

# Immersion and Invariance Adaptive Control with $\sigma$ -modification for Air-breathing Hypersonic Vehicles\*

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**Abstract**—A novel immersion and invariance (I&I) adaptive control with  $\sigma$ -modification is proposed for flexible air-breathing hypersonic vehicles in the paper. Based on the framework of backstepping, the controller is a combination of I&I adaptive control and  $\sigma$ -modification. The control system can not only deal with the significant aerodynamic parameter uncertainties in AHVs model, but also guarantee the stability of the adaptive law in the presence of unavoidable nonparametric uncertainties. With the introduction of  $\sigma$ -modification, the adaptive law is designed away from the problem of parameter drift. Moreover, the structured design process and taking amplitude/rate constraints of states and actuators into account make the control system feasible to engineering practice. The stability of the closed-loop system is demonstrated by Lyapunov theory and the effectiveness of the method is illustrated by numerical simulations.

**Index Terms**—immersion and invariance; adaptive control; hypersonic vehicles;  $\sigma$ -modification.

## I. INTRODUCTION

Air-breathing hypersonic vehicles (AHVs) have been a research hotspot and frontier in the aerospace community during the last decade. Because of the extremely fast speed and large flight range, it is potential to be an available access to the economically reusable space shuttle and prompt global strike, and is recognized as game-changing technologies [1]. The successes of the NASA X-41 and USAF X-51A indicate that sustainable hypersonic flight will be a reality in the foreseeable future.

However, it is a great challenge to design control systems for AHVs at this stage [2]. The complex mechanism of hypersonic flight brings great difficulties to the vehicles modeling, which results in significant uncertainties in the current AHVs mathematical models. Moreover, both the integrated engine-airframe configuration adopted in AHVs and severe requirements of flight speed/altitude needed for scramjets working lead to the strong couplings between propulsive thrust, aerodynamic forces and structural flexibility. These complex plant characteristics including strong nonlinearity and fast time-varying, which are distinct to conventional

aircraft, are insurmountable obstacles for the existed linear flight control system design methods which either can not meet the requirements or are too costly to implement in large flight envelope. Therefore, many nonlinear control methods are investigated in AHVs flight control system design.

In [3], pure sliding mode controller (SMC) and adaptive SMC are designed based on feedback linearization framework for a generic hypersonic vehicle model respectively. Simulations illustrate that the pure SMC is robust to parameter uncertainties, but the price is large controller gains and control chattering. In adaptive SMC case, by contrast, there is a prominent performance improvement in terms of smaller control effort and free of control chattering. It is indicated that the combination of adaptive control and robust control will reduce controller gains and improve control performance. Actually, the adaptive robust control has been the mainstream research ideas for AHVs control. In [2], a robust adaptive control, which is a combination of adaptive dynamic inversion and adaptive backstepping, is proposed for AHVs where the Lyapunov synthesis approach is adopted for adaptive law designing like in [3]. Whereas, there is not a generic or systematic technique to choose a Lyapunov function for adaptive control system design. Particularly, for some complex nonlinear systems, like AHVs, the design may be complicated. In [4], an immersion and invariance (I&I) [5] adaptive control system, which bypasses the issue of choosing Lyapunov function at the adaptive law design level.

When adaptive control is applied to practical cases, another issue called parameter drift which is the unpredictable instability phenomena caused by nonparametric uncertainty and measurement noise, must be dealt with [6]. Though many robust adaptive control schemes are proposed for AHVs [7], [8] and good tracking performances are illustrated by simulations, as far as I know, few work addresses the two issues together. What's more, it appears that no attempt has been made to design I&I adaptive laws with  $\sigma$ -modification [9] in AHVs control. Motivated by this, a new I&I adaptive control scheme with  $\sigma$ -modification is proposed in this paper, aiming to design an implementable adaptive robust control system for AHVs.

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## II. HYPERSONIC VEHICLE MODEL

The longitudinal dynamics of flexible AHVs can be described by [2]

$$\dot{V} = (T \cos \alpha - D)/m - g \sin \gamma \quad (1a)$$

$$\dot{h} = V \sin \gamma \quad (1b)$$

$$\dot{\gamma} = (L + T \sin \alpha)/(mV) - (g \cos \gamma)/V \quad (1c)$$

$$\dot{\alpha} = -(L + T \sin \alpha)/(mV) + Q + (g \cos \gamma)/V \quad (1d)$$

$$\dot{Q} = M/I_{yy} \quad (1e)$$

$$\ddot{\eta}_i = -2\zeta_i\omega_i\dot{\eta}_i - \omega_i^2\eta_i + N_i, \quad i = 1, 2, 3. \quad (1f)$$

The expressions of the aerodynamic forces, pitch moment, generalized forces, and the corresponding aerodynamic coefficients can refer to [10].

