

# Optimal Allocation of Ad Inventory in Real-Time Bidding Advertising Markets

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**Abstract**— With the rapid development of big data analytics in online marketing, real-time bidding (RTB) has emerged as a promising business model in recent years and now becomes one of the major online advertising channels. Based on analysis of Web Cookies, RTB platforms are able to precisely identify the features and preferences of target audiences visiting publishers' websites, and forward the information to competing advertisers submitting bids for their best-matched audience in real-time ad auctions. As the supplier of ad impressions, publishers typically have multiple channels to sell their ad impressions (i.e., ad inventory), making their strategies for allocating ad inventory one of the most critical research problems. In this paper, we strive to study publishers' optimal strategy of allocating ad inventory across online channel of RTB-based auctions and offline channel prevailingly realized in the form of guaranteed contracts. Considering the ad reserve price as the control variable, we establish the optimization model. We also explicitly take the default penalty in offline channels into consideration, so as to balance the short-term online revenue and long-term offline revenue. In our work, we analyze altogether three kinds of strategies for publishers to allocate their ad inventory in pursuit of the optimal strategy, and validate our model and analysis via computational experiments. We find that there is no dominant strategy that can outperform others in all cases, and interestingly, publishers using the hybrid-channel strategy do not always gain more revenues than those using the single-channel strategy.

**Keywords**—real-time bidding; guaranteed contract; publisher; ad inventory allocation; ad impression

## I. INTRODUCTION

Real-time bidding (RTB) is a popular ad delivery channel in online display advertising markets. Widely publicized by its ability of big data driven user profiling and precision marketing, RTB has witnessed a rapid development in recent years, and is expected to be the standard business model of all online digital media[1]. Instead of using the traditional “media buying” or “ad-slot buying” pattern, RTB advertising has evolved to use more fine-grained “impression-level buying”

pattern, which can help realize precise audience targeting and dynamic ad resource allocation, improving advertisers' promotion performance and efficiency. In RTB markets, ad impressions (or ad inventory) are traded via programmatic instantaneous auctions on a per-impression basis. As such, RTB has the potential of a real-time control and management of online ad impressions.

In RTB markets, publishers play the key role as ad inventory suppliers. The decision on appropriately allocating ad inventory across multiple ad delivery channels not only affects their own revenue, but also definitely imposes great influence on supply-demand balance and market structure of the RTB ecosystems. As such, ad inventory allocation is widely considered in the literature as a critical and challenging decision for publishers [2, 3]. With the continuous development and popularization, RTB has now evolved to be a preferred channel for publishers to sell ad inventory; At the same time, publishers also have alternative options to allocate a portion of ad inventory to individual deals with specific advertisers or agencies through offline guaranteed contracts. Therefore, in case when an ad impression arrives, the publisher must make a real-time decision on whether to send the impression to competing advertisers in online RTB auctions, or to assign it to a specific advertiser with an offline guaranteed contract of a certain amount ad impressions.

The key to ad inventory allocation is predicting the price of ad impressions from both the online and offline channels. To date, the existing works mainly deal with online ad allocations. For instance, Balseiro et al. formulated the inventory allocation strategy as a stochastic control problem [4], and designed an efficient policy for online ad allocations. Walsh et al. proposed an approach that can automatically partition ad inventory into abstract channels, and developed a suite of techniques based on column and constraint generation so as to tackle the channel explosion [5]. Fernandez-Tapia focused on the optimization of RTB-based ad inventory buying, and provided a baseline

framework permitting to obtain explicit mathematical relations between the different macroscopic variables [6].

