WA-ICP Algorithm for Tackling Ambiguous Correspondence

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Abstract

This paper proposes a weighted average iterative closest point (ICP) algorithm (WA-ICP) to register multiple 3D surfaces. The new algorithm aims to tackle ambiguous correspondence and clearly improves the efficiency and reliability of 3D surface registration. We developed the WA-ICP theory, implemented the algorithm and evaluated its performance on two typical groups of surface samples. The performance of our new method is compared with both traditional ICP and weighted ICP. We also integrate a more elaborate scheme using normals for comparison. Our experiments show that WA-ICP has a good quality of convergence rate especially in the early iterations. The algorithm is also validated to be robust to low resolution and noise perturbation of 3D surfaces. Despite its simple form, the proposed WA-ICP clearly shows improved performance and some robust characteristics.

1. Introduction

Registration is a widely applied technique in the field of computer vision and a fundamental process for applications such as 3D reconstruction and augmented reality [1]. The goal of surface registration is to estimate a 3D transformation so that two surfaces can be placed in the same coordinate system with correspondences optimally matched. We name one of the two surfaces as model, which is the target object of aligning, while the other as data, which is expected to be registered to the model. ICP algorithm [2] is one of the most popular techniques for registration between 3D rigid surfaces which are represented by point sets. [3] offers a recent survey on ICP algorithm. ICP can be simply seen as a two-stage problem of correspondence estimation and rigid transformation calculation.

In this paper, we propose a new weighted ICP algorithm, dubbed WA-ICP to solve ambiguous correspondence which means a pair of matching points does not represent the same point in the real world. The main contribution of our work is that we use a new weighting approach to establish correspondence. We also test WA-ICP on specific point sets. The new method clearly shows improved performance and some robust characteristics.

2. Related work

Traditional ICP only utilizes the geometric information of the point sets and does not require any feature extraction. The simplicity of the algorithm makes it the dominant method for 3D point sets registration. We will give brief introduction to ICP algorithm and its variants, since the concept of ICP plays an important role within the new method we propose in this paper.

2.1. ICP algorithm

ICP tries to find the rigid transformation between two surfaces, which can be represented as a 3×3 rotation matrix $\mathbf{R}$ and a 3×1 translation vector $\vec{t}$. Let $\vec{x}_i$ be a point from the data point set $\{\vec{x}_i\}, i = 1,\ldots,N_x$ and its corresponding point, which is found based on closest distance, is the one in the model point set $\{\vec{y}_j\}, j = 1,\ldots,N_y$ with the same index. The error metric is defined as the sum of squared distance between all correspondences, as shown in Eq. (1). Then an optimization method is applied to minimize the error function. The obtained transformation through optimization in turn acts on the data points and the same procedure is repeated in the next round. The process iterates until termination conditions are met.

$$\text{err} = \sum_{i=1}^{N_x} \left\| \vec{y}_i - \mathbf{R}\vec{x}_i - \vec{t} \right\|^2 \tag{1}$$

Besl and Neil [2] proved that ICP algorithm could at least converge to local minimum. The optimization can be achieved by two mature closed-form methods which are singular value decomposition (SVD) approach [4] and quaternion-based approach [2].

2.2. ICP extensions

ICP algorithm is a fine registration method whose performance depends strongly on the initial pose estimation of the two surfaces. Many so called coarse registration aim at getting good initial states to avoid ICP going to local minima. [5] provides a good summary of the coarse registration. ICP has been evolving over time to adapt to

978-1-4799-6100-9/15/$31.00 ©2015 IEEE

extensive situations and get improvements in speed and accuracy. Rusinkiewicz and Levoy [6] introduced a detailed classification of ICP variants. Establishing correspondence is the most important phase due to the fact that it not only dominates the total elapsed time but also affects the final aligning accuracy.