The model is comprised of five rigid-body states  $\mathbf{x} = [V, h, \gamma, \alpha, Q]^T$ , six flexible states  $\boldsymbol{\eta} = [\eta_1, \dot{\eta}_1, \eta_2, \dot{\eta}_2, \eta_3, \dot{\eta}_3]^T$  and three control inputs  $\mathbf{u} = [\phi, \delta_e, \delta_c]^T$ . The controlled output is  $\mathbf{y} = [V, h]^T$ . And the reference command is denoted by  $\mathbf{y}_c = [V_c, h_c]^T$ . The control object is to design robust adaptive control system for AHVs to achieve stable tracking of velocity and altitude reference trajectories in the presence of significant parametric and nonparametric uncertainties.

The model above is used for simulation to validate the performance of the control system. For control system design, the model is simplified from the following two points [4]. First, the flexible dynamics are removed and their effects are taken as internal disturbances. Then, the canard is ganged with the elevator as  $\delta_c = k_{ec}\delta_e$ , where the coefficient  $k_{ec}$  is a negative constant [11], then the control inputs are reduced as  $\mathbf{u} = [\phi, \delta_e]^T$ .

It's assumed that all aerodynamic coefficients, i.e.,  $C_T^{(\cdot)}$ ,  $C_M^{(\cdot)}$ ,  $C_L^{(\cdot)}$  and  $C_D^{(\cdot)}$  are uncertain which are modeled as  $C_j = C_j^*(1 + \Delta C_j)$ , where  $C_j^*$  and  $\Delta C_j$  are nominal value and uncertainty respectively. In the paper, a maximum variation for each coefficient is set within 40%, namely,  $|\Delta C_j| \leq 0.4$ .

## III. ADAPTIVE CONTROL SYSTEM DESIGN

AHVs are MIMO, high-order and nonlinear systems. To begin with, the model (1) is decomposed into velocity subsystem and altitude subsystem from the control viewpoint. Furthermore, the altitude system is taken as a sequentiallyum install centos-release-scl loop structure, and the backstepping method is applied for controller design of each order subsystem. The block diagram of the control system structure is shown as Fig. 1. Note that each controller has a amplitude and rate constraints filter [12] as output to take the physical constraints of states into account.

### A. Velocity subsystem controller

Velocity subsystem is a 1st-order system. Define the velocity tracking error as  $\tilde{V} \triangleq V - V_c$ . Substituting the expresses of aerodynamic forces and moment into (1a), we can arrange the dynamics of velocity tracking error as

$$\dot{\tilde{V}} = \boldsymbol{\theta}_{v1}^T \boldsymbol{\varphi}_{v1} + \boldsymbol{\theta}_{v2}^T \boldsymbol{\varphi}_{v2} \phi - g \sin \gamma - \dot{V}_c, \quad (2)$$

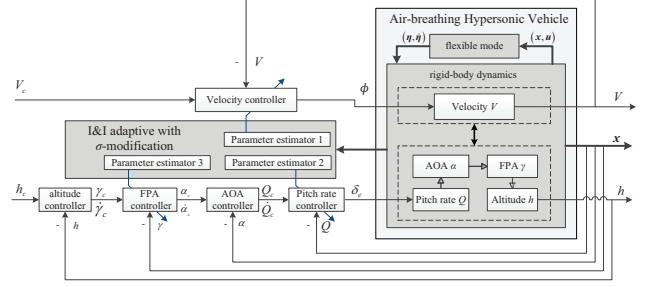


Fig. 1. Block diagram of the control system.

where  $\boldsymbol{\theta}_{v1} \in \mathbb{R}^9$ ,  $\boldsymbol{\theta}_{v2} \in \mathbb{R}^4$  are vectors of unknown parameters and

$$\begin{aligned} \boldsymbol{\varphi}_{v1} &= \frac{\bar{q}S}{m} [\alpha^3 \cos \alpha, \alpha^2 \cos \alpha, \alpha \cos \alpha, \cos \alpha, -\alpha^2, -\alpha, \\ &\quad -\delta_e^2, -\delta_e, -1]^T \\ \boldsymbol{\theta}_{v1} &= [C_T^3, C_T^2, C_T^1, C_T^0, C_D^{\alpha^2}, C_D^{\alpha}, C_D^{\delta_e^2} + k_{ec}^2 C_D^{\delta_e^2}, \\ &\quad C_D^{\delta_e} + k_{ec} C_D^{\delta_e}, C_D^0]^T \\ \boldsymbol{\varphi}_{v2} &= \bar{q}S \cos \alpha [\alpha^3, \alpha^2, \alpha, 1]^T / m \\ \boldsymbol{\theta}_{v2} &= [C_T^{\phi \alpha^3}, C_T^{\phi \alpha^2}, C_T^{\phi \alpha}, C_T^{\phi}]^T. \end{aligned}$$

