# Task-Space Adaptive Dynamic Modularity Control of Free-Floating Space Manipulators 

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#### Abstract

In this paper, we investigate the task-space adaptive control problem for free-floating space manipulators with uncertain kinematics and dynamics and with an unmodifiable inner joint control loop. The existence of an unmodifiable inner joint control loop makes most torque-based control algorithms in the literature inapplicable. We propose a dynamic modularity (DM) approach to resolve this problem, and this is hopeful for bridging the potential gap between the advanced control theory for (free-floating) space manipulators and practical engineering applications. Adaptive outer loop controllers are developed and shown to be able to guarantee the convergence of the task-space tracking errors. The performance of the proposed DM approach is shown by a numerical simulation.


## I. Introduction

The on-orbit servicing (OOS) with the aid of robot manipulators has recently become active in the space industry and academic community (see, e.g., [1], [2], [3]). The research on the system composed of a spacecraft and one or multiple manipulators mounted on it (known as space manipulator), however, dates back to the quite early years, covering the system kinematics, dynamics, and control (see, e.g., [4], [5], [6], [7]), and we may also note that the concept of space manipulator has already spread quite a lot (e.g., tethered space robots [8], [9]). The expectation of space manipulators to maneuver various classes of objects flexibly and intelligently motivates the sustaining research on developing adaptive controllers for space manipulators [10], [11], [12], [13], [14], [15], and the study of similar systems using adaptive algorithms also appears in the context of robots on a moving platform [16].

Dealing with the control of complex nonlinear systems is challenging, and one desirable situation that we may intuitively expect is the separation or relative independence of different system loops (i.e., achieving certain degree of modularity). One such case occurs in the adaptive control of fixed-base robots [17], [18]. Specifically, the kinematic and

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dynamic loops are separated and performance improvement is achieved in [17] by employing an adaptive kinematic regressor matrix and the inverse Jacobian feedback (unlike the results in [19], [20], [21], [22]), and a dynamic modularity (DM) approach is proposed in [18] to accommodate the users' needs and the interest of the robot manufacturers where the designs of the outer loop (for users) and the inner loop (for robot manufacturers) can be performed independently and more desirably the stability of the whole system can still be rigorously guaranteed. This motivates us to extend the DM approach in [18] to free-floating space manipulators (FFSMs), which have important applications in space activities and show particular advantages in terms of safe manipulation and energy conservation [5], [23]. The control scheme in [12] is torque-based and thus inapplicable to FFSMs that do not allow the torque input design [for instance, in many robotic applications, it is well known that the joint velocity (or position) command rather than joint torque can be directly specified]; in addition, the performance of the transpose Jacobian control used in [12] is not guaranteed to be satisfactory, as is stated in [24]. Due to the similar reason, the application of the inverse-dynamicsbased adaptive controller in [14] to free-floating systems with a manipulator that does not admit the joint torque design is also challenging.

In this paper, we propose a DM approach for FFSMs with uncertain kinematics and dynamics, which can be considered as an extension of [25], [18] to the case of robots with a freefloating base. Specifically, we propose a design approach for the joint velocity (or position) command as well as parameter adaptation laws (which can also be referred to as adaptive outer loop controller) to address the dynamic effects and parametric uncertainties of the system consisting of both the manipulator and its free-floating base. It is shown that the proposed adaptive outer loop controllers can ensure the stability and convergence of the FFSM system with an inner unmodifiable PD/PID (proportional-derivative/proportional-integral-derivative) controller. In addition, the feedback control as well as the kinematic parameter adaptation law used here can be interpreted as adaptive inverse-Jacobianlike control (extending the results in [26], [27], [17] to address the case of a free-floating base), which tends to yield the improved performance in comparison with the adaptive transpose Jacobian controller in [12] (see, e.g., [17]). From an application perspective, thanks to the achieved dynamic modularity, the proposed approach can be applied to most engineering space robotic systems with an unmodifiable inner control loop (e.g., the ETS-VII space manipulator).

Furthermore, the proposed dynamic modularity control approach may seem more promising in that it would potentially decrease the cost of development of high-performance space robotic systems due to the dynamic modularity and that it might also help promote the commercialization of space robotic projects.

## II. Kinematics and Dynamics of Free-Floating Space Manipulators

Consider a FFSM with zero initial linear and angular momenta. Let $x \in R^{n}$ be the position of the end-effector of the FFSM in the task space (e.g., Cartesian space or image space), and it is relevant to the manipulator joint position and spacecraft attitude via a nonlinear mapping (see, e.g., [5], [28]). The end-effector velocity $\dot{x}$ can be written as [5], [28]

$$
\begin{equation*}
\dot{x}=J_{b}\left(\epsilon_{b}, q_{m}\right) \omega_{b}+J_{m}\left(\epsilon_{b}, q_{m}\right) \dot{q}_{m} \tag{1}
\end{equation*}
$$

where $q_{m} \in R^{n}$ is the manipulator joint position, $\epsilon_{b} \in R^{4}$ is the Euler parameter vector that corresponds to the spacecraft attitude matrix (see, e.g., [29]), $\omega_{b} \in R^{3}$ is the angular velocity of the spacecraft with respect to the inertial frame expressed in the spacecraft frame, and $J_{b}\left(\epsilon_{b}, q_{m}\right) \in R^{n \times 3}$ and $J_{m}\left(\epsilon_{b}, q_{m}\right) \in R^{n \times n}$ are the Jacobian matrices. For conciseness, $J_{b}\left(\epsilon_{b}, q_{m}\right)$ and $J_{m}\left(\epsilon_{b}, q_{m}\right)$ are denoted by $J_{b}$ and $J_{m}$ in the sequel, respectively. The kinematics (1) has the following linearity-in-parameters property [19], [26].

