Exploring New Mechanisms for Demand-Side Platforms in Real Time Bidding Markets*

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***Abstract*—When online advertising met the big data technology, programmatic buying has become more and more popular, in which Real Time Bidding (RTB) is regarded as one of the most important formats of programmatic buying advertising. In RTB advertising markets, there is a two-stage auction process for each ad impression, in which Demand-Side Platforms (DSPs) adopt a two-stage resale model to get their revenues. Thus, for each DSP, how to design effective auction mechanisms in the two-stage auction process so as to get higher revenues for both itself and its advertisers has become a critical issue. This paper aims to study this issue, and propose a new bidding and pricing mechanism for the DSP. We also utilize the computational experiment approach to evaluate our proposed mechanism, and the experimental results show that our new mechanism can improve the revenues of both the DSP and the advertisers.**

Keywords*—*real time bidding, two-stage resale model, demand side platform, pricing mechanism, bidding mechanism

I. Introduction

With the integration of Internet and advertisement, keyword advertising and display advertising have become two most important advertising formats [1–9]. As one of the most important display advertisement form, Real Time Bidding (RTB) emerged in recent years with the rapid development of Internet and big data technology [10, 11]. Due to its precision targeting ability and higher marketing effect [12, 13], RTB advertising has attracted an increasing number of advertisers and publishers.

In RTB markets, Demand Side Platforms (DSPs) play as intermediaries between the advertisers and the supply side publishers, and each ad request will be sold to the advertiser via a two-stage auction process. Once receiving an ad request from the Ad Exchange (AdX), the DSP will start the first stage auction among its bidding advertisers, and find the winner with the highest bid. The second stage auction is run by the AdX among all the bidding DSPs, and the DSP with the highest bid wins the ad impression triggered by the ad request in the second stage auction, and resells it to the winning advertiser on it to get the intermediate fees [14–16]. In this two-stage resale model, DSPs must make two key decisions [16]. The first one is how much to bid on the AdX (i.e., the bidding mechanism), and the second one is how much to charge its winning advertiser (i.e., the pricing mechanism). Thus, the mechanism designing problem for the two-stage resale model has become an important issue for DSPs, which can greatly affect the revenues of both the advertisers and the DSPs.

In practical RTB markets, DSPs usually adopt the second price mechanism in its first stage local auction, i.e., the advertiser with the highest bid wins in the first stage auction, and he/she needs to pay the second highest bid among all the bidding advertisers on the DSP, if the DSP wins in the second stage auction [16]. As such, DSPs usually participate in the second-stage auction with this second-highest bid, in order to get a nonnegative revenue from the two-stage resale process. Under such mechanisms, when the highest bid of the advertisers are much higher than the second-highest bid, a large number of ad impressions may be missed. Thus, such bidding and pricing mechanisms are obvious non-optimal for both DSPs and advertisers [17].

In this paper, we aim to explore new mechanisms for DSPs in the two-stage resale model of RTB advertising, considering that the revenues of both the winning advertiser and the DSP can be improved comparing with those in the commonly used mechanism. Furthermore, considering the great advantages and successful applications of the computational experiments approach in solving complex socioeconomic systems [18–20], we utilize the computational experiment approach and design some computational experiments to evaluate our proposed mechanisms, and the experimental results show that our new mechanisms can greatly increase the revenues for both the winning advertiser and the DSP.

The rest of this paper is arranged as follows: In Section II, we introduce the process of the two-stage model in RTB advertising, and the commonly used mechanisms by DSPs. In Section III, we propose a new mechanism, and compare it with the commonly used mechanisms. In Section IV, we utilize a computational experiment approach to solve the proposed

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model, and design some computational experiments to evaluate our proposed mechanisms. Section V concludes this paper.

II. The Two-stage Model

A. Two-stage Resale Model

In RTB advertising, there is a two-stage auction process for each ad request, which enables DSPs to get their revenues according to a two-stage resale model. The detailed procedure of the two-stage resale process is shown in Fig. 1, which can be described as follows [15, 16]:

(1) Once an ad request arrives, each DSP will identify the interests and characteristics of the user behind the ad request, and start the first-stage auction asking for bids from all eligible advertisers registered on it.

