Revenue Models for Demand Side Platforms in Real Time Bidding Advertising

Rui Qin, Xiaochun Ni, Yong Yuan*, Juanjuan Li
The State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China
Qingdao Academy of Intelligent Industries, Qingdao, China
Beijing Engineering Research Center of Intelligent Systems and Technology, Beijing, China
Email: {rui.qin, xiaochun.ni}@ia.ac.cn, yong.yuan{Corresponding author}, juanjuan.li}@ia.ac.cn

Fei-Yue Wang, Fellow, IEEE
The State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China
Qingdao Academy of Intelligent Industries, Qingdao, China
Research Center of Military Computational Experiments and Parallel System, National University of Defense Technology, Changsha, China
Email: feiyue.wang@ia.ac.cn

Abstract—Real time bidding (RTB) has become an emerging online advertising with the development of Internet big data in recent years. In the whole RTB ecosystem, the Demand Side Platform (DSP) plays a central role, and it realizes the programmatic and accurate buying of the advertisements for the advertisers via a two-stage auction. In RTB business logics, DSP plays as an intermediary between the advertisers and the center platform. Due to the principle-agent relationship between the advertisers and the DSP, the DSP aims not only to maximize the revenue for the advertisers, but also gain its revenue in this process. So far there are two revenue modes for DSP, namely the two-stage resale model and the commission model, respectively. In this paper, we mainly consider the revenue model for DSP in RTB advertising market. We aim to study the properties of the two revenue models, and compare the revenues for the DSP and the advertisers under these two models. We also provide an example to illustrate our proposed models and their properties. The results show that under small ratio of the commission, the advertisers are more likely to choose the commission model, but the DSP is more likely to choose the two-stage resale model, and set a larger weight, while under large ratio of the commission, the advertisers are more likely to choose the two-stage resale model, but the DSP is more likely to choose the commission model. Our research work highlights the importance of the revenue model on the revenues of the advertisers and the DSP, and is intended to provide a useful reference for DSPs in RTB advertising markets.

Keywords: real time bidding, demand side platform, revenue model, two-stage resale, strategy optimization

I. INTRODUCTION

With the development and popularization of Internet big data and programmatic buying, Real time bidding (RTB) has become one of the most popular online advertising forms in recent years [1, 3, 8, 9].

In general, RTB advertising is a complex auction process since there are multiple participants in the whole RTB ecosystem. In practice, the Demand Side Platform (DSP) plays as an intermediary between the advertisers and the AdExchange platform (AdX) [2, 5], and helps the advertisers gain their revenues via a two-stage auction process [7]. The first stage auction is among all the bidding advertisers on the DSP, and the advertiser with the highest bid wins in the first stage auction. The second stage auction is run by the AdX, and it aims to decide the final winning advertiser for the ad impression.

Due to the principle-agent relationship between the advertisers and the DSP, the purpose of the DSP is not only to maximize the revenue for the advertisers, but also gain its revenue from the auction process. Based on the two-stage auction, there are two revenue models for the DSP to gain its revenue. The first one is called the two-stage resale model [4], in which the DSP participates in the second stage auction according to the bids of its advertisers, and resells the ad impression to the winning advertiser on it if it wins in the second stage auction. In this model, the DSP gets its revenue from the price difference between the fee charged from its winning advertiser and its payment to the AdX. The second one is called the commission model, in which the winning advertiser in the first stage participates in the second stage auction, and pays some commission to the DSP.

In RTB industry, the two revenue models have been widely adopted by the DSP companies. However, which revenue model is better for the advertisers and the DSP? What are the disadvantages and the advantages of each revenue model? As far as we know, there are few relevant discussions in the literature.

This paper aims to study the two revenue models, and establish relevant optimization models for them to discuss the properties of the two revenue models. Besides, we also compare the revenues for the advertisers and the DSP under the two revenue models in some special cases. To better illustrate the two revenue models, we provide an example, and the results show that the two revenue models can bring different revenues for the advertisers and the DSP, and in different cases, the advertisers and the DSP may have different preferences to the two revenue models.

