

Optimizing the Revenue for Ad Exchanges in Header Bidding Advertising Markets

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Abstract—With the ever-growing popularization of Real Time Bidding (RTB) advertising, the Ad Exchange (AdX) platform has long enjoyed a dominant position in the RTB ecosystem due to its unique role in bridging publishers and advertisers in the supply and demand sides, respectively. A novel technology called header bidding emerged in the recent one or two years, however, is widely believed to have the potential of challenging this dominant position. Compared with RTB markets, header bidding establishes a priority sub-market allowing bidding partners of the publisher submit their bids before the ad impression delivered to the open AdX platform, resulting in a decreased winning probability and revenue for the AdX. As such, there is a critical need for the AdX to tackle this challenge so as to better coexist with header bidding platforms. This need motivates our research. We utilize stochastic programming approach and establish a stochastic optimization model with risk constraints to optimize the pricing strategy for the AdX, considering that the highest bids from the bidding partners can be characterized by random variables. We study the equivalent forms of our proposed model in case when the randomness is characterized by uniform or normal random variables. With the computational experiment approach, we validate our proposed model, and the experimental results indicate that both the risk tolerance of the AdX and the distribution of randomness of the highest bid from the bidding partners can greatly affect the optimal strategy and the corresponding optimal revenue of the AdX. Our work highlights the importance of the risk level of the AdX and the distribution of the randomness generated by the partners to the decision making process of the AdXs in header bidding markets.

Keywords: header bidding, real time bidding, ad Exchange, stochastic optimization model, risk level

I. INTRODUCTION

With the ever-growing popularization of Real Time Bidding (RTB) advertising, the online advertising industry has entered into a new era of big-data-driven programmatic buying. As a central player in RTB markets, Ad Exchange (AdX) platforms serve as a middleman bridging publishers in the supply side and advertisers in the demand side, dispatch large numbers of ad impressions generated from publishers' landing pages to the best-matched advertisers, and in this way make profits of intermediary fee from a proportion of publishers' revenue. Due to this tight coupling and mutual dependency, AdXs'

marketing strategies, especially their pricing strategies of the intermediary commission, directly affect publishers' revenues.

The emergence and rapid development of header bidding technology, however, has been witnessed to greatly challenge AdXs' dominant position in the RTB ecosystems. Technically speaking, header bidding can offer publishers an alternative choice by establishing a priority sub-market allowing publishers directly sell ad impressions to their allied bidding partners, in case when these partners bid higher than that can be sold in the open AdX markets. Obviously, header bidding has the potential of profoundly reshaping the traditional RTB business model. Using header bidding, publishers might enjoy lowered dependency with AdXs, enhanced diversity in sales channels, and in turn improved advertising revenues, and thus are increasingly willing to embrace this novel technology. To date, header bidding has become an important and effective tool in publishers' advertising arsenal. As reported by BI Intelligence, nearly 70% of top publishers in the U.S. market have adopted header bidding in their programmatic process¹. OpenX, one of the programmatic ad-tech providers, reported that header bidding has experienced a 300% growth in both the first-quarter of 2016² and the year of 2015, which accounts for 80% of the company's overall growth³. Furthermore, publishers who adopted OpenX as their header bidding partner have witnessed a 20–50% lift in their revenues¹. Another advertising company named Index Exchange claimed that 80% of its total revenue comes from header bidding³. As a counter-example, Rubicon Project company saw its stock fell 32% and thus lost 200 million dollars in a Wednesday, primarily because it failed to respond quickly to the header bidding trend³.

Just as every coin has two sides, AdXs, on the other hand, might suffer from significantly decreased pricing power and revenue, as well as growing challenges to their dominant

¹<http://www.businessinsider.com/header-bidding-gains-momentum-drives-up-publisher-ad-revenue-2016-5>

²<http://www.mediapost.com/publications/article/274230>

³<https://adexchanger.com/platforms/great-header-bidding-shake-begun/amp/>

position in RTB markets with header bidding. As reported by AdExchanger, many programmatic-focused publishers who use header bidding saw Google's share of revenue declined from 90% to just 40-50%⁴. Therefore, from the perspective of AdXs, one might naturally ask a question that how to effectively formulate their advertising strategies, so as to tackle this challenge and better coexist with the novel header bidding platforms.

