2017 IEEE International Conference on Systems, Man, and Cybernetics (SMC)
Banff Center, Banff, Canada, October 5-8, 2017

The Impact of Reserve Price on Publisher Revenue in Real-Time Bidding Advertising Markets

Juanjuan Li, Xiaochun Ni, Yong Yuan*, Rui Qin, Xiao Wang
The State Key Laboratory of Management and Control for Complex Systems, Institute of Automation,
Chinese Academy of Sciences, Beijing, China
Qingdao Academy of Intelligent Industries, Qingdao, China
Beijing Engineering Research Center of Intelligent Systems and Technology, Institute of Automation,
Chinese Academy of Sciences, Beijing, China
{juanjuan.li, xiaochun.ni, yong.yuan(Corresponding author), rui.qin, x.wang}@ia.ac.cn,

Fei-Yue Wang, *Fellow*, *IEEE*
The State Key Laboratory of Management and Control for Complex Systems, Institute of Automation,
Chinese Academy of Sciences, Beijing, China
Qingdao Academy of Intelligent Industries, Qingdao, China
Research Center of Military Computational Experiments and Parallel Systems, National University of Defense Technology, Changsha, China
feiyue.wang@ia.ac.cn

Abstract— **With the rapid development of big data analytics in online marketing, real-time bidding (RTB) has emerged as a promising business model in recent years, and now becomes one of the major online advertising channels. Based on analysis of Web Cookies, RTB platforms are able to precisely identify the features and preferences of target audiences visiting publishers' websites, and forward the generated ad impressions to competing advertisers who submit bids for their best-matched audience in real-time ad auctions. In RTB markets, reserve price serves as an important tuner to exclude advertisers with low estimated values, and hence can guarantee a desirable result for the publisher from ad impression auctions. In this paper, we strive to study publishers' strategy on the reserve price, and probe the impact of reserve price on their revenues. We first analyze the ad impression auction under a direct auction mechanism. We then introduce the reserve price and study its impact on publishers' revenues under an indirect auction mechanism, and our research findings indicate that a rational positive reserve price will always improve publishers' revenues even if it is not optimal. Also, the optimal reserve price is figured out based on the advertisers' bid distributions for publishers' revenue maximization. Finally, experiments using empirical log data from real-world RTB markets are designed to validate our model and analysis, and the results provide strong support to our theoretical analysis. The experimental results also indicate that although the number of bids does not impose any influence on the optimal reserve price, it has significant impacts on publishers' revenues.**

Keywords—RTB; reserve price; revenue maximization; publisher; ad impression

I. INTRODUCTION

In recent years, real-time bidding (RTB) has developed to be an important sales channel in online display advertising markets. Enabled with big-data-driven programmatic buying capability, RTB can complete the user profiling and identification process within several milliseconds, and thus realize the real-time precision marketing. Instead of using the traditional "media buying" or "ad-slot buying" patterns, RTB advertising has evolved to the more fine-grained "audience buying" pattern, which helps facilitate precise audience targeting for advertisers in the demand side as well as dynamic ad resource allocation for publishers in the supply side, and can therefore greatly improve the promotion performance and market efficiency. In RTB markets, ad impressions (a.k.a., ad inventory) are traded via programmatic instantaneous auctions on a per-impression basis. As such, RTB has the potential of the real-time control and management of online ad impressions.

In RTB markets, publishers play key role as the suppliers of ad inventory, and must make decisions on allocating appropriate ad impressions to the second-price auctions conducted by the intermediating Ad Exchanges (AdXs). In case when publishers send an ad impression to an AdX, the auction will be accomplished automatically within 10-100 milliseconds. Due to this real-time property, publishers typically have weak or even no control of their ad impressions in RTB auctions. Therefore, it is important for publishers to determine the reserve prices of their ad impressions submitted to AdXs, and those ad impressions with bidding prices lower than the reserve prices will not be sold. From the perspective of microscopic auction sessions, setting reserve prices will help guarantee the sales prices of ad impressions, and in turn determine advertisers' payments and publishers' revenue. From the perspective of system-wide RTB ecosystems, the reserve price will impose great influence on supply-demand balance and the market structure. Therefore, there is a critical need to study the reserve price of ad impressions, as well as its important role in RTB advertising markets.