Traditional ICP algorithm uses the closest Euclidean distance to establish correspondences and it takes these correspondences into account equally in the error function. In order to improve accuracy and robustness of registration, some modified strategies are proposed. Instead of merely using the positions to establish correspondence, several studies attempt to integrate additional information, such as normals, curvatures and colors, to find more reliable matching pair [7].

Further, the original error function (1) taking each correspondence equally is not sufficiently reliable and it may increase the effect of low-quality correspondences on the error metric which will result in poor accuracy of registration. Therefore, the idea to assign different weights for correspondences is proposed [8]. Similar to the notifications in traditional ICP, the modified algorithm optimizes Eq. (2) which intuitively introduces weight w_{ij} to measure the certainty in the contribution of each correspondence to the error metric. Rusinkiewicz and Levoy [6] summarized several weighting functions and concluded that the effect of weights on convergence rate is small. Besides, the modified algorithm could show the property to decrease the risk to converge to local minima [9]. A framework based on evolutionary game theory is proposed to get reasonable weights for correspondences [10]. In our paper, we apply this approach for obtaining reliable weights of weighted ICP. SoftAssign [11] and EM-ICP [12] are the extensions of Eq. (2) and they all lead to the enhancement of registration.

$$\mathrm{err} = \sum_{j=1}^{N_y} \sum_{i=1}^{N_x} w_{ij} \left\| \vec{y}_j - \mathbf{R}\vec{x}_i - \vec{t} \right\|^2 \qquad (2)$$

In the next section, we will focus on more details about the interpretation of WA-ICP we propose.

3. Methods

WA-ICP algorithm will be introduced and explored thoroughly in the following subsections.

3.1. Basic ideas of WA-ICP

Traditional ICP executes under the assumption that a pair of correspondence will be the same in the final registered surface. However, it is hard or even impossible to find the exact correspondence due to inhomogeneous sources of surface data in practice. For example, 3D scans cannot guarantee the same physical point will be captured in different views. Furthermore, point sets could also be obtained from different imaging modalities, like the same organ surface reconstructions from both 3D endoscope and preoperative CT segmentation in computer-aided surgery [13]. These cases cause correspondence ambiguity which can be illustrated in Figure 1. Each data point (in red) does not exactly correspond to a model point (in blue) after performing ICP. The existence of ambiguous correspondence has not gotten sufficient attention before and it is easy to trap registration into local optima, especially when the two point sets have significantly different resolutions.

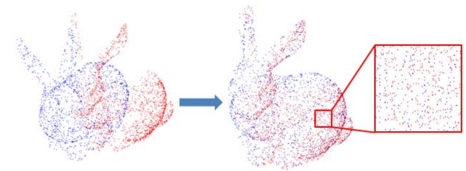


Figure 1. An illustration of ambiguous correspondences in original registration.

Weighted ICP is not going to solve ambiguous correspondence. It also potentially considers one data point to match to one model point exactly as correspondence but assign a probability to it. Inspired by weighted ICP, we come up with an idea to properly deal with the ambiguity by modifying correspondence calculation in ICP. Our goal is to find the corresponding point to a given data point as accurately as possible. Since we could only obtain discrete points rather than complete mesh model, the most naive approach is to use weighted average of the entire model points to calculate corresponding point. This process could be simply illustrated in Figure 2, where the corresponding point is not any of the three points in model set but is defined as the combination of them. Therefore, the error function should be constructed as in Eq. (3).