Our work aims to preliminarily answer this research question, from the aspect of optimizing AdXs' pricing strategy of intermediary fee. We view this pricing strategy as an important control variable that should be carefully tuned, so as to find an appropriate proportion to divide the sales revenue of ad impressions between AdXs and publishers. Intuitively, on one hand, setting a larger proportion of commission from publishers may lead to an increased revenue for the AdX, while at the same time risk decreasing AdX's winning probability in single auction sessions (i.e., due to possibly returning a price lower than publishers' reserve price), and in the long run losing its edge as a dominant player in RTB ecosystems. On the other hand, setting a smaller proportion may help increase AdX's winning probability in auctions, at the cost of lowered revenue both in each ad request and the entire marketing operations. As such, there is a critical need for the AdX to make tradeoffs and find the optimal commission proportion, targeting at maintaining its overall stability, profitability and effectiveness in RTB ecosystems.

In this paper, we strive to study this optimization problem faced by the AdX in RTB markets with header bidding. As we aforementioned, AdXs' decreased winning probability may lead to a lowered dependency between AdXs and publishers, and in turn raise AdXs' risk of losing their dominant market position. So, we can characterize this risk by a function of AdXs' winning probability, and utilize the stochastic programming approach [1] to establish a stochastic optimization model for AdXs under risk constraints, considering that the highest bidding of publishers' bidding partners is randomly distributed. When the randomness is characterized by uniform or normal random variables, we study the equivalent forms of our proposed model. We also utilize the computational experiment approach [6] to validate our model, and the experimental results show that in case without risk constraints, the revenue of the AdX has a tendency of a rise first followed by a decline. Moreover, in case with risk constraints, the optimal proportion and the corresponding optimal revenue increase with the risk level until it reaches a threshold, and then become stablized when the risk level is higher than the threshold in each case.

The remainder of this paper is organized as follows. In Section II, we first introduce the bidding process of header bidding, and then we state our problem and establish a stochastic optimization model with risk constraints. After that, we study the equivalent model in some special cases. In Section III, we design computational experiments to validate our proposed

model. Section IV discusses the managerial insights of our research findings. Section V concludes.

II. THE OPTIMIZATION MODEL OF AdX IN HEADER BIDDING

A. The Auction Process with Header Bidding

Typically, in RTB models [3, 4, 5], publishers directly forward ad impressions to the AdX. As the unique intermediary agency linking publishers to advertisers, AdXs can to a large extent determine publishers' revenue via their pricing strategies, and thus enjoy a dominant position in the RTB ecosystems. In header bidding markets, however, AdXs may face severe competition and in turn reduced pricing power since publishers' cooperative partners are given priority to submit their bids for each ad impression before it is delivered to the AdX, as is shown in Fig. 1.

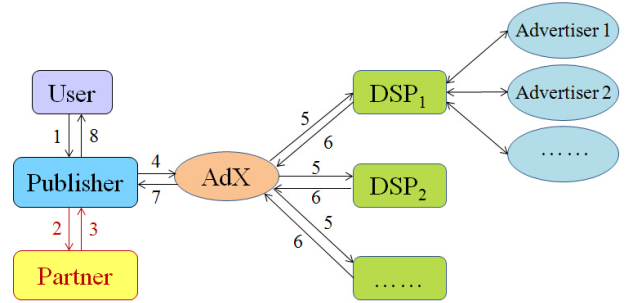


Fig. 1. The bidding process of header bidding advertising

The bidding process of header bidding advertising can be described as follows:

- (1) When a user visits a webpage of the publisher, an ad impression is generated.
- (2) The publisher will first request its cooperative bidding partners to bid for this ad impression.
- (3) The highest bid among all bidding partners is returned to the publisher.
- (4) The publisher then delivers the ad impression to the open AdX platform, seeking even higher bids from advertisers than its partner's bid.
- (5) The AdX starts to call for bids from all the registered Demand Side Platforms (DSPs), who bid on behalf of their advertisers.
- (6) The DSPs compete and submit their bids to the AdX.
- (7) The AdX determines the highest bid among all DSPs. In case when DSPs outbid partners, the AdX will deliver the winning DSP's advertisement and the resulting cost to the publisher. Otherwise, the partner will be the winner.
- (8) The publisher displays the winner's advertisement to the user.

