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Adaptive tracking control of uncertain switched stochastic nonlinear systems

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Abstract In this paper, a tracking control problem is investigated for a class of switched stochastic nonlinear uncertain systems with unknown dead-zone input. By using the common Lyapunov function method and backstepping technique, a common controller and an uniform adaptive mechanism are constructed, and a novel adaptive fuzzy control scheme is developed. It is proved that the proposed control method can guarantee that all the signals in the closed-loop system are bounded in probability and the tracking error is convergent to a neighborhood of the origin under arbitrary switching. Finally, simulation results are provided to show the effectiveness of the proposed control scheme.

Keywords Adaptive fuzzy control · Backstepping technique · Stochastic switched nonlinear systems · Dead-zone nonlinearities

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1 Introduction

Over the last decades, switched systems have drawn considerable attention, due to their wide application in engineering practice such as circuit and power systems, aircraft control systems, robot manipulators and multi-agent systems [1–4]. Switched systems exhibit switching among a set of subsystems according to changing environmental factors. As stated in [5], a common Lyapunov function (CLF) approach is an effective tool for the stability analysis of switched systems under arbitrary switching. By using a CLF method, some remarkable achievements have been obtained for switched linear systems (see [6–10]). At the same time, by employing backstepping technique, several interesting results for switched nonlinear systems in strict-feedback form have been reported in [11–13]. However, the nonlinear functions in the above-mentioned switched systems are required to be completely known.

To control unknown nonlinear systems, the fuzzy logic system (FLS) and neural network (NN) are often used to approximate unknown nonlinear functions of the controlled systems. It has been proved in [14,15] that the FLS and NN can approximate any nonlinear continuous function on a closed set. By combining the universal approximation of FLS (or NN) and the adaptive control method, some remarkable adaptive fuzzy or neural schemes were proposed for uncertain nonlinear systems in triangular form without switchings in [16–27]. Recently, by combining the CLF method, the above adaptive backstepping approaches have been developed for switched nonlinear uncertain systems and many excellent results have been obtained for uncertain nonlinear systems in triangular form under arbitrary switching (see [28–31]). Unfortunately, the aforementioned works do not consider the effect of stochastic disturbance on switched systems. Now, there are very few results for switched stochastic nonlinear systems. The stabilization problem of switched stochastic nonlinear systems was investigated in [32]. Although some progress has been made, the nonlinear terms and the stochastic disturbance terms of switched stochastic systems are required to be completely known. If such a prior knowledge of switched stochastic systems is not available, the approach becomes infeasible. In addition, the work in [32] does not take into consideration the effect of dead zone on system.

It is generally known that dead zone is one of the most important nonsmooth nonlinearities in practical applications. For example, the stiction and dry friction usually bring dead-zone effects in electromechanical systems [33]. The presence of dead zone in actuator may deteriorate the system's performances and lead to the instability of the system if it is ignored [34]. To reduce the dead-zone effect on the system, two approaches are often adopted. The first one is to utilize the inverse of the dead zone to minimize the effects of the dead zone [34]. The second one is to model the dead zone as a combination of a linear term and a disturbance-like term (see, [35,36]). It should be noted that the existing results on dead zone are confined to the nonswitched systems, and the switched systems with dead zone are not addressed.

Based on the above discussion, this article aims to solve the tracking problem of switched stochastic uncertain nonlinear systems in strict-feedback form with dead-zone input under arbitrary switching. In this paper, the fuzzy logic systems are adopted to identify the unknown stochastic nonlinear system. By utilizing a CLF approach and the backstepping technique, an adaptive fuzzy control scheme is developed. Compared with the existing results, the main contributions of this paper are summarized as follows.

1. This paper investigates the tracking problem of more general switched nonlinear systems, i.e., switched stochastic nonlinear uncertain systems with unknown dead-zone input. This is different from the existing works on switched nonlinear systems, where the random disturbance and dead-zone input are ignored in [28–31], and the uncertainty and actuator nonlinearity are not taken into account in [32].
2. By constructing a common virtual control function and an uniform adaptive mechanism at each step of the backstepping design, a novel adaptive fuzzy control scheme is proposed. By constructing the dead-zone compensation, the proposed control scheme can guarantee the control performance of the switched stochastic system and reduce the influence of the dead zone on the control performance under arbitrary switching.

The rest of this paper is organized as follows. The preliminaries and problem formulation are described in Sect. 2. A novel adaptive fuzzy control scheme is presented in Sect. 3. A simulation example is given to show the effectiveness of the main result in Sect. 4. Finally, a conclusion is made in Sect. 5.

