for a generic hypersonic vehicle. In [6], a nonlinear robust adaptive controller is proposed for AHVs using the so-called nonlinear sequential loop closure approach which is an application of backstepping

method in essence. These two control frameworks, feedback linearization and backstepping, are the mainstream schemes of hypersonic vehicle control system design.

As far as we know, seldom work refers to AHV finite-time control until now. Compared with Lyapunov stability, finite-time stability is characterized by faster convergence, higher control precision and stronger robustness to model uncertainties and disturbances [7]. It makes great senses to apply finite-time stability theory to the robust and adaptive control system design of AHVs. In [7], the definition of finite-time-stable equilibrium is given and Lyapunov finite-time stability theory is established, which are theoretical foundation of finite-time control system design. The most active methodologies for finite-time stabilizing controller design are homogeneous system theory [8] and terminal sliding mode control (TSMC) [9]. Finite-time stable controller design based on homogeneous system theory derives from such a theorem that a homogeneous system is finite-time stable if and only if it is asymptotically stable and has a negative degree of homogeneity [8]. The method involves many complex mathematical concepts and is difficult to apply to nonlinear systems with uncertainty and adaptive control systems, because it's an intricate work to determine whether they are homogeneous. Therefore, its application is confined to simple linear systems, for example, the double integrator. By contrast, TSMC is more understandable and widely used. Different from conventional sliding mode control, TSMC not only can reach sliding surface in finite-time, but also can make states reach the equilibrium point in finite-time by designing the nonlinear sliding manifold.

Like conventional linear SMC, TSMC is robust to matched model imprecision, and meanwhile has the disadvantage of chattering which must be avoided in the high-performance control systems. The most studied approach to overcome the shortcoming is to combine SMC with adaptive control method. Recently, adaptive immersion and invariance (I&I) approach [10] attracts much more attention compared to Lyapunov adaptive method. The design of I&I adaptive estimators is independent of the controller design, which provides a structured design procedure of the control system. Further the I&I approach will introduce a cross-term of states and estimation errors which makes the stability analysis easier [11].

In this paper, an adaptive I&I TSMC system is designed for AHV to achieve states finite-time stable. In the next section, the AHVs' model and related preliminary is introduced. The detailed design procedure of the control system is expounded in Section III. The simulation results and

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its analysis are stated in Section IV. The last section is conclusions of the paper.

II. HYPERSONIC VEHICLE MODEL AND PRELIMINARY

A. Hypersonic Vehicle Model

Assuming a flat Earth, the longitudinal dynamics of flexible AHVs can be described by [6]

$$\dot{V} = (T\cos\alpha - D)/m - g\sin\gamma \tag{1a}$$

$$\dot{h} = V\sin\gamma \tag{1b}$$

$$\dot{\gamma} = (L + T\sin\alpha)/(mV) - (g\cos\gamma)/V \tag{1c}$$

$$\dot{\alpha} = -(L + T\sin\alpha)/(mV) + Q + (g\cos\gamma)/V \tag{1d}$$

$$\dot{Q} = M/I_{yy} \tag{1e}$$

$$\ddot{\eta}_i = -2\zeta_i\omega_i\dot{\eta}_i - \omega_i^2\eta_i + N_i, \ i = 1, 2, 3. \tag{1f}$$

And the approximate expressions of the aerodynamic forces and moments, namely, thrust T, lift L, drag D, pitch moment M and generalized forces N_i, are

$$
\begin{aligned}
T &\approx \bar{q}S\big[C_{T,\phi}(\phi) + C_T(\alpha) + \boldsymbol{C}_T^\eta\boldsymbol{\eta}\big] \\
L &\approx \bar{q}SC_L(\alpha, \boldsymbol{\delta}, \boldsymbol{\eta}) \\
D &\approx \bar{q}SC_D(\alpha, \boldsymbol{\delta}, \boldsymbol{\eta}) \\
M &\approx z_TT + \bar{q}\bar{c}SC_M(\alpha, \boldsymbol{\delta}, \boldsymbol{\eta}) \\
N_i &\approx \bar{q}S\big(N_i^{\alpha^2}\alpha^2 + N_i^\alpha\alpha + N_i^{\delta_c}\delta_c + N_i^0 + \boldsymbol{N}_i^\eta\boldsymbol{\eta}\big), i = 1, 2, 3,
\end{aligned}
\tag{2}
$$

where $\bar{q} = \frac{1}{2}\rho V^2$, and

$$
\begin{aligned}
C_{T,\phi}(\alpha) &= C_T^{\phi\alpha^3}\alpha^3 + C_T^{\phi\alpha^2}\alpha^2 + C_T^{\phi\alpha}\alpha + C_T^\alpha \\
C_T(\alpha) &= C_T^3\alpha^3 + C_T^2\alpha^2 + C_T^1\alpha + C_T^0 \\
C_M(\alpha, \boldsymbol{\delta}, \boldsymbol{\eta}) &= C_M^{\alpha^2}\alpha^2 + C_M^\alpha\alpha + C_M^{\delta_e}\delta_e + C_M^{\delta_c}\delta_c + C_M^0 + \boldsymbol{C}_M^\eta\boldsymbol{\eta} \\
C_L(\alpha, \boldsymbol{\delta}, \boldsymbol{\eta}) &= C_L^\alpha\alpha + C_L^{\delta_e}\delta_e + C_L^{\delta_c}\delta_c + C_L^0 + \boldsymbol{C}_L^\eta\boldsymbol{\eta} \\
C_D(\alpha, \boldsymbol{\delta}, \boldsymbol{\eta}) &= C_D^{\alpha^2}\alpha^2 + C_D^\alpha\alpha + C_D^{\delta_e^2}\delta_e^2 + C_D^{\delta_e}\delta_e \\
&\quad + C_D^{\delta_c^2}\delta_c^2 + C_D^{\delta_c}\delta_c + C_D^0 + \boldsymbol{C}_D^\eta\boldsymbol{\eta} \\
\boldsymbol{C}_j^\eta &= \big[C_j^{\eta_1}, 0, C_j^{\eta_2}, 0, C_j^{\eta_3}, 0\big], \ j = T, M, L, D \\
\boldsymbol{N}_i^\eta &= \big[N_i^{\eta_1}, 0, N_i^{\eta_2}, 0, N_i^{\eta_3}, 0\big], i = 1, 2, 3,
\end{aligned}
\tag{3}
$$