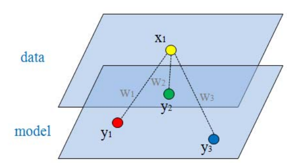


Figure 2. An illustration of obtaining corresponding point to a given data point in WA-ICP.

$$\mathrm{err} = \sum_{i=1}^{N_x} \left\| \left(\sum_{j=1}^{N_y} w_{ij}\vec{y}_j \Big/ \sum_{j=1}^{N_y} w_{ij} \right) - \mathbf{R}\vec{x}_i - \vec{t} \right\|^2 \qquad (3)$$

3.2. Interpretation of WA-ICP

The theoretical basis for our method can be seen as obtaining the best corresponding point via interpolation. Although it is hard to prove the convergence of the proposed algorithm, we can switch to traditional ICP whose convergence has been proven if the value of Eq. (3) is greater than the previous iteration. The above strategy is going to make sure error function (3) never increases. The elapsed time of WA-ICP is comparable to the original ICP because both of them need to compute distance from a data point to all model points and the similar strategies could be adopted to accelerate them. Despite the fact that lots of methods could speed up k nearest neighborhood searching, we find that using weighted average of the nearest k model points in WA-ICP could not reveal significant improvement, so we do not adopt it in this paper.

Although Eq. (3) is similar to Eq. (2), it is actually different. Setting $w_{ij} / \sum_{j=1}^{N_y} w_{ij} = w'_{ij}$, we get $\sum_{j=1}^{N_y} w'_{ij} = 1$. Eq. (3) can be rewritten as

$$\sum_{i=1}^{N_x} \left\| \sum_{j=1}^{N_y} w'_{ij} \bar{y}_j - \sum_{j=1}^{N_y} w'_{ij} \left(\mathbf{R} \bar{x}_i + \bar{t} \right) \right\|^2 = \sum_{i=1}^{N_x} \left\| \sum_{j=1}^{N_y} w'_{ij} \left(\bar{y}_j - \mathbf{R} \bar{x}_i - \bar{t} \right) \right\|^2$$

$$= \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} w'^2_{ij} \left\| \bar{y}_j - \mathbf{R} \bar{x}_i - \bar{t} \right\|^2 + 2 \sum_{i=1}^{N_x} \sum_{\substack{j,k=1 \\ j \neq k}}^{N_y} w'_{ij} \left(\bar{y}_j - \mathbf{R} \bar{x}_i - \bar{t} \right) \cdot w'_{ik} \left(\bar{y}_k - \mathbf{R} \bar{x}_i - \bar{t} \right)$$

As can be seen from the above derivation, the difference between weighted ICP and WA-ICP is that the later one has cross terms. These cross terms will contribute to the improved performance which will be validated with experiments and comparisons in the next section.

3.3. Discussion about weighting functions

In principle, the value of weight turns to be inversely correlated to the spatial distance between data and model point. Therefore, we consider weighting strategies as shown from Eq. (4) to Eq. (6) in which Euclidean distance is used. The choice of weighting function and parameters is crucial to the performance of WA-ICP. In the experimental section, we found Eq. (4) is more widely applicable, so we use Eq. (4) and drop the others in all the experiments. Furthermore, setting α to 4 performs well to our experiments.

$$\text{weight} = \frac{1}{\text{dist}^{\alpha}} \tag{4}$$

$$\text{weight} = e^{-\alpha \cdot \text{dist}^{\beta}} \tag{5}$$

$$\text{weight} = 1 - \frac{\text{dist}}{\text{dist}_{\max}} \tag{6}$$

Let us briefly conclude the characteristics of WA-ICP below: (1) using weighted average of model points to obtain corresponding point to each data point; (2) the optimal form of weighting function is inversely proportional to the spatial distance between data and model point. In the next section, we will give experimental results of WA-ICP and compare its performance with other existing methods.

4. Experiments

In this section, we design experiments to validate the efficiency of the proposed algorithm. First, we examine the effect of WA-ICP in tackling ambiguous correspondence. Further, we compare WA-ICP with other algorithms and study the robustness of WA-ICP.

4.1. Experimental setup

As the performance of registration is data-dependent, there are no unified point sets to test algorithms. We use two groups of point sets which can cover most of the cases that ICP algorithm may apply to. Each point set contains about 2000 points which are sampled from the famous Stanford Bunny [14] and transformed within the convergence basin. An assumption that convergence could be achieved without coarse registration is made in our experiments, so that we can just focus on fine registration. We perform two experiments which are partial surface registration and whole surface registration. Their initial states are shown in Figure 4(a) and Figure 5(a) respectively.