(2) Denote the highest bid and the second highest bid on the DSP as b_1 and b_2, respectively, and the value of the highest-bid advertiser for the ad impression as v_1. In the second-stage auction, the DSP submits a bid d_1 in the second-stage auction, and the highest bid price of all the other DSPs is d_2. If $d_1 > d_2$, then the DSP will win the ad impression with cost d_2 according to the second price mechanism of the AdX.

(3) If the DSP wins in the second stage auction, it will resell the ad impression back to its winning advertiser. The winning advertiser needs to pay c_1 to the DSP and get revenue $v_1 - c_1$, while the DSP needs to pay d_2 and get revenue $c_1 - d_2$ from the ad impression.

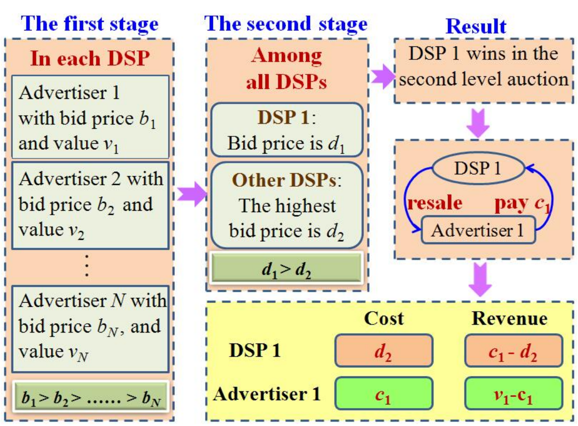

Fig. 1. The two-stage resale process in RTB advertising

For simplicity, we can assume that the reserve price on the AdX is $\rho = 0$. Suppose the advertisers are bidding truthfully, i.e., $b_1 = v_1$, then according to the above two-stage resale process, the revenue of the winning advertiser and the DSP can be computed by

$$r_1(c_1, d_1) = \begin{cases} b_1 - c_1, & \text{if } d_2 < d_1 \\ 0, & \text{other} \end{cases} \tag{1}$$

and

$$r_2 = \begin{cases} c_1 - d_2, & \text{if } d_2 < d_1 \\ 0, & \text{other,} \end{cases} \tag{2}$$

respectively.

B. Mechanisms in Two-stage Resale Model

According to the procedure of the two-stage resale model, how much to charge the winning advertiser (e.g., c_1) and how much to bid in the second stage auction (e.g., d_1) are two crucial issues faced by DSPs, which can greatly affect the revenues of the advertisers and DSPs. In RTB advertising, the most commonly used mechanism is the Pre-award Vickrey action mechanism (PRE) mechanism [14], where the winning advertiser needs to pay the second highest bid b_2 to the DSP if he/she obtains the ad impression. The DSP submits the second highest bid b_2 of the advertisers in the second stage auction in order to get non-negative revenues for each ad impression. Thus, in this mechanism, we have $c_1 = b_2$ and $d_1 = b_2$.

According to (1) and (2), the revenues of the advertiser and the DSP in the PRE mechanism can be computed as

$$r_1^{\text{PRE}} = \begin{cases} b_1 - b_2, & \text{if } d_2 < b_2 \\ 0, & \text{other} \end{cases} \tag{3}$$

and

$$r_2^{\text{PRE}} = \begin{cases} b_2 - d_2, & \text{if } d_2 < b_2 \\ 0, & \text{other,} \end{cases} \tag{4}$$

respectively.

Obviously, in the PRE mechanism, c_1 is decided before the second stage auction, and when $b_2 < d_2 < b_1$, the revenues of both the advertiser and the DSP are 0 in the PRE mechanism.

Considering the drawbacks of the PRE mechanism, Stavrogiannis et al. [17] studied the case that the cost c_1 charged from the advertiser can be determined after the second stage auction, and proposed a Post-award Vickrey auction mechanism (POST), where the DSP submits the highest bid b_1 of the advertisers in the second stage auction, and if the DSP wins with cost d_2, the winning advertiser on it needs to pay the maximum of b_2 and d_2 to the DSP. That is, $c_1 = \max\{b_2, d_2\}$ and $d_1 = b_1$.