The estimates of  $\boldsymbol{\theta}_{v1}$  and  $\boldsymbol{\theta}_{v2}$  are defined as  $\hat{\boldsymbol{\theta}}_{v1} + \boldsymbol{\beta}_{v1}$  and  $\hat{\boldsymbol{\theta}}_{v2} + \boldsymbol{\beta}_{v2}$  respectively. It's different with Lyapunov synthesis adaptive approach in terms of  $\boldsymbol{\beta}_{(\cdot)}$  which are introduced by I&I adaptive method. The additional term  $\boldsymbol{\beta}_{(\cdot)}$  provides extra design freedom for shaping the dynamics of estimation errors, that has great advantage over the traditional adaptive control which has little knowledge about that. Then, the estimation errors can be defined as

$$\tilde{\boldsymbol{\theta}}_v \triangleq \hat{\boldsymbol{\theta}}_v + \boldsymbol{\beta}_v - \boldsymbol{\theta}_v, \quad (3)$$

where  $\tilde{\boldsymbol{\theta}}_v = [\tilde{\boldsymbol{\theta}}_{v1}^T, \tilde{\boldsymbol{\theta}}_{v2}^T]^T$ ,  $\hat{\boldsymbol{\theta}}_v = [\hat{\boldsymbol{\theta}}_{v1}^T, \hat{\boldsymbol{\theta}}_{v2}^T]^T$ ,  $\boldsymbol{\beta}_v = [\boldsymbol{\beta}_{v1}^T, \boldsymbol{\beta}_{v2}^T]^T$  and  $\boldsymbol{\theta}_v = [\boldsymbol{\theta}_{v1}^T, \boldsymbol{\theta}_{v2}^T]^T$ .

To get the nominal control command, the controller can be designed as

$$\phi_{cd} = \frac{-k_v \tilde{V} - (\hat{\boldsymbol{\theta}}_{v1} + \boldsymbol{\beta}_{v1})^T \boldsymbol{\varphi}_{v1} + g \sin \gamma + \dot{V}_c}{(\hat{\boldsymbol{\theta}}_{v2} + \boldsymbol{\beta}_{v2})^T \boldsymbol{\varphi}_{v2}}, \quad (4)$$

where  $k_v$  is a positive constant.

Taking account of the dynamics of the scramjet, the desired command cannot be guaranteed to always be reached, especially during the transient process. Thus, the feasible control command  $\phi_c$  and its time derivative  $\dot{\phi}_c$  can be generated by passing  $\phi_{cd}$  through the constraint command filter. It can be assumed that  $\phi = \phi_c$  as  $\phi_c$  is physically realizable by the actuator.

Define the compensated velocity tracking error as [13]

$$\tilde{V} \triangleq \tilde{V} - \xi_v, \quad (5)$$

where

$$\dot{\xi}_v \triangleq -k_v \xi_v + (\hat{\theta}_{v2} + \beta_{v2})^T \varphi_{v2}(\phi_c - \phi_{cd}). \quad (6)$$

It's obvious that the compensated velocity tracking error will converge to the velocity tracking error when the constraints of actuator are not in effect in view of the definitions. Substituting (2) and (6) into the differentiation of (5), we can write the dynamics of compensated velocity tracking error as

$$\dot{\bar{V}} = -k_v \bar{V} - \tilde{\theta}_v \Phi_v, \quad (7)$$

where  $\Phi = [\varphi_{v1}^T, \varphi_{v2}^T \phi]^T$ .

Differentiating (3) and using (7), yields

$$\dot{\hat{\theta}}_v = \dot{\theta}_v + \frac{\partial \beta_v}{\partial \bar{V}} [-k_v \bar{V} - \tilde{\theta}_v \Phi_v]. \quad (8)$$

In view of the dynamics of estimation errors, the adaptive law is designed as

$$\begin{cases} \dot{\hat{\theta}}_v = \frac{\partial \beta_v}{\partial \bar{V}} (k_v \bar{V}) - r_v \sigma_v (\hat{\theta}_v + \beta_v) \\ \frac{\partial \beta_v}{\partial \bar{V}} = r_v \Phi_v, \end{cases} \quad (9)$$

where  $\sigma_v$  is a small constant,  $r_v$  is a positive diagonal matrix and  $r_v = \text{diag}[r_{v1} I_{9 \times 9}, r_{v2} I_{4 \times 4}]$ ,  $r_{v1} > 0$ ,  $r_{v2} > 0$ .

The term including  $\sigma_v$  in the adaptive law is brought in by the so-called  $\sigma$ -modification which ensures the update law bounded in the existence of modeling errors. As known, in practical control engineering, the model for control system design exists not only parametric uncertainties, but also nonparametric uncertainties, such as unmodeled dynamics and measurement noises. However, both conventional and I&I adaptive control may excite parameter drift phenomenon in adaptive law and lead to estimations divergence and instability of the closed-loop system under the circumstances of nonparametric uncertainties. It is intolerable and must be carefully addressed in AHVs flight control for the significant uncertainties of the model. Therefore, the adaptive law which is a combination of I&I adaptive control and  $\sigma$ -modification is designed here. And the important role of  $\sigma$ -modification will be shown in the stability analysis section.