In RTB practice, publishers will set a reserve price for each ad impression, prescribing the lowest price at which they are willing to sell the ad impression. The reserve price can serve as a good control variable for ad inventory allocation across multiple channels. For instance, in case when publishers decide to assign some low-quality ad impressions to the offline channel, they will possibly set high reserve prices for the remaining ad impressions sold through RTB channel for better profits. Unlike in other advertising formats, advertisers in RTB markets only submit bids for the best matched impressions generated by users from their targeting niche markets. As such, both the supply and demand of ad impressions are random variables [7], and all decisions must be done in a real-time fashion [8]. Radovanovic et al. [9] presented a dynamic algorithm for ad inventory pricing to maximize publishers' revenue, which adjusts iterative price in the direction of the gradient of an appropriately constructed Lagrangian relaxation. Fridgeirsdottir et al. [10] investigated the optimal pricing strategy for ad inventory when impressions and clicks are uncertain, and found that the general heuristics to convert between the CPC and CPM pricing schemes may be misleading as it may cause a great amount of revenue loss for publishers. Yuan et al. [7] reported an empirical study and live test of the reserve price optimization problem in RTB markets from an operational environment, and examined several commonly adopted algorithms for setting the reserve price. The results suggest that the proposed game theoretic OneShot algorithm performs the best and the superiority is significant in most cases.

From a research perspective, the existing research efforts focus mainly on single-channel pricing of ad inventory, while the research of cross-channel ad inventory allocation is still far from enough. The underlying pricing models in these channels are essentially different, posing great challenges for publishers in allocating their limited ad impressions. For instance, in the channel of guaranteed contracts, the price for bulk impressions is typically predetermined through offline negotiations between publishers and (usually big-brand) advertisers. On the contrary, a real-time auction-based pricing scheme on a per-impression basis is used in online RTB channel. From the perspective of market practice, the allocation strategy of ad inventory will not only affect the publishers' revenues, but also in a system level plays a key role in improving the accuracy and performance of the display advertising ecosystems. Therefore, there is a critical need to study the cross-channel allocation of ad inventory.

Our research is targeted at filling in this important gap. This paper focuses on ad inventory allocation across both the offline guaranteed contract and online RTB channel. We establish an optimization model considering the reserve price as the control variable. We take the default penalty in the offline channel into consideration with the aim of balancing the short-term online revenues and long-term offline revenues. Also, view sequence of the ads has been described as an important parameter for bid prediction of the publisher. Then, we analyze all three potential strategies that the publisher will apply to allocate ad inventory in detail, and then propose the solution process to the model. Finally, computational experiments are conducted to validate our model and analysis.

The remainder of this paper is organized as follows. In Section II, we briefly state the ad inventory allocation problem, formulate it as an optimization model, and propose the solution process through analyzing all three possible strategies. Section III conducts computational experiments to validate our model and analysis. Section IV discusses the management insights of our research. Section V concludes.

## II. THE MODEL OF AD INVENTORY ALLOCATION

### A. Problem Statement

Typically, publishers are willing to sell the ad impressions through multiple channels in order for risk avoidance and revenue maximization. Once a user visits a publisher's webpage, an ad impression is triggered and the publisher must make an instant decision whether to allocate it to the offline channel of guaranteed contracts or online RTB markets. As is shown in Figure 1, the allocation decision is affected by multiple factors including supply of ad impressions, demand in offline and online channels, advertisers' target audience, CPM and bids.

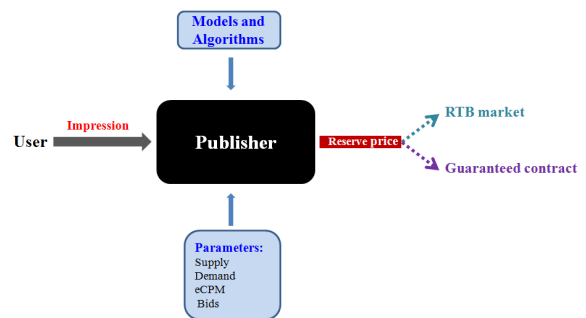


Fig. 1. Ad inventory allocation

In practice, the demand and CPM of ad impressions will be specified as a consensus between publishers and advertisers in offline guaranteed contracts. When publishers successfully accomplish the contract requirement (e.g., a certain number of ad impressions allocated to offline advertisers), they will get the revenue predetermined in the contracts. Otherwise, they have to accept the punitive loss subtracted from the contracted revenue. As for the online RTB advertising, advertisers decide to bid for an impression only in case when the user behind the impression falls into their target audience group. Advertisers cannot exactly predict the bids from competing advertisers and the sold prices of ad impressions, thus they submit bids based on their valuation of the ad impression. According to the second-price scheme of RTB auctions, once an impression comes, if there is no advertiser interested in it, we consider the highest bid and second highest bid are all zero; and if there is only one advertiser bidding for it, then his/her bid is the highest bid and the second highest bid is set to be zero.