Property 1: The kinematics (1) depends linearly on a constant kinematic parameter vector $a_{k}$, which gives rise to

$$
\begin{equation*}
J_{b} \psi_{1}+J_{m} \psi_{2}=Z\left(\epsilon_{b}, q_{m}, \psi\right) a_{k} \tag{2}
\end{equation*}
$$

where $\psi_{1} \in R^{3}, \psi_{2} \in R^{n}, \psi=\left[\psi_{1}^{T}, \psi_{2}^{T}\right]^{T}$, and $Z\left(\epsilon_{b}, q_{m}, \psi\right)$ is the kinematic regressor matrix.

The equations of motion of the FFSM taking into consideration the actuator model can be written as [10], [30]

$$
M\left(q_{m}\right) \ddot{q}+C\left(q_{m}, \dot{q}\right) \dot{q}+\left[\begin{array}{c}
0_{3}  \tag{3}\\
B \dot{q}_{m}
\end{array}\right]=\left[\begin{array}{c}
0_{3} \\
K u
\end{array}\right]
$$

where $\dot{q}_{m}$ is the manipulator joint velocity, $\dot{q}=\left[\omega_{b}^{T}, \dot{q}_{m}^{T}\right]^{T}$, $M\left(q_{m}\right)=\left[\begin{array}{cc}M_{b b} & M_{b m} \\ M_{b m}^{T} & M_{m m}\end{array}\right]$ is the inertia matrix, $C\left(q_{m}, \dot{q}\right)=$ $\left[\begin{array}{cc}C_{b b} & C_{b m} \\ C_{m b} & C_{m m}\end{array}\right]$ is the Coriolis and centrifugal matrix, $M_{b b} \in$
$R^{3 \times 3}$ is the inertia matrix of the spacecraft, $M_{b m} \in R^{3 \times n}$ is $R^{3 \times 3}$ is the inertia matrix of the spacecraft, $M_{b m} \in R^{3 \times n}$ is the coupled inertia matrix between the spacecraft and the manipulator, $M_{m m} \in R^{n \times n}$ is the inertia matrix of the manipulator, $C_{b b} \in R^{3 \times 3}$ is the Coriolis and centrifugal matrix of the spacecraft, $C_{b m} \in R^{3 \times n}$ is the coupled Coriolis and centrifugal matrix between the spacecraft and the manipulator, $C_{m b} \in R^{n \times 3}$ is the coupled Coriolis and centrifugal matrix between the manipulator and the spacecraft, $C_{m m} \in R^{n \times n}$ is the Coriolis and centrifugal matrix of the manipulator, $B \in R^{n \times n}$ is a diagonal positive definite matrix, $K \in R^{n \times n}$ is a diagonal positive definite matrix, and $u \in R^{n}$ is the armature voltage. Three wellrecognized properties associated with (3) are listed as follows (see, e.g., [5], [10], [31]).

Property 2: The inertia matrix $M\left(q_{m}\right)$ is symmetric and uniformly positive definite.

Property 3: The Coriolis and centrifugal matrix $C\left(q_{m}, \dot{q}\right)$ can be suitably chosen so that $\dot{M}\left(q_{m}\right)-2 C\left(q_{m}, \dot{q}\right)$ is skewsymmetric.

Property 4: The dynamics (3) depends linearly on a constant dynamic parameter vector $a_{d}$, which yields

$$
\begin{align*}
& M_{b b} \dot{\zeta}_{1}+M_{b m} \dot{\zeta}_{2}+C_{b b} \zeta_{1}+C_{b m} \zeta_{2} \\
& =Y_{b}\left(q_{m}, \dot{q}, \zeta, \dot{\zeta}\right) a_{d}  \tag{4}\\
& M_{b m}^{T} \dot{\zeta}_{1}+M_{m m} \dot{\zeta}_{2}+C_{m b} \zeta_{1}+C_{m m} \zeta_{2}+B \zeta_{2} \\
& =Y_{m}\left(q_{m}, \dot{q}, \zeta, \dot{\zeta}\right) a_{d} \tag{5}
\end{align*}
$$

where $\zeta_{1} \in R^{3}$ and $\zeta_{2} \in R^{n}$ are differentiable vectors, $\zeta=$ $\left[\zeta_{1}^{T}, \zeta_{2}^{T}\right]^{T}, \dot{\zeta}$ is the derivative of $\zeta$, and $Y_{b}\left(q_{m}, \dot{q}, \zeta, \dot{\zeta}\right)$ and $Y_{m}\left(q_{m}, \dot{q}, \zeta, \dot{\zeta}\right)$ are regressor matrices.