In the POST mechanism, according to (1) and (2), the revenues of the advertiser and the DSP are

$$r_1^{\text{POST}} = \begin{cases} b_1 - b_2, & \text{if } d_2 < b_2 \\ b_1 - d_2, & \text{if } b_2 \le d_2 < b_1 \\ 0, & \text{other} \end{cases} \tag{5}$$

and

$$r_2^{\text{POST}} = \begin{cases} b_2 - d_2, & \text{if } d_2 < b_2 \\ 0, & \text{other,} \end{cases} \tag{6}$$

respectively.

Comparing the revenues of the advertiser and the DSP in the PRE mechanism and the POST mechanism, it is obvious that when $d_2 < b_2$, we have

$$r_1^{\text{PRE}} = r_1^{\text{POST}}, r_2^{\text{PRE}} = r_2^{\text{POST}}, \tag{7}$$

and when $b_2 \le d_2 < b_1$, we have

$$r_1^{\text{PRE}} < r_1^{\text{POST}}, r_2^{\text{PRE}} = r_2^{\text{POST}}. \tag{8}$$

Thus, the POST mechanism represents an improved version of the PRE mechanism from the perspective of the advertiser. However, compared with the PRE mechanism, the POST mechanism can not improve the revenue for the DSP, who thus has no incentives to choose the POST mechanism since it is more complicated than the PRE mechanism.

III. New Mechanisms

In this section, we aim to explore new mechanisms, which can improve the revenues of both the advertiser and the DSP comparing with the commonly used PRE mechanism.

According to (1) and (2), the total revenue of the advertiser and the DSP is

$$\begin{aligned} r &= r_1 + r_2 \\ &= \begin{cases} b_1 - d_2, & \text{if } d_2 < d_1 \\ 0, & \text{other,} \end{cases} \end{aligned} \tag{9}$$

which is only determined by the bidding mechanism d_1. Thus, we first study the optimal bidding mechanism, which can be stated in the following theorem.

Theorem 1. *Suppose the highest and the second highest bids on the DSP are b_1 and b_2, respectively. The bid of the DSP in the second stage auction is d_1, and the highest bid of all the other DSPs is d_2. Then the optimal bidding mechanism for the DSP is to bid $d_1 = b_1$ in the second stage auction.*

Proof: If $d_1 < b_1$, then for any $d_1 < d_2 < b_1$, we have $r = 0$, which is smaller than the total revenue $b_1 - d_2$ under the case $d_1 = b_1$.

If $d_1 > b_1$, then for any $b_1 < d_2 < d_1$, we have $r = b_1 - d_2 < 0$, which is smaller than the total revenue 0 under the case $d_1 = b_1$.

Thus, the optimal bidding mechanism for the DSP is to bid $d_1 = b_1$ in the second stage auction. ■

When $d_1 = b_1$, the revenue of the advertiser and the DSP are

$$r_1 = \begin{cases} b_1 - c_1, & \text{if } d_2 < b_1 \\ 0, & \text{other} \end{cases} \tag{10}$$

and

$$r_2 = \begin{cases} c_1 - d_2, & \text{if } d_2 < b_1 \\ 0, & \text{other,} \end{cases} \tag{11}$$

respectively.

In the following, we seek for the optimal pricing mechanism c_1, aiming to improve the revenues of both the advertiser and the DSP. The optimal pricing mechanism is given in the following theorem.

Theorem 2. *Suppose the highest and the second highest bids on the DSP are b_1 and b_2, respectively. The bid of the DSP in the second stage auction is d_1, and the highest bid of all the other DSPs is d_2. Then the optimal pricing mechanism for the DSP is to charge $c_1 = c_1(\omega)$ from its winning advertiser, where*

$$c_1(\omega) = \begin{cases} b_2, & \textit{if } d_2 < b_2 \\ \omega d_2 + (1-\omega) b_1, & \textit{if } b_2 \le d_2 < b_1 \\ 0, & \textit{other,} \end{cases} \tag{12}$$

and $\omega \in [0, 1]$ is a weight factor.