In our model, we optimize the allocation strategy of ad inventory in the system level, instead of the per-impression level. A reserve price will be set for all ad impressions. Using specific model and algorithm, publishers can set the reserve price to effectively control the allocation of ad impressions. For example, if publishers predict that the reserve price is higher than the highest bid that an ad impression can win in online

RTB channel, it will be allocated to offline channel of guaranteed contracts. Otherwise, it will be allocated to RTB advertising markets. An increased reserve price can help control more low-quality ad impressions allocated to the offline channel and the remaining high-quality ad impressions allocated in the online RTB channel with better revenue realized.

Generally speaking, in online RTB markets, publishers are faced with great challenge in setting an optimal reserve price to control the ad inventory allocation. Over-pricing may lead to a large proportion of ad impressions unsold in online RTB channel and be wasted. On the contrary, under-pricing will lead to these ad impressions undersold in RTB markets while possibly fail to accomplish the offline contract. Our model and the following analysis are dedicated to solve this dilemma.

### B. The Model

We consider a scenario of  $M$  users sequentially visiting a publisher's webpage with  $N$  ad slots. When a user opens the webpage,  $N$  ad impressions will be generated, and one for each ad slot. So, the overall supply of impressions is  $S = M \times N$ .

Generally, for a specific ad impression  $e_i^j$  generated by user  $i$  in ad slot  $j$ , there are altogether three possible cases in allocating and selling it to advertisers. Namely, it can be assigned to the offline channel of guaranteed contracts, or to the online RTB channel and successfully be sold, or to the RTB channel but failed to be sold due to high reserve price and in turn has to be wasted [4]. With the purpose of revenue maximization, publishers usually allow to "effectively" waste some of their ad impressions.

Regarding the offline channel of guaranteed contracts, the demand  $d'$  and CPM  $c$  of impressions are all predetermined. If  $e_i^j$  is allocated to complete the contract, the publisher will obtain a revenue of  $c$ . In case that the number of ad impressions requested in the contract is satisfied, the total contract revenue will be  $d'c$ . Otherwise, in case that the contract is not fulfilled, the publisher will be faced with a default penalty and obtain a revenue of  $r' = d'c - \alpha f(s', d')$ , where  $\alpha > 0$  denotes the penalty factor and  $s'$  is the actual supply for the contract. If  $s' \geq d'$ , we have  $f(s', d') = 0$ .

If the impression  $e_i^j$  is allocated to the online RTB market, three scenarios might occur in the ad auctions in terms of the reserve price denoted by  $h$ . First, in case when both the highest and second-highest bids are no less than the reserve price, the impression will be sold at a price equal to the second highest bid, according to the second-price scheme of RTB auctions. Second, if the reserve price is set in the interval between the highest and second-highest bids, the impression will be sold at the reserve price. Third, if both the highest and second-highest bids are less than the reserve price, the impression will fail to be sold and has to be wasted.

The publisher cannot exactly predict advertisers' bids for ad impressions in online RTB auctions, but can derive a rough estimation according to his/her experience, skills or analysis from historical data. Therefore, in our model, we can reasonably

assume that the publisher's decision on ad inventory allocation is made based on the predicted bids  $b_i^j$  of advertisers.

Practically, due to the limitation of webpage size and users' view scope, not all the ads on the webpage can be viewed at the first sight. Some ads at the bottom of the webpage might be exposed to users at the second, third view or even later. The view sequence poses great influence on the value of ad impressions, since the ad on the top of the webpage typically has a higher click-through rate and conversion rate, thus has a larger value for both advertisers and publishers. Therefore, the publisher should adjust his/her expectation of bids to be  $b_i^j(k)$ , where  $k$  represents the index of view sequence,  $k = 1, 2, \dots, K$ , and a smaller  $k$  leads to a higher  $b_i^j(k)$ .