The remainder of this paper is organized as follows. In Section II, we first introduce the two revenue models, and then establish relevant optimization models for them. In Section III, we study the properties of the two revenue models in some
special case. In Section IV, we provide an example to illustrate our proposed models and properties. Section V concludes this paper.

II. REVENUE MODEL OF DSP IN RTB

A. Problem Statement

In RTB advertising, there exists a two-stage auction process, and the first stage auction is on the DSP, while the second stage auction is on the AdX. As an intermediary, the DSP platform has two kinds of ways to gain its revenue. The first way is according to a two-stage resale model (see Figure 1), in which the DSP participates in the second stage auction, and resells the ad impression to its winning advertiser if it wins [4]. In this way, the profit of the DSP is the difference between the payment of its winning advertiser to its payment to the AdX. The second way is to let the winning advertiser on it participate in the second stage auction, and get some commissions from the advertiser it he/she wins in the second stage auction (see Figure 2).

B. Two-stage Resale Model

We consider the case with only one ad impression, and there are multiple DSPs.

In the first stage, suppose the highest and the second highest bids on the DSP are $b_1$ and $b_2$, respectively, and the DSP adopts the second price mechanism, i.e., the advertiser with the highest bid $b_1$ wins, and he/she only needs to pay the second highest bid $b_2$ [2]. Since submitting their true valuations is a weakly dominant strategy for advertisers in second price mechanism [6], we can assume that the value of the impression to each advertiser is equal to his/her bid.

In the second stage, the DSP bids $d_1$ on the AdX to participate in the auction for the ad impression. Suppose the highest bid of the other DSPs is $d_2$, then if $d_1 > d_2$, the DSP will win the ad impression, and it needs to pay $d_2$ to the AdX according to the second price mechanism of the AdX.

Thus, if $d_1 > d_2$, the revenue of the winning advertiser is $b_1 - b_2$, and the revenue of the DSP is $b_2 - d_2$.

Since the DSP cannot obtain the precise highest bid of other DSPs, but some information of its distributions may be estimated from the historical bidding data, we can assume $d_2$ is a random variable distributed in $[a_1, a_2]$ with distribution function $f(x)$, where $a_1 < b_2 < b_1 < a_2$. Thus, the expected revenues of the advertiser and the DSP can be computed by

$$V_1 = \int_{a_1}^{d_1} (b_1 - b_2) f(x) dx$$

and

$$V_2 = \int_{a_1}^{d_1} (b_2 - x) f(x) dx,$$

respectively.

In the two-stage resale processes, the DSP not only needs to consider its own revenue, but also needs to consider the revenue of its advertisers in order to keep enough advertisers, thus, the aim of the DSP is to find an optimal $d_1$, which can get a balance between the DSP’s own revenue and the advertisers’ revenue.

If we induce a weight variable $\lambda \in [0, 1]$ to represent the degree of the DSP to consider its own revenue, then $1 - \lambda$ can be defined as the degree that the DSP considers winning advertiser’s revenue.

Thus, the optimization model of the DSP under the two-stage resale model can be formulated as

$$\max_{d_1} \quad V = (1 - \lambda) V_1 + \lambda V_2$$

s.t.  \hspace{1cm} V_1 \geq 0 \hspace{1cm} \text{and} \hspace{1cm} V_2 \geq 0.$$

where $V_1 \geq 0$ and $V_2 \geq 0$ represent that both the DSP and the winning advertiser should have a nonnegative revenue.

C. Commission Model

In the commission model, suppose the ratio of the commission charged by the DSP from its winning advertiser is $\alpha \in [0, 1]$. If the winning advertiser wins in the second stage auction, he/she should pay $d_2$ to the AdX. Thus, the revenue of the winning advertiser is $(b_1 - d_2)(1 - \alpha)$, and the revenue of the DSP is $(b_1 - d_2)\alpha$. 