B. Problem Statement and Notations

As can be seen in Section II-A, there is a two-stage auction process in header bidding advertising markets. We consider the case that there is only 1 ad impression. Suppose the intrinsic reserve price set by the publisher is ρ , which will be directly delivered to the AdX if there is no partner bidding higher

⁴<https://adexchanger.com/ad-exchange-news/appnexus-strikes-back-against-googles-attempt-to-end-header-bidding/>

than it. Otherwise, if the highest bids from bidding partners $d > \rho$, an updated reserve price $\rho' = d$ will be claimed by the publisher and delivered to the AdX. That is,

$$\rho' = \max\{\rho, d\}. \quad (1)$$

Once receiving the ad impression with the reserve price ρ' from the publisher, the AdX starts an auction with its own reserve price $\beta(\lambda)$ among all the DSPs registered on it. As a reward of its intermediating efforts, the AdX will charge a proportion λ from the advertising costs. As such, the reserve price of the AdX [2] will be set as

$$\beta(\lambda) = \frac{\rho'}{1-\lambda}, \lambda \in [0, 1). \quad (2)$$

Suppose the highest and second highest bids for the ad impression on the AdX are b_1 and b_2 , respectively. The highest-bid advertiser wins the ad impression, if and only if $b_1 \geq \beta(\lambda)$.

If $b_2 \geq \beta(\lambda)$, the winning advertiser needs to pay b_2 to the AdX according to the well-known Generalized Second Price (GSP) auction mechanism, and the AdX returns the advertising cost $b_2(1-\lambda)$ to the publisher. Thus, the revenue of the AdX is

$$r(\lambda) = b_2\lambda. \quad (3)$$

If $b_1 \geq \beta(\lambda) > b_2$, then the winning advertiser needs to pay $\beta(\lambda)$ to the AdX, who returns the advertising cost $\beta(\lambda)(1-\lambda) = \rho'$ to the publisher. Thus, the revenue of the AdX from the ad impression is

$$r(\lambda) = \beta(\lambda) - \rho' = \frac{\lambda\rho'}{1-\lambda}. \quad (4)$$

According to the above discussions, it is obvious that the proportion λ of the AdX can not only determine whether the AdX could win each ad impression, but also its revenue from the ad impression when it wins. Thus, how to choose the best proportion λ to maximize the revenues becomes an important decision-making problem faced by the AdX.

Due to the complex dynamics of the advertising markets, the AdX can hardly predict the exact value of the highest bid d from the bidding partners. However, an estimation of the distribution of d may be easily obtained from analyzing the Web log of historical ad impressions. Thus, we can model d as a random variable. Moreover, if the winning probability of the AdX is low, the AdX will risk losing its dominant position in the auction. Thus, we can model the risk of the AdX as a function of the winning probability of the AdX.

In the following section, we will utilize the stochastic programming approach and establish a stochastic optimization model for the AdX under risk constraints.

C. Stochastic Optimization Model with Risk Constraints

Suppose that the highest bid d of the bidding partners is a random variable distributed in $[a_1, a_2]$, with the probability distribution function $f(x)$. In real-world markets, the possible relation between the highest bid d and the reserve price ρ may be $d < \rho$ or $d \geq \rho$, and the relations of d with b_1 and b_2 may be $d < b_2$, $b_2 \leq d < b_1$ or $d > b_1$. Moreover, it is obvious that $b_2 < \rho < b_1$ is a simplified case of $\rho < b_2$ according to (3) and (4). Thus, in this paper, we only consider a general case

$a_1 < \rho < b_2 < b_1 < a_2$, and other cases will be discussed analogously in an extension version of this work.

Since d is a random variable distributed in $[a_1, a_2]$, according to (1) and (2), we have

$$\rho' = \begin{cases} \rho, & \text{if } x \in [a_1, \rho) \\ x, & \text{if } x \in [\rho, a_2] \end{cases} \quad (5)$$

and

$$\beta(\lambda) = \begin{cases} \frac{\rho}{1-\lambda}, & \text{if } x \in [a_1, \rho) \\ \frac{x}{1-\lambda}, & \text{if } x \in [\rho, a_2]. \end{cases} \quad (6)$$

Obviously, we have

$$\beta(\lambda) \geq \frac{\rho}{1-\lambda}. \quad (7)$$

Thus, if $\rho/(1-\lambda) > b_1$, i.e., $\lambda > 1 - \rho/b_1$, then the revenue of the AdX from the ad impression is 0. Therefore, in the following, we only need to consider the case of $\lambda \leq 1 - \rho/b_1$.