where $\boldsymbol{\delta} = [\delta_e, \delta_c]^T$, $\boldsymbol{\eta} = [\eta_1, \dot{\eta}_1, \eta_2, \dot{\eta}_2, \eta_3, \dot{\eta}_3]^T$.

This model consists of five rigid-body state variables $\boldsymbol{x} = [V, h, \gamma, \alpha, Q]^T$, six flexible states $\boldsymbol{\eta} = [\eta_1, \dot{\eta}_1, \eta_2, \dot{\eta}_2, \eta_3, \dot{\eta}_3]^T$ and three control inputs $\boldsymbol{u} = [\phi, \delta_e, \delta_c]^T$, which affect flight states through aerodynamics forces and moments. The controlled output is selected as $\boldsymbol{y} = [V, h]^T$. And the reference command is denoted by $\boldsymbol{y}_{ref} = [V_{ref}, h_{ref}]^T$. The main relevant variables and parameters above are given in Table I.

TABLE I. Nomenclature

V	Vehicle velocity	$\bar{q}$	Dynamic pressure
h	Flight altitude	ρ	Air density
γ	Flight-path angle	S	Reference area
α	Angle-of-attack	$\bar{c}$	Mean aerodynamic chord
Q	Pitch rate	z_T	Thrust moment arm
ϕ	Fuel-to-air ratio	m	Vehicle mass
δ_e	Elevator deflection	I_{yy}	Moment of inertia
δ_c	Canard deflection	η_i	ith generalized elastic coordinate

The model described above, is used for simulation to validate the performance of control system. For control

system design, the model is simplified from the following two points [12]. First, the flexible dynamics are removed and their effects are taken as internal disturbances. Then, the canard is ganged with the elevator as $\delta_c = k_{ec}\delta_e$, where the coefficient k_{ec} is a negative constant [13].

It's assumed that all aerodynamic coefficients, i.e., $C_T^{(\cdot)}$, $C_M^{(\cdot)}$, $C_L^{(\cdot)}$ and $C_T^{(\cdot)}$ are uncertain which are modeled as $C_j = C_j^*(1 + \Delta C_j)$, where C_j^* and ΔC_j are nominal value and uncertainty respectively. In the paper, a maximum variation for each coefficient is set within 40%, namely, $|\Delta C_j| \le 0.4$. The control object is to achieve finite-time stable tracking of velocity and altitude reference trajectories in the presence of significant parametric uncertainty.

B. Preliminary

Consider the following nonlinear second-order system

$$
\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = f(\boldsymbol{x}) + g(\boldsymbol{x})u = \boldsymbol{\varphi}_1(\boldsymbol{x})^T\boldsymbol{\theta}_1 + \boldsymbol{\varphi}_2(\boldsymbol{x})^T\boldsymbol{\theta}_2u
\end{cases},
\tag{4}
$$

where $\boldsymbol{x} = [x_1, x_2]^T$, $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ can be linear parameterized and $g(\boldsymbol{x}) \ne 0$, $\boldsymbol{\varphi}_1(\boldsymbol{x})$ and $\boldsymbol{\varphi}_2(\boldsymbol{x})$ are known vector-functions while $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are unknown constant vectors with proper dimensionality. The problem is to design an adaptive I&I non-singular TSMC law to make the origin of (4) globally finite-time stable.

According to the adaptive I&I method [12], let the estimates of $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ as $\hat{\boldsymbol{\theta}}_1 + \boldsymbol{\beta}_1$ and $\hat{\boldsymbol{\theta}}_2 + \boldsymbol{\beta}_2$ respectively. The additional terms $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ afford extra design degree of freedom for shaping the dynamics of estimation errors which are

$$\boldsymbol{z} = \hat{\boldsymbol{\theta}} + \boldsymbol{\beta} - \boldsymbol{\theta}, \tag{5}$$

where $\boldsymbol{z} = \big[\boldsymbol{z}_1^T, \boldsymbol{z}_2^T\big]^T$, $\hat{\boldsymbol{\theta}} = \big[\hat{\boldsymbol{\theta}}_1^T, \hat{\boldsymbol{\theta}}_2^T\big]^T$, $\boldsymbol{\beta} = \big[\boldsymbol{\beta}_1^T, \boldsymbol{\beta}_2^T\big]^T$ and $\boldsymbol{\theta} = \big[\boldsymbol{\theta}_1^T, \boldsymbol{\theta}_2^T\big]^T$.

