

Model Predictive Control Implementation and Simulation for Urban Traffic Networks

Chao Guo

Qingdao Academy of Intelligent Industries
Qingdao, China
Beijing Engineering Research Center of Intelligent Systems and Technology
Institute of Automation, Chinese Academy of Sciences
Beijing, China
Email: guochao@casc.ac.cn

Gang Xiong (Correspondence Author), Fenghua Zhu

Cloud Computing Center, Chinese Academy of Sciences
Songshan Lake, Dongguan, China
Qingdao Center for Intelligent Systems and Technology
Institute of Automation, Chinese Academy of Sciences
Qingdao, China

Mei Zhang

South China University of Technology
Guangzhou, China

Abstract—In this paper, we consider the problem of designing traffic network signal control systems for congested urban road networks, aiming to relieve traffic congestion and improve the utilization of the existing traffic infrastructures. A Model Predictive Control (MPC) method is introduced which is based on a microscopic store-and-forward modeling (SFM) paradigm. Moreover, a preliminary simulation of urban traffic flow management is implemented with the help of MATLAB/SIMULINK and PARAMICS. The results demonstrate the efficiency and feasibility of the MPC signal control method, which can take all the operational constraints into consideration easily. And the MPC control framework can also be applied for other infrastructure systems whose characteristics are common intelligent, from a generic point of view.

Keywords—Traffic infrastructures; Urban traffic control system; Model predictive control; Microscopic traffic simulation

I. INTRODUCTION

Along with the development of the transportation infrastructures, traffic networks become larger and more complex. Especially, the capacity of urban roads saturates during rush hours, resulting in congestion. These terrible traffic jams would cause large delays, resulting in higher travel costs and negative impact on the environment, e.g., noise and pollution. Apparently, the capacity of roads reaches its limits, while the traditional traffic management is getting less effective.

To tackle these problems, network wide traffic control systems that can coordinate the whole network and improve the utilization of the existing traffic infrastructures [1] are highly required. At the same time, various sensing, communication and management components are increasingly being equipped in roads networks, opening a way to the application of developed control methods to improve system performance. A number of systems using optimization control

methods emerged in the 1980s, such as SCOOT [2] and PRODDYN.

One of possible solutions is Model Predictive Control [3] [4], based on prediction and rolling horizon approaches. MPC method can handle constraints on inputs, states, and outputs, in an explicit way. The ability to perform online optimization is also attractive. The advantages help it receive on-going interest from researchers in both the industrial and academic communities. Its application in complex industrial process [5] to deal with uncertainty of real system and no exact model conditions, had a great success since the 1970s. Nowadays, the developed strategy is increasingly implemented in large-scale networks, nonlinear systems, such as, supply chain management and traffic control systems.

MPC based ramp metering and speed limit in highway traffic management came up in the early 2000s [6]. In this paper, we investigate the application of MPC in urban traffic control systems. The purpose is to improve capacity of roads and make traffic system perform more reliably and efficiently.

A simple traffic flow model is needed to design the control system. We choose the macroscopic store-and-forward model [7], which is well-suited for online control and state space approach. Some essential concepts and philosophy of MPC are presented below. The constraints and optimal approaches are also discussed. Then, we simulated the control system in MATLAB and PARAMICS environment to test and validate the MPC algorithm. The implement framework is also presented.

Simulation results demonstrate the effectiveness of the system compared with traditional fixed time traffic control. MPC strategy is suitable to manage the urban traffic systems.

Recently, the performance of other infrastructure networks is also pushed to its limits just like traffic networks. Their unreliability and inefficiency will have fatal effect on our life and society. A lot of characteristics are common to these large-scale networks. Inspired by the MPC based urban traffic

management, we can put forward some control frameworks in the sense of systems theory. We need establish a theory which provides common problem solving strategies from a generic point of view.

II. MODELING

A. Traffic flow models

Traffic flow models can be distinguished into three types, namely, macroscopic, microscopic and mesoscopic, according to the level of model details. An overview of existing traffic models is given in [8]. The more exhaustive a model is, the higher computational complexity the system has. Macroscopic models use aggregated variables to describe traffic flows. It means that they are suited well for online control where the prediction will be implemented online in an optimization operation.

To get a balanced trade-off between control accuracy and computational efforts, we use a store-and-forward traffic flow model. The strategy enables mathematical description of the traffic flow, using state space method. It's a development result of Gazis and Potts in 1963. It makes application of highly efficient control methods available, such as MPC. This model forms the basis of our model-based predictive control system.