Substituting (9) into (8), the estimation error dynamics are rewritten as

$$\dot{\tilde{\theta}}_v = -r_v \Phi_v \tilde{\theta}_v^T \Phi_v - r_v \sigma_v (\hat{\theta}_v + \beta_v). \quad (10)$$

### B. Altitude subsystem controller

The Altitude subsystem is a 4th-order system, whose dynamics can be rewritten as

$$\begin{aligned} \dot{h} &= V \sin \gamma \\ \dot{\gamma} &= \theta_{\gamma 1}^T \varphi_{\gamma 1} + \theta_{\gamma 2}^T \varphi_{\gamma 2} \alpha - \frac{g}{V} \cos \gamma \\ \dot{\alpha} &= Q - \dot{\gamma} \\ \dot{Q} &= \theta_{q1}^T \varphi_{q1} + \theta_{q2}^T \varphi_{q2} \delta_e, \end{aligned} \quad (11)$$

where  $\theta_{\gamma 1} \in \mathbb{R}^{10}$ ,  $\theta_{\gamma 2} \in \mathbb{R}$ ,  $\theta_{q1} \in \mathbb{R}^{11}$ ,  $\theta_{q2} \in \mathbb{R}$  are unknown constants and

$$\begin{aligned} \varphi_{\gamma 1} &= \frac{\bar{q}S}{mV} [\delta_e, 1, \alpha^3 \phi \sin \alpha, \alpha^2 \phi \sin \alpha, \alpha \phi \sin \alpha, \\ &\quad \phi \sin \alpha, \alpha^3 \sin \alpha, \alpha^2 \sin \alpha, \alpha \sin \alpha, \sin \alpha]^T \\ \theta_{\gamma 1} &= [C_L^{\delta_e} + k_{ec} C_L^{\delta_e}, C_L^0, C_T^{\phi \alpha^3}, C_T^{\phi \alpha^2}, C_T^{\phi \alpha}, C_T^{\phi}, C_T^3, C_T^2, C_T^1, C_T^0]^T \\ \varphi_{\gamma 2} &= \bar{q}S/(mV), \quad \theta_{\gamma 2} = C_L^\alpha \\ \varphi_{q1} &= \frac{\bar{q}S}{I_{yy}} [z_T \phi \alpha^3, z_T \phi \alpha^2, z_T \phi \alpha, z_T \phi, z_T \alpha^3, z_T \alpha^2, \\ &\quad z_T \alpha, z_T, \bar{c} \alpha^2, \bar{c} \alpha, \bar{c}]^T \\ \theta_{q1} &= [C_T^{\phi \alpha^3}, C_T^{\phi \alpha^2}, C_T^{\phi \alpha}, C_T^{\phi}, C_T^3, C_T^2, C_T^1, C_T^0, C_M^{\alpha^2}, C_M^\alpha, C_M^0]^T \\ \varphi_{q2} &= \bar{q} \bar{c} S / I_{yy}, \quad \theta_{q2} = C_M^{\delta_e} + k_{ec} C_M^{\delta_e}. \end{aligned}$$

The estimates of unknown constants are defined as  $\hat{\theta}_{\gamma 1} + \beta_{\gamma 1}$ ,  $\hat{\theta}_{\gamma 2} + \beta_{\gamma 2}$ ,  $\hat{\theta}_{q1} + \beta_{q1}$  and  $\hat{\theta}_{q2} + \beta_{q2}$  respectively. Thus the estimation errors are

$$\tilde{\theta}_\gamma \triangleq \hat{\theta}_\gamma + \beta_\gamma - \theta_\gamma \quad (12a)$$

$$\tilde{\theta}_q \triangleq \hat{\theta}_q + \beta_q - \theta_q, \quad (12b)$$

where  $\tilde{\theta}_\gamma = [\tilde{\theta}_{\gamma 1}^T, \tilde{\theta}_{\gamma 2}^T]^T$ ,  $\hat{\theta}_\gamma = [\hat{\theta}_{\gamma 1}^T, \hat{\theta}_{\gamma 2}^T]^T$ ,  $\beta_\gamma = [\beta_{\gamma 1}^T, \beta_{\gamma 2}^T]^T$ ,  $\theta_\gamma = [\theta_{\gamma 1}^T, \theta_{\gamma 2}^T]^T$ ,  $\tilde{\theta}_q = [\tilde{\theta}_{q1}^T, \tilde{\theta}_{q2}^T]^T$ ,  $\hat{\theta}_q = [\hat{\theta}_{q1}^T, \hat{\theta}_{q2}^T]^T$ ,  $\beta_q = [\beta_{q1}^T, \beta_{q2}^T]^T$  and  $\theta_q = [\theta_{q1}^T, \theta_{q2}^T]^T$ .

The control object of the subsystem is to design elevator  $\delta_e$  to achieve altitude trajectory tracking. In view of (11), the first equation, i.e., altitude dynamics, does not satisfy the parametric strict-feedback form, but it is deterministic, namely, there is no uncertainty between altitude and FPA. Therefore, the common practice is to design a command converter which converts altitude reference command to FPA reference directly [4]. Then, the altitude subsystem that is 4th-order originally can be reduced into a 3th-order system to which the backstepping will be applied because it is parametric strict-feedback.