Select the non-singular terminal sliding surface as [14]

$$s = x_1 + k_1^{-1}x_2^{\alpha_1}, \tag{6}$$

where $k_1 > 0$ and $\alpha_1 = p/q$, p and q are positive odd integers and $q < p < 2q$. It indicates that when $s = 0$, x_1 and x_2 will converge to zero in finite-time. Replacing the unknown parameters of (4) with their estimates, the control law can be designed as

$$u = \frac{-\frac{k_1}{\alpha_1}x_2^{2-\alpha_1} - \lambda s - k_2|s|^{\alpha_2}\text{sign}(s) - \boldsymbol{\varphi}_1^T(\hat{\boldsymbol{\theta}}_1 + \boldsymbol{\beta}_1)}{\boldsymbol{\varphi}_2^T(\hat{\boldsymbol{\theta}}_2 + \boldsymbol{\beta}_2)}. \tag{7}$$

Substituting (7) into the system (4), one can get the closed-loop system

$$
\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = -\frac{k_1}{\alpha_1}x_2^{2-\alpha_1} - \boldsymbol{\Phi}^T\boldsymbol{z} - \lambda s - k_2|s|^{\alpha_2}\text{sign}(s)
\end{cases},
\tag{8}
$$

where $\boldsymbol{\Phi} = \big[\boldsymbol{\varphi}_1(\boldsymbol{x})^T, \boldsymbol{\varphi}_2(\boldsymbol{x})^Tu\big]^T$.

Substituting (7) into the time derivative of (6), the dynamics of the sliding surface can be written as

$$\dot{s} = a\Big[-\boldsymbol{\Phi}^T\boldsymbol{z} - \lambda s - k_2|s|^{\alpha_2}\text{sign}(s)\Big], \tag{9}$$

where $a = \frac{\alpha_1}{k_1} x_2^{\alpha_1 - 1}$ and $a > 0$ for $x_2 \neq 0$. Substituting (9) into the derivative of (5) yields the dynamics of the estimation errors

$$\dot{z} = \dot{\hat{\theta}} - \frac{\partial \beta}{\partial s} \left\{ a \left[\Phi^T z + \lambda s + k_2 |s|^{\alpha_2} \sin(s) \right] \right\}. \quad (10)$$

Then, by the adaptive I&I method, the adaptive law could be designed as

$$\begin{cases} \dot{\hat{\theta}} = \dfrac{\partial \beta}{\partial s} \left\{ a \left[\lambda s + k_2 |s|^{\alpha_2} \operatorname{sign}(s) \right] \right\} \\ \dfrac{\partial \beta}{\partial s} = r \Phi \end{cases}, \quad (11)$$

where r is a positive definite diagonal matrix with proper dimensionality. Substituting (11) into (10), the dynamics of the estimation errors can be rewritten as

$$\dot{z} = -a r \Phi \Phi^T z, \quad (12)$$

Below, the stability of the closed-loop system and finite-time reach of states will be proved. Define a Lyapunov function candidate

$$W(s, z) = \frac{1}{2} s^2 + \frac{1}{2} \lambda^{-1} z^T r^{-1} z, \quad (13)$$

by Young's inequality, whose derivative is

$$\dot{W} \leq -a \left[\frac{1}{2} \lambda s^2 + k_2 |s|^{\alpha_2 + 1} + \frac{1}{2} \lambda^{-1} \left(\Phi^T z \right)^2 \right] \leq 0. \quad (14)$$

Obviously, $\dot{W}$ is negative semidefinite, therefore the function W is non-increasing and bounded. An invariant set of the closed-loop system is

$$E = \left\{ (s, z) \mid x_2 = 0 \text{ and } s = 0, \Phi^T z = 0 \right\}. \quad (15)$$

The trajectory of the closed-loop system cannot stay on the line $x_2 = 0$ unless $x_1 = 0$. In another word, the closed-loop system cannot reach and stay on $x_2 = 0$, except for the origin. Therefore, the invariant set (15) can be further reduced as

$$E' = \left\{ (s, z) \mid s = 0, \Phi^T z = 0 \right\}. \quad (16)$$

According to the LaSalle's invariance principle, it yields $\lim_{t \to \infty} \left(s, \Phi^T z \right) = 0$. In another word, the closed-loop system, whose phase portrait is shown as Fig. 1, is asymptotically stable.
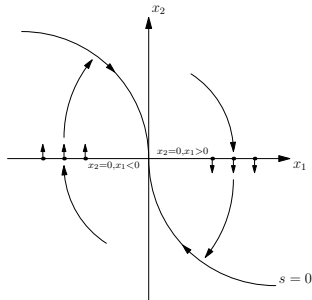

Fig. 1. Phase portrait of the closed-loop system

From the above conclusion that the state s is asymptotically stable, one can get that $s\dot{s} < 0$ if $s \neq 0$, namely, $\dot{s} \operatorname{sign}(s) < 0$ if $s \neq 0$ which leads to

$$a \left[-\Phi^T z \operatorname{sign}(s) - \lambda |s| - k_2 |s|^{\alpha_2} \right] < 0. \quad (17)$$

As explained before, $a = 0 \, (x_2 = 0)$ cannot make the states stop. Thus (17) can be rewritten as

$$-\Phi^T z \operatorname{sign}(s) - \lambda |s| - k_2 |s|^{\alpha_2} < 0. \quad (18)$$

Then, there must exit a constant b, which $0 < b < 1$, satisfying

$$-\Phi^T z \operatorname{sign}(s) < b \left(\lambda |s| + k_2 |s|^{\alpha_2} \right). \quad (19)$$

To show the finite-time stability of the state s, choose another Lyapunov function candidate $W_2 = |s|$. One gets

$$\dot{W}_2 = \dot{s} \operatorname{sign}(s) < -a(1 - b)\lambda |W_2| - a(1 - b) k_2 |W_2|^{\alpha_2}. \quad (20)$$

According to the Lyapunov finite-time stability theory, the state s is finite-time stable.