Proof: Let $r_1 \ge r_1^{\text{PRE}}$, then for any $d_2 < b_2$, we have

$$b_1 - c_1 \ge b_1 - b_2, \tag{13}$$

which concludes that $c_1 \le b_2$. For any d_2, $b_2 \le d_2 < b_1$, we have

$$b_1 - c_1 \ge 0, \tag{14}$$

which concludes that $c_1 \le b_1$.

Let $r_2 \ge r_2^{\text{PRE}}$, then for any $d_2 < b_2$, we have

$$c_1 - d_2 \ge b_2 - d_2, \tag{15}$$

which concludes that $c_1 \ge b_2$. For any d_2, $b_2 \ge d_2 < b_1$, we have

$$c_1 - d_2 \ge 0, \tag{16}$$

which concludes that $c_1 \ge d_2$.

According the the above analysis, we have $c_1 = b_2$ when $d_2 < b_2$, and $d_2 \le c_1 \le b_1$ when $b_2 \le d_2 < b_1$.

Let

$$c_1(\omega) = \begin{cases} b_2, & \text{if } d_2 < b_2 \\ \omega d_2 + (1-\omega) b_1, & \text{if } b_2 \le d_2 < b_1 \\ 0, & \text{other,} \end{cases} \tag{17}$$

where $\omega \in [0, 1]$ is a weight factor, then setting $c_1 = c_1(\omega)$ and $d_1 = b_1$ can improve the revenues of both the advertiser and the DSP. ■

According to Theorem 1 and Theorem 2, we can get a new mechanism, which can be stated in the following corollary.

Corollary 1. *Suppose the highest and the second highest bids on the DSP are b_1 and b_2, respectively. The bid of the DSP in the second stage auction is d_1, and the highest bid of all the other DSPs is d_2. Then the optimal mechanism for the DSP is to set $d_1 = b_1$ and $c_1 = c_1(\omega)$, where $c_1(\omega)$ is defined in (12).*

According to Corollary 1, the revenues of the advertiser and the DSP in the new mechanism can be computed as

$$r_1^{\text{NEW}}(\omega) = \begin{cases} b_1 - b_2, & \text{if } d_2 < b_2 \\ \omega(b_1 - d_2), & \text{if } b_2 \le d_2 < b_1 \\ 0, & \text{other} \end{cases} \tag{18}$$

and

$$r_2^{\text{NEW}}(\omega) = \begin{cases} b_2 - d_2, & \text{if } d_2 < b_2 \\ (1-\omega)(b_1 - d_2), & \text{if } b_2 \le d_2 < b_1 \\ 0, & \text{other,} \end{cases} \tag{19}$$

respectively.

A. Comparisons of Mechanisms

In this section, we compare the proposed mechanism with the PRE and POST mechanisms. We first study the properties of the new mechanism, which can be stated in the following theorem.

Theorem 3. *The POST mechanism is a special case of our proposed new mechanism, and when $\omega = 1$, the new mechanism is degenerated to the POST mechanism.*

Proof: When $\omega = 1$, according to (12)–(19), we have

$$c_1^{\text{NEW}}(1) = \begin{cases} b_2, & \text{if } d_2 < b_2 \\ d_2, & \text{if } b_2 \le d_2 < b_1 \\ 0, & \text{other,} \end{cases} \tag{20}$$

$$r_1^{\text{NEW}}(1) = \begin{cases} b_1 - b_2, & \text{if } d_2 < b_2 \\ b_1 - d_2, & \text{if } b_2 \le d_2 < b_1 \\ 0, & \text{other} \end{cases} \tag{21}$$