We employ the Root-Mean-Square (RMS) error which is defined in Eq. (7) for evaluation. The correspondence is found based on the closest spatial distance to assess all the different methods in a common measurement. We use our own MATLAB-based implementation of the algorithms in all the experiments.

$$\text{err} = \sqrt{\frac{\sum_{i=1}^{N_x} \left\| \bar{y}_i - \bar{x}_i \right\|^2}{N_x}} \tag{7}$$

4.2. Effect of WA-ICP

After performing partial registration, the visualization of correspondences in ICP and WA-ICP are shown in Figure 3(a) and (b) respectively. In these figures, red points denote transformed data points and black points are the corresponding points established using closest model point in ICP while weighted average of model points in WA-ICP. It is easy to notice that the average distance between transformed data points and their corresponding points in WA-ICP is smaller compared with the case in ICP. The results clearly indicate the new method can find better corresponding points and is effective in solving ambiguous problem.

The reason why weighting function (4) could outperform other forms is that its shape is compatible with the process of registration. Big distance between two surfaces in the early iterations of registration results in weighting function locating in relatively gentle area. The value of weights to a data point presented in Figure 3(c) means that most model points contribute to the calculation of corresponding point. On the contrary, when the two surfaces are quite close in the final iterations, weighting function (4) is located near zero and its steep shape makes the choice of model points more selective. The value of weights presented in Figure 3(d) means using only minority of model points to obtain correspondence which is quite close to traditional ICP.

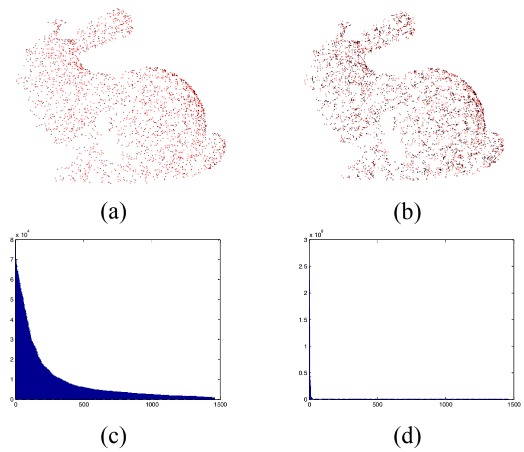


Figure 3. Visualization of correspondences and ordered weights. (a) Correspondences after performing ICP. (b) Correspondences after performing WA-ICP. (c) All the ordered model weights to a data point in the first iteration of WA-ICP. (d) All the ordered model weights to a data point in the final iteration of WA-ICP.

4.3. Comparison of registration algorithms

WA-ICP is compared with other algorithms in this subsection. Besides traditional ICP, strategy using normals in conjunction with positions to obtain closest model point is also adopted. Albarelli et al. [10] proposed the framework which used evolutionary game theory to obtain the weights for correspondences. We follow this idea in our own weighted ICP, because it surpasses other types of weights in accuracy. Matching candidates are established using only positions and normals rather than feature descriptors. Traditional ICP, ICP integrating normals (N-ICP), weighted ICP using game framework to compute weights (W-ICP) and the new WA-ICP are compared in our experiments. After the same number of iterations, the experimental results of partial registration and whole registration are given in Figure 4 and Figure 5 respectively. We sum up the experimental results in Table 1 for clear comparison. But we do not report results when the algorithms fail to register.

We can see from the figures that the convergence rate of WA-ICP is relatively fast. This apparently happens during the early stages of the iteration when the error declines rapidly. This indicates that the number of iterations required to converge is relatively reduced. The reason why WA-ICP could reveal this property is that the method takes more model points into consideration. More points will definitely have strong influence on the calculation of translation especially when the two surfaces are far apart during the early stages of registration. Therefore, WA-ICP shows a good quality to pull together the two far-apart clouds quickly.