According to the distribution function of d , the probability of $d < \rho$ can be computed by

$$\Pr\{d < \rho\} = \int_{a_1}^{\rho} f(x)dx. \quad (8)$$

When $x \in [a_1, \rho)$, the AdX will win the ad impression, and when $x \in [\rho, a_2]$, the AdX will win the ad impression only if $b_1 \geq x/(1-\lambda)$, i.e., $x \leq b_1(1-\lambda)$. Thus, the winning probability of the AdX for the ad impression can be computed by

$$p(\lambda) = \Pr\{d < \rho\} + \int_{\rho}^{b_1(1-\lambda)} f(x)dx = \int_{a_1}^{b_1(1-\lambda)} f(x)dx. \quad (9)$$

In the following, we compute the revenue of the AdX, which can be divided into the following two cases:

1) In case of $b_2 < \rho/(1-\lambda) \leq b_1$, i.e., $1 - \rho/b_2 < \lambda \leq 1 - \rho/b_1$, the AdX will get revenue $\lambda\rho/(1-\lambda)$ when $x \in [a_1, \rho)$, and $\lambda x/(1-\lambda)$ when $x \in [\rho, a_2]$. Thus, when $x \in [a_1, a_2]$, the expected revenue of the AdX can be computed by

$$E[r_1(\lambda)] = \frac{\lambda\rho}{1-\lambda} \int_{a_1}^{\rho} f(x)dx + \frac{\lambda}{1-\lambda} \int_{\rho}^{b_1(1-\lambda)} x f(x)dx. \quad (10)$$

2) In case of $\rho/(1-\lambda) \leq b_2$, i.e., $\lambda \leq 1 - \rho/b_2$, when $x \in [a_1, \rho)$, the AdX will get revenue $b_2\lambda$. When $x/(1-\lambda) \leq b_2$, i.e., $x \in [\rho, b_2(1-\lambda))$, the AdX will get revenue $b_2\lambda$. When $b_2 < x/(1-\lambda) \leq b_1$, i.e., $x \in (b_2(1-\lambda), b_1(1-\lambda)]$, the revenue of the AdX is $\lambda x/(1-\lambda)$. Thus, when $x \in [a_1, a_2]$, the expected revenue of the AdX can be computed by

$$E[r_2(\lambda)] = b_2\lambda \int_{a_1}^{b_2(1-\lambda)} f(x)dx + \frac{\lambda}{1-\lambda} \int_{b_2(1-\lambda)}^{b_1(1-\lambda)} x f(x)dx. \quad (11)$$

According to the above discussions, if we define

$$I_1(\lambda) = \begin{cases} 1, & \text{if } 1 - \frac{\rho}{b_2} < \lambda \leq 1 - \frac{\rho}{b_1} \\ 0, & \text{other} \end{cases} \quad (12)$$

and

$$I_2(\lambda) = \begin{cases} 1, & \text{if } 0 < \lambda \leq 1 - \frac{\rho}{b_2} \\ 0, & \text{other,} \end{cases} \quad (13)$$

then the expected value of the AdX from the ad impression can be computed by

$$E[r(\lambda)] = I_1(\lambda)E[r_1(\lambda)] + I_2(\lambda)E[r_2(\lambda)]. \quad (14)$$

Since the winning probability of the AdX is $p(\lambda)$, and the AdX will have the risk of losing the dominant position in the auction if its winning probability is low, we can define the risk of the AdX as $1 - p(\lambda)$. If the risk level of the AdX is $\alpha \in [0, 1]$, then the risk constraint becomes

$$1 - p(\lambda) \leq \alpha. \quad (15)$$

Therefore, we can formulate the following stochastic optimization model for the AdX under risk constraints

$$\begin{aligned} \max_{\lambda \in (0,1)} V(\lambda) &= I_1(\lambda)E[r_1] + I_2(\lambda)E[r_2] \\ \text{s.t. } 1 - p(\lambda) &\leq \alpha. \end{aligned} \quad (16)$$

D. The Equivalent Forms

In this section, we study the equivalent form of model (16) in the cases that d is characterized by a uniform random variable or a normal random variable.

Theorem 1. Suppose d is a uniform random variable distributed in $[a_1, a_2]$, where $a_1 < \rho < b_2 < b_1 < a_2$. Then model (16) becomes

$$\begin{aligned} \max_{\lambda \in [0,1]} I_1(\lambda)E[r_1(\lambda)] + I_2(\lambda)E[r_2(\lambda)] \\ \text{s.t. } \lambda \leq 1 - \frac{(1-\alpha)a_2 + \alpha a_1}{b_1}, \end{aligned} \quad (17)$$

where

$$E[r_1(\lambda)] = \frac{\lambda\rho(\rho - a_1)}{2(1-\lambda)(a_2 - a_1)} + \frac{b_1^2\lambda(1-\lambda)}{2(a_2 - a_1)}, \quad (18)$$

$$E[r_2(\lambda)] = \frac{(b_1^2 + b_2^2)\lambda(1-\lambda) - 2a_1b_2\lambda}{2(a_2 - a_1)}. \quad (19)$$