1) *FPA controller*: Define the FPA tracking error as  $\tilde{\gamma} \triangleq \gamma - \gamma_c$ , whose dynamics is

$$\dot{\tilde{\gamma}} = \theta_{\gamma 1}^T \varphi_{\gamma 1} + \theta_{\gamma 2}^T \varphi_{\gamma 2} \alpha - \frac{g}{V} \cos \gamma - \dot{\gamma}_c. \quad (13)$$

To get the nominal virtual control command, the stabilizing function can be designed as

$$\alpha_{cd} = \frac{-k_\gamma \tilde{\gamma} - (\hat{\theta}_{\gamma 1} + \beta_{\gamma 1})^T \varphi_{\gamma 1} + \frac{g \cos \gamma}{V} + \dot{\gamma}_c}{(\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2}}, \quad (14)$$

where  $k_h > 0$ . Pass  $\alpha_{cd}$  through the command filter to produce the magnitude and rate limited command signal  $\alpha_c$  and its derivative  $\dot{\alpha}_c$ .

Define the compensated FPA tracking error as

$$\bar{\gamma} = \tilde{\gamma} - \xi_\gamma, \quad (15)$$

where

$$\dot{\xi}_\gamma = -k_\gamma \xi_\gamma + (\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} (\alpha_c - \alpha_{cd} + \xi_\alpha), \quad (16)$$

and  $\xi_\alpha$  will be defined in the next section. Substituting (13), (14) and (16) into the differential of (15), yields the dynamics of the compensated FPA tracking error as

$$\dot{\bar{\gamma}} = -k_\gamma \bar{\gamma} - \tilde{\theta}_\gamma^T \Phi_\gamma + (\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} \bar{\alpha}, \quad (17)$$

where  $\bar{\alpha}$  is the compensated AOA tracking error that will be defined in the next section.

Substituting (17) into the derivative of (12a), the dynamics of the estimation errors can be written as

$$\dot{\hat{\theta}}_\gamma = \dot{\theta}_\gamma + \frac{\partial \beta_\gamma}{\partial \bar{\gamma}} \left[ -k_\gamma \bar{\gamma} - \tilde{\theta}_\gamma^T \Phi_\gamma + (\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} \bar{\alpha} \right] \quad (18)$$

The I&I adaptive law with  $\sigma$ -modification can be designed as

$$\begin{cases} \dot{\hat{\theta}}_\gamma = \frac{\partial \beta_\gamma}{\partial \bar{\gamma}} \left[ -k_\gamma \bar{\gamma} + (\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} \bar{\alpha} \right] \\ \quad - r_\gamma \sigma_\gamma (\hat{\theta}_\gamma + \beta_\gamma) \\ \frac{\partial \beta_\gamma}{\partial \bar{\gamma}} = r_\gamma \Phi_\gamma, \end{cases} \quad (19)$$

where  $\sigma_\gamma$  is a small constant,  $r_\gamma$  is a positive diagonal matrix and  $r_\gamma = \text{diag}[r_{\gamma 1} I_{10 \times 10}, r_{\gamma 2}]$ ,  $r_{\gamma 1} > 0$ ,  $r_{\gamma 2} > 0$ .

Substituting (19) into (18), the dynamics of the estimation error can be rewritten as

$$\dot{\tilde{\theta}}_\gamma = -r_\gamma \Phi_\gamma \tilde{\theta}_\gamma^T \Phi_\gamma - r_\gamma \sigma_\gamma (\hat{\theta}_\gamma + \beta_\gamma). \quad (20)$$

2) *AOA controller*: Define the AOA tracking error as  $\tilde{\alpha} \triangleq \alpha - \alpha_c$ , whose dynamics is

$$\dot{\tilde{\alpha}} = \dot{\alpha} - \dot{\alpha}_c. \quad (21)$$

The nominal virtual command trajectory for the pitch rate can be designed as

$$Q_{cd} = -k_\alpha \tilde{\alpha} + \dot{\gamma} + \dot{\alpha}_c - (\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} \bar{\gamma}, \quad (22)$$

where  $k_\alpha > 0$ . The feasible virtual command  $Q_c$  and its derivate  $\dot{Q}_c$  can be produced by passing  $Q_{cd}$  through the magnitude and rate constraints filter.

Define the compensated AOA tracking error as

$$\bar{\alpha} = \tilde{\alpha} - \xi_\alpha, \quad (23)$$

where

$$\dot{\xi}_\alpha = -k_\alpha \xi_\alpha + Q_c - Q_{cd} + \xi_q, \quad (24)$$

and  $\xi_q$  will be defined in the next section. Differentiating (23) and using (21), (22) and (24), gives the dynamics of the compensated AOA tracking error as

$$\dot{\bar{\alpha}} = -k_\alpha \bar{\alpha} - (\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} \bar{\gamma} + \bar{Q}. \quad (25)$$

3) *Pitch rate controller*: Define the pitch rate tracking error  $\bar{Q} \triangleq Q - Q_c$ , whose dynamics is

$$\dot{\bar{Q}} = \theta_{q1}^T \varphi_{q1}^T + \theta_{q2}^T \varphi_{q2}^T \delta_e - \dot{q}_c. \quad (26)$$