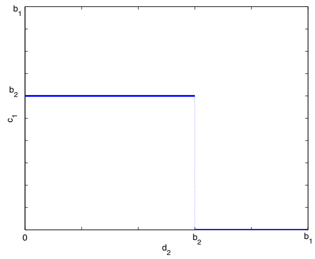

Fig. 2. The pricing mechanism in the PRE mechanism

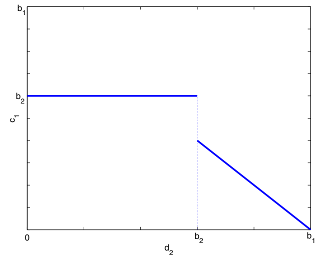

Fig. 3. The pricing mechanism in the POST mechanism

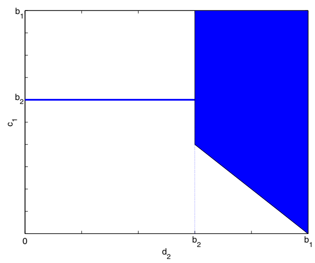

Fig. 4. The pricing mechanism in the new mechanism

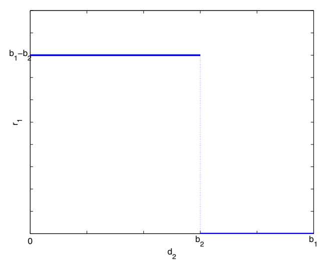

Fig. 5. The revenue of the winning advertiser in the PRE mechanism

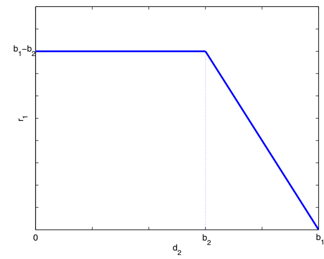

Fig. 6. The revenue of the winning advertiser in the POST mechanism

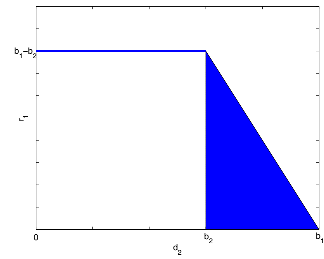

Fig. 7. The revenue of the winning advertiser in the new mechanism

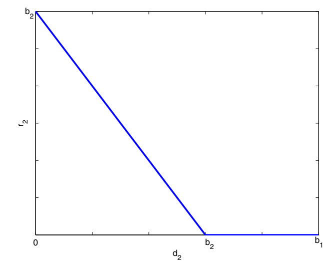

Fig. 8. The revenue of the DSP in the PRE and POST mechanism

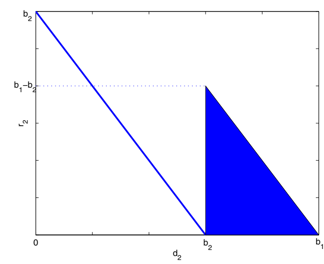

Fig. 9. The revenue of the DSP in the new mechanism

and

$$r_2^{\text{NEW}}(1) = \begin{cases} b_2 - d_2, & \text{if } d_2 < b_2 \\ 0, & \text{other.} \end{cases} \tag{22}$$

Thus, we have

$$\begin{aligned} c_1^{\text{NEW}}(1) &= c_1^{\text{POST}}, \\ r_1^{\text{NEW}}(1) &= r_1^{\text{POST}}, \\ r_2^{\text{NEW}}(1) &= r_2^{\text{POST}}, \end{aligned} \tag{23}$$

i.e., the new mechanism becomes the POST mechanism, which illustrates that the POST mechanism is a special case of our proposed new mechanism. ■

In the following, we make a detailed and intuitional comparison of the three mechanisms from the aspect of the pricing mechanism and the revenues for the DSP and the winning advertiser. The comparisons of the pricing mechanism in the three mechanisms are given in Fig. 2–Fig. 4, respectively, and the revenues of the advertiser and the DSP in the three mechanisms are given in Fig. 5–Fig. 9, respectively. From these figures, it is obvious that our proposed new mechanism has a better performance in improving the revenues of both the advertiser and the DSP.

IV. Computational Experiments

As RTB markets is a complex social-economic system, and the essential data can not be obtained due to the privacy of RTB business model, it is extremely difficult or even impossible to evaluate our proposed mechanisms with online field experiments. Fortunately, with the proposal of the innovative ACP theory [18] and its successful applications [21, 22], computational experiments approach have been regarded as an alternative and effective approach in dealing with such complex socio-economic problems.

Thus, in this section, we utilize the computational experiments approach to evaluate the superiority of our proposed new mechanisms. For comparison purpose, we adopt the PRE and the POST mechanisms as baseline mechanisms. Our proposed mechanism is denoted as New mechanism, in which the weight ω is randomly generated from $[0, 1]$.