Proof. Since d is a uniform random variable in $[a_1, a_2]$, the probability distribution function of d is

$$f(x) = \frac{1}{a_2 - a_1}, x \in [a_1, a_2]. \quad (20)$$

Thus, according to (10) and (11), the revenue of the AdX from the ad impression is

$$\begin{aligned} E[r_1(\lambda)] &= \frac{\lambda\rho}{1-\lambda} \int_{a_1}^{\rho} \frac{1}{a_2 - a_1} dx + \frac{\lambda}{1-\lambda} \int_{\rho}^{b_1(1-\lambda)} \frac{x}{a_2 - a_1} dx \\ &= \frac{\lambda\rho(\rho - a_1)}{2(1-\lambda)(a_2 - a_1)} + \frac{b_1^2\lambda(1-\lambda)}{2(a_2 - a_1)} \end{aligned} \quad (21)$$

when $1 - \rho/b_2 < \lambda \leq 1 - \rho/b_1$, and

$$\begin{aligned} E[r_2(\lambda)] &= b_2\lambda \int_{\mu-3\sigma}^{b_2(1-\lambda)} \frac{1}{a_2 - a_1} dx + \frac{\lambda}{1-\lambda} \int_{b_2(1-\lambda)}^{b_1(1-\lambda)} \frac{x}{a_2 - a_1} dx \\ &= \frac{(b_1^2 + b_2^2)\lambda(1-\lambda) - 2a_1b_2\lambda}{2(a_2 - a_1)} \end{aligned} \quad (22)$$

when $\lambda \leq 1 - \rho/b_2$.

According to (9), the winning probability of the AdX from the ad impression is

$$p(\lambda) = \int_{a_1}^{b_1(1-\lambda)} \frac{1}{a_2 - a_1} dx = \frac{b_1(1-\lambda) - a_1}{a_2 - a_1}, \quad (23)$$

and the risk constraint becomes

$$1 - \frac{b_1(1-\lambda) - a_1}{a_2 - a_1} \leq \alpha, \quad (24)$$

thus, we have

$$\lambda \leq 1 - \frac{(1-\alpha)a_2 + \alpha a_1}{b_1}. \quad (25)$$

□

In the following, we discuss the equivalent form of model (16) when d is a normal random variable satisfying $d \sim \mathcal{N}(\mu, \sigma^2)$. For similarity, we can take the distribution interval of d as $[\mu - 3\sigma, \mu + 3\sigma]$, since its confidence level is 0.9974 according to the properties of normal distribution.

Theorem 2. Suppose d is a normal random variable in $[\mu - 3\sigma, \mu + 3\sigma]$ satisfying $d \sim \mathcal{N}(\mu, \sigma^2)$, where $\mu - 3\sigma < \rho < b_2 < b_1 < \mu + 3\sigma$. Then, model (16) becomes

$$\begin{aligned} \max_{\lambda \in [0,1]} I_1(\lambda)E[r_1(\lambda)] + I_2(\lambda)E[r_2(\lambda)] \\ \text{s.t. } \Phi\left(\frac{b_1(1-\lambda) - \mu}{\sigma}\right) \geq 1 - \alpha + \Phi(-3). \end{aligned} \quad (26)$$

where

$$\begin{aligned} E[r_1(\lambda)] &= \frac{\lambda}{1-\lambda} ((\rho - \mu)\Phi\left(\frac{\rho - \mu}{\sigma}\right) + \mu\Phi\left(\frac{b_1(1-\lambda) - \mu}{\sigma}\right) - \rho\Phi(-3)) \\ &\quad + \frac{\lambda\sigma}{\sqrt{2\pi}(1-\lambda)} (\exp(-\frac{(b_1(1-\lambda) - \mu)^2}{2\sigma^2}) - \exp(-\frac{(\rho - \mu)^2}{2\sigma^2})), \end{aligned} \quad (27)$$

$$\begin{aligned} E[r_2(\lambda)] &= \frac{\lambda}{1-\lambda} ((b_2(1-\lambda) - \mu)\Phi\left(\frac{b_2(1-\lambda) - \mu}{\sigma}\right) \\ &\quad + \mu\Phi\left(\frac{b_1(1-\lambda) - \mu}{\sigma}\right) - b_2(1-\lambda)\Phi(-3)) \\ &\quad + \frac{\lambda\sigma}{\sqrt{2\pi}(1-\lambda)} (\exp(-\frac{(b_1(1-\lambda) - \mu)^2}{2\sigma^2}) - \exp(-\frac{(b_2(1-\lambda) - \mu)^2}{2\sigma^2})). \end{aligned} \quad (28)$$