## III. Adaptive Outer Loop Control

In this section, we investigate the adaptive outer loop controller design for the FFSM system given by (1) and (3) with $u$ being specified by the PD control action (the case of PID control is discussed later)

$$
\begin{equation*}
u=-K_{D}\left(\dot{q}_{m}-\dot{q}_{m c}\right)-K_{P}\left(q_{m}-q_{m c}\right) \tag{6}
\end{equation*}
$$

where $q_{m c}$ and $\dot{q}_{m c}$ act as the joint position and velocity commands, respectively, and $K_{D}$ and $K_{P}$ are diagonal positive definite matrices and typically unknown to the user. This is a major difference between our work and most results in the literature (e.g., [11], [12], [13], [14]), and it is well known that an inner PD or PID controller is typically adopted in most industrial/commercial robots (see, e.g., [32], [33]) and in most space robotic applications (e.g., the ETS-VII space manipulator). The control objective is to realize the asymptotic end-effector trajectory tracking, i.e., $x-x_{d} \rightarrow 0$ as $t \rightarrow \infty$, where $x_{d} \in R^{n}$ denotes the desired trajectory and it is assumed that $x_{d}, \dot{x}_{d}$, and $\ddot{x}_{d}$ are all bounded. For facilitating the controller design in the sequel, we rewrite (6) as

$$
\begin{equation*}
u=-K_{D}\left[\left(\dot{q}_{m}-\dot{q}_{m c}\right)+\mathcal{K}_{P}\left(q_{m}-q_{m c}\right)\right] \tag{7}
\end{equation*}
$$

where $\mathcal{K}_{P}=K_{D}^{-1} K_{P}=\operatorname{diag}\left[w_{P}\right]$ with $w_{P}$ being an $n$ dimensional vector.

We first define a spacecraft reference velocity $\omega_{b r}$ by

$$
\begin{equation*}
\hat{M}_{b b} \dot{\omega}_{b r}+\hat{M}_{b m} \ddot{q}_{m r}+\hat{C}_{b b} \omega_{b r}+\hat{C}_{b m} \dot{q}_{m r}=K_{b}\left(\omega_{b}-\omega_{b r}\right) \tag{8}
\end{equation*}
$$

where $K_{b}$ is a symmetric positive definite matrix, $\dot{q}_{m r}$ and $\ddot{q}_{m r}$ will be defined later, and the matrices $\hat{M}_{b b}, \hat{M}_{b m}, \hat{C}_{b b}$, and $\hat{C}_{b m}$ are obtained by replacing the parameter $a_{d}$ in $M_{b b}, M_{b m}, C_{b b}$, and $C_{b m}$ with its estimate $\hat{a}_{d}$, respectively. The definition of $\omega_{b r}$ given by (8) is based on [12], and the difference is that here the estimated transpose Jacobian feedback is no longer needed. The interesting point may lie in the fact that the above definition, although for the case of task-space trajectory tracking, is the same as the one for the case of joint-space trajectory tracking in [34]. Using $\omega_{b r}$ defined by (8), we define a sliding vector

$$
\begin{equation*}
s_{b}=\omega_{b}-\omega_{b r} \tag{9}
\end{equation*}
$$

Next, following [12], we define a manipulator joint reference velocity

$$
\begin{equation*}
\dot{q}_{m r}=\hat{J}_{m}^{-1}\left(\dot{x}_{r}-\hat{J}_{b} \omega_{b r}\right) \tag{10}
\end{equation*}
$$

where $\dot{x}_{r}=\dot{x}_{d}-\alpha \Delta x$ with $\alpha$ being a positive design constant and $\Delta x=x-x_{d}$, and $\hat{J}_{b}$ and $\hat{J}_{m}$ are the estimated Jacobian matrices and are obtained by replacing $a_{k}$ in $J_{b}$ and $J_{m}$ with its estimate $\hat{a}_{k}$, respectively. Differentiating (10) with respect to time yields

$$
\begin{equation*}
\ddot{q}_{m r}=\hat{J}_{m}^{-1}\left(\ddot{x}_{r}-\hat{J}_{b} \dot{\omega}_{b r}-\dot{\hat{J}}_{b} \omega_{b r}-\dot{\hat{J}}_{m} \dot{q}_{m r}\right) \tag{11}
\end{equation*}
$$

We now define a sliding vector

$$
\begin{equation*}
s_{m}=\dot{q}_{m}-\dot{q}_{m r} \tag{12}
\end{equation*}
$$

From (9) and (12) and using (1), (10), and Property 1, we have that

$$
\begin{align*}
J_{b} s_{b}+J_{m} s_{m} & =\dot{x}-J_{m} \dot{q}_{m r}-J_{b} \omega_{b r} \\
& =\Delta \dot{x}+\alpha \Delta x+Z\left(\epsilon_{b}, q_{m}, \dot{q}_{r}\right) \Delta a_{k} \tag{13}
\end{align*}
$$

and equation (13) can further be written as

$$
\begin{equation*}
\Delta \dot{x}=-\alpha \Delta x-Z\left(\epsilon_{b}, q_{m}, \dot{q}_{r}\right) \Delta a_{k}+J s \tag{14}
\end{equation*}
$$