2 Preliminaries and problem statement

2.1 Stochastic stability

Consider the following stochastic nonlinear system:

$$\mathrm{d}x = f(x,t)\mathrm{d}t + h(x,t)\mathrm{d}w, \tag{1}$$

where $x \in R^n$ stands for the state variable, $f : R^n \times R^+ \to R^n$, $h : R^n \times R^+ \to R^{n\times r}$ are continuous functions, w represents an independent $r-$dimension standard Brownian motion defined on the complete probability space $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ with Ω representing a sample space, F being a $\sigma-$field, $\{F_t\}_{t\geq 0}$ representing a filtration and P representing a probability measure.

Definition 1 ([37]) For twice continuously differentiable function $V(x,t)$, define a differential operator L as follows:

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2} Tr\left\{ h^T \frac{\partial^2 V}{\partial x^2} h \right\}, \tag{2}$$

where Tr represents a trace of the matrix.

Remark 1 The second-order differential $\frac{\partial^2 V}{\partial x^2}$ in It$\hat{o}$ correction term $\frac{1}{2} Tr\{h^T \frac{\partial^2 V}{\partial x^2} h\}$ will make the construction of common virtual control functions and uniform adaptive mechanisms for uncertain switched stochastic nonlinear systems much more difficult than that of the switched deterministic systems.

Lemma 1 ([22]) *Suppose that there exist a function* $V(x,t) \in C^{2,1}$*, two positive constants* c *and* b*,* $\kappa_\infty-$*functions* α_1 *and* α_2*, such that*

$$\begin{cases} \alpha_1(\|x\|) \le V(x,t) \le \alpha_2(\|x\|) \\ LV \le -cV(x,t) + b, \end{cases} \tag{3}$$

for $\forall x \in R^n$ *and* $\forall t > 0$*. Then, there exists an unique strong solution of system (1) for each* $x_0 \in R^n$ *and the system is bounded in probability.*

Lemma 2 (Young's inequality [38]) *For* $\forall(x,y) \in R^2$*, the following inequality holds*:

$$xy \le \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\varepsilon^q}|y|^q,$$

where $\varepsilon > 0$*,* $p > 1, q > 1$*, and* $(p-1)(q-1) = 1$*.*

Lemma 3 ([39]) *Consider the dynamic system of the form*:

$$\dot{\hat{\theta}}(t) = -\gamma\hat{\theta}(t) + \kappa\rho(t), \tag{4}$$

where γ *and* κ *are positive constants and* $\rho(t)$ *is a positive function, and then, for* $\forall t \ge t_0$ *and any given bounded initial condition* $\hat{\theta}(t_0) \ge 0$*,* $\theta(t) \ge 0$*.*

2.2 Problem formulation

Consider the following switched stochastic nonlinear uncertain systems with dead-zone input:

$$\begin{cases} \mathrm{d}x_i = (x_{i+1} + f_{i,\sigma(t)}(\bar{x}_i))\mathrm{d}t + g^T_{i,\sigma(t)}(\bar{x}_i)\mathrm{d}w, \quad 1 \le i \le n-1, \\ \mathrm{d}x_n = (u_{\sigma(t)} + f_{n,\sigma(t)}(\bar{x}_n))\mathrm{d}t + g^T_{n,\sigma(t)}(\bar{x}_n)\mathrm{d}w, \\ y = x_1. \end{cases} \tag{5}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i (i = 1, 2, \ldots, n)$ denotes the state vector and $y \in R$ is system output. $\sigma(t) : [0, \infty) \to M = \{1, 2, \ldots, m\}$ stands for a piecewise continuous switching signal. $\sigma(t) = k(k \in M)$ implies that the kth subsystem is active. w is defined in (1). $f_{i,k}(.) : R^i \to R$ and $g_{i,k}(.) : R^i \to R^r (i = 1, 2, \ldots, n)$ are unknown smooth nonlinear functions. $u_{\sigma(t)} \in R$ represents the output of the dead zone, which can be expressed as the following form:

$$u_{\sigma(t)} = \begin{cases} m_{r,\sigma(t)}(v_{\sigma(t)} - b_{r,\sigma(t)}), & v_{\sigma(t)} \ge b_{r,\sigma(t)} \\ 0, \quad b_{l,\sigma(t)} < v_{\sigma(t)} < b_{r,\sigma(t)} \\ m_{l,\sigma(t)}(v_{\sigma(t)} - b_{l,\sigma(t)}), & v_{\sigma(t)} \le b_{l,\sigma(t)}. \end{cases} \tag{6}$$

where $v_{\sigma(t)}$ represents the input of the dead-zone characteristic; $m_{r,\sigma(t)}$ and $m_{l,\sigma(t)}$ are the right slope and the left slope of the dead zone; $b_{l,\sigma(t)}$ and $b_{r,\sigma(t)}$ stand for the breakpoints of the input nonlinearity.