Therefore, the payment to the publisher from the impression  $e_i^j(k)$  will be:

$$p_i^j = \begin{cases} 0, & h > \hat{b}_i^j(k) \\ \tilde{b}_i^j(k), & h \leq \tilde{b}_i^j(k) \\ h, & \tilde{b}_i^j(k) < h \leq \hat{b}_i^j(k) \end{cases}$$

Where  $\hat{b}_i^j(k), \tilde{b}_i^j(k)$  are the highest and second-highest bids expected by the publisher, respectively. The revenue from the online RTB market for the publisher will be

$$r = \sum_{i=1}^M \sum_{j=1}^N p_i^j;$$

and the revenue of the publisher from the offline guaranteed contracts is  $r' = d'c - \alpha f(s', d')$ , where

$$s' = \sum_{i=1}^M \sum_{j=1}^N z_i^j, \text{ and } z_i^j = \begin{cases} 1, & p_i^j = 0 \\ 0, & p_i^j > 0 \end{cases}.$$

If  $s' \geq d'$ , we will get  $r' = d'c$ .

Consequently, the payoff function for the ad inventory allocation problem will be:

$$R = r + r'$$

The optimization model of ad impression allocation across the offline guaranteed contracts and the online RTB markets can be established as follows:

$$\begin{aligned} \max \quad & R = r + r' \\ \text{s.t.} \quad & r = \sum_{i=1}^M \sum_{j=1}^N p_i^j \\ & r' = d'c - \alpha f(s', d') \\ & s' = \sum_{i=1}^M \sum_{j=1}^N z_i^j \\ & p_i^j = \begin{cases} 0, & h > \hat{b}_i^j(k) \\ \tilde{b}_i^j(k), & h \leq \tilde{b}_i^j(k) \\ h, & \tilde{b}_i^j(k) < h \leq \hat{b}_i^j(k) \end{cases} \\ & z_i^j = \begin{cases} 1, & p_i^j = 0 \\ 0, & p_i^j > 0 \end{cases} \\ & c > 0, h \geq 0, 0 \leq \alpha \leq 1 \\ & \hat{b}_i^j(k) \geq 0, \tilde{b}_i^j(k) \geq 0 \end{aligned} \quad (1)$$

### C. The Analysis and Solution

Intuitively, there are altogether three kinds of strategies for publishers to allocate their ad inventory with the aim of maximizing their total revenue, namely, allocating all ad impressions to the online RTB channel (Strategy-1), or satisfying the requested amount of impressions in the offline guaranteed contracts and allocating the remaining impressions to RTB channel (Strategy-2), or assigning less than  $d'$  impressions to the offline channel while spending more impressions to the online RTB channel in pursuit of better online revenues (Strategy-3). Obviously, Strategy-1 is a single-channel allocation strategy while Strategy-2 and Strategy-3 are hybrid cross-channel strategies.

First, we consider Strategy-1 in which  $s' = 0$ . Since the offline revenue is fixed as  $r'(0) = d'c - \mathcal{C}(0, d')$  in this case, the ad inventory allocation problem turns to maximizing the online revenue, and the model (1) can be simplified as:

$$\begin{aligned} \max \quad & r = \sum_{i=1}^M \sum_{j=1}^N p_i^j \\ \text{s.t.} \quad & p_i^j = \begin{cases} 0, & h > \hat{b}_i^j(k) \\ \tilde{b}_i^j(k), & h \leq \tilde{b}_i^j(k) \\ h, & \tilde{b}_i^j(k) < h \leq \hat{b}_i^j(k) \end{cases} \\ & \hat{b}_i^j(k) \geq 0, \tilde{b}_i^j(k) \geq 0 \\ & h \geq 0 \end{aligned} \quad (2)$$

The solution for the model (2) can be depicted as follows:

- For a certain impression, the optimal reserve price  $h_i^{j*}(k)$  must be either the highest bid  $\hat{b}_i^j(k)$  or the second highest bid  $\tilde{b}_i^j(k)$ . If the reserve price is higher than the highest bid, the impression will fail to be sold and the publisher wins nothing; Otherwise, if the reserve price is lower than the second highest bid, the revenue gained from the impression still can be improved.