Therefore, the proposed adaptive I&I non-singular TSMC method achieves the finite-time stability of the states of the system (4) and asymptotic stability of the unknown parameters' estimation. It will be applied to the altitude control of HSV in the next section to evaluate its performance.

III. Control System Design

A. Velocity Subsystem Controller

Let the velocity tracking error be $\tilde{V} = V - V_{ref}$. Substituting the expressions of T and D and the aerodynamic coefficients into (1a), one gets the dynamics of the velocity tracking error as

$$\dot{\tilde{V}} = \varphi_{V2}^T \theta_{V2} \phi + \varphi_{V1}^T \theta_{V1} - g \sin \gamma - \dot{V}_{ref}, \quad (21)$$

where $\theta_{V1} \in \mathbb{R}^9$, $\theta_{V2} \in \mathbb{R}^4$ are vectors of uncertain parameters and

$$\varphi_{V1} = \frac{\bar{q}S}{m} \left[\alpha^3 \cos \alpha, \alpha^2 \cos \alpha, \alpha \cos \alpha, \cos \alpha, -\alpha^2, -\alpha, \right.$$
$$\left. -\delta_e^2, -\delta, -1 \right]^T$$
$$\theta_{V1} = \left[C_T^3, C_T^2, C_T^1, C_T^0, C_D^{\alpha^2}, C_D^{\alpha}, C_D^{\delta_e^2} + k_{ec}^2 C_D^{\delta_c^2}, \right.$$
$$\left. C_D^{\delta_e} + k_{ec} C_D^{\delta_c}, C_D^0 \right]^T$$
$$\varphi_{V2} = \bar{q}S \cos \alpha \left[\alpha^3, \alpha^2, \alpha, 1 \right]^T / m$$
$$\theta_{V2} = \left[C_T^{\phi\alpha^3}, C_T^{\phi\alpha^2}, C_T^{\phi\alpha}, C_T^{\phi} \right]^T.$$

θ_{V1} and θ_{V2} are regarded as unknown constants required to estimate online with $\hat{\theta}_{V1} + \beta_{V1}$ and $\hat{\theta}_{V2} + \beta_{V2}$ respectively according to the adaptive I&I approach. Define the estimation errors as

$$z_V = \hat{\theta}_V + \beta_V - \theta_V, \quad (22)$$

where $z_V = \left[z_{V1}^T, z_{V2}^T \right]^T$, $\hat{\theta}_V = \left[\hat{\theta}_{V1}^T, \hat{\theta}_{V2}^T \right]^T$, $\beta_V = \left[\beta_{V1}^T, \beta_{V2}^T \right]^T$ and $\theta_V = \left[\theta_{V1}^T, \theta_{V2}^T \right]^T$.

Design the sliding surface as

$$s_V = \dot{\tilde{V}} + \lambda_V \tilde{V} + k_V |\tilde{V}|^{\alpha_V} \operatorname{sign}(\tilde{V}), \quad (23)$$

where $\lambda_V, k_V > 0$ and $0 < \alpha_V < 1$ are constants to be designed. Assuming $s_V = 0$, one could get the control law as

$$\phi = \frac{\begin{aligned} -\lambda_V \tilde{V} - k_V |\tilde{V}|^{\alpha_V} \operatorname{sign}(\tilde{V}) + g \sin \gamma \\ + \dot{V}_{ref} - \varphi_{V1}^T (\hat{\theta}_{V1} + \beta_{V1}) \end{aligned}}{\hat{\theta}_{V2} + \beta_{V2}}. \quad (24)$$

Substituting the control law (24) into (23), the velocity tracking error dynamics can be rewritten as

$$\dot{\tilde{V}} = -\boldsymbol{\Phi}_V^T \boldsymbol{z}_V - \lambda_V \tilde{V} - k_V |\tilde{V}|^{\alpha_V} \operatorname{sign}(\tilde{V}), \qquad (25)$$

where $\boldsymbol{\Phi}_V = \left[\boldsymbol{\varphi}_{V1}^T, \boldsymbol{\varphi}_{V2}^T \phi\right]^T$. Substituting (25) into the derivative of (22) yields the dynamics of the estimation errors

$$\dot{\boldsymbol{z}}_V = \dot{\hat{\boldsymbol{\theta}}}_V - \frac{\partial \boldsymbol{\beta}_V}{\partial \tilde{V}} \left[\boldsymbol{\Phi}_V^T \boldsymbol{z}_V + \lambda_V \tilde{V} + k_V |\tilde{V}|^{\alpha_V} \operatorname{sign}(\tilde{V}) \right]. \qquad (26)$$