where $\dot{q}_{r}=\left[\omega_{b r}^{T}, \dot{q}_{m r}^{T}\right]^{T}, s=\left[s_{b}^{T}, s_{m}^{T}\right]^{T}, \Delta a_{k}=\hat{a}_{k}-a_{k}$, and $J=\left[\begin{array}{ll}J_{b} & J_{m}\end{array}\right]$. The formulation (13) and (14) is made by introducing a kinematic regressor matrix $Z\left(\epsilon_{b}, q_{m}, \dot{q}_{r}\right)$ that depends on the reference velocity $\dot{q}_{r}$ (in contrast to [12]), which extends the results in [26], [27], [17] to consider the case of a free-floating base.

The joint velocity command is given as

$$
\begin{align*}
\dot{q}_{m c}+\hat{\mathcal{K}}_{P} q_{m c}= & \dot{q}_{m r}+\hat{\mathcal{K}}_{P} q_{m r}+\operatorname{diag}\left[\hat{w}_{i}, i=1, \ldots, n\right] \\
& \times\left[Y_{m}\left(q_{m}, \dot{q}, \dot{q}_{r}, \ddot{q}_{r}\right) \hat{a}_{d}\right] \tag{15}
\end{align*}
$$

where

$$
\begin{equation*}
q_{m r}=q_{m r}(0)+\int_{0}^{t} \dot{q}_{m r}(\sigma) d \sigma \tag{16}
\end{equation*}
$$

$\hat{w}_{i}$ is the estimate of the inverse of the $i$-th diagonal entry of $K_{D}^{*}=K K_{D}$ which is denoted by $w_{i}, i=1, \ldots, n$, and $\mathcal{K}_{P}=\operatorname{diag}\left[\hat{w}_{P}\right]$ with $\hat{w}_{P}$ being the estimate of $w_{P} . w_{i}$ can be explicitly expressed as $w_{i}=\left(k_{i i} k_{D, i i}\right)^{-1}$ with $k_{i i}$ denoting the $i$-th diagonal entry of $K$ and $k_{D, i i}$ denoting the $i$-th diagonal entry of $K_{D}, i=1, \ldots, n$. The adaptation laws for the estimated parameters $\hat{a}_{k}, \hat{a}_{d}, \hat{w}=\left[\hat{w}_{1}, \ldots, \hat{w}_{n}\right]^{T}$, and $\hat{w}_{P}$ are given as

$$
\begin{align*}
& \dot{\hat{a}}_{k}=\Gamma_{k} Z^{T}\left(\epsilon_{b}, q_{m}, \dot{q}_{r}\right) \Delta x  \tag{17}\\
& \dot{\hat{a}}_{d}=-\Gamma_{d} Y^{T}\left(q_{m}, \dot{q}, \dot{q}_{r}, \ddot{q}_{r}\right) s  \tag{18}\\
& \dot{\hat{w}}=-\Lambda \operatorname{diag}\left[Y_{m}\left(q_{m}, \dot{q}, \dot{q}_{r}, \ddot{q}_{r}\right) \hat{a}_{d}\right] s_{m}  \tag{19}\\
& \dot{\hat{w}}_{P}=\Lambda_{P} \operatorname{diag}\left[q_{m c}-q_{m r}\right] s_{m} \tag{20}
\end{align*}
$$

where $\Gamma_{k}$ and $\Gamma_{d}$ are symmetric positive definite matrices, $\Lambda$ and $\Lambda_{P}$ are diagonal positive definite matrices, $s=$ $\left[s_{b}^{T}, s_{m}^{T}\right]^{T}$, and
$Y\left(q_{m}, \dot{q}, \dot{q}_{r}, \ddot{q}_{r}\right)=\left[Y_{b}^{T}\left(q_{m}, \dot{q}, \dot{q}_{r}, \ddot{q}_{r}\right) \quad Y_{m}^{T}\left(q_{m}, \dot{q}, \dot{q}_{r}, \ddot{q}_{r}\right)\right]^{T}$.

Substituting (8) into the upper portion of (3) and using Property 4 gives

$$
\begin{align*}
& M_{b b} \dot{s}_{b}+M_{b m} \dot{s}_{m}+C_{b b} s_{b}+C_{b m} s_{m} \\
& =-K_{b} s_{b}+Y_{b}\left(q_{m}, \dot{q}, \dot{q}_{r}, \ddot{q}_{r}\right) \Delta a_{d} \tag{21}
\end{align*}
$$

where $\Delta a_{d}=\hat{a}_{d}-a_{d}$. Combining (15), (7), and the lower portion of (3) gives (using Property 4)