A. Experimental Scenario

We consider the experimental scenario that there are two DSPs in the whole RTB market, and there are 100 independent ad impressions during a given period. The highest bid and the second highest bid for the 100 ad impressions on the DSP is randomly generated from $[15, 20]$ and $[5, 10]$, respectively, and the highest bid on the other DSP is randomly generated from $[2, 20]$. According to the above data, the randomly generated experimental data are shown in Fig. 10.

B. Experimental Result

The revenues for the winning advertiser and the DSP from each ad impression are shown in Fig. 11 and Fig. 12, respectively. The total revenues of the advertisers and the DSP from the 100 ad impressions are shown in Fig. 13 and Fig. 14, respectively.

From Fig. 11–Fig. 14, we can obtain the following experimental results:

(1) For each ad impression, the revenue of the winning advertiser is the highest in POST mechanism, and the lowest in the PRE mechanism.

(2) For each ad impression, the revenue of the DSP in the New mechanism is higher than those in the PRE mechanism and the POST mechanism, and the revenue

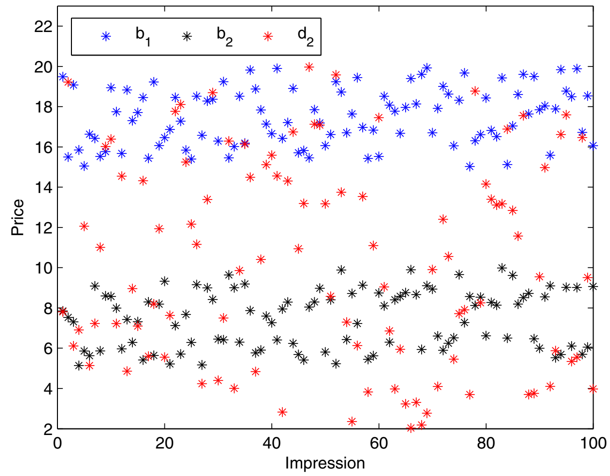

Fig. 10. The randomly generated b_1, b_2 and d_2 for the 100 ad impressions

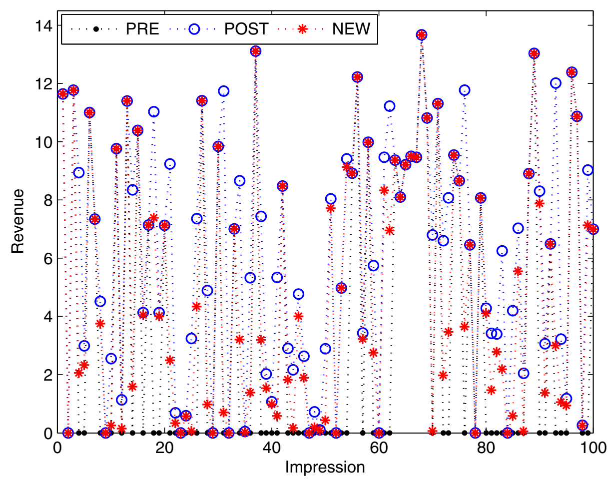

Fig. 11. Comparisons of the revenues of the winning advertisers from each ad impression in the three mechanisms

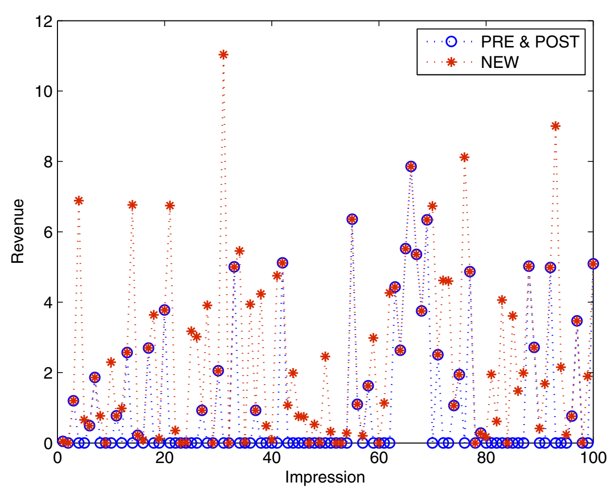

Fig. 12. Comparisons of the revenues of the DSP from each ad impression in the three mechanisms