Due to the "second-price" mechanism of RTB auctions, the reserve price has no direct influence on advertisers' bidding behaviour, and thus it is still a weakly dominant strategy to bid one's own private value [1]. Since the sales price of an ad impression must be no less than its reserve price, no advertiser with his/her value lower than the reserve price can make positive profits in the RTB advertising market. A positive reserve price results in the exclusion of some lower-valued advertisers, thus improving the sales price of the winning advertiser; meanwhile, a high reserve price may increase the risk that the ad impression fail to be sold, which will lead to possible loss of the publisher's revenue. In RTB practice, how to estimate the potential gain and loss caused by setting the reserve price is usually beyond the ability of most publishers. Therefore, for publishers, it is a challenging task awaiting further research to set proper reserve prices, so as to balance profits and risks and maximize revenue.

The basis of setting reserve prices is predicting advertisers' values and bids, especially the winning bids. These parameters not only determine the threshold prices, but also affect reserve price's impact on the auction results. Myerson [2] showed that the optimal reserve price can be calculated from the bid distribution. In RTB practice, however, the partially observable ad exchanges make it very challenging to explore the pattern of winning prices [3]. Wu et al. [4] studied the prediction of winning prices in RTB auctions when only partial features and the winning price of historical winning bids were able to be observed. Li & Guan [5] extracted various features regarding the nature of the ad requests to make the winning rate and winning price prediction. Cui et al. [6] proposed a general divide-and-conquer approach to forecast the bid distribution for any advertising campaign in RTB markets. To date, current research efforts related to bid prediction focus on bidding strategy optimization from the perspective of advertisers [7,8], which can be good references for publishers to forecast advertisers' bids in RTB markets.

For publishers, one possible way to determine reserve prices is to set them as the opportunity costs of RTB auctions, i.e., the maximum reachable prices over all other channels for an ad impression. In general, this way needs online algorithms endogenizing the system-wide demand and supply adaptively [9], and the decisions must be done in a real-time fashion [10]. In some way, setting reserve prices can be viewed as ad inventory pricing with the purpose of risk-aware revenue maximization [11]. Fridgeirsdottir et al. [12] investigated the optimal pricing strategy for ad inventory when impressions and clicks are uncertain, and found that the general heuristics to convert between the CPC and CPM pricing schemes may be misleading as it may cause a great amount of revenue loss for publishers. In the research of [13], reserve price is formulated as a control variable of the ad inventory allocation. An empirical study and live test of the reserve price optimization problem in RTB markets from an operational environment was conducted in [9] to examine several commonly adopted algorithms for setting reserve prices. The results suggest that the proposed game theoretic OneShot algorithm performs the best and the superiority is significant in most cases. To summarize, these existing research efforts focus mainly on reserve price optimization, but fail to provide a straightforward evidence whether or not setting reserve prices is an effective means for publishers to improve their revenues, even in case that the reserve price is not optimal in RTB markets.

Our paper mainly focuses on the impact of the reserve price on publishers' revenue and the reserve price optimization in the ad impression auctions. We first study publishers' revenue maximization using a direct auction mechanism, and then reserve price will be introduced to examine its impact on publishers' revenue maximization with an indirect auction mechanism, under which the optimal reserve price will be figured out. We finally conduct experiments to validate our model and analysis, using empirical log data released by the largest programmatic buying platform in China.

The remainder of this paper is organized as follows. In Section II, we briefly state our research problem. Section III formulates the ad impression auction model, and studies the related revenue maximization issue under a direct auction mechanism. Section IV introduces reserve price, studies its impact on publishers' revenue, and finds the optimal reserve price. Section V conducts experiments to validate our model and analysis. Section VI discusses the managerial insights of our research. Section VII concludes.