By the adaptive I&I method, the adaptive law can be designed as

$$\begin{cases} \dot{\hat{\boldsymbol{\theta}}}_V = \dfrac{\partial \boldsymbol{\beta}_V}{\partial \tilde{V}} \left[\lambda_V \tilde{V} + k_V |\tilde{V}|^{\alpha_V} \operatorname{sign}(\tilde{V}) \right] \\ \dfrac{\partial \boldsymbol{\beta}_V}{\partial \tilde{V}} = \boldsymbol{r}_V \boldsymbol{\Phi}_V \end{cases}, \qquad (27)$$

where $\boldsymbol{r}_V = \operatorname{diag}\left[r_{V1} I_{9\times 9}, r_{V2} I_{4\times 4} \right]$, $r_{V1} > 0$ and $r_{V2} > 0$. Substituting (27) into (26), the dynamics of the estimation errors become

$$\dot{\boldsymbol{z}}_V = -\boldsymbol{r}_V \boldsymbol{\Phi}_V \boldsymbol{\Phi}_V^T \boldsymbol{z}_V. \qquad (28)$$

The asymptotic stability and finite-time stability of the closed-loop control system can be demonstrated by Lyapunov function candidates $W(\tilde{V}, \boldsymbol{z}_V) = \frac{1}{2}\tilde{V}^2 + \frac{1}{2}\lambda_V^{-1}$ and $W_2 = |\tilde{V}|$ respectively, which is similar to the procedure shown in Section II. One can get the state is finite-time stable and $\boldsymbol{\Phi}_V^T \boldsymbol{z}_V$ is asymptotic stable.

B. (α, Q) Subsystem Controller

Substituting the aerodynamic forces and moment into the (1d) and (1e), the dynamics of (α, Q) subsystem is written as

$$\begin{cases} \dot{\alpha} = Q - \dot{\gamma} \\ \dot{Q} = \boldsymbol{\varphi}_{Q2}^T \boldsymbol{\theta}_{Q2} \delta_e + \boldsymbol{\varphi}_{Q1}^T \boldsymbol{\theta}_{Q1} \end{cases}, \qquad (29)$$

where $\boldsymbol{\theta}_{Q1} \in \mathbb{R}^{11}$, $\boldsymbol{\theta}_{Q2} \in \mathbb{R}$ are uncertain parameters, and

$$\boldsymbol{\varphi}_{Q1} = \frac{\bar{q}S}{I_{yy}} \left[z_T \phi \alpha^3, z_T \phi \alpha^2, z_T \phi \alpha, z_T \phi, z_T \alpha^3, z_T \alpha^2, \right.$$
$$\left. z_T \alpha, z_T, \bar{c}\alpha^2, \bar{c}\alpha, \bar{c} \right]$$
$$\boldsymbol{\theta}_{Q1} = \left[C_T^{\phi \alpha^3}, C_T^{\phi \alpha^2}, C_T^{\phi \alpha}, C_T^{\phi}, C_T^3, C_T^2, C_T^1, C_T^0, \right.$$
$$\left. C_M^{\alpha^2}, C_M^{\alpha}, C_M^0 \right]$$
$$\boldsymbol{\varphi}_{Q2} = \bar{q}\bar{c}S/I_{yy}, \quad \boldsymbol{\theta}_{Q2} = C_M^{\delta_e} + k_{ec} C_M^{\delta_c}.$$

The estimates of $\boldsymbol{\theta}_{Q1}$ and $\boldsymbol{\theta}_{Q2}$ are define as $\hat{\boldsymbol{\theta}}_{Q1} + \boldsymbol{\beta}_{Q1}$ and $\hat{\boldsymbol{\theta}}_{Q2} + \boldsymbol{\beta}_{Q2}$ respectively. Thus the estimation errors are

$$\boldsymbol{z}_Q = \hat{\boldsymbol{\theta}}_Q + \boldsymbol{\beta}_Q - \boldsymbol{\theta}_Q, \qquad (30)$$

where $\boldsymbol{z}_Q = \left[\boldsymbol{z}_{Q1}^T, \boldsymbol{z}_{Q2}^T \right]^T$, $\hat{\boldsymbol{\theta}}_Q = \left[\hat{\boldsymbol{\theta}}_{Q1}^T, \hat{\boldsymbol{\theta}}_{Q2}^T \right]^T$, $\boldsymbol{\beta}_Q = \left[\boldsymbol{\beta}_{Q1}^T, \boldsymbol{\beta}_{Q2}^T \right]^T$ and $\boldsymbol{\theta}_Q = \left[\boldsymbol{\theta}_{Q1}^T, \boldsymbol{\theta}_{Q2}^T \right]^T$.

The adaptive I&I non-singular TSMC method introduced in the Section II can be applied to this second-order subsystem. First, define $e_1 = \alpha - \alpha_{ref}$ and $e_2 = \dot{e}_1 = Q - \dot{\gamma} - \dot{\alpha}_{ref}$. Through a variable substitution, the original system (29) can be transformed into a form of integral chain

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = \boldsymbol{\varphi}_{Q2}^T \boldsymbol{\theta}_{Q2} \delta_e + \boldsymbol{\varphi}_{Q1}^T \boldsymbol{\theta}_{Q1} - \ddot{\gamma} - \ddot{\alpha}_{ref}. \end{cases} \qquad (31)$$

The AOA is an intermediate virtual control variable here. The demanded AOA is defined as α_{ref} which will be generated by the outer-loop of the altitude subsystem.