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Fig. 1. The process of the two-stage resale model

Fig. 2. The process of the commission model
When \( d_2 \) is a random variable distributed in \([a_1, a_2]\) with distribution function \( f(x) \), the expected revenues of the advertiser and the DSP can be computed by
\[
V_1' = \int_{a_1}^{b_1} (b_1 - x)(1 - \alpha)f(x)dx
\]  
(4)
and
\[
V_2' = \int_{a_1}^{b_1} (b_1 - x)\alpha f(x)dx.
\]  
(5)
respectively.

### III. Properties of the Two Revenue Models

In this section, we study the properties of the two revenue models, and compare the revenues for the advertisers and the DSP under the two models, considering the highest bid of other DSPs can be characterized by uniform random variable.

#### A. Two-stage Resale Model

When the highest bid of the other DSPs is characterized by a uniform random variable, the revenues of the advertiser and the DSP can be given in the following theorem.

**Theorem 1.** When \( d_2 \) is a uniform random variable in \([a_1, a_2]\), for any given \( \lambda \in [0, 1] \), the optimal solution and optimal value of the model (3) are
\[
d_1^* = \begin{cases} 
2b_2 - b_1 + \frac{b_1-b_2}{\lambda}, & \text{if } \lambda \in [\frac{b_1-b_2}{b_1-a_1}, 1] \\
2b_2 - a_1, & \text{other}
\end{cases}
\]  
(6)
and
\[
V = \begin{cases} 
\frac{(\lambda(2b_2-a_1-b_1)+b_1-b_2)(\lambda^2(b_1-a_1)+(2-3\lambda)(b_1-b_2))}{2\lambda(a_2-a_1)}, & \text{if } \lambda \in [\frac{b_1-b_2}{b_1-a_1}, 1] \\
\frac{2(b_1-b_2)(b_2-a_1)}{a_2-a_1}, & \text{other}
\end{cases}
\]  
(7)
respectively, and the corresponding revenues of the winning advertiser and the DSP are
\[
V_1 = \begin{cases} 
\frac{(b_1-b_2)(\lambda(2b_2-a_1-b_1)+b_1-b_2)}{\lambda(a_2-a_1)}, & \text{if } \lambda \in [\frac{b_1-b_2}{b_1-a_1}, 1] \\
\frac{2(b_1-b_2)(b_2-a_1)}{a_2-a_1}, & \text{if } \lambda \in [0, \frac{b_1-b_2}{b_1-a_1}]
\end{cases}
\]  
(8)
and
\[
V_2 = \begin{cases} 
\frac{(\lambda(2b_2-a_1-b_1)+b_1-b_2)(\lambda(b_1-a_1)-b_1+b_2)}{2\lambda(a_2-a_1)}, & \text{if } \lambda \in [\frac{b_1-b_2}{b_1-a_1}, 1] \\
0, & \text{if } \lambda \in [0, \frac{b_1-b_2}{b_1-a_1}]
\end{cases}
\]  
(9)
respectively.

**Proof.** When \( d_2 \) is a uniform random variable in \([a_1, a_2]\), i.e., the distribution function of \( d_2 \) is
\[
f(x) = \frac{1}{a_2-a_1}, x \in [a_1, a_2].
\]  
(10)
According to (1) and (2), the revenues of the winning advertiser and the DSP in the two-stage resale model are
\[
V_1 = \int_{a_1}^{b_1} (b_1 - b_2)\frac{1}{a_2-a_1}dx
\]  
(11)
and
\[
V_2 = \int_{a_1}^{b_1} (b_2 - x)\frac{1}{a_2-a_1}dx
\]  
(12)
\[
= \frac{(d_1-a_1)(2b_2-(d_1+a_1))}{a_2-a_1},
\]
respectively.