$$
\begin{align*}
& M_{b m}^{T} \dot{s}_{b}+M_{m m} \dot{s}_{m}+C_{m b} s_{b}+C_{m m} s_{m} \\
& = \\
& \quad-\left(K_{D}^{*}+B\right) s_{m}-K_{P}^{*}\left[\int_{0}^{t} s_{m}(\sigma) d \sigma+\delta_{0}\right] \\
& \quad+K_{D}^{*} \operatorname{diag}[\Delta w] Y_{m}\left(q_{m}, \dot{q}, \dot{q}_{r}, \ddot{q}_{r}\right) \hat{a}_{d} \\
& \quad-K_{D}^{*} \operatorname{diag}\left[q_{m c}-q_{m r}\right] \Delta w_{P}  \tag{22}\\
& \quad+Y_{m}\left(q_{m}, \dot{q}, \dot{q}_{r}, \ddot{q}_{r}\right) \Delta a_{d}
\end{align*}
$$

where $\delta_{0}=q_{m}(0)-q_{m r}(0)$ is a constant vector, $\Delta w=$ $\hat{w}-w$ with $w=\left[w_{1}, \ldots, w_{n}\right]^{T}, \Delta w_{P}=\hat{w}_{P}-w_{P}$, and $K_{P}^{*}=K K_{P}$. Let us write (21) and (22) compactly as

$$
\begin{align*}
M & \left(q_{m}\right) \dot{s}+C\left(q_{m}, \dot{q}\right) s \\
= & -K^{*} s-\left[0_{3}^{T},\left[K_{P}^{*}\left(\int_{0}^{t} s_{m}(\sigma) d \sigma+\delta_{0}\right)\right]^{T}\right]^{T} \\
& +\left[0_{3}^{T},\left(K_{D}^{*} \operatorname{diag}[\Delta w] Y_{m}\left(q_{m}, \dot{q}, \dot{q}_{r}, \ddot{q}_{r}\right) \hat{a}_{d}\right)^{T}\right]^{T} \\
& -\left[0_{3}^{T},\left(K_{D}^{*} \operatorname{diag}\left[q_{m c}-q_{m r}\right] \Delta w_{P}\right)^{T}\right]^{T} \\
& +Y\left(q_{m}, \dot{q}, \dot{q}_{r}, \ddot{q}_{r}\right) \Delta a_{d} \tag{23}
\end{align*}
$$

where $K^{*}=\operatorname{diag}\left[K_{b}, K_{D}^{*}+B\right]$.
We are presently ready to formulate the following theorem.
Theorem 1: Suppose that $\hat{\mathcal{K}}_{P}$ is uniformly positive definite. Then the adaptive outer loop controller given by (15), (17), (18), (19), and (20) for the FFSM system given by (1) and (3) under the inner PD controller (6) ensures the convergence of the task-space tracking errors, i.e., $\Delta x \rightarrow 0$ and $\Delta \dot{x} \rightarrow 0$ as $t \rightarrow \infty$.

Proof: Consider the Lyapunov-like function candidate

$$
\begin{align*}
V_{1}= & \frac{1}{2} s^{T} M\left(q_{m}\right) s+\frac{1}{2}\left[\int_{0}^{t} s_{m}(\sigma) d \sigma+\delta_{0}\right]^{T} K_{P}^{*} \\
& \times\left[\int_{0}^{t} s_{m}(\sigma) d \sigma+\delta_{0}\right]+\frac{1}{2} \Delta w^{T} K_{D}^{*} \Lambda^{-1} \Delta w \\
& +\frac{1}{2} \Delta w_{P}^{T} K_{D}^{*} \Lambda_{P}^{-1} \Delta w_{P}+\frac{1}{2} \Delta a_{d}^{T} \Gamma_{d}^{-1} \Delta a_{d} \tag{24}
\end{align*}
$$

and its derivative along the trajectories of (23), (18), (19), and (20) can be written as (using Property 3)

$$
\begin{equation*}
\dot{V}_{1}=-s^{T} K^{*} s \leq 0 \tag{25}
\end{equation*}
$$

This leads us to immediately obtain that $s \in \mathcal{L}_{2} \cap \mathcal{L}_{\infty}$, $\int_{0}^{t} s_{m}(\sigma) d \sigma \in \mathcal{L}_{\infty}, \hat{w} \in \mathcal{L}_{\infty}, \hat{w}_{P} \in \mathcal{L}_{\infty}$, and $\hat{a}_{d} \in \mathcal{L}_{\infty}$.

Due to the well-recognized fact that $J$ is bounded, we obtain that $J s \in \mathcal{L}_{2}$. Hence, there must exist a positive constant $\ell_{M}$ such that $\int_{0}^{t} s^{T}(\sigma) J^{T}(\sigma) J(\sigma) s(\sigma) d \sigma \leq \ell_{M}$ for $\forall t \geq 0$. Consider the following quasi-Lyapunov function
candidate

$$
\begin{align*}
V_{2}= & \frac{1}{2} \Delta x^{T} \Delta x+\frac{1}{2} \Delta a_{k}^{T} \Gamma_{k}^{-1} \Delta a_{k} \\
& +\frac{1}{2 \alpha}\left[\ell_{M}-\int_{0}^{t} s^{T}(\sigma) J^{T}(\sigma) J(\sigma) s(\sigma) d \sigma\right] \tag{26}
\end{align*}
$$

where the use of the last term in $V_{2}$ follows the standard practice (see, e.g., [35, p. 118]). Differentiating $V_{2}$ with respect to time along the trajectories of (14) and (17) yields

$$
\begin{equation*}
\dot{V}_{2}=-\alpha \Delta x^{T} \Delta x+\Delta x^{T} J s-\frac{1}{2 \alpha} s^{T} J^{T} J s \tag{27}
\end{equation*}
$$