Then, select the terminal sliding surface as

$$s_Q = e_1 + k_{Q1}^{-1} e_2^{\alpha_{Q1}}, \qquad (32)$$

where $k_{Q1} > 0$ and $\alpha_{Q1} = p/q$, p and q are positive odd integers and $q < p < 2q$. The controller is designed as

$$\delta_e = \frac{\begin{aligned} &-\frac{k_{Q1}}{\alpha_{Q1}} e_2^{2-\alpha_{Q1}} - k_{Q2}|s_Q|^{\alpha_{Q2}} \operatorname{sign}(s_Q) \\ &- \lambda_Q s_Q - \boldsymbol{\varphi}_{Q1}^T \left(\hat{\boldsymbol{\theta}}_{Q1} + \boldsymbol{\beta}_{Q1} \right) + \ddot{\gamma} + \ddot{\alpha}_{ref} \end{aligned}}{\boldsymbol{\varphi}_{Q2}^T \left(\hat{\boldsymbol{\theta}}_{Q2} + \boldsymbol{\beta}_{Q2} \right)}, \qquad (33)$$

where λ_Q, k_{Q2} and α_{Q2} are strictly positive constants. Substituting (33) into the derivative of (32), the dynamics of the sliding surface is

$$\dot{s}_Q = a_Q \left[-\boldsymbol{\Phi}_Q^T \boldsymbol{z}_Q - \lambda_Q s_Q - k_{Q2}|s_Q|^{\alpha_{Q2}} \operatorname{sign}(s_Q) \right], \qquad (34)$$

where $\boldsymbol{\Phi}_Q = \left[\boldsymbol{\varphi}_{Q1}^T, \boldsymbol{\varphi}_{Q2}^T \delta_e \right]^T$ and $a_Q = \frac{\alpha_{Q1}}{k_{Q1}} e_2^{\alpha_{Q1}-1}$. Substituting (34) into the derivative of (30) yields the dynamics of the estimation errors dynamics

$$\dot{\boldsymbol{z}}_Q = \dot{\hat{\boldsymbol{\theta}}}_Q - \frac{\partial \boldsymbol{\beta}_Q}{\partial s_Q} \left[a_Q \left(\boldsymbol{\Phi}_Q^T \boldsymbol{z}_Q + \lambda_Q s_Q + k_{Q2}|s_Q|^{\alpha_{Q2}} \operatorname{sign}(s_Q) \right) \right]. \qquad (35)$$

Therefore, the adaptive I&I law are designed as

$$\begin{cases} \dot{\hat{\boldsymbol{\theta}}}_Q = \dfrac{\partial \boldsymbol{\beta}_Q}{\partial s_Q} \left[a_Q \left(\lambda_Q s_Q + k_{Q2}|s_Q|^{\alpha_{Q2}} \operatorname{sign}(s_Q) \right) \right] \\ \dfrac{\partial \boldsymbol{\beta}_Q}{\partial s_Q} = \boldsymbol{r}_Q \boldsymbol{\Phi}_Q \end{cases}, \qquad (36)$$

where $\boldsymbol{r}_Q = \operatorname{diag}\left(r_{Q1} I_{11\times 11}, r_{Q2} \right)$ and $r_{Q1}, r_{Q2} > 0$. Substituting (36) into (35), the estimation errors dynamics can be rewritten as

$$\dot{\boldsymbol{z}}_Q = -a \boldsymbol{r}_Q \boldsymbol{\Phi}_Q \boldsymbol{\Phi}_Q^T \boldsymbol{z}_Q. \qquad (37)$$

With the control law (33) and adaptive law (36), the state α will converge to α_{ref} in finite-time and the adaptive terms $\boldsymbol{\Phi}_Q^T \boldsymbol{z}_Q$ converge to zero asymptotically.

C. (h, γ) Subsystem Controller

This subsystem is comprised of (1b) and (1c). It's obvious that there is no uncertainty between the flight altitude h and the FPA γ. To capitalize on that, the common practice is to convert altitude command h_{ref} to FPA command γ_{ref} directly by a PI controller [15]. The PI controller is practical, but its performance is not good in the whole flight envelop of AHVs, existing apparent contradiction between rapidity and accuracy.

An integral TSMC is adopted here to achieve precisely finite-time tracking in the command conversion. First, Design the sliding surface as

$$s_h = \dot{\tilde{h}} + \int_0^t \left(\lambda_{h1} \tilde{h} + k_{h1}|\tilde{h}|^{\alpha_{h1}} \operatorname{sign}(\tilde{h}) \right) \mathrm{d}t, \qquad (38)$$

where $\tilde{h} = h - h_{ref}$ is altitude tracking error and $\lambda_{h1}, k_{h1} > 0$, $0 < \alpha_{h1} < 1$. Choose a fast terminal sliding mode type reaching law, one gets the command conversion controller

$$\gamma_{ref} = \arctan \left[\frac{\dot{\tilde{h}} - \lambda_{h1}\tilde{h} - k_{h1}|\tilde{h}|^{\alpha_{h1}}\operatorname{sign}(\tilde{h}) - \lambda_{h2}s_h - k_{h2}|s_h|^{\alpha_{h2}}\operatorname{sign}(s_h)}{V} \right], \quad (39)$$

where the function $\arcsin$ has been replaced by the function $\arctan$ to avoid singular value.

With this operation, the (h, γ) subsystem which originally is a second-order system is simplified to a first-order system. So the fast TSMC used in the velocity subsystem is applied herein. And the following design procedure is similar to that of the first subsystem.