- Pick out all the highest bids and the second highest bids as the solution space  $H$  for the optimal reserve price, and compute online revenue  $r$  by summing up the revenue from each impression under different reserve prices from the solution space.

- Compare all online revenues under different reserve price to find the maximized  $r^*$  and determine the optimal reserve price  $h^* = \arg \max_{h \in H} (r^*)$ . The maximized gross revenue  $R^* = r^* + r'(0)$ .

If Strategy-2 is adopted,  $d'$  impressions will be allocate to the offline channel with the revenue realized as  $r' = d'c$ . Then, the problem will be transferred into maximizing the online revenue from the remaining impressions. In order to guarantee the offline demand, the reserve price of  $d'$  impressions should exceed the highest bids submitted by advertisers. We first sort all the highest bids  $\hat{b}_i^j(k)$  in ascending order  $\hat{b}_1, \hat{b}_2, \dots, \hat{b}_{d'}, \dots, \hat{b}_{MN}$ , and find the  $d'$ th one  $\hat{b}_{d'}$ . Thus, the reserve price setting for Strategy-2 should ensure  $h > \hat{b}_{d'}$ . Then, under the condition that

the first  $d'$  impressions must be allocated to the offline channel, we find the optimal reserve price for revenue maximization of the remaining  $S - d'$  impressions. In reference to solution process of model (2), we will get an optimal reserve price  $h^\#$ , and a corresponding number  $s^\#$  of impressions are sold in RTB markets. If  $S - d' > s^\#$ , then  $(S - d') - s^\#$  impressions are effectively wasted. Then, the maximized total revenue for the publisher adopting Strategy-2 is

$$R^\# = d'c + \sum_{i=1}^M \sum_{j=1}^N p_i^j(h^\#).$$

Regarding Strategy-3, suppose  $s'$  ad impressions are predetermined to supply for the offline channel, and  $s' \in \{1, 2, \dots, d' - 1\}$ . The offline revenue is  $r' = d'c - \mathcal{C}(s', d')$ . Given a specific  $s'$ , we can figure out the corresponding maximized online revenue  $r^*(s')$  and the optimal reserve price  $h^*(s')$  using the solution process of Strategy-2. Then, a further adjustment of offline supply should be made to avoid invalid wastes, since the optimal reserve price  $h^*(s')$  will restrict a corresponding amount of impressions  $\lambda[h^*(s')]$  failing to be sold through RTB channel, which should all be allocated to guaranteed contract for total revenue maximization. Thus, the adjusted offline supply should be  $\lambda[h^*(s')]$  and offline revenue will be  $d'c - \mathcal{C}(\lambda[h^*(s')], d')$ . Consequently, the maximized total revenue under the supply  $s'$  for the guaranteed contract can be found as  $R^*(s') = d'c - \mathcal{C}(\lambda[h^*(s')], d') + r^*(s')$ . If the specific penalty function is formulated, the maximized revenue can be figured out. Therefore, the revenue maximization problem has been transformed into finding the optimal supply for the offline channel. Given the penalty function, we can easily find the optimal offline supply  $s^*$  and the maximized revenue  $R^*$ .

Comparing  $R^*$ ,  $R^\#$  and  $R^*$ , we can finally identify the optimal ad inventory allocation strategy for maximizing the publisher's revenue.

### III. COMPUTATIONAL EXPERIMENTS

In this section, we will conduct computational experiments to validate our model of ad inventory allocation, and try to find the optimal reserve price for maximizing the publisher's total revenue. Computational experiments are designed to deal with the difficulties of model intractability and the lack of empirical data from marketing practice.

First, we consider a randomly generated experiment scenario with 200 users and 5 ad slots. Therefore, 1000 impressions are supplied. For each impression, a highest bid and a second highest bid is randomly generated. All the second highest bids are uniformly distributed in  $[0, 15]$ , and accordingly, all the highest bids are distributed in  $[0, 30]$ .

We conduct thousands of independent computational experiments, for the purpose of drawing generalized conclusions for the optimal allocation strategy under different bid series.