To get the nominal reference command for the elevator, the controller is designed as

$$\delta_{ecd} = \frac{-k_q \bar{Q} - (\hat{\theta}_{q1} + \beta_{q1})^T \varphi_{q1} + \dot{q}_c - \bar{\alpha}}{(\hat{\theta}_{q2} + \beta_{q2})^T \varphi_{q2}}, \quad (27)$$

where  $k_q > 0$ . Pass  $\delta_{ecd}$  to generate achievable control signal  $\delta_{ec}$  through a magnitude and rate constraints filter which is the model of the elevator. As  $\delta_{ec}$  is feasible by physical actuator, it is reasonable to assume that  $\delta_e = \delta_{ec}$ . Define the compensated pitch rate tracking error as

$$\bar{Q} = \bar{Q} - \xi_q, \quad (28)$$

where

$$\dot{\xi}_q = -k_q \xi_q + (\hat{\theta}_{q2} + \beta_{q2})^T \varphi_{q2} (\delta_{ec} - \delta_{ecd}). \quad (29)$$

Substituting (26), (27) and (29) into the derivative of (28), one can get the dynamics of the compensated pitch rate tracking error as

$$\dot{\bar{Q}} = -k_q \bar{Q} - \tilde{\theta}_q \Phi_q - \bar{\alpha}. \quad (30)$$

Differentiating (12b) and using (30), the dynamics of the estimation errors can be written as

$$\dot{\hat{\theta}}_q = \dot{\theta}_q + \frac{\partial \beta_q}{\partial \bar{Q}} (-k_q \bar{Q} - \tilde{\theta}_q \Phi_q - \bar{\alpha}). \quad (31)$$

The I&I adaptive law with  $\sigma$ -modification can be designed as

$$\begin{cases} \dot{\hat{\theta}}_q = \frac{\partial \beta_q}{\partial \bar{Q}} (k_q \bar{Q} + \bar{\alpha}) - r_q \sigma_q (\hat{\theta}_q + \beta_q) \\ \frac{\partial \beta_q}{\partial \bar{Q}} = r_q \Phi_q, \end{cases} \quad (32)$$

where  $\sigma_q$  is a small constant,  $r_q$  is a positive diagonal matrix and  $r_q = \text{diag}[r_{q1} I_{11 \times 11}, r_{q2}]$ ,  $r_{q1} > 0$ ,  $r_{q2} > 0$ .

Substituting (32) into (31), the dynamics of the estimation error can be rewritten as

$$\dot{\tilde{\theta}}_q = -r_q \Phi_q \tilde{\theta}_q^T \Phi_q - r_q \sigma_q (\hat{\theta}_q + \beta_q). \quad (33)$$

### C. Stability analysis

The adaptive control system for hypersonic vehicles model is comprised of (4), (9), (14), (19) and (22), (27). Considering the nonparametric uncertainties, the dynamics of the closed-loop system are described as

$$\begin{cases} \dot{V} = -k_v \bar{V} - \tilde{\theta}_v \Phi_v + \Delta_v \\ \dot{\hat{\theta}}_v = -r_v \Phi_v \tilde{\theta}_v^T \Phi_v - r_v \sigma_v (\hat{\theta}_v + \beta_v) \\ \dot{\bar{\gamma}} = -k_\gamma \bar{\gamma} - \tilde{\theta}_\gamma^T \Phi_\gamma + (\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} \bar{\alpha} + \Delta_\gamma \\ \dot{\hat{\theta}}_\gamma = -r_\gamma \Phi_\gamma \tilde{\theta}_\gamma^T \Phi_\gamma - r_\gamma \sigma_\gamma (\hat{\theta}_\gamma + \beta_\gamma) \\ \dot{\bar{\alpha}} = -k_\alpha \bar{\alpha} - (\hat{\theta}_{\gamma 2} + \beta_{\gamma 2})^T \varphi_{\gamma 2} \bar{\gamma} - \bar{Q} + \Delta_\alpha \\ \dot{\bar{Q}} = -k_q \bar{Q} - \tilde{\theta}_q \Phi_q - \bar{\alpha} + \Delta_q \\ \dot{\hat{\theta}}_q = -r_q \Phi_q \tilde{\theta}_q^T \Phi_q - r_q \sigma_q (\hat{\theta}_q + \beta_q), \end{cases} \quad (34)$$

where  $\Delta(\cdot)$  are additive modeling errors with unknown bounds. Assume that  $\|\Delta_i\| \leq \eta_i$ ,  $\eta_i$  are positive constants,  $i = v, \gamma, \alpha, q$ . To analyze the stability of the whole closed-loop system, a composite Lyapunov function is defined as

$$W = \frac{1}{2} \left( \bar{V}^2 + k_v^{-1} \tilde{\theta}_v^T r_v^{-1} \tilde{\theta}_v + \bar{\gamma}^2 + k_\gamma^{-1} \tilde{\theta}_\gamma^T r_\gamma^{-1} \tilde{\theta}_\gamma + \bar{\alpha}^2 + \bar{Q}^2 + k_q^{-1} \tilde{\theta}_q^T r_q^{-1} \tilde{\theta}_q \right), \quad (35)$$