From the constraints of the model (3), we have
\[
\begin{cases}
(b_1-b_2)(d_1-a_1) \\ a_2-a_1 \geq 0,
(d_1-a_1)(2b_2-(d_1+a_1)) \\ a_2-a_1 \geq 0,
\end{cases}
\]  
(13)
which concludes that
\[
a_1 \leq d_1 \leq 2b_2 - a_1.
\]  
(14)
Thus, model (3) becomes
\[
\begin{cases}
\max \quad V = (1-\lambda)\frac{(b_1-b_2)(d_1-a_1)}{a_2-a_1} \\
+ \lambda\frac{(d_1-a_1)(2b_2-(d_1+a_1))}{a_2-a_1}
\end{cases}
\]  
(15)
s.t. \( a_1 \leq d_1 \leq 2b_2 - a_1 \)

In the following, we seek for the optimal solution of model (15). Differentiate \( V \) with \( d_1 \), we have
\[
\frac{\partial V}{\partial d_1} = \frac{(1-\lambda)b_1-(1-2\lambda)b_2-d_1}{a_2-a_1}
\]  
(16)
and
\[
\frac{\partial^2 V}{\partial d_1^2} = -\frac{\lambda}{a_2-a_1} < 0,
\]  
(17)
from which we can obtain that \( V \) is a convex function.

Let \( \frac{\partial V}{\partial d_1} = 0 \), then we have
\[
d_1^* = 2b_2 - b_1 + \frac{b_1-b_2}{\lambda}.
\]  
(18)
Obviously, we have \( d_1^* > b_2 \).

1) If \( d_1^* \leq 2b_2 - a_1 \), i.e., \( \lambda \in [\frac{b_1-b_2}{b_1-a_1}, 1] \), then \( d_1^* = d_1 \), and the optimal value of model (3) is
\[
V_{\text{max}} = \frac{(\lambda(2b_2-a_1-b_1)+b_1-b_2)(\lambda^2(b_1-a_1)+(2-3\lambda)(b_1-b_2))}{2\lambda(a_2-a_1)}
\]  
(19)
The corresponding \( V_1 \) and \( V_2 \) are
\[
V_1 = \frac{(b_1-b_2)(\lambda(2b_2-a_1-b_1)+b_1-b_2)}{\lambda(a_2-a_1)}
\]  
(20)
and
\[
V_2 = \frac{(\lambda(2b_2-a_1-b_1)+b_1-b_2)(\lambda(b_1-a_1)-b_1+b_2)}{2\lambda(a_2-a_1)}
\]  
(21)
respectively.

2) If \( d_1^* > 2b_2 - a_1 \), i.e., \( \lambda \in [0, \frac{b_1-b_2}{b_1-a_1}] \), then \( d_1^* = 2b_2 - a_1 \), and the optimal value of model (3) is
\[
V_{\text{max}} = \frac{2(b_1-b_2)(b_2-a_1)}{a_2-a_1}.
\]  
(22)
The corresponding \( V_1 \) and \( V_2 \) are
\[
V_1 = \frac{2(b_1-b_2)(b_2-a_1)}{a_2-a_1}
\]  
(23)
and
\[
V_2 = 0,
\]  
(24)
respectively.

□
B. Commission Model

When the highest bid of other DSPs is characterized by a uniform random variable, the revenues of the advertiser and the DSP in the commission model are given in the following theorem.

**Theorem 2.** When \( d_2 \) is a uniform random variable in \([a_1, a_2]\), for any given \( \alpha \in [0, 1] \), the revenues of the winning advertiser and the DSP are

\[
V_1' = \frac{(b_1 - a_1)^2(1 - \alpha)}{2(a_2 - a_1)} \tag{25}
\]

and

\[
V_2' = \frac{\alpha(b_1 - a_1)^2}{2(a_2 - a_1)}, \tag{26}
\]

respectively.

**Proof.** When \( d_2 \) is a uniform random variable in \([a_1, a_2]\) with distribution function

\[
f(x) = \frac{1}{a_2 - a_1}, x \in [a_1, a_2],
\]

according to (4) and (5), the revenues of the winning advertiser and the DSP in the commission model are

\[
V_1' = \int_{a_1}^{b_1} f(x)(b_1 - x)(1 - \alpha) \frac{1}{a_2 - a_1} \, dx
\[
= \frac{(b_1 - a_1)^2(1 - \alpha)}{2(a_2 - a_1)} \tag{28}
\]

and

\[
V_2' = \int_{a_1}^{b_1} \frac{\alpha(b_1 - x)}{a_2 - a_1} \, dx
\[
= \frac{\alpha(b_1 - a_1)^2}{2(a_2 - a_1)}, \tag{29}
\]

respectively.