II. RESEARCH PROBLEM

In RTB advertising markets, once a user visiting to a publisher's webpage triggers an ad impression, the publisher must make an instant decision, whether or not, to send it to the AdX with a reserve price. Essentially, reserve price is a threshold, indicating the lowest price the publisher is willing to accept for selling the ad impression. If the auction ends with no bid higher than the reserve price, the ad impression will fail to be sold. Therefore, the reserve price is set to exclude advertisers with over-low values and also ensure the ad impression be sold at a good price.

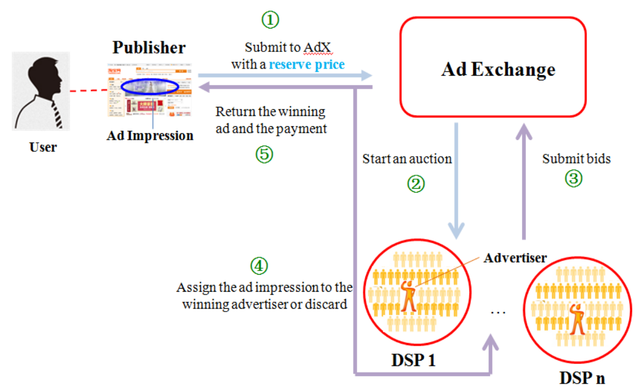

Fig.1 The auction process in RTB markets

Figure 1 describes the auction process of the ad impression. If there is no reserve price, the ad impression will be sold even if there is only one bid exceeding zero. When the reserve price is brought into the second-price auction, the auction result will be greatly different under the situation that there is no more than one bid higher than the reserve price. Consequently, the publisher's revenue will be affected by the reserve price. On one hand, the reserve price can help the publisher to ensure the ad impression be sold at an acceptable price, which is in favour of his/her revenue improvement; On the other hand, the reserve price may also increase the risk of failing to sell the ad impression, which will result in potential losses of his/her revenue. Therefore, a rational reserve price is very important for maximizing the publisher's revenue.

The reserve price will not only be influenced by the publisher's own valuation of an ad impression, but also by his/her expectation of advertisers' values. Since RTB adopts the ad-impression-based "audience-buying" pattern, each ad impression, even with the same audience behind it, may differ in value among advertisers. Over-pricing may lead to a large proportion of ad impressions unsold and hence be wasted. On the contrary, under-pricing will lead to ad impressions sold with over-low prices. Therefore, it is very challenging for the publisher to predict the bids received by the ad impression, and make proper decisions in setting reserve prices so as to improve his/her revenue.

In what follows, we establish a second-price auction model to examine the impact of reserve price on the publisher's revenue, and find the optimal reserve price for the publisher.

III. THE AD IMPRESSION AUCTION MODEL

Suppose the publisher and advertisers in the RTB advertising markets are all risk-neutral. Whenever an ad impression is generated, the publisher will send it to the AdX with corresponding information for the purpose of selling it to one of the potential advertisers through a second-price auction. For the RTB auction, n advertisers who identify the user as target audience will participate in the auction. If the advertiser wins the auction, he/she will gain the opportunity to display an ad to the user and pay for it.

In RTB advertising markets, the publisher usually ensures that all advertisers are symmetric to receive the same noisy signal. After receiving the information about the ad impression including the target audience, ad slot, webpage URL and so on, the advertiser i will formulate an independent value w_i about the ad impression.

Although RTB usually adopts the second-price scheme for ad impression auction, some leading AdXs (e.g., Google, Facebook, etc) still claim that their mechanisms can encourage advertisers to bid with their true valuations and also realize the maximization of social efficiency or social revenues. Therefore, it is rational for us to assume that the auction mechanism in RTB markets is incentive-compatible [14, 15]. Under this assumption, advertisers will submit their true valuations to Demand Side Platforms (DSPs) and AdXs, and then receive RTB auction results and the corresponding payments.