Let the FPA tracking error be $\tilde{\gamma} = \gamma - \gamma_{ref}$ whose dynamics is

$$\dot{\tilde{\gamma}} = \boldsymbol{\varphi}_{\gamma 2}^T \boldsymbol{\theta}_{\gamma 2} \alpha + \boldsymbol{\varphi}_{\gamma 1}^T \boldsymbol{\theta}_{\gamma 1} - \frac{g}{V} - \dot{\gamma}_{ref}, \quad (40)$$

where $\boldsymbol{\theta}_{\gamma 1} \in \mathbb{R}^{10}$, $\boldsymbol{\theta}_{\gamma 2} \in \mathbb{R}$ are uncertain parameters and

$$\boldsymbol{\varphi}_{\gamma 1} = \frac{\bar{q}S}{mV} \big[\delta_e, 1, \alpha^3\phi\sin\alpha, \alpha^2\phi\sin\alpha, \alpha\phi\sin\alpha,$$
$$\phi\sin\alpha, \alpha^3\sin\alpha, \alpha^2\sin\alpha, \alpha\sin\alpha, \sin\alpha \big]$$
$$\boldsymbol{\theta}_{\gamma 1} = [C_L^{\delta_e} + k_{ec}C_L^{\delta_c}, C_L^0, C_T^{\phi\alpha^3}, C_T^{\phi\alpha^2}, C_T^{\phi\alpha},$$
$$C_T^{\phi}, C_T^3, C_T^2, C_T^1, C_T^0]$$
$$\boldsymbol{\varphi}_{\gamma 2} = \bar{q}S/(mV), \quad \boldsymbol{\theta}_{\gamma 2} = C_L^{\alpha}.$$

The estimators are define as $\hat{\boldsymbol{\theta}}_{\gamma 1} + \boldsymbol{\beta}_{\gamma 1}$ and $\hat{\boldsymbol{\theta}}_{\gamma 2} + \boldsymbol{\beta}_{\gamma 2}$ respectively. Thus the estimation errors are

$$\boldsymbol{z}_{\gamma} = \hat{\boldsymbol{\theta}}_{\gamma} + \boldsymbol{\beta}_{\gamma} - \boldsymbol{\theta}_{\gamma}, \quad (41)$$

where $\boldsymbol{z}_{\gamma} = \begin{bmatrix} \boldsymbol{z}_{\gamma 1}^T, \boldsymbol{z}_{\gamma 2}^T \end{bmatrix}^T$, $\hat{\boldsymbol{\theta}}_{\gamma} = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{\gamma 1}^T, \hat{\boldsymbol{\theta}}_{\gamma 2}^T \end{bmatrix}^T$, $\boldsymbol{\beta}_{\gamma} = \begin{bmatrix} \boldsymbol{\beta}_{\gamma 1}^T, \boldsymbol{\beta}_{\gamma 2}^T \end{bmatrix}^T$ and $\boldsymbol{\theta}_{\gamma} = \begin{bmatrix} \boldsymbol{\theta}_{\gamma 1}^T, \boldsymbol{\theta}_{\gamma 2}^T \end{bmatrix}^T$.

The control law is designed as

$$\alpha_{ref} = \frac{-\lambda_{\gamma}\tilde{\gamma} - k_{\gamma}|\tilde{\gamma}|^{\alpha_{\gamma}}\operatorname{sign}(\tilde{\gamma}) + \frac{g}{V}\cos\gamma + \dot{\gamma}_{ref} - \boldsymbol{\varphi}_{\gamma 1}^T(\hat{\boldsymbol{\theta}}_{\gamma 1} + \boldsymbol{\beta}_{\gamma 1})}{\hat{\boldsymbol{\theta}}_{\gamma 2} + \boldsymbol{\beta}_{\gamma 2}}. \quad (42)$$

where $\lambda_{\gamma}, k_{\gamma} > 0$ and $0 < \alpha_{\gamma} < 1$.

As mentioned before, the AOA is an virtual control variable. According to the time-scale separation principle, in series system, like the altitude subsystem, the tracking errors of the inner-loop is assumed as zero when designing the outer-loop controller. Differentiating (41) yields

$$\dot{\boldsymbol{z}}_{\gamma} = \dot{\hat{\boldsymbol{\theta}}}_{\gamma} - \frac{\partial \boldsymbol{\beta}_{\gamma}}{\partial \tilde{\gamma}} \left[\boldsymbol{\Phi}_{\gamma}^T \boldsymbol{z}_{\gamma} + \lambda_{\gamma}\tilde{\gamma} + k_{\gamma}|\tilde{\gamma}|^{\alpha_{\gamma}}\operatorname{sign}(\tilde{\gamma}) \right]. \quad (43)$$

The I&I adaptive law is selected as

$$\begin{cases} \dot{\hat{\boldsymbol{\theta}}}_{\gamma} = \dfrac{\partial \boldsymbol{\beta}_{\gamma}}{\partial \tilde{\gamma}} \left[\lambda_{\gamma}\tilde{\gamma} + k_{\gamma}|\tilde{\gamma}|^{\alpha_{\gamma}}\operatorname{sign}(\tilde{\gamma}) \right] \\ \dfrac{\partial \boldsymbol{\beta}_{\gamma}}{\partial \tilde{\gamma}} = \boldsymbol{r}_{\gamma}\boldsymbol{\Phi}_{\gamma} \end{cases}, \quad (44)$$

where $\boldsymbol{r}_{\gamma} = \operatorname{diag}\left[r_{\gamma 1}I_{10\times10}, r_{\gamma 2} \right]$ and $r_{\gamma 1}, r_{\gamma 2} > 0$. Substituting (44) into (43), the dynamics of the estimation errors are written as

$$\dot{\boldsymbol{z}}_{\gamma} = -\boldsymbol{r}_{\gamma}\boldsymbol{\Phi}_{\gamma}\boldsymbol{\Phi}_{\gamma}^T\boldsymbol{z}_{\gamma}. \quad (45)$$

With the controllers (40) and (42), the tracking errors $\tilde{h}$ and $\tilde{\gamma}$ reach zero in finite-time. And $\boldsymbol{\Phi}_{\gamma}^T\boldsymbol{z}_{\gamma}$ is asymptotic stable.