From the standard result concerning the basic inequalities, we have that

$$
\begin{equation*}
\Delta x^{T} J s \leq \frac{\alpha}{2} \Delta x^{T} \Delta x+\frac{1}{2 \alpha} s^{T} J^{T} J s \tag{28}
\end{equation*}
$$

using which, we obtain from (27) that

$$
\begin{equation*}
\dot{V}_{2} \leq-\frac{\alpha}{2} \Delta x^{T} \Delta x \leq 0 \tag{29}
\end{equation*}
$$

The result given by (29) immediately yields the conclusion that $\Delta x \in \mathcal{L}_{2} \cap \mathcal{L}_{\infty}$ and $\hat{a}_{k} \in \mathcal{L}_{\infty}$. From (10), we obtain that $\hat{J}_{b} \omega_{b r}+\hat{J}_{m} \dot{q}_{m r}=\dot{x}_{r} \in \mathcal{L}_{\infty}$. Due to the result that $\hat{J}_{b} s_{b}+\hat{J}_{m} s_{m} \in \mathcal{L}_{\infty}$, we then obtain that $\hat{\dot{x}}=\hat{J}_{b} \omega_{b}+\hat{J}_{m} \dot{q}_{m} \in$ $\mathcal{L}_{\infty}$ where $\hat{\dot{x}}$ is the estimate of $\dot{x}$. According to [6], the angular momentum conservation equation can be written as $R_{b}\left(M_{b b} \omega_{b}+M_{b m} \dot{q}_{m}\right)=0$ with $R_{b}$ being the attitude matrix of the spacecraft with respect to the inertial frame, and then we obtain that $\hat{\dot{x}}=\left(\hat{J}_{m}-\hat{J}_{b} M_{b b}^{-1} M_{b m}\right) \dot{q}_{m}$. If the matrix $\hat{J}_{m}-\hat{J}_{b} M_{b b}^{-1} M_{b m}$ is nonsingular, $\dot{q}_{m}=\left(\hat{J}_{m}-\right.$ $\left.\hat{J}_{b} M_{b b}^{-1} M_{b m}\right)^{-1} \hat{\dot{x}} \in \mathcal{L}_{\infty}$ and thus $\omega_{b} \in \mathcal{L}_{\infty}$ based on the above angular momentum conservation equation. From the kinematics (1), we obtain that $\dot{x} \in \mathcal{L}_{\infty}$ and hence $\Delta \dot{x} \in \mathcal{L}_{\infty}$ and $\ddot{x}_{r} \in \mathcal{L}_{\infty}$. Therefore, $\Delta x$ is uniformly continuous. From the properties of square-integrable and uniformly continuous functions [36, p. 232], we obtain that $\Delta x \rightarrow 0$ as $t \rightarrow \infty$.

The fact that $\dot{q} \in \mathcal{L}_{\infty}$ and $s \in \mathcal{L}_{\infty}$ implies that $\dot{q}_{r} \in \mathcal{L}_{\infty}$. Then, we obtain from (17) that $\dot{a}_{k} \in \mathcal{L}_{\infty}$, giving rise to the boundedness of $\dot{\hat{J}}_{b}$ and $\dot{\hat{J}}_{m}$. Equations (8) and (11) can be rewritten compactly as

$$
\underbrace{\left[\begin{array}{cc}
\hat{M}_{b b} & \hat{M}_{b m}  \tag{30}\\
\hat{J}_{b} & \hat{J}_{m}
\end{array}\right]}_{H}\left[\begin{array}{c}
\dot{\omega}_{b r} \\
\ddot{q}_{m r}
\end{array}\right]=\left[\begin{array}{c}
-\hat{C}_{b b} \omega_{b r}-\hat{C}_{b m} \dot{q}_{m r}+K_{b} s_{b} \\
\ddot{x}_{r}-\dot{\hat{J}}_{b} \omega_{b r}-\hat{\hat{J}}_{m} \dot{q}_{m r}
\end{array}\right]
$$

From the standard matrix theory, the invertibility of $H$ is equivalent to that of $H^{*}=\left[\begin{array}{cc}\hat{M}_{b b} & \hat{M}_{b m} \\ 0_{n \times 3} & \hat{J}_{m}-\hat{J}_{b} \hat{M}_{b b}^{-1} \hat{M}_{b m}\end{array}\right]$ and further that of $\hat{M}_{b b}$ and $\hat{J}_{m}-\hat{J}_{b} \hat{M}_{b b}^{-1} \hat{M}_{b m}$. Therefore, if $\hat{M}_{b b}$ and $\hat{J}_{m}-\hat{J}_{b} \hat{M}_{b b}^{-1} \hat{M}_{b m}$ are invertible, we obtain from (30) that $\dot{\omega}_{b r} \in \mathcal{L}_{\infty}$ and $\ddot{q}_{m r} \in \mathcal{L}_{\infty}$. Equation (15) can further be written as