With a specific w_i, an advertiser will have an expected probability $\beta_i(w_i)$ to win the ad impression. In RTB auctions, the concrete value of an advertiser typically is private information, but the aggregated distribution of all advertisers' values is usually common knowledge, which can be defined as independently distributed F_i on $[0,W_i]$, and the corresponding density f_i is strictly positive.

In case of single ad impression auctions, we can compute the payment for the publisher from advertiser i as

$$\begin{aligned} P_i &= \beta_i(w_i) \times E(w_i | \overline{w}_j < w_i) \\ &= w_i \beta_i(w_i) - \int_0^{w_i} \beta_i(x_i) dx_i \end{aligned} \tag{1}$$

Here, $\overline{w}_j$ is the highest valuation of the remaining competitive advertisers, and only $\overline{w}_j < w_i$ is satisfied can the advertiser i win the auction. If $w_i = 0$, we have $P_i = 0$, which means the losing advertisers do not need to pay extra fees.

As a result, the total revenues expected by the publisher are the sum of the *ex-ante* expected payment from each advertiser, which is equal to

$$\begin{aligned} R &= \sum_{i=1}^{n} E(P_i) \\ &= \sum_{i=1}^{n} \int_0^{W_i} P_i f(w_i) dw_i \\ &= \int_W (\sum_{i=1}^{n} Z_i(w_i)\beta_i(w_i)) f(w) dw \end{aligned} \tag{2}$$

where we have $Z_i(w_i) = w_i - (1 - F_i(w_i)) / f_i(w_i)$ representing the virtual valuation of advertiser i with value w_i. It is rational here to consider that $Z_i(w_i)$ increases with w_i, since each rational advertiser will not undervalue the ad impression with higher value than that with lower values.

In the market, only the advertisers with positive virtual valuations are possible to win the ad impression. A higher valuation will result in a higher winning probability, thus returning more revenue to the publisher. Then, revenues of the publisher are optimized when $\sum_{i=1}^{n} Z_i(w_i)\beta_i(w)$ is maximized for all advertisers. Since there is only one ad impression, the advertiser with highest valuation will win the ad display opportunity.

IV. RESERVE PRICE'S IMPACT ON PUBLISHER'S REVENUE

Actually, in RTB advertising markets, the publisher prefers to set a reserve price r to control the sales of ad inventory. Therefore, we inject it into the above model to further discuss the ad impression auction with an indirect mechanism.

If the bid is less than the reserve price, the advertiser cannot get the ad impression and does not need to pay for it; if the bid is no less than the reserve price, the one with the highest valuation will win the ad-display opportunity with the payment equal to the second highest bid or the reserve price.

We define the reserve price that is no less than the second highest bid as an "effective" reserve price. Generally, setting a reserve price less than the second highest bid cannot exert any impact on ad impression allocations and then publishers' payoffs; setting a reserve price between the highest and second highest bids will increase the sales price of the ad impression and publishers' revenues; and setting a reserve price higher than the highest bids will result in the failure of selling the ad impression. For an ad impression, among all the "effective" reserve prices, there exists an "optimal" one to ensure the publisher to get maximal revenue.

In the second-price auction, the reserve price has no direct influence on the bidding behaviours of all advertisers [16], but will influence the auction results and final payoffs. Under the reserve price r, the winning probability of advertiser i should be updated to be $\kappa_i(w_i, r)$, and we have

$$\kappa_i(w_i, r) = \begin{cases} 0, & w_i < r \\ \beta_i(w_i), & w_i \geq r \end{cases} \tag{3}$$

Thus, we adjust the payment for the publisher from advertiser i to be

$$P_i(w_i,r) = w_i\kappa_i(w_i,r) - \int_0^{w_i}\kappa_i(x_i,r)dx_i \quad (4)$$

As for $P_i(w_i,r)$, we have the following analysis:

- If $w_i < r$, we have $P_i(w_i,r) = 0$. If the advertiser i has the value less than the reserve price, he/she will lose the auction and does not need to pay for it. This implies a trivial fact that "rational" reserve price will never exceed the highest bid.
- If $w_i = r$, we have $P_i(w_i,r) = r\beta_i(r)$. If the advertiser i wins the ad impression auction, he/she should pay the reserve price r, since the second-highest bid must be below r.
- If $w_i > r$, we have $P_i(w_i,r) = r\beta_i(r) + \int_r^{w_i} x_i f_i(x_i)dx_i$. If the advertiser i wins the auction, he/she will pay at the price of the larger one between the second-highest bid and the reserve price.

Therefore, the *ex-ante* expected payment for the publisher from the advertiser i should be

$$\begin{aligned} E[P_i(w_i,r)] &= \int_r^{W_i} P_i(w_i,r)f_i(w_i)dw_i \\ &= r(1-F_i(r))\beta_i(r) + \int_r^{W_i} w_i(1-F_i(w_i))f_i(w_i)dw_i \end{aligned} \quad (5)$$

In RTB advertising markets, if the ad impression fails to be sold, it will be wasted and the publisher can get nothing from it. Therefore, there is no extra revenue for the publisher from owning the ad impression but not selling it, that is, the value or opportunity cost of the ad impression for him/her is 0. Obviously, the publisher is not willing to set a reserve price below 0. Then the total expected payoff of the publisher under the reserve price r should be

$$R' = \sum_{i=1}^{n} E[P_i(w_i,r)] \quad (6)$$

Differentiate R' with respect to r, we can obtain

$$\begin{aligned} dR'/dr &= \sum_{i=1}^{n}[1-F_i(r) - rf_i(r)]\beta_i(r) \\ &= \sum_{i=1}^{n}[1-rZ_i(r)](1-F_i(r))\beta_i(r) \end{aligned} \quad (7)$$

The optimal point should satisfy the condition $dR'/dr = 0$. Therefore, we have

$$\sum_{i=1}^{n}[1-r^*Z_i(r^*)] = 0 \quad (8)$$

where r^* is the optimal reserve price.

Based on the assumption that all advertisers have an independent identical distribution of valuations, we compute equation (8) to get

$$Z(r^*) = r^* - \frac{1-F(r^*)}{f(r^*)} = 0 \quad (9)$$

Solving the equation (8), we will get the optimal reserve price r^*.

Substituting r^* into the equation (6), the corresponding maximal expected revenue R'^* for the publisher will be calculated. It is easy to prove that $R'^* > R^*$.

If $r = 0$, we have the derivative of R' equal to zero. Since $Z(r)$ and r are bounded, the expected payment under the rational reserve price will attain a local minimum R at zero. Therefore, we have $R' > R$, which means a rational positive reserve price has positive impact on the publishers' revenue acquisition. Accordingly, the revenue maximizing publisher should always set a reserve price more than his/her opportunity cost.

V. EXPERIMENTS

In this section, we will design experiments to verify the above analysis about the impact of reserve price on the publisher's revenue. The experimental dataset is the Web log data released by one of the largest programmatic buying platforms in China, i.e., iPinyou.com.cn, which comprises records of more than 10 million bids and 3 million ad impressions of 4 representative advertisers. After data cleansing and de-noising, the experimental data can ensure that one bid ID only matches one ad impression. Here, we define set A and B to describe the ad impressions they bid and the ad impressions they win, respectively.

During this period, publishers set reserve prices for all ad impressions. First, we identify the records of ad impressions won by these 4 advertisers satisfying $P = r$, and find that 24.9% ad impressions are paid at the reserve price, which means the publishers' revenues gained from these ad impressions are distinctly improved on account of setting reserve price.