whose dynamics along (34) is

$$\begin{aligned} \dot{W} = & \bar{V} \dot{\bar{V}} + k_v^{-1} \tilde{\theta}_v^T r_v^{-1} \dot{\tilde{\theta}}_v + \bar{\gamma} \dot{\bar{\gamma}} + k_\gamma^{-1} \tilde{\theta}_\gamma^T r_\gamma^{-1} \dot{\tilde{\theta}}_\gamma \\ & + \bar{\alpha} \dot{\bar{\alpha}} + \bar{Q} \dot{\bar{Q}} + k_q^{-1} \tilde{\theta}_q^T r_q^{-1} \dot{\tilde{\theta}}_q \\ \leq & -k_v \bar{V}^2 - \bar{V} \tilde{\theta}_v^T \Phi_v + \|\bar{V}\| \eta_v - k_v^{-1} (\tilde{\theta}_v^T \Phi_v)^2 \\ & - k_v^{-1} \sigma_v \tilde{\theta}_v^T (\hat{\theta}_v + \beta_v) - k_\gamma \bar{\gamma}^2 - \bar{\gamma} \tilde{\theta}_\gamma^T \Phi_\gamma + \|\bar{\gamma}\| \eta_\gamma \\ & - k_\gamma^{-1} (\tilde{\theta}_\gamma^T \Phi_\gamma)^2 - k_\gamma^{-1} \sigma_\gamma \tilde{\theta}_\gamma^T (\hat{\theta}_\gamma + \beta_\gamma) - k_\alpha \bar{\alpha}^2 \\ & + \|\bar{\alpha}\| \eta_\alpha - k_q \bar{Q}^2 - \bar{Q} \tilde{\theta}_q^T \Phi_q + \|\bar{Q}\| \eta_q \\ & - k_q^{-1} (\tilde{\theta}_q^T \Phi_q)^2 - k_q^{-1} \sigma_q \tilde{\theta}_q^T (\hat{\theta}_q + \beta_q). \end{aligned} \quad (36)$$

By inequalities

$$\begin{aligned} \|\bar{V}\| \eta_v & \leq \frac{1}{2} \bar{V}^2 + \frac{1}{2} \eta_v^2 \\ -\bar{V} \tilde{\theta}_v^T \Phi_v & \leq \frac{k_v}{2} \bar{V}^2 + \frac{k_v^{-1}}{2} (\tilde{\theta}_v^T \Phi_v)^2 \\ -\tilde{\theta}_v^T (\hat{\theta}_v + \beta_v) & \leq -\frac{1}{2} \|\tilde{\theta}_v\|^2 + \frac{1}{2} \|\theta_v\|^2, \end{aligned} \quad (37)$$

when  $k_v, k_\gamma, k_q > 2$ ,  $k_\alpha > 1$ , one can get

$$\dot{W} \leq -\eta W + M, \quad (38)$$

where

$$\begin{aligned} M = & \frac{1}{2} (k_v^{-1} \sigma_v \|\theta_v\|^2 + k_\gamma^{-1} \sigma_\gamma \|\theta_\gamma\|^2 + k_q^{-1} \sigma_q \|\theta_q\|^2 \\ & + \eta_v^2 + \eta_\gamma^2 + \eta_\alpha^2 + \eta_q^2) \\ \eta = & \min \left[ k_v, k_\gamma, k_q, \frac{\sigma_v}{\lambda_{\max}(r_v^{-1})}, \frac{\sigma_\gamma}{\lambda_{\max}(r_\gamma^{-1})}, \frac{\sigma_q}{\lambda_{\max}(r_q^{-1})} \right]. \end{aligned}$$

The symbol  $\lambda_{\max}(\cdot)$  means the maximum eigenvalue of the matrix. Notice that the  $\sigma$  terms in (36) introduce the square terms of estimation errors, namely  $\|\tilde{\theta}_i\|^2$ ,  $i = v, \gamma, q$ , to (37). It implies that for  $W \geq W_0 = M/\eta$ ,  $\dot{W} \leq 0$ . Thus, the closed loop adaptive control system is uniformly ultimately bounded, namely,  $\bar{V}, \tilde{\theta}_v, \bar{\gamma}, \tilde{\theta}_\gamma, \bar{\alpha}, \bar{Q}, \tilde{\theta}_q \in \mathcal{L}_\infty$ .

#### IV. SIMULATIONS

To validate the effectiveness and performance of proposed method, two numerical simulations are conducted. One is performed with significant parametric uncertainties, to verify the tracking performance of the control system. The other is with significant parameter uncertainties and process noises, to test the robustness of the system. The initial flying state is set as  $\mathbf{x}_0 = [V_0, h_0, \gamma_0, \alpha_0, Q_0]^T = [7846.4 \text{ ft/s}, 85000 \text{ ft}, 0 \text{ rad}, 0.0219 \text{ rad}, 0 \text{ rad/s}]^T$  with  $\boldsymbol{\eta}_0 = [0.594, 0, -0.0976, 0, -0.0335, 0]^T$  and the initial control inputs are  $\mathbf{u}_0 = [\phi_0, \delta_{e0}]^T = [0.12, 0.12 \text{ rad}]^T$  [2]. To

increase fidelity, the dynamic characteristics of actuators are taken into account.