B. Store-and-forward modeling

The roads network can be described as sets of links $z \in Z$ and junctions $j \in J$. For each signalized junction j , there are sets of incoming I_j and outgoing O_j links. Fig.1. shows urban roads containing two neighboring junctions M and N , where $z \in I_N$ and $z \in O_M$.

We define some essential variables as follows:

- $x_z(k)$: number of vehicles in link z , practically, the length of queue at step k , the state variable;
- $g_{j,i}$: the green time of stage i at junction j , the control input;
- S_z : saturation flow of link z ;
- $t_{w,z}$: turning rate towards link z from the links w that enter junction M ;
- C_j : the cycle time of junction j ;
- T : the discrete time step, control interval;
- v_z : the set of stages where link z has right of way;
- k : the discrete time index, $k = 0, 1, 2, \dots$;
- j : the junction identifier;
- i : the stage identifier.

For simplicity, we assume that the cycle times C_j for all junctions $j \in J$ are equal and fixed, namely $C_j = C$, which is usual.

Then, the dynamics of link z are represented by the conservation equation:

$$x_z(k+1) = x_z(k) + T \left[(1 - \tau_z) \sum_{w \in I_M} t_{w,z} \frac{S_w \sum_{i \in v_w} g_{M,i}(k)}{C} - \frac{S_z \sum_{i \in v_z} g_{N,i}(k)}{C} \right] \quad (1)$$

The saturation flow S_z of link $z \in Z$ means the outflow capacity of the link during its green time. Actually, it's assumed to be known and constant in practice, calculated by another approach or using a standard value. The turning rates $t_{w,z}$ where $w \in I_j$ and $z \in O_j$, are also set using a statistical value or estimated in real-time.

And if we assume $T = C$ and replace the second and third term with some simplified variables, equation (1) can be described by:

$$x_z(k+1) = x_z(k) + T[p_z(k) - q_z(k) + d_z(k) - e_z(k)] \quad (2)$$

where $p_z(k)$ and $q_z(k)$ are the inflow and outflow of link z in the sample time $[kT, (k+1)T]$, respectively; $d_z(k)$ and $e_z(k)$ are the demand and the exit flow in the link z , respectively. The exit flow $e_z(k)$ can be estimated by $s_z(k) = \tau_z p_z(k)$ where the exit rates τ_z are known usually.

Note that the vehicles are considered as passenger car unit (PCU) resulting from appropriate transformation. And the modeled flow is an average of real flow for each period which avoids the considering of red-green switching in a cycle to reduce computational efforts. The outflow is represented by:

$$q_z(k) = \frac{S_z \sum_{i \in v_z} g_{N,i}(k)}{C} \quad (3)$$

Equation (1) is linear scalar equation for describing a given link. We are supposed to change for state space model to define a whole traffic network, representing the real network characteristics. Organize all interconnected conservation equations for each link in a state space form:

$$\mathbf{x}(k+1) = \mathbf{x}(k) + \mathbf{B}\mathbf{g}(k) + \mathbf{d}(k) \quad (4)$$

where $\mathbf{x}(k)$ is the state vector representing numbers of vehicles in each link; \mathbf{B} is a constant coefficient matrix of proper dimensions representing the network characteristics, like topology and turning rates; $\mathbf{g}(k)$ is the control vector representing all green time settings; $\mathbf{d}(k)$ is the disturbance.

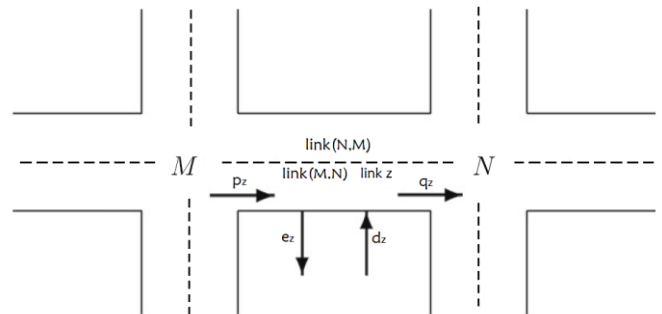


Fig.1. A representation of urban roads using links and junctions

The coefficient matrix \mathbf{B} can be constructed according to (1).

III. MPC BASED TRAFFIC CONTROL SYSTEM

A. Model predictive control for traffic networks

MPC is an online model-based predictive control approach in which a prediction model and optimization are used to determine the control actions that optimize a given performance criterion over a given prediction horizon subject to given constraints.

The strategy has outstanding performance in complex dynamic systems where it's difficult to obtain an exhaustive model. On the other hand, plenty of constraints can be considered explicitly.