For all bids submitted by these 4 advertisers, 70.1% of them do not win in the auction, which we define as set C, and $C = A - B$. For ad impressions in C, they are won by other advertisers. Further analysis confirms that none of the ad impressions in C receives paying price less than the reserve price, and only 0.002% ad impressions are paid at the reserve price, and 99.8% ad impressions are sold at the price higher than the reserve price, which means reserve price exerts extremely slight influence on their loss of ad impressions, but does not create a negative impact on the publishers' revenues.

The above results of data analysis provide a solid support to the positive impact of the reserve price on publishers' revenues.

For the ad impressions gained by these 4 advertisers, only 17.9% revenues are generated under the effective reserve price where the paying price is equal to the reserve price, while the remaining 82.1% still can be improved by setting more effective reserve prices. Figure 2 depicts the reserve prices and sales prices of 10 thousands ad impressions randomly sampled from the experimental dataset. From the comparison of reserve price and sales price, we can find that the vast majority of reserve prices are set far below the sales price, and under these situations, the reserve price cannot effectively influence the final sales price and publishers' revenue acquisition.

Fig. 2 Comparison between reserve price and sales price

For the ad impressions lost by these 4 advertisers, 98.4% of revenues are generated under the situation that the sales price is higher than the reserve price; therefore, increasing the reserve price properly can help improve publishers' revenues.

In practice, publishers do not set an optimal or even effective reserve price to improve their revenues in the vast majority of cases. Therefore, it is urgently necessary to study the reserve price optimization to help publishers maximize their revenues.

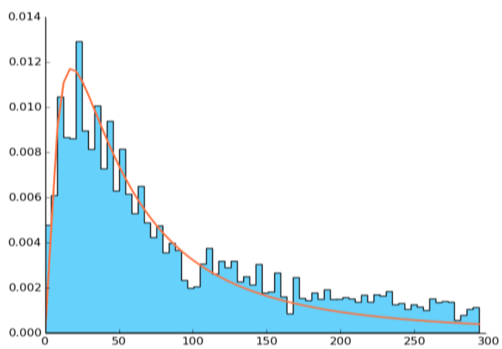

Fig. 3 The winning bid prices fits log normal distribution

From the real-world data, we find that the winning bidding prices from the de-noised experimental dataset fit with a log normal distribution on $[0,300]$ as shown in figure 3, which can be further confirmed using a Kolmogorov-Smirnov test. The set of winning bids is the random sample of bids from all advertisers, and it is reasonable to consider the bids also fit with the log normal distributions (Cui et al., 2011).

From figure 3, we can get that:

$$f(w)=\frac{1}{1.071w\sqrt{2\pi}}e^{-\frac{(\ln w-4.033)^2}{2(1.071)^2}}$$

Using the equation (9), we can compute the optimal reserve price as $r^* \approx 93.55$, which has no relation to the number of bids.

Based on the distribution, we randomly generate new experimental datasets to examine the impact of reserve price on publishers' revenue under different numbers of bids. For each number, we conduct 1000 independent experiments.

First, we study the impact of the optimal reserve price, and the experimental result is shown in figure 4. Here, increase rate represents the effect of optimal reserve price on the publisher's maximal revenue, and is computed by the equation $(R'-R)/R*100\%$. From figure 4, we can draw the following conclusions:

(1) Even the optimal reserve price will not change with the number of bids; its impact on the publisher's revenue is still greatly influenced by the number of bids.

(2) We observe that increase rate keeps larger than zero under different bid numbers, which means the optimal reserve price will always raise the publisher's revenue.

(3) Also, the tendency of increase rate changing with bid numbers decreases sharply to reach an inflection point, and then followed with a gentle descent. When the ad impression is bid by 2 advertisers, setting an optimal reserve price will raise the revenue by 55.7%, setting an optimal reserve price will only raise the revenue by 1.5% under 10 advertisers, and when the bid number reaches 22, the increase rate is only 0.012%. It implies that the optimal reserve price can exert stronger influence on the publisher's revenue when the bid numbers is very small; while in case of large number of bids, the effect is very trivial.