By now, the design of the entire adaptive I&I TSMC system is accomplished. It has been demonstrated that every subsystem is finite-time stable and there is no coupling between each other, thus the whole control system is finite-time stable.

IV. SIMULATION

Simulation experiments are conducted to verify the control system designed in Section III. The initial conditions are set as [12]. To increase fidelity, the dynamic characteristics of actuators are taken into account [12]. The reference commands are set as step signals, which will be filtered by second-order prefilters [6], in velocity and altitude with amplitude of 2153.6 ft/s and 15000 ft respectively, which is a relatively aggressive maneuver.

Two representative simulations are studied here. One is conducted on the nominal model, namely without parameter uncertainty, to validate the tracking performance of the control system. The other is performed with significant parameter uncertainty to assess the robustness. The results of the first simulation are shown in Figs. $2 \sim 5$. Fig. 2 illustrates the velocity and altitude tracking errors reach zero fast and accurately, which indicates the proposed adaptive I&I TSMC has good tracking performance. Fig. 3 shows the PFA and AOA could track the respective virtual control inputs well. The response of flexible modes and actuators are shown in Fig. 4 which indicates the flexible modes are stable. And the control outputs not only meet the amplitude and rate constraints set up before, but also are continuous and smooth without saturation. So the designed controllers are physically feasible. Fig. 5 shows the I&I adaptive estimators are stable and convergent to zero.
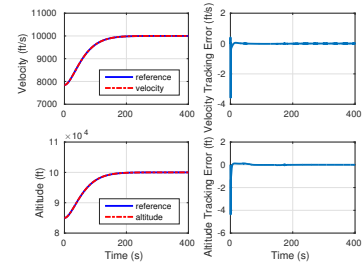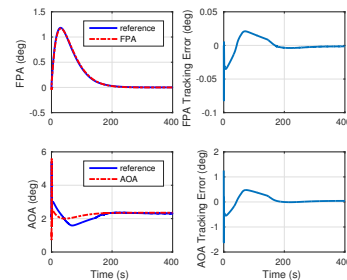

Fig. 2. Response of the velocity and altitude
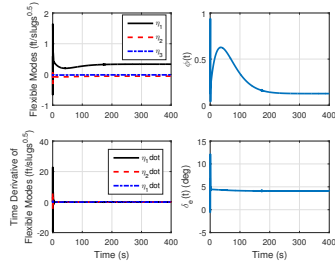

Fig. 3. Response of the FPA and AOA

Fig. 4. Response of flexible modes and actuators

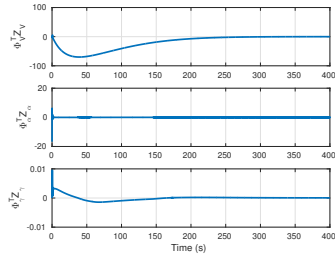Fig. 5. Response of the adaptive terms

V. Conclusions

In this paper, a continuous adaptive I&I TSMC system is designed for AHVs to achieve finite-time stability of states with parameter uncertainty. The control system, a combination of adaptive I&I theory and terminal sliding mode control methodology, is characterized by structured design procedure, strong robustness to disturbance and parametric uncertainty and finite-time convergence of states. To begin with, the AHV model is decomposed into several first-order and second-order subsystems by backstepping and time-scale separation principle. Then, for each first-order subsystem, a fast TSM controller combined with I&I adaptive estimator is designed to make the finite-time tracking a reality. For the second-order subsystem, an adaptive I&I non-singular TSM controller is designed. Last, simulation results indicate the fast and accurate tracking performance of the designed control system in the present of parameter uncertainty.

The second simulation results are listed as Fig. 6∼7. Here a uncertain condition of $\Delta C_L = \Delta C_T = -40\%$ and $\Delta C_D = \Delta C_M = 40\%$ are considered, which means a 40% decrease in the lift and thrust while a 40% increase in the drag and pitch moment. Note it's a quite tough situation for flight control. Compared to the first case, there is little degradation of the tracking performance. In Fig. 6, tracking performance of the velocity and altitude is still fast and accurate, and their initial tracking errors are only slightly bigger than Fig. 2. The intermediate variables and flexible modes are stable shown by Fig. 7.
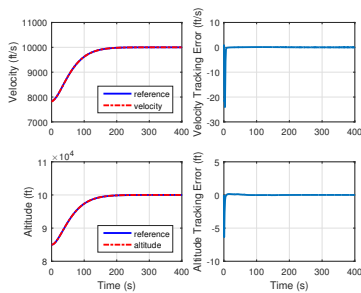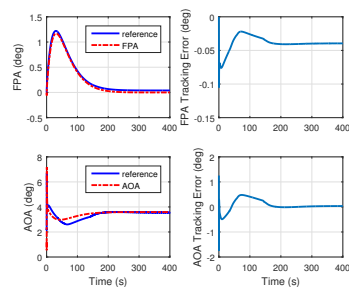
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Fig. 6. Response of the velocity and altitude with uncertainty

Fig. 7. Response of the FPA and AOA with uncertainty