There are three pivotal steps to achieve a MPC system. Model based prediction: design a model to describe the controlled process, and predict output of the system with the consideration of history and input. Rolling horizon optimization [9]: repeat the optimization using a sliding time window and the receding horizon is finite. Feedback control: correct the prediction procedure errors and disturbance rejection. The closed-loop path makes the system more accurate and stable. A schematic framework of MPC structure is given in Fig.2.

Using a receding horizon approach, only the first step of the optimized control actions over a control horizon is applied. And next optimization is started again with the prediction horizon shifted one time step further. Fig.3. depicts the philosophy of receding horizon. After a control signal is applied, a measurement of states is made and the controller computed new control signals over $t = (k + N_c)T$ via prediction of the system and online optimization from kT to $(k + N_p)T$. At the next control step $(k + 1)$, this procedure is repeated. Note that during a control interval control signals are taken to be constant. The aim of the controller is to find the control signals that result in an optimal behavior of the traffic flows evaluated by a costs function. In our tasks, the minimum of the number of vehicles waiting in line is the objective.

Predictive horizon N_p has great influence on the stability and rapidity of control system. Small values may result in inability to get satisfied control signals to approach expected states through rolling horizon optimization, or even cause oscillation, while a large one couldn't meet the demand of rapidity. In general, a shorter prediction horizon is usually sufficient, which reduces complexity, and makes the real-time

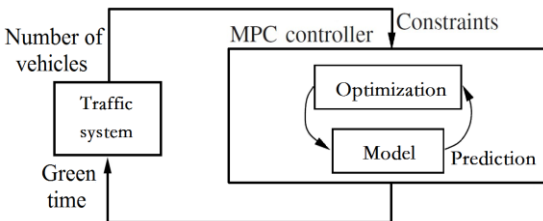


Fig.2. Schematic view of MPC structure

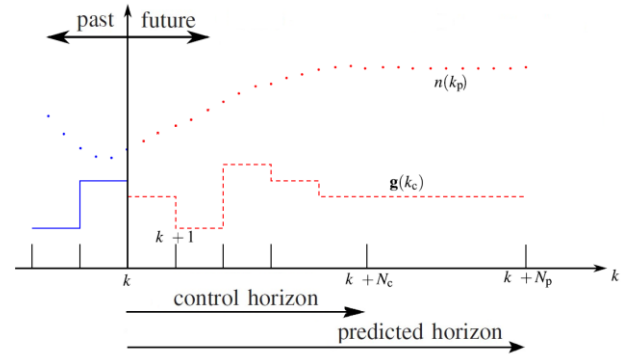


Fig.3. Receding horizon

application of MPC feasible. Control horizon N_c reflects the steps of computed control signals to undertake an optimization task, which means we need enough steps to obtain an accurate control performance with the price of robustness. Note that it's shorter than N_p , generally.

Actually, it's difficult to consider the selection of N_p and N_c directly. Trial-and-error solution or simulation is required. And the initial values can be accessed by Heuristic reasoning approach.

When handling large networks, the computational efforts become too heavy for a centralized controller. The network should be separated into some sub-networks that are controlled by separate controllers [10] [11]. In these cases, multi-agent MPC structure [12] can be used. This structure can also improve robustness and reliability with a reduction of communication delays.

B. Constraints

Based on the SFM discussed in previous section, there are some constraints that have to be considered.

The state variable, namely, queues are subjected to the length of a link between two junctions.

$$0 \leq x_z(k) \leq x_{z,\max}, \quad \forall z \in Z \quad (5)$$

where $x_{z,\max}$ is the maximum admissible numbers of vehicle in link z . The limitation can avoid oversaturation during rush hours obviously.

The constraint of control input, namely green time holds at junction j in stage i

$$\sum_{i=1}^{N_j} g_{j,i}(k) \leq C - L_j, \quad \forall j \in J \quad (6)$$

where N_j is the number of stages at junction j . L_j is the fixed lost time determined by the geometry at junction j .

In addition, the upper and lower bounds of green time is given by

$$g_{j,\min} \leq g_{j,i} \leq g_{j,\max}, \quad \forall j \in J \quad (7)$$

where $g_{j,\min}$ and $g_{j,\max}$ represents the minimum and maximum permissible time at junction j respectively, to set enough

green time for pedestrian phase.

C. Optimization approaches

Optimization method applied could directly influence the performance of control system. An appropriate solution which leads to a balanced trade-off between outstanding results and online computations is required. We consider a LQ optimization [13] which simplifies real-time calculations.