Although W-ICP in partial registration and N-ICP in whole registration could also reveal a better performance, they result in higher computational cost due to the process of computing normals and performing game theory. Besides, WA-ICP can achieve convergence on both groups without losing its properties, which indicates that it can apply to broad situations.

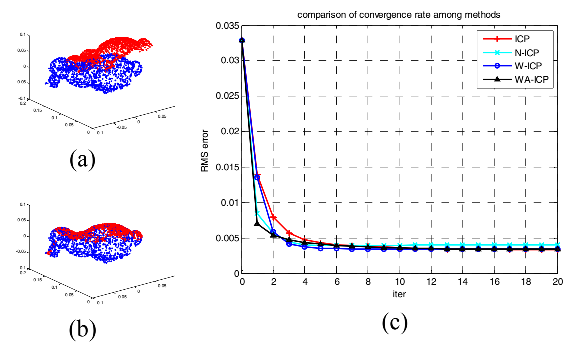


Figure 4. Results for partial surface registration. (a) Initial states before registration. (b) Finally converged states after registration. (c) Comparison of convergence rate.

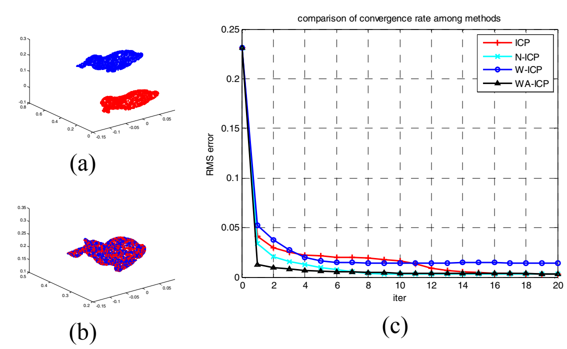


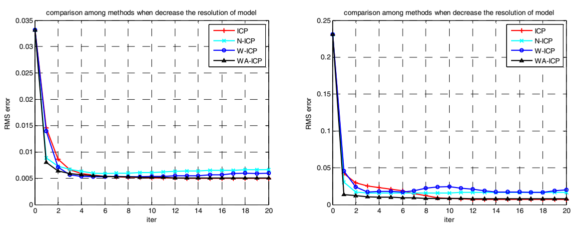
Figure 5. Results for whole surface registration. (a) Initial states before registration. (b) Finally converged states after registration. (c) Comparison of convergence rate.

Table 1. Summary of comparison among four registration methods.

	Partial surface registration				Whole surface registration			
Comparing items	ICP	N-ICP	W-ICP	WA-ICP	ICP	N-ICP	W-ICP	WA-ICP
Final RMS error	0.0034	0.0040	0.0034	0.0034	0.0033	0.0032	0.0142	0.0036
First error decline in percentage	57.4%	74.2%	58.7%	78.7%	82.1%	85.2%	77.3%	94.5%
Number of iterations needed	13	/	6	11	19	10	/	14

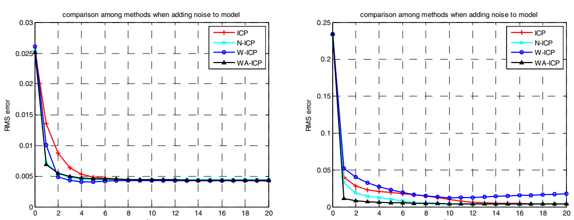
4.4. Robustness of algorithms

Robustness we are going to evaluate here is the ability of algorithm to resist change of input data without losing its characteristics and performance. The experiments we design to test the robustness are stated below. Robustness is demonstrated through reducing resolution and adding noise to model sets. By means of random down sampling, we reduce the first model set to half while the second to a quarter. Another test is to perturb both model sets through adding noise which obeys uniformed distribution with a standard deviation of 0.01 units in each direction. The experimental results on models reducing resolution and adding noise are shown in Figure 6 and Figure 7.