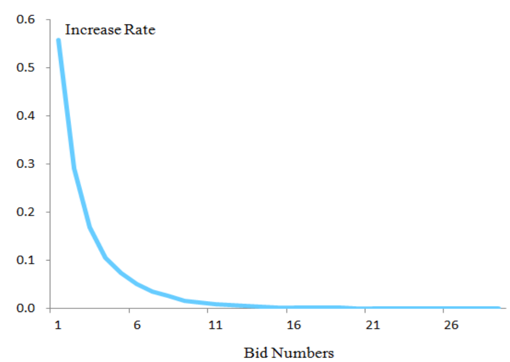

Fig.4 Variation of average revenue increase rate under optimal reserve price with different bid numbers

Then, we use the same sample datasets to study the impact on the revenue of different reserve prices. The experimental result is shown in figure 5, from which we can get that:

(1) Under all positive reserve prices no more than 300, the publisher's revenue is always increasing, compared with the case without reserve price. This means a rational positive reserve price can exert positive impacts on publishers' revenues.

(2) With the increase of bid numbers, the increase rate of publisher's revenues caused by setting reserve prices decreases, which means that the reserve price exerts less effect when facing with a larger amount of bids.

(3) Over-low or over-high reserve prices will not help the publisher to raise revenues to the most extent. The greatest increase rate of the publisher's revenue is realized under the optimal reserve price.

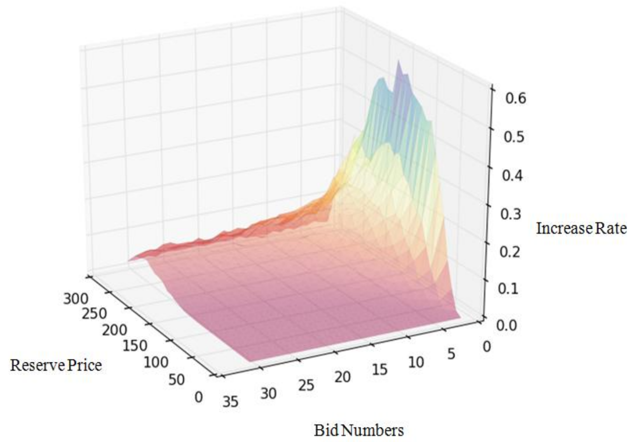

Fig.5 Variation of average revenue increase rate under different bid numbers and reserve prices

VI. MANAGERIAL INSIGHTS

Our research can offer useful managerial insights for setting publishers' reserve prices in RTB advertising markets. On one hand, reserve price will help exclude low-valued advertisers, and thus can help avoid undervaluing ad impressions so as to gain higher payments from the sold ad impressions. On the other hand, reserve price adds the risks that the ad impressions fail to be sold through the RTB channel since the highest bids may be restricted to be invalid. Therefore, it is very challenging for publishers to decide whether and how to set reserve price. Our research provides a straight forward support for them to set reserve price. A rational positive reserve price, even if it is not optimal, will help improve publishers' revenues, and an optimal reserve price will help them to gain maximal revenues.

From our theoretical analysis, we find that the optimal reserve price will not be influenced by the number of advertisers, but the experimental analysis has shown that the impact of reserve price is affected by the number of advertisers. The more advertisers bid for the ad impressions, the less impact the reserve price can exert on publishers' revenue maximization. Therefore, we get a counter-intuitive suggestion for publishers that they should pay more attention to the reserve price of the long-tail ad impressions that receive a small amount of bids, rather than those of popular ad impressions with large amount of bids.