The simulation results of the first case are listed as Figs. 2~5, where a parameter uncertain condition of  $\Delta C_L = \Delta C_T = -40\%$  and  $\Delta C_D = \Delta C_M = 40\%$  are considered which means a 40% decrease in the lift and thrust while a 40% increase in the drag and pitch moment. Note that it's a quite tough situation for flight control. The closed-loop system has good velocity and altitude tracking characteristics illustrated by Fig. 2. The internal states, including FPA, AOA and pitch rate, also can tracing the intermediate virtual command fast and smoothly shown by Fig. 3. Fig. 4 shows that the adaptive estimators are converge to a small neighborhood of zero which agree with the  $\sigma$ -modification. The flexible modes are stable and converge to constants while the output of the actuators are smooth and meet the constraints shown by Fig. 5.

The results of the second case are presented as Fig. 6~7, where a model process noises are considered in (1a) and (1b) together with the same parametric uncertainties as the first case. And the process noises are set as additive zero-mean Gaussian white noise whose variance is 100. With the significant parametric and nonparametric uncertainties, the closed-loop system still represent good tracking performance shown as Fig. 6. Though there exist slight jitter, the adaptive estimators are converge to a small neighborhood of zero and free of parameter drift shown as Fig. 7.

#### V. CONCLUSION

Focusing on the unpredictable instability phenomena of conventional adaptive control, a new robust adaptive control system is designed for AHVs in the paper. Based on I&I theory and  $\sigma$ -modification, the closed-loop system is free from parameter drift and robust to nonparametric uncertainties and unmodeled dynamics. The structured design process and the consideration of states and actuators constraints make it easy to design and engineering implement. Theoretical analysis and simulation results demonstrate the effectiveness and robustness of the proposed method.

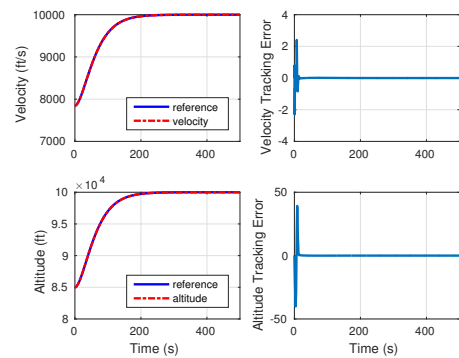


Fig. 2. Response of velocity and altitude with parametric uncertainty

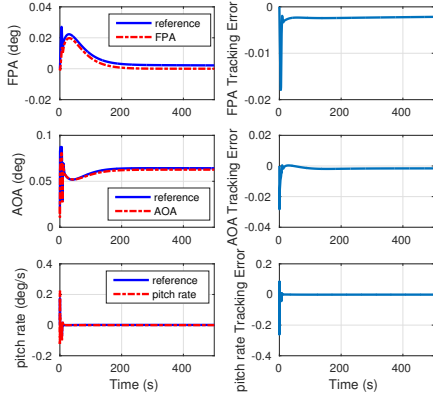


Fig. 3. Response of intermediate states with parametric uncertainty

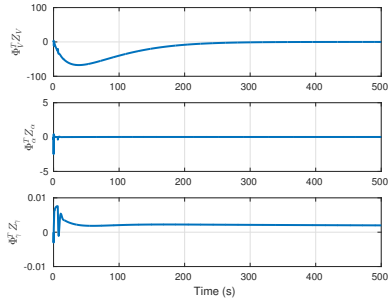


Fig. 4. Response of adaptive law with parametric uncertainty

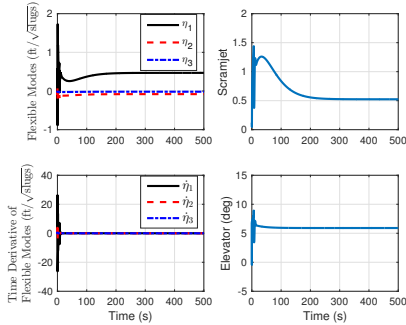


Fig. 5. Outputs of actuators with parametric uncertainty

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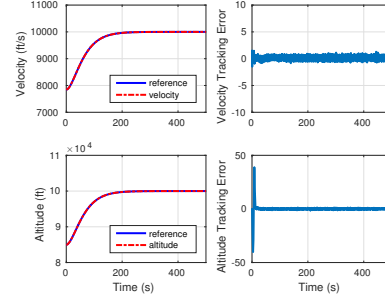


Fig. 6. Response of outputs with parametric uncertainty and process noise

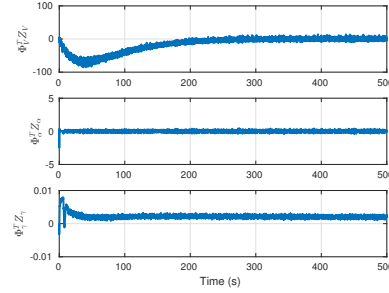


Fig. 7. Response of adaptive law with parametric uncertainty and process noise

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