(a) Partial registration (b) Whole registration

Figure 6. Comparison of registering results for reducing resolutions of model set.



(a) Partial registration (b) Whole registration

Figure 7. Comparison of registering results for perturbing model set by noise.

Although the converged value of RMS error influenced by the changed point sets is increased, we can see that the faster error decline in the early stages is not affected by the change of model set. The reason is that WA-ICP using all the model points to compute corresponding point will weaken the influence caused by the change of model sets. The results indicate that WA-ICP has some robust properties that are not sensitive to the change of model set by reducing resolution and adding noise.

5. Conclusions

In this paper we present an improved version of ICP algorithm whose innovation point is to compute correspondence through weighted average of the entire model points. The optimal form of weighting function in our experiments is inversely proportional to the distance. We use mathematical interpolation to explain the essence of the algorithm.

Furthermore, we use two groups of point sets to validate our algorithm and compare the results with traditional ICP and weighted ICP. Due to the ambiguous nature of the correspondences, our method may be a more appropriate approach than other existing algorithms. WA-ICP can improve the rate of convergence and it is robust towards resolution reduction and noise perturbation.

References

[1] J. Geng, "Structured-light 3D surface imaging: a tutorial," Advances in Optics and Photonics, 3(2):128-160, 2011.

[2] P. J. Besl and N. D. McKay, "A Method for Registration of 3-D Shapes," IEEE Transactions on Pattern Analysis and Machine Intelligence, 14(2):239-256, 1992.

[3] B. Bellekens et al., "A Survey of Rigid 3D Pointcloud Registration Algorithm," in Fourth International Conference on Ambient Computing, Applications, Services and Technologies, Rome, Italy, 2014.

[4] K. S. Arun, T. S. Huang and S. D. Blostein, "Least-Squares Fitting of Two 3-D Point Sets," IEEE Transactions on Pattern Analysis and Machine Intelligence, PAMI-9(5):698-700, 1987.

[5] Y. Díez et al., "A Qualitative Review on 3D Coarse Registration Methods," ACM Computing Surveys(CSUR), 47(3), 2015.

[6] S. Rusinkiewicz and M. Levoy, "Efficient Variants of the ICP Algorithm," In Third International Conference on 3-D Digital Imaging and Modeling, 145-152, 2001.

[7] G. C. Sharp, S. W. Lee and D. K. Wehe, "ICP registration using invariant features," IEEE Transactions on Pattern Analysis and Machine Intelligence, 24(1): 90-102, 2002.

[8] U. Castellani and A. Bartoli, "3D Shape Registration," in 3D Imaging, Analysis and Applications, 221-264, Springer, 2012.

[9] A. Marinov et al., "Weighted ICP Algorithm for Alignment of Stars from Scanned Astronomical Photographic Plates," Serdica Journal of Computing, 6(1):101-110, 2012.

[10] A. Albarelli, E. Rodolà and A. Torsello, "Fast and accurate surface alignment through an isometry-enforcing game," Pattern Recognition, 48(7):2209-2226, 2015.

[11] S. Gold et al., "New algorithms for 2D and 3D point matching: pose estimation and correspondence," Pattern Recognition, 31(8):1019-1031, 1998.

[12] S. Granger and X. Pennec, "Multi-scale EM-ICP: A Fast and Robust Approach for Surface Registration," in Seventh European Conference on Computer Vision, 2353:418-432, 2002.

[13] L. M. Su et al., "Augmented Reality During Robot-assisted Laparoscopic Partial Nephrectomy: Toward Real-Time 3D-CT to Stereoscopic Video Registration," Urology, 73(4):896-900, 2009.

[14] G. Turk and M. Levoy, "Zippered polygon meshes from range images," in 21st annual conference on Computer graphics and interactive techniques, 311-318, 1994.