Proof. Since d is a normal random variable satisfying $d \sim \mathcal{N}(\mu, \sigma^2)$, the probability distribution function of d is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(x-\mu)^2}{2\sigma^2}). \quad (29)$$

According to (10) and (11), the revenue of the AdX from the ad impression is

$$\begin{aligned} E[r_1(\lambda)] &= \frac{\lambda\rho}{1-\lambda} \int_{\mu-3\sigma}^{\rho} \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(x-\mu)^2}{2\sigma^2}) dx \\ &\quad + \frac{\lambda}{1-\lambda} \int_{\rho}^{b_1(1-\lambda)} x \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(x-\mu)^2}{2\sigma^2}) dx \\ &= \frac{\lambda}{1-\lambda} ((\rho - \mu)\Phi\left(\frac{\rho - \mu}{\sigma}\right) + \mu\Phi\left(\frac{b_1(1-\lambda) - \mu}{\sigma}\right) - \rho\Phi(-3)) \\ &\quad + \frac{\lambda\sigma}{\sqrt{2\pi}(1-\lambda)} (\exp(-\frac{(b_1(1-\lambda) - \mu)^2}{2\sigma^2}) - \exp(-\frac{(\rho - \mu)^2}{2\sigma^2})) \end{aligned} \quad (30)$$

when $1 - \rho/b_2 < \lambda \leq 1 - \rho/b_1$, and

$$\begin{aligned} E[r_2(\lambda)] &= b_2\lambda \int_{\mu-3\sigma}^{b_2(1-\lambda)} \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(x-\mu)^2}{2\sigma^2}) dx \\ &\quad + \frac{\lambda}{1-\lambda} \int_{b_2(1-\lambda)}^{b_1(1-\lambda)} x \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(x-\mu)^2}{2\sigma^2}) dx \\ &= \frac{\lambda}{1-\lambda} ((b_2(1-\lambda) - \mu)\Phi\left(\frac{b_2(1-\lambda) - \mu}{\sigma}\right) \\ &\quad + \mu\Phi\left(\frac{b_1(1-\lambda) - \mu}{\sigma}\right) - b_2(1-\lambda)\Phi(-3)) \\ &\quad + \frac{\lambda\sigma}{\sqrt{2\pi}(1-\lambda)} (\exp(-\frac{(b_1(1-\lambda) - \mu)^2}{2\sigma^2}) - \exp(-\frac{(b_2(1-\lambda) - \mu)^2}{2\sigma^2})) \end{aligned} \quad (31)$$

when $\lambda \leq 1 - \rho/b_2$.

According to (9), the winning probability of the AdX from the ad impression is

$$\begin{aligned} p(\lambda) &= \int_{\mu-3\sigma}^{b_1(1-\lambda)} \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(x-\mu)^2}{2\sigma^2}) dx \\ &= \Phi\left(\frac{b_1(1-\lambda) - \mu}{\sigma}\right) - \Phi(-3), \end{aligned} \quad (32)$$

and the risk constraint becomes

$$1 - \left(\Phi\left(\frac{b_1(1-\lambda) - \mu}{\sigma}\right) - \Phi(-3)\right) \leq \alpha, \quad (33)$$

thus, we have

$$\Phi\left(\frac{b_1(1-\lambda) - \mu}{\sigma}\right) \geq 1 - \alpha + \Phi(-3). \quad (34)$$

□

When analyzing model (17) and (26), it is rather difficult to derive their accurate numerical solutions, due to the complexity of the objective function of model (17) and the existence of the standard normal distribution function $\Phi(\cdot)$ in both the objective function and constraints of model (26). Thus, in the following section, we will resort to the computational experiment approach to validate our proposed models [6].

III. COMPUTATIONAL EXPERIMENTS

In this section, we design computational experiment scenarios to validate our proposed model, due to lacking high-quality data in the newly emerging header bidding markets.