We first analyze Strategy-1. In case of this strategy, the ad inventory allocation problem has been simplified into finding an optimal reserve price for online revenue maximization. Figure 2 shows the experimental results. In all experiments, the total revenues of Strategy-1 for the publisher have a tendency that increase with the reserve price to reach a peak and then decrease all the way down. The results indicate that over-low reserve price will lead to underselling ad impressions, and over-high reserve price will result in many ad impression unsold. In both cases, the publishers' revenue will be damaged.

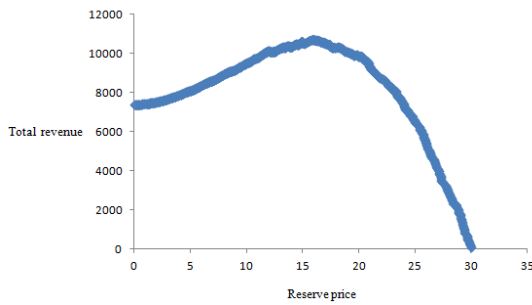


Fig. 2. The reserve price and total revenues of Strategy-1

For Strategy-2, the experimental results are shown in Figure 3 and Figure 4, which indicate that:

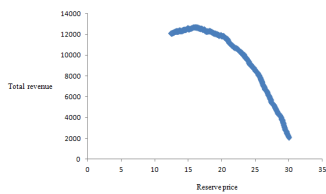


Fig. 3. The reserve price and total revenues of Strategy-2 when  $d' \leq \beta$

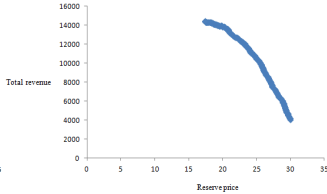


Fig. 4. The reserve price and total revenues of Strategy-2 when  $d' > \beta$

(1) Corresponding with the optimal point in Figure 2, we find that a certain number of ad impressions are effectively wasted in the RTB markets. We denote the number as  $\beta$ . Then, we can conclude that if  $d' \leq \beta$ , the optimal reserve price of Strategy-1 is also the optimal one of Strategy-2; and if  $d' > \beta$ , the optimal reserve price of Strategy-2 is greater than that of Strategy-1.

(2) The offline CPM  $c$  is the deterministic influence for the publisher to choose Strategy-1 or Strategy-2 for revenue maximization.

From the above experiments, we know that when  $d' \leq \beta$ , the optimal reserve price and maximized revenue of Strategy-3 is also the same with those in Strategy-2, respectively. So, we design experiments to analyze Strategy-3 under the condition that  $d' > \beta$ . For the offline channel of guaranteed contracts, demand  $d'$  is given as 400 and the CPM  $c$  is given as 10. The offline payoff function is formulated as  $r' = d'c - \alpha(d' - s')$ , and we set  $\alpha = c = 10$ .

The experimental results are shown in Figure 5, and we can find that:

(1) There exists a threshold of offline supply of ad impressions, and all the predetermined supply amounts no more than the threshold will result in the same optimal total revenues and actually the publisher will allocate the threshold number of ad impressions to the offline channel of guaranteed contracts.

(2) Figure 5 (b) and (c) show that the total revenue increases with the arise of the adjusted offline supply and reserve price, which means Strategy-3 is inferior of Strategy-2 under the offline payoff function we set.

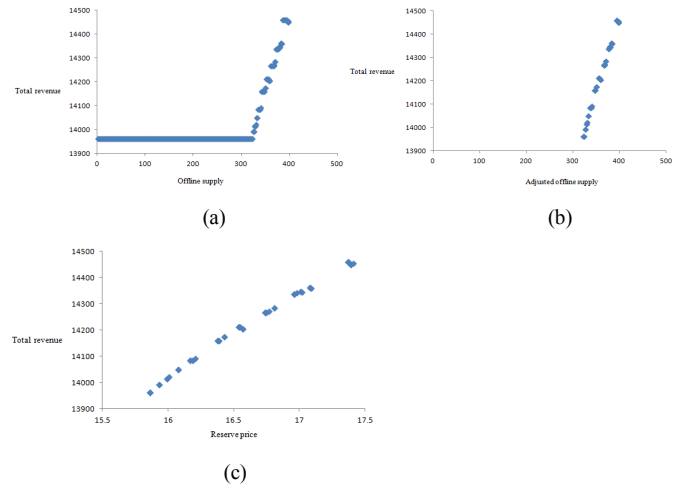


Fig. 5 (a) The offline supply and total revenue of Strategy-3; (b) The adjusted offline supply and total revenue of Strategy-3; (c) The reserve price and total revenue of Strategy-3

Moreover, we try to change the penalty factors to make further investigation of Strategy-3. The experimental results are shown in Figure 6, from which we can draw the following conclusions:

(1) Different penalty factors do not change the tendency of the total revenue's changing with different offline supply, but influence the optimal reserve price and maximal revenues.