VII. CONCLUSION AND FUTURE WORKS

Reserve price is a critical decision for publishers to sell ad impressions through RTB channel, which not only influence auction result and revenue acquisition from the perspective of participants, but also influence supply-demand balance from the perspective of the market. In this paper, we first study the publisher's revenue maximization using a direct auction mechanism. Then, reserve price is introduced to examine its impact on the publisher's revenue maximization under an indirect auction mechanism. Finally, based on empirical Web log data, we conduct experiments to validate our research findings. The conclusion of our paper can be summarized as: 1) a rational positive reserve price will always help publishers increase revenues; 2) based on advertisers' bid distribution, we can figure out the optimal reserve price which will result in maximal revenues for publishers; 3) the impact of reserve price on the publishers' revenue is affected by the number of bids even if it does not exert any influence on the optimal reserve price.

In future work, we plan to extend this paper to consider reserve price setting under the scenario with asymmetric advertisers; and also the joint optimization of reserve price and ad impression allocation using parallel computing method [17].

ACKNOWLEDGMENT

This work is partially supported by National Natural Science Foundation of China (#71472174, #71232006, #61533019, #61233001, #71402178, #61603381).

REFERENCES

[1] Krishna V. Auction Theory [M], Elsevier, 2010.

[2] Myerson R B. Optimal Auction Design[J]. Mathematics of Operations Research, 1978, 6(1):58-73.

[3] Ghosh A, Rubinstein B I P, Vassilvitskii S, et al. Adaptive bidding for display advertising[C]// International Conference on World Wide Web. ACM, 2009:251-260.

[4] Wu C H, Yeh M Y, Chen M S. Predicting Winning Price in Real Time Bidding with Censored Data[C]// The, ACM SIGKDD International Conference. ACM, 2015:1305-1314.

[5] Li X, Guan D. Programmatic Buying Bidding Strategies with Win Rate and Winning Price Estimation in Real Time Mobile Advertising[M]// Advances in Knowledge Discovery and Data Mining. Springer International Publishing, 2014:447-460.

[6] Cui Y, Zhang R, Li W, et al. Bid landscape forecasting in online ad exchange marketplace[C]// ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, San Diego, Ca, Usa, August. DBLP, 2011:265-273.

[7] Lee K C, Jalali A, Dasdan A. Real Time Bid Optimization with Smooth Budget Delivery in Online Advertising[J]. Computer Science, 2013:1-9.

[8] Fernandeztapia J, Guéant O, Lasry J M. Optimal Real-Time Bidding Strategies[J]. Mathematics, 2015.

[9] Yuan Y, Wang F, Li J, et al. A survey on real time bidding advertising[C]// IEEE International Conference on Service Operations and Logistics, and Informatics. IEEE, 2014:418-423.

[10] Yuan S, Wang J, Chen B, et al. An empirical study of reserve price optimisation in real-time bidding[C]// Proceedings of the 20th ACM SIGKDD international conference on Knowledge discovery and data mining. ACM, 2014:1897-1906.

[11] Muthukrishnan S. Ad Exchanges: Research Issues[M]// Internet and Network Economics. Springer Berlin Heidelberg, 2009:1-12.

[12] Radovanovic A, Heavlin W D. Risk-aware revenue maximization in display advertising[C]// Proceedings of the 21st international conference on World Wide Web. ACM, 2012:91-100.

[13] Balseiro S, Feldman J, Mirrokni V, et al. Yield optimization of display advertising with ad exchange[J]. Management Science, 2011, 60(12):27-28.

[14] Hurwicz L. The design of mechanisms for resource allocation[J]. The American Economic Review, 1973, 63(2): 1-30.

[15] Nisan N, et al. Algorithmic game theory. Vol. 1. Cambridge: Cambridge University Press, 2007.

[16] Yuan Y, Wang F. and Zeng D. Competitive Analysis of Bidding Behavior on Sponsored Search Markets[J]. IEEE Transactions on Computational Social Systems, Accepted.

[17] Wang F. Software-defined systems and knowledge automation: a parallel paradigm shift from Newton to Merton[J]. Acta Automatica Sinica, 2015, 41(1):1-8