Remark 2 If the stochastic disturbance terms $g^T_{i,\sigma(t)}(\bar{x}_i)dw$ and the dead zone in (6) are ignored, then the switched systems (5) are the same to the plants investigated in [28–31]. If the functions $f_{i,\sigma(t)}(\bar{x}_i))$ and $g^T_{i,\sigma(t)}(\bar{x}_i)$ are known and the dead zone is not considered, the switched systems (5) are the same to the controlled system in [32]. Therefore, the switched systems considered in this manuscript are more general.

In this section, the following assumption is required.

Assumption 1 Parameters $m_{r,\sigma(t)}$, $m_{l,\sigma(t)}$ are unknown positive constants. There exist positive constants b_m and b_M such that $0 < b_m \le \min\{m_{l,\sigma(t)}, m_{r,\sigma(t)} \mid \sigma(t) \in M\} \le \max\{m_{l,\sigma(t)}, m_{r,\sigma(t)} \mid \sigma(t) \in M\} \le b_M$.

According to [35] and [36], the output of the dead zone (6) can be expressed as the following form:

$$u_{\sigma(t)} = m_{\sigma(t)}v_{\sigma(t)} + \bar{d}_{\sigma(t)} \tag{7}$$

where

$$m_{\sigma(t)} = \begin{cases} m_{r,\sigma(t)}, & v_{\sigma(t)} > 0, \\ m_{l,\sigma(t)}, & v_{\sigma(t)} \le 0. \end{cases} \tag{8}$$

$$\bar{d}_{\sigma(t)} = \begin{cases} -m_{r,\sigma(t)}b_{r,\sigma(t)}, & v_{\sigma(t)} \ge b_{r,\sigma(t)} \\ -m_{\sigma(t)}v_{\sigma(t)}, & b_{l,\sigma(t)} < v_{\sigma(t)} < b_{r,\sigma(t)} \\ -m_{l,\sigma(t)}b_{l,\sigma(t)}, & v_{\sigma(t)} \le b_{l,\sigma(t)}. \end{cases} \tag{9}$$

According to Assumption 1 and (9), we get

$$|\bar{d}_{\sigma(t)}| \le \bar{d}^* \tag{10}$$

where $\bar{d}^* = b_M \max_{\sigma(t)\in M}\{|b_{r,\sigma(t)}|, |b_{l,\sigma(t)}|\}$ is a positive constant.

2.3 Fuzzy logic systems

In this paper, a fuzzy logic system will be used to approximate a continuous function $f(x)$ defined on some compact set Ω. Adopt the singleton fuzzifier, the product inference and the center-average defuzzifier to deduce the following fuzzy rules:

R^l : If x_1 is F_1^l and ... and x_n is F_n^l,
Then y is G^l, $l = 1, 2, \ldots, N$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ and $y \in R$ are the input and the output of the fuzzy system, respectively, F_i^l and G^l are fuzzy sets in R, and N is the number of the rules. By employing the singleton function, the center-average defuzzification and the product inference [14], the output of the fuzzy system is

$$y(x) = \frac{\sum_{l=1}^{N} \Phi_l \prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} [\prod_{i=1}^{n} \mu_{F_i^l}(x_i)]}$$

where

$$\Phi_l = \max_{y \in R} \mu_{G^l}(y), \quad \Phi = (\Phi_1, \Phi_2, \ldots, \Phi_N)^T.$$

Let

$$\xi_l(x) = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} [\prod_{i=1}^{n} \mu_{F_i^l}(x_i)]}$$

and $\xi(x) = (\xi_1(x), \xi_2(x), \ldots, \xi_N(x))^T$. Furthermore, the fuzzy logic system can be rewritten as

$$y(x) = \Phi^T \xi(x). \tag{11}$$

Lemma 4 ([14]) *Let $f(x)$ be a continuous function defined on a compact set Ω. Then, for $\forall \varepsilon > 0$, there exists a fuzzy logic system (11) such that*

$$\sup_{x \in \Omega} |f(x) - \Phi^T \xi(x)| \leq \varepsilon. \tag{12}$$

The objective of this paper is to design a common adaptive controller such that the system output y tracks a reference signal y_d in the sense of mean quartic value and all closed-loop signals are bounded in probability under arbitrary switchings.

Assumption 2 The reference signal $y_d(t)$ and its time derivatives up to the nth order are continuous and bounded.

Remark 3 Compared with the existing works on switched stochastic nonlinear systems in [32], this paper focuses on the tracking problem. Note that if $y_d(t) = 0$, then the tracking problem is equivalent to the stabilization problem in [32]. Therefore the tracking problem in this paper is more general and interesting than the stabilization problem in [32], and it is a challenging work.