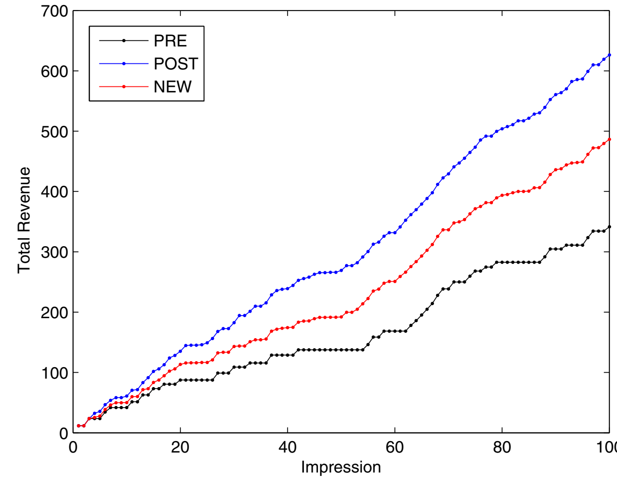

Fig. 13. Comparisons of the total revenues of the advertisers from the 100 ad impressions in the three mechanisms

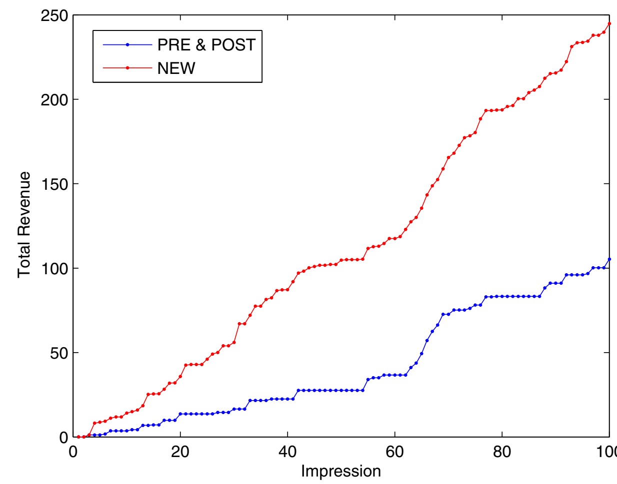

Fig. 14. Comparisons of the total revenues of the DSP from the 100 ad impressions in the three mechanisms

of the DSP in the POST mechanism is equal to that in the PRE mechanism.

(3) The total revenues of the advertisers in the PRE, POST and NEW mechanisms are 341.410, 626.220, 486.683, respectively, which illustrates that the total revenues of the advertisers is the highest in the POST mechanism, and the lowest in the PRE mechanism.

(3) The total revenue of the DSP in the PRE, POST and NEW mechanisms are 105.330, 105.330, 244.867, respectively, which illustrates that the total revenues of the DSP is the highest in the New mechanism, and the total revenues of the DSP in the PRE mechanism and the POST mechanism are the same.

C. Analysis of the Results

In this section, we aim to compare the three mechanisms based on the experimental results. First, compared with the PRE mechanism, the POST mechanism can improve the revenue of each winning advertiser and the total revenue of the advertisers, however, the revenue of the DSP can not be improved. Thus, there is no incentive for the DSP to increase its bid from b_2 to b_1 in the second stage.

Second, our proposed new mechanism can improve the revenues of both the advertisers and the DSP comparing with the PRE mechanism. Thus, our proposed mechanism is the most effective one among the three mechanisms.

V. Conclusions and Future Work

This paper studied the mechanisms for DSPs in the two-stage resale model in the RTB advertising market. With the aim of improving the revenues of both the advertiser and the DSP compared with the commonly used PRE mechanism, we proposed a new pricing and bidding mechanism, and analyzed the superiorities of the new mechanism. With the computational experiment approach, we validate the effectiveness of the proposed new mechanism.

In our future work, we will extend our work from the following aspects: (a) Explore the mechanisms for the random cases; (b) Study the social welfare of RTB ecosystem under the new mechanisms.

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