$$
\begin{align*}
& \dot{q}_{m c}-\dot{q}_{m r}+\hat{\mathcal{K}}_{P}\left(q_{m c}-q_{m r}\right) \\
& \quad=\operatorname{diag}\left[\hat{w}_{i}, i=1, \ldots, n\right] Y_{m}\left(q_{m}, \dot{q}, \dot{q}_{r}, \ddot{q}_{r}\right) \hat{a}_{d} \tag{31}
\end{align*}
$$

and since $\hat{\mathcal{K}}_{P}$ is uniformly positive definite and bounded, we obtain that $q_{m c}-q_{m r} \in \mathcal{L}_{\infty}$ and $\dot{q}_{m c}-\dot{q}_{m r} \in \mathcal{L}_{\infty}$
from the standard linear system theory. On the other hand, $\int_{0}^{t} s_{m}(\sigma) d \sigma=q_{m}-q_{m}(0)-\left[q_{m r}-q_{m r}(0)\right] \in \mathcal{L}_{\infty}$ and therefore $q_{m}-q_{m r} \in \mathcal{L}_{\infty}$, which then implies that $q_{m}-q_{m c} \in \mathcal{L}_{\infty}$. Due to the result that $\dot{q}_{m r} \in \mathcal{L}_{\infty}$, we obtain that $\dot{q}_{m c} \in \mathcal{L}_{\infty}$. From (23) and using Property 2, we obtain that $\dot{s} \in \mathcal{L}_{\infty}$, and consequently, $\dot{\omega}_{b} \in \mathcal{L}_{\infty}$ and $\ddot{q}_{m} \in \mathcal{L}_{\infty}$. This immediately leads to the result that $\ddot{x} \in \mathcal{L}_{\infty}$ according to the differentiation of the kinematics (1). Thus, $\Delta \ddot{x} \in \mathcal{L}_{\infty}$, implying that $\Delta \dot{x}$ is uniformly continuous. According to Barbalat's Lemma [31], we obtain the result that $\Delta \dot{x} \rightarrow 0$ as $t \rightarrow \infty$.

Remark 1: The proposed controller requires that the estimated inertia matrix $\hat{M}_{b b}$, the estimated generalized Jacobian matrices $\hat{J}_{m}-\hat{J}_{b} M_{b b}^{-1} M_{b m}$ and $\hat{J}_{m}-\hat{J}_{b} \hat{M}_{b b}^{-1} \hat{M}_{b m}$ (i.e., the estimated versions of the generalized Jacobian matrix in, e.g., [4], [6]), and the estimated Jacobian matrix $\hat{J}_{m}$ are all invertible, which is the same as the case in [12]. The parameter projection algorithms [37] can be adopted to fulfill this requirement in the adaptation process (see, e.g., [38], [19], [20]). Furthermore, the condition that $\hat{\mathcal{K}}_{P}$ is uniformly positive definite can also be straightforwardly guaranteed by using the projection algorithms [37] since $\hat{\mathcal{K}}_{P}$ is diagonal.

## IV. Adaptive Outer Loop Control With an Inner PID Controller

In this section, we present the adaptive outer loop controller design as an inner PID controller is embedded in the space robotic system. In this case, the armature voltage $u$ takes the following form

$$
\begin{align*}
u= & -K_{D}\left(\dot{q}_{m}-\dot{q}_{m c}\right)-K_{P}\left(q_{m}-q_{m c}\right) \\
& -K_{I} \int_{0}^{t}\left[q_{m}(\sigma)-q_{m c}(\sigma)\right] d \sigma \tag{32}
\end{align*}
$$

where $K_{D}, K_{P}$, and $K_{I}$ are diagonal positive definite matrices. Due to the incorporation of the integral action, the previous outer loop controller generally cannot ensure the stability of the system. To this end, we introduce the following quantity based on the joint reference velocity given by (10)

$$
\begin{equation*}
\dot{q}_{m r}^{*}=\dot{q}_{m r}-K_{c}\left(q_{m}-q_{m r}\right) \tag{33}
\end{equation*}
$$

where $K_{c}$ is a diagonal positive definite matrix. The joint velocity command is now defined as

$$
\begin{align*}
& \dot{q}_{m c}+\hat{\mathcal{K}}_{P} q_{m c}+\hat{\mathcal{K}}_{I} \int_{0}^{t}\left[q_{m c}(\sigma)-q_{m r}(\sigma)\right] d \sigma \\
& =\dot{q}_{m r}^{*}+\hat{\mathcal{K}}_{P} q_{m r}+\operatorname{diag}\left[\hat{w}_{i}, i=1, \ldots, n\right] \\
& \quad \times\left[Y_{m}\left(q_{m}, \dot{q}, \dot{q}_{r}^{*}, \ddot{q}_{r}^{*}\right) \hat{a}_{d}\right] \tag{34}
\end{align*}
$$

where $\dot{q}_{r}^{*}=\left[\omega_{b r}^{T}, \dot{q}_{m r}^{* T}\right]^{T}, \ddot{q}_{r}^{*}$ is the derivative of $\dot{q}_{r}^{*}$, and $\hat{\mathcal{K}}_{I}=\operatorname{diag}\left[\hat{w}_{I}\right]$ with $\hat{w}_{I}$ being an $n$-dimensional vector. The
adaptation laws for $\hat{a}_{d}, \hat{w}, \hat{w}_{P}$, and $\hat{w}_{I}$ are given as