Our objective is the minimization of numbers of vehicles in queue. A quadratic costs function satisfying (4) and constraints (6) (7) is given by

$$J = \sum_{k=1}^{N_p} (\|x(k)\|_Q^2 + \|g(k)\|_R^2) \quad (8)$$

where N_p is the length of the predictive horizon. Q and R are appropriate dimensional, nonnegative and diagonal weighting matrices which could influence the stability of the closed loop. So the parameters should be chosen to coordinate performance of the system. Value of $R = rI$ could be set by a trial-and-error procedure, although control results are insensitive for a large range of r , where I is the unit matrix.

But the LQ method may lead to a suboptimal solution and couldn't consider the limits of state variables exhaustively. Therefore it's attractive to explore more complex solutions which can handle the problem well. Many intelligent optimization approaches like generic algorithms, particle swarm optimization or simulated annealing can be used.

IV. SIMULATION AND RESULTS

A. Roads networks modeling

We used PARAMICS [14] to establish a microscopic model of roads networks based on real traffic parameters. It's a suite of high performance software used to model and simulate the movement and behavior of individual vehicles on urban and highway road networks. It consists of Modeler, Programmer, Analyzer, Estimator and other tools.

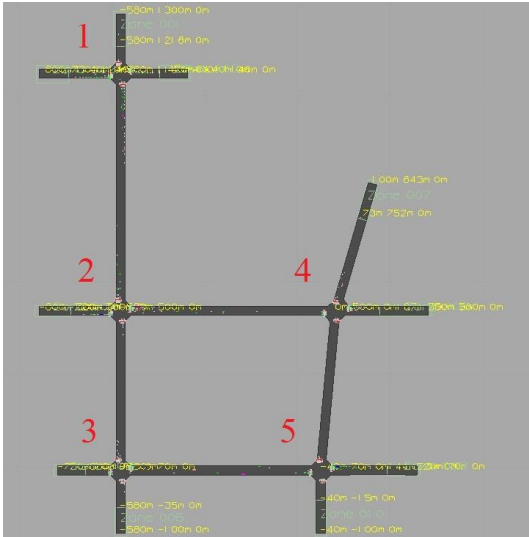


Fig.4. Model of the test roads network

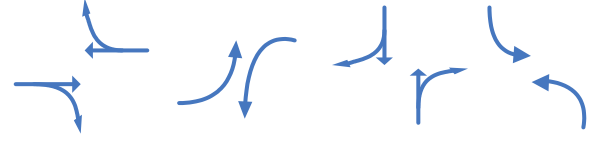


Fig.5. Four phases traffic flow management at a junction

The Programmer provides us with plenty of API functions to access and modify the core model and simulation parameters. The Estimator can help us calibrate the OD matrices to be used in Modeler simulation, based on observed count data. And the Analyzer tool is designed to display and report statistics that are relevant in a format that is both visual and flexible for reporting and further analysis.

The distance between several junctions is short, less than 800 meters in general, which means the coordinate operation of traffic networks is reasonable. The test network is selected from Tianhe district of Guangzhou by appropriate simplification. It consists of 5 signalized junctions and 12 links (Fig.4.): $J = \{1, \dots, 5\}$, $Z = \{1, \dots, 12\}$. We compare the performance of MPC based system and fixed time control system. Assume that there is no ramp in links J and the lost time in each phase is fixed. At the same time, the pedestrian phases are ignored. Several vital parameters are discussed below.

The cycle times for all junctions $j \in J$ satisfies $36\text{sec} \leq C \leq 120\text{sec}$ in practice. We consider $C = 120\text{sec}$ and $T = C$ is taken as a control interval. A standard value for the minimum of green time is $g_{j,\min} = 6\text{sec}$, so the constraint is represented by:

$$6\text{sec} \leq g_{j,i} \leq 102\text{sec}, \quad \forall j \in J \quad (9)$$

The weighting matrix $R = \text{diag}(0.01)$ for LQ approach is applied. Fig.5. shows the pass priority for all junctions using a four-phase traffic flow control. And related control parameters are listed in TABLE I.

B. Simulation approach

We used traffic network modeler and simulator PARAMICS, multi-paradigm numerical computing environment MATLAB and Visual C++ programming language to implement a MPC based system simulation.