(2) Under different penalty factors, the superiority between Strategy-2 and Strategy-3 is different. Lower default penalty allows the publisher to suffer some offline loss to maximize the total revenue, for the reason that the revenue gained from the RTB advertising markets can compensate the loss from breaking the guaranteed contract.

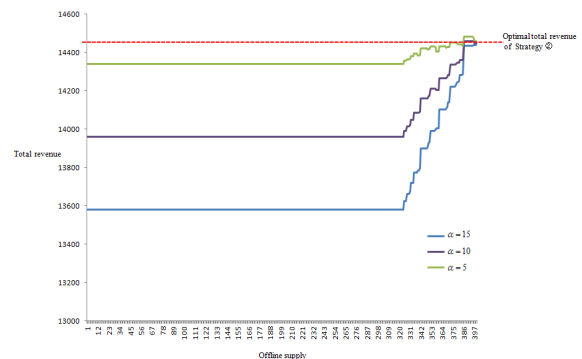


Fig. 6 The comparison of total revenues under different penalty factors

Given all the parameters and deterministic payoff functions, the publisher can find the optimal reserve price and allocation strategy to optimize ad inventory allocation. Both offline revenue and online revenue will not always increase with the increasing of reserve price. As for the online revenue, there exists a threshold, before which it can be improved by increasing the reserve price. However, in case when the reserve price exceeds the threshold, the online revenue will decrease sharply. As for the offline revenue, when the reserve price is set so as to control more than offline demand of ad impressions to be allocated to offline channel, the offline revenue will still keep on the promised one. Each strategy shows its priority under different situations, the publisher should choose the corresponding optimal allocation strategy according to different situations. Higher offline CPM motivates the publisher to allocate more to offline guaranteed contract, while lower offline penalty motivates the publisher to allocate more impressions to online RTB advertising markets.

#### IV. MANAGEMENT INSIGHTS

Our research findings can offer useful managerial insights for publishers' ad inventory allocation decisions in RTB advertising markets. Intuitively, better revenues may be expected from the ad inventory selling through hybrid channels than single channel for publishers. However, our research offers a different conclusion that although hybrid channels provide more choices and help diversify risks; it does not always result in better revenue. Instead, we prove that there is no dominant strategy for the publisher under all situations, and all ad inventory allocation strategies show their superiority given different parameters. Since the ad impressions generated by users and bids are proposed by advertisers, publishers cannot make any change on them; so they should strive for good offline demand, CPM and penalty through effective negotiation, which will not only influence offline revenues, but still have great impact on total revenues.

In practice, publishers allocate ad inventory across offline and online channels on the basis of their valuation of impressions, while we provide an actionable solution for them to take RTB advertisers' bids into consideration and set an optimal reserve price to control the ad inventory allocation effectively. Also, our research provides a feasible solution for publishers to decide the sales channel from the perspective of system-level ad inventory allocation, which helps maximize their total revenue.

#### V. CONCLUSIONS AND FUTURE WORK

Ad inventory allocation is an important decision for the publisher, which not only plays a key role for revenue maxi-

mization, but also influences the supply-demand balance and market structure in RTB advertising market. In this paper, we establish an optimization model taking the reserve price of ad impressions as the control variable of ad inventory allocation across both the offline channel of guaranteed contracts and the online channel of RTB markets. Computational experiments are conducted to validate our model and analysis.

In our future work, we are planning to extend this paper to consider the ad inventory allocation under uncertain supply and demand in the competitive environment. We also plan to compare our work with the existing ad inventory allocation strategies to explore new revenue optimization models.

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