C. Comparisons of the Revenues in the Two Revenue Models

Based on the above analysis, we will compare the revenues of the advertisers and the DSP under the two revenue models.

1) Comparing the Revenues of the Advertisers

According to (8) and (25), the revenues of the advertisers under the two revenue models are shown in Figure 3 and Figure 4, respectively.

![Figure 3](image-url)  
**Fig. 3.** The revenue of the advertiser in the two-stage resale model

From Figure 3 and Figure 4, it is obvious that

\[
V_{1,\text{min}} = \frac{(b_1 - b_2)(b_2 - a_1)}{a_2 - a_1}, V_{1,\text{max}} = \frac{2(b_1 - b_2)(b_2 - a_1)}{a_2 - a_1},
\]

and

\[
V_1' = 0, V_1' = \frac{(b_1 - a_1)^2}{2(a_2 - a_1)}. \tag{31}
\]

Since

\[
V_{1,\text{max}} \leq \frac{2((b_1 - b_2 + b_2 - a_1)^2}{a_2 - a_1} = V_1', \tag{32}
\]

and when \( b_1 - b_2 = b_2 - a_1 \), i.e., \( 2b_2 = b_1 + a_1 \), we have \( V_{1,\text{max}} = V_1' \). Otherwise, we have \( V_{1,\text{max}} < V_1' \).

a) If \( 2b_2 = b_1 + a_1 \), then for any \( \lambda \in [0, \frac{b_1 - b_2}{b_1 - a_1}] \), we have

\[
V_1 \geq V_1', \forall \alpha \in [0, 1]. \tag{33}
\]

For \( \lambda \in (\frac{b_1 - b_2}{b_1 - a_1}, 1] \), if

\[
\alpha < 1 - \frac{2(b_1 - b_2)(\lambda(2b_2 - a_1 - b_1) + b_1 - b_2)}{(b_1 - a_1)^2}, \tag{34}
\]

then we have \( V_1 < V_1' \), and if

\[
\alpha \geq 1 - \frac{2(b_1 - b_2)(\lambda(2b_2 - a_1 - b_1) + b_1 - b_2)}{(b_1 - a_1)^2}, \tag{35}
\]

we have \( V_1 \geq V_1' \).

b) If \( 2b_2 \neq b_1 + a_1 \), then for any \( \lambda \in [0, \frac{b_1 - b_2}{b_1 - a_1}] \), if

\[
\alpha < 1 - \frac{2(b_1 - b_2)(b_2 - a_1)}{(b_1 - a_1)^2}, \tag{36}
\]

then we have \( V_1 \geq V_1' \), and if

\[
\alpha \geq 1 - \frac{2(b_1 - b_2)(b_2 - a_1)}{(b_1 - a_1)^2}, \tag{37}
\]

we have \( V_1 < V_1' \).

For \( \lambda \in (\frac{b_1 - b_2}{b_1 - a_1}, 1] \), if

\[
\alpha < 1 - \frac{2(b_1 - b_2)(\lambda(2b_2 - a_1 - b_1) + b_1 - b_2)}{(b_1 - a_1)^2}, \tag{38}
\]
then we have $V_1 < V'_1$, and if
\[
\alpha \geq 1 - \frac{2(b_1 - b_2)(\lambda(2b_2 - a_1 - b_1) + b_1 - b_2)}{\lambda(b_1 - a_1)^2}, \quad (39)
\]
we have $V_1 \geq V'_1$.

2) Comparing the Revenues of the DSP

According to (9) and (26), the revenues of the DSP under the two models are shown in Figure 5 and Figure 6, respectively.