A. Computational Experiment Scenario

We consider a randomly generated experiment scenario with 1 publisher and 1 AdX in the market, and the values for the parameters are randomly generated according to the assumption $a_1 < \rho < b_2 < b_1 < a_2$ in our proposed model: the highest and second highest bids on the AdX are randomly

generated in $[5.00, 10.00]$, the reserve price of the publisher is fixed at $\rho = 4$, and the highest bid from the partner is distributed in $[2.00, 11.00]$.

For the sake of comparison, we consider the following three cases in our experiments:

- Baseline: In order to evaluate the effect of header bidding on the revenue of the AdX, we utilize a typical RTB scenario without header bidding as a baseline.
- Case-Uniform: The highest bid from the bidding partner is uniformly distributed in $[2.00, 11.00]$, i.e., $d \sim \mathcal{U}[2.00, 11.00]$.
- Case-Normal: The highest bid from the bidding partner is normally distributed in $[2.00, 11.00]$ with $\mu = 6.5$ and $\sigma = 1.5$, i.e., $d \sim \mathcal{N}(6.5, 1.5^2)$.

B. Experimental Results and Analysis

In order to evaluate the effect of the bidding partners on the revenue of the AdX, we construct a computational experiment with randomly generated 100 ad impressions, and conduct 1000 independent computational experiments, aiming to draw general conclusions.

We first consider the case without risk constraints. The revenues and the winning probability of the AdX under the three cases are given in Fig. 2–Fig. 3, respectively, from which we can draw the following conclusions:

- (1) For each of the three cases, there exists a threshold. When λ is smaller than this threshold, the revenue of the AdX can be improved by increasing λ . When λ is larger than this threshold, the revenue of the AdX will have a tendency of decrease, though the winning probabilities of the AdX are keeping decreased with the increasing of λ for all three cases. One possible reason may be that before the threshold, the positive effect of increasing λ on the average revenue is larger than its negative effect on the winning probability. The results illustrate that charging a larger proportion might not always make the AdX better off, no matter there are bidding partners or not.
- (2) The thresholds in the three cases are quite different, and the one in Baseline is larger than others in Case-Uniform and Case-Normal, which illustrates that the existence of bidding partners can greatly affect the optimal proportion of the AdX. Moreover, the threshold in Case-Uniform is larger than that in Case-Normal. One possible reason may be that the positive effect of increasing λ on the average revenue can be balanced out more quickly by its negative effect on the winning probability in Case-Normal than that in Case-Uniform, since the winning probability of the AdX in Case-Normal decreases faster than that in Case-Uniform. The results also illustrate that the distribution of the highest bid of the bidding partners can greatly affect the optimal proportion of the AdX.
- (3) For all possible λ s, both the revenues and the winning probabilities of the AdX are higher in Baseline than those in Case-Uniform and Case-Normal, which illustrates that the existence of the bidding partners can greatly decrease

the winning probability and thus the revenues of the AdX, possibly due to the increasing competition for the AdX.

- (4) Both the revenues and the winning probabilities of the AdX in Case-Uniform are lower than those in Case-Normal with a smaller λ , while higher than that in Case-Normal when λ is larger. This may be caused by the different distribution characteristics of the two random distributions.

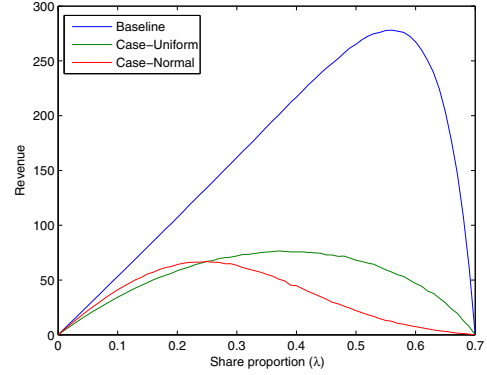


Fig. 2. The revenues of the AdX in the three cases under different λ

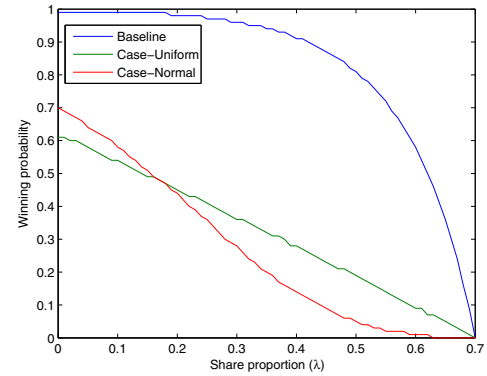


Fig. 3. The winning probability of the AdX in the three cases under different λ

Moreover, when considering risk constraints, the optimal proportions (λ^*) and the corresponding optimal revenue of the AdX under different risk levels in the three cases are given in Fig. 4–Fig. 5, respectively, from which we can draw the following conclusions:

- (1) For each case, there exists a threshold. Both the optimal proportion and optimal revenue of the AdX increase with the risk level before the threshold, while become stabilized when the risk level is higher than the threshold, which illustrates that when the risk tolerance of the AdX is weak (e.g., less than the threshold), it has great effects on both the optimal proportion and the optimal revenue of the AdX. When the risk tolerance is strong enough (e.g., higher than the threshold), the risk level will not influence the optimal proportion and the optimal revenue of the AdX any more.
- (2) When the risk tolerance of the AdX is small, there exists no optimal proportion and optimal revenue in Case-Uniform and Case-Normal, but the optimal proportion

and optimal revenue exist in Baseline for any risk tolerance, which illustrates that the existence of the bidding partners and the resulting competition can force the AdX to increase its risk tolerance.

- (3) For all possible risk levels, both the optimal proportion and optimal revenue of the AdX are higher in Baseline than that in Case-Uniform and Case-Normal. One possible reason may be that the existence of the bidding partners and the resulting competition limit the optimal proportion and optimal revenue of the AdX.
- (4) Both the optimal proportion and the optimal revenue of the AdX in Case-Uniform are lower than those in Case-Normal under a lower λ , while higher than that in Case-Normal under a higher λ , which may be caused by the different distribution characteristics of the two random distributions.

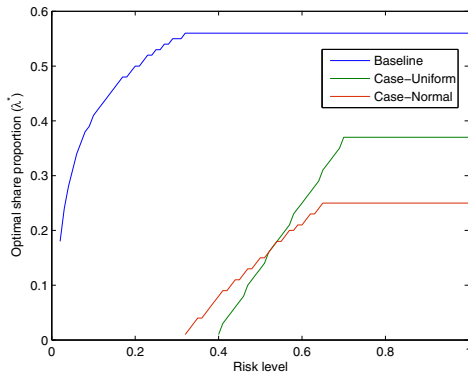


Fig. 4. The optimal λ of the AdX under different risk levels in the three cases

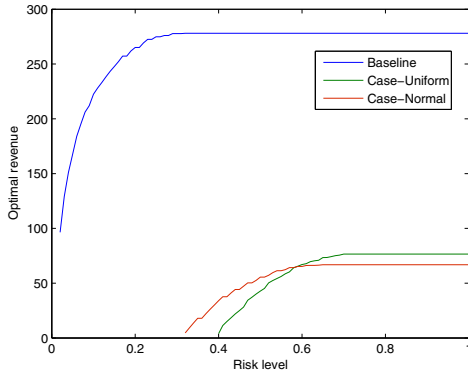


Fig. 5. The optimal revenue of the AdX under different risk levels in the three cases

IV. MANAGERIAL INSIGHTS

Our research findings can offer useful managerial insights for AdX's decision making in the novel header bidding markets. On one hand, the risk tolerance can greatly affect the optimal decision and thus the optimal revenue of the AdX. Due to the competition from the bidding partners, if the AdX has a low risk tolerance, it can hardly win in header bidding markets. Thus, the AdX should increase its risk tolerance in pursuit of more revenue.

On the other hand, the random distributions of the highest bids from the bidding partners can greatly affect the optimal

decision of the AdX. When facing two cases with different random distributions, the AdX with a lower risk tolerance can set a higher proportion in case of normal distribution than that of uniform distribution, while the AdX with a higher risk tolerance can set a higher proportion in case of uniform distribution than that of normal distribution.

V. CONCLUSIONS AND FUTURE WORKS

In header bidding advertising, how to charge an optimal proportion of the advertising cost (i.e., the intermediary fee) is an important issue in AdXs' revenue model. In this paper, we established a stochastic optimization model with risk constraints to seek for the optimal proportion of the AdX under given risk levels. We also studied the equivalent forms of the proposed model when the randomness can be characterized by uniform or normal random variable. With the computational experiment approach, we evaluated our proposed model, and our research findings show that the AdX should better consider both its risk level and the random distribution from the bidding partners when optimizing its proportion decisions.

To our knowledge, this paper represents the first attempt to study the decision making problems of the AdX in header bidding markets. In our future work, we are planning to extend this paper from the following aspects: (a) Studying the games played by the AdX and the publishers, and analyzing the resulting equilibrium; (b) Exploring the dynamic adjusting strategies of the AdX with the parallel dynamic programming approach [7].

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