$$
\begin{align*}
& \dot{\hat{a}}_{d}=-\Gamma_{d} Y^{T}\left(q_{m}, \dot{q}, \dot{q}_{r}^{*}, \ddot{q}_{r}^{*}\right) \xi  \tag{35}\\
& \dot{\hat{w}}=-\Lambda \operatorname{diag}\left[Y_{m}\left(q_{m}, \dot{q}, \dot{q}_{r}^{*}, \ddot{q}_{r}^{*}\right) \hat{a}_{d}\right] \xi_{m}  \tag{36}\\
& \dot{\hat{w}}_{P}=\Lambda_{P} \operatorname{diag}\left[q_{m c}-q_{m r}\right] \xi_{m}  \tag{37}\\
& \dot{\hat{w}}_{I}=\Lambda_{I} \operatorname{diag}\left[\int_{0}^{t}\left[q_{m c}(\sigma)-q_{m r}(\sigma)\right] d \sigma\right] \xi_{m} \tag{38}
\end{align*}
$$

where $\xi_{m}=\dot{q}_{m}-\dot{q}_{m r}^{*}=s_{m}+K_{c}\left[\int_{0}^{t} s_{m}(\sigma) d \sigma+\delta_{0}\right], \xi=$ $\left[s_{b}^{T}, \xi_{m}^{T}\right]^{T}$, and $\Lambda_{I}$ is a diagonal positive definite matrix. The adaptation law for $\hat{a}_{k}$ remains the same as (17).

Theorem 2: Suppose that $K_{c}$ is chosen so that $\mathcal{M}=$ $\left(K_{D}+K^{-1} B\right) K_{c}+K_{P}-K_{I} K_{c}^{-1}$ is positive semidefinite and that the system

$$
\begin{equation*}
\ddot{z}^{*}+\hat{\mathcal{K}}_{P} \dot{z}^{*}+\hat{\mathcal{K}}_{I} z^{*}=0 \tag{39}
\end{equation*}
$$

with $z^{*} \in R^{n}$ is uniformly exponentially stable. Then the adaptive outer loop controller given by (34), (17), (35), (36), (37), and (38) for the FFSM system given by (1) and (3) under the inner PID controller (32) ensures the convergence of the task-space tracking errors, i.e., $\Delta x \rightarrow 0$ and $\Delta \dot{x} \rightarrow 0$ as $t \rightarrow \infty$.

Following similar steps as in [18] and in the proof of Theorem 1, we can complete that of Theorem 2.

## V. Simulation Results

In this section, we use the simulation example in [14] [i.e., a two-DOF (degree-of-freedom) space manipulator moving in a plane] to show the system performance under the proposed adaptive outer loop control. We here consider the case of using an inner PD controller. The sampling periods for the inner and outer loops are set as 0.5 ms (fast) and 20 ms (slow), respectively.

The gains for the inner PD controller are set as $K_{D}=$ $10.0 I_{2}$ and $K_{P}=20.0 I_{2}$, and the diagonal matrices $K$ and $B$ are set as $K=\operatorname{diag}[100.0,60.0]$ and $B=$ diag $[12.0,6.0]$, respectively. The controller parameters for the outer control loop are chosen as $K_{b}=200.0, \alpha=$ $15.0, \Lambda=0.001 I_{2}, \Lambda_{P}=500.0 I_{2}, \Gamma_{d}=120.0 I_{10}$, and $\Gamma_{k}=160.0 I_{4}$. The initial kinematic and dynamic parameter estimates are set as $\hat{a}_{k}(0)=[0.6,1.2,2.8,2.9]^{T}$ and $\hat{a}_{d}(0)=[1.8,1.8,1.8,1.8,1.8,155.0,50.0,10.0,0.0,0.0]^{T}$, and the initial values of $\hat{w}$ and $\hat{w}_{P}$ are set as $\hat{w}(0)=$ $[0,0]^{T}$ and $\hat{w}_{P}(0)=[1.0,1.0]^{T}$, respectively. The desired trajectory in the task space is given as $x_{d}=[2.85+$ $0.25 \cos (0.8 \pi t),-0.38+0.25 \sin (0.8 \pi t)]^{T}$. The simulation results are shown in Fig. 1 and Fig. 2.

## VI. Conclusion

The purpose of our study here is to develop task-space adaptive outer loop controllers for free-floating space manipulators with uncertain kinematics and dynamics so as to approach the objective of dynamic modularity for space robotic systems. The proposed outer loop controllers are in the form of joint velocity (or position) command, which is


Fig. 1. Position tracking errors.


Fig. 2. Joint velocity commands.
dynamically and adaptively generated to address both the dynamic effects of the system and the parametric uncertainties, and the inner joint servoing controller is assumed to take the form of PD or PID control. It is shown that the taskspace tracking errors converge to zero asymptotically. The performance of the proposed control approach is shown by numerical simulation results.

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