From Figure 5 and Figure 6, we can obtain the following results:

- For any $\lambda \in [0, \frac{b_1 - b_2}{b_1 - a_1}]$, we have $V_2 \leq V'_2$, $\forall \alpha \in [0, 1]$.  
  \[
  V_2 \leq V'_2, \forall \alpha \in [0, 1]. \qquad (40)
  \]

- For any $\lambda \in (\frac{b_1 - b_2}{b_1 - a_1}, 1]$, if
  \[
  \alpha \geq \frac{\lambda(2b_2 - a_1 - b_1) + b_1 - b_2)(\lambda(b_1 - a_1) - b_1 + b_2)}{2\lambda(a_2 - a_1)},
  \]
  we have $V_2 \leq V'_2$, and if
  \[
  \alpha < \frac{\lambda(2b_2 - a_1 - b_1) + b_1 - b_2)(\lambda(b_1 - a_1) - b_1 + b_2)}{2\lambda(a_2 - a_1)},
  \]
  we have $V_2 > V'_2$.

IV. AN ILLUSTRATIVE EXAMPLE

In this section, we provide an example to illustrate the comparisons for the revenues of the advertisers and the DSP in the two revenue models.

**Example 1.** Let $b_1 = 10$, $b_2 = 4$, $a_1 = 2$, $a_2 = 12$, then according to (8) and (9), the revenues of the advertisers and the DSP in the two-stage resale model can be computed as

\[
V_1 = \begin{cases} \frac{3\lambda^2}{2} - 2.4, & \text{if } \lambda \in [0.75, 1] \\ 2.4, & \text{if } \lambda \in [0, 0.75) \end{cases}
\]

and

\[
V_2 = \begin{cases} 0.45 - 0.05(\frac{\lambda}{2} - 5)^2, & \text{if } \lambda \in [0.75, 1] \\ 0, & \text{if } \lambda \in [0, 0.75) \end{cases}
\]

respectively. According to (25) and (26), the revenues of the winning advertisers and the DSP in the commission model can be computed as $V'_1 = 3.2 - 3.2\alpha$ and $V'_2 = 3.2\alpha$, respectively.

In the following, we first compare the revenues of the advertisers in the two revenue models.

Since $2b_2 \neq b_1 + a_1$, then for any $\lambda \in [0, 0.75]$, if $\alpha \geq 0.375$, we have $V_1 \geq V'_1$, and if $\alpha > 0.375$, we have $V_1 < V'_1$. For $\lambda \in (0.75, 1]$, if $\alpha < 1.75 - 1.152/\lambda$, then we have $V_1 < V'_1$, and if $\alpha \geq 1.75 - 1.152/\lambda$, we have $V_1 \geq V'_1$. The result is shown in Figure 7.

![Fig. 7. The comparisons of the revenues of the advertiser in the two revenue models under different $\lambda$ and $\alpha$](image)

Next, we compare the revenues of the DSP in the two revenue models.

For any $\lambda \in [0, 0.75]$, we have $V_2 \leq V'_2$ for $\forall \alpha \in [0, 1]$. For any $\lambda \in (0.75, 1]$ if

\[
\alpha \geq \frac{(3 - 2\lambda)(4\lambda - 3)}{5\lambda},
\]

then we have $V_2 \leq V'_2$, and if

\[
\alpha < \frac{(3 - 2\lambda)(4\lambda - 3)}{5\lambda},
\]

then we have $V_2 > V'_2$. The result is shown in Figure 8.

From Figure 7 and Figure 8, we can obtain the following results:
characterized by a uniform random variable, we compared the revenues for the advertisers and the DSP under the two revenue models. We also provide an example to illustrate our proposed models and properties, and the results show that different values of the parameters can result in different preferences of the advertisers and the DSP to the two revenue models.

This work is a preliminary study on the comparison of different revenue models in RTB advertising. In our future work, we intend to extend the paper from the following aspects: a) Comparing the two revenue models in more general cases; b) Explore more appropriate revenue models in RTB advertising markets.

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