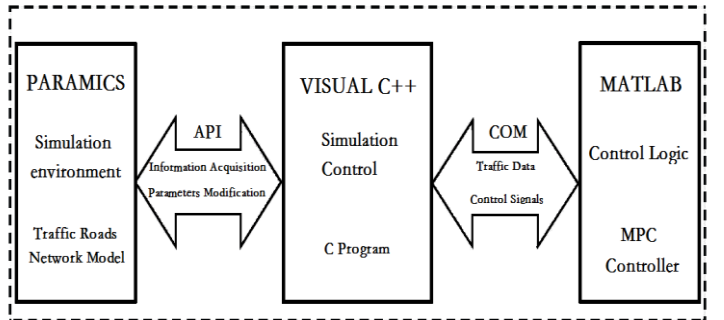


Fig.6. Simulation framework

TABLE I. Control parameters

Control parameters	Physical meaning	Values
T	Control interval	2 min
C	Cycle time	2 min
N_p	Prediction horizon	10 min (5 cycles)
N_c	Control horizon	4 min (2 cycles)
$T_{simulation}$	Simulation time	60 min
$g_{j,min}$	Minimum of green time	6 s
$g_{j,max}$	Maximum of green time	102 s
t_s	Simulation time step	0.5 s

TABLE II. Simulation results

Strategy	Evaluation criteria	Normal traffic	Heavier traffic	Improvement		
Fixed time control	Avg. number of vehicles in queue	54	61	+13%	/	/
	Avg. delay time per vehicle	67	75	+12%	/	/
MPC based control	Avg. number of vehicles in queue	48	49	+2%	-11%	-20%
	Avg. delay time per vehicle	54	54	+0%	-19%	-28%

The control algorithm is achieved in MATLAB using a MPC toolbox, instead of a C program, due to long developing time and heavy computational efforts. The optimization problem is also solved using quadratic functions of MATLAB. Besides, a C application is developed to control the simulation process and the data transfer. The program obtains traffic information via QPG Get Functions which let user query the value and state of objects, and transfers it to MATLAB. Then, control signals computed by controller are returned to the simulation model. Actually, the above-mentioned process is completed by a plugin in DLL form which will be loaded when the Modeler is working. The simulation processing framework is depicted in Fig.6.

Several simulation parameters are specified as follows. The free flow speed on test roads is 50 km/h. Demand factor is 100% which defines the dynamic demand for current simulation. Queuing speed is specified as 7.2 km/h. And mean driver reaction time is set as 1 s. OD matrix could be estimated according to measured traffic flow data with a help of the Estimator.

C. Simulation results

To investigate the applicability and effectiveness of the MPC based control strategy, we applied different control approaches in the same simulation environment mentioned above and compared their evaluation criteria.

Two evaluation criteria are used for comparison: average number of vehicles in queue and average delay time (s). In addition, we generated normal traffic flow and heavier traffic flow where a 15% additional flow is created in the experiment. The simulation results for fixed time control and MPC strategy are presented in TABLE II.

As the results show, both strategies perform well in normal traffic condition while MPC method is a little better leading to a reduction of both evaluation criteria. However, there is a big difference in a heavier traffic condition. MPC strategy is superior resulting in a large improvement while the performance of fixed time strategy gets worse.

We expect the minimum of vehicles waiting in queue and relieve the congestion in urban traffic networks. Obviously, MPC based strategy provides a good solution. The average number of vehicles in queue will almost not increase when intensive flow is added. But the criteria of fixed time strategy grows greatly which means it cannot adapt the heavy traffic condition.

The MPC strategy is effective and applicable in urban traffic networks to deal with the congestion problem. It's advisable to improve performance of the existing traffic infrastructures using MPC.

V. CONCLUSIONS AND FUTURE WORK

Currently, traffic networks are becoming more complex and traditional signal control strategy cannot work effectively anymore. Terrible traffic jams occur frequently around the world causing higher and higher social costs. More Efficient and reliable management of traffic flow is extremely needed. MPC strategy can predict the system behaviors in advance and avoid the unexpected situations with a consideration of constraints explicitly. It's advisable to use MPC to solve the traffic problem mentioned above.

We designed a MPC based signal control system combining store-and-forward traffic flow model and receding horizon optimization to relieve traffic congestion. And simulations with an objective of minimizing the number of vehicles in queue are implemented to validate the applicability and effectiveness of the management system. As results show, MPC based strategy perform well to deal with traffic congestion problem, especially in case of heavy traffic flow in rush hours.

The safe, reliable and efficient operation of large-scale infrastructure systems, like electricity supply systems and Internet, is of crucial importance for the functioning of the whole society and our daily life. Just like urban traffic networks, the performance of other infrastructure systems is pushed to its limits. Therefore, future work will deal with the exploration of reasonable methods, such as MPC strategy, to make infrastructure systems more intelligent. Besides, systems should be controlled by separate controllers when handling large networks. The investigation of distributed control systems is meaningful.

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