3 Adaptive control design

To develop the backstepping design, define the following coordinate transformation:

$$z_1 = y - y_d, \quad z_i = x_i - \alpha_{i-1}, \quad i = 2, \ldots, n. \tag{13}$$

where α_{i-1} represents an intermediate control function to be determined later. Define a vector function as $\bar{y}_d^{(i)} = [y_d, y_d^{(1)}, \ldots, y_d^{(i)}]^T$, $i = 1, 2, \ldots, n$, where $y_d^{(i)}$ denotes the ith time derivative of y_d.

In each step of the backstepping design, a fuzzy logic system $\Phi_i(X_{i,k})$ will be employed to approximate an unknown function $\bar{f}_{i,k}$. For $i = 1, 2, \ldots, n$, define a constant as follows $\theta_i = \max\{\|\Phi_{i,k}\|^2 : k \in M\}$, denote $\hat{\theta}_i$ as the estimation of θ_i; then, the estimation error is $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$.

Step 1 For stochastic switched systems (5), the error dynamic of z_1 is

$$dz_1 = (x_2 + f_{1,k}(x_1) - \dot{y}_d)dt + g_{1,k}^T dw. \tag{14}$$

Consider a stochastic Lyapunov function candidate as

$$V_1 = \frac{z_1^4}{4} + \frac{\tilde{\theta}_1^2}{2r_1}, \tag{15}$$

where $r_1 > 0$ is a design constant.

According to (2), (13) and (14), we have

$$LV_1 = z_1^3(z_2 + \alpha_1 + f_{1,k} - \dot{y}_d) + \frac{3}{2} z_1^2 g_{1,k}^T g_{1,k} - \frac{\tilde{\theta}_1 \dot{\hat{\theta}}_1}{r_1}. \tag{16}$$

Applying Lemma 2, the following inequalities hold:

$$\frac{3}{2} z_1^2 g_{1,k}^T g_{1,k} \leq \frac{3}{4} l_1^{-2} z_1^4 \|g_{1,k}\|^4 + \frac{3}{4} l_1^2, \tag{17}$$

$$z_1^3 z_2 \leq \frac{3z_1^4}{4} + \frac{z_2^4}{4} \tag{18}$$

where l_1 represents a positive design constant. Substituting (17) and (18) into (16) yields

$$LV_1 \leq z_1^3 \alpha_1 + z_1^3 \bar{f}_{1,k} + \frac{3}{4} l_1^2 + \frac{z_2^4}{4} - \frac{\tilde{\theta}_1 \dot{\hat{\theta}}_1}{r_1}. \tag{19}$$

where $\bar{f}_{1,k} = f_{1,k} + \frac{3}{4} z_1 + \frac{3}{4} l_1^{-2} z_1 \|g_{1,k}\|^4 - \dot{y}_d$.

According to Lemma 4, for $\forall \varepsilon_{1,k} > 0$, there exists a fuzzy logic system $\Phi_{1,k}^T \xi_{1,k}(X_1)$ such that

$$\bar{f}_{1,k} = \Phi_{1,k}^T \xi_{1,k}(X_1) + \delta_{1,k}(X_1), \quad |\delta_{1,k}(X_1)| \leq \varepsilon_{1,k} \tag{20}$$

where $X_1 = (x_1, y_d, \dot{y}_d)$.

Applying Young's inequality and $\xi_{1,k}^T\xi_{1,k} \le 1$, we have

$$z_1^3\bar{f}_{1,k} \le \frac{z_1^6\theta_1}{2a_{1,k}^2} + \frac{1}{2}a_{1,k}^2 + \frac{3}{4}z_1^4 + \frac{\varepsilon_{1,k}^4}{4}, \tag{21}$$

where $a_{1,k}$ represents a positive parameter.

Define $a_{1,\min} = \max\{a_{1,k} : k \in M\}$ and $a_{1,\max} = \max\{a_{1,k} : k \in M\}$ and choose the virtual control signal and the adaptation law as

$$\alpha_1 = -\left(\lambda_1 + \frac{3}{4}\right) z_1 - \frac{\hat{\theta}_1 z_1^3}{2a_{1,\min}^2}, \tag{22}$$

$$\dot{\hat{\theta}}_1 = \frac{r_1 z_1^6}{2a_{1,\min}^2} - \gamma_1\hat{\theta}_1, \hat{\theta}_1(0) \ge 0 \tag{23}$$

where λ_1 and γ_1 are positive design constants. According to Lemma 3 and Eq. (23), $\hat{\theta}_1(t) \ge 0$ for $\forall t > t_0$, therefore

$$\frac{\hat{\theta}_1 z_1^6}{2a_{1,k}^2} \le \frac{\hat{\theta}_1 z_1^6}{2a_{1,\min}^2}. \tag{24}$$

Denote $\varepsilon_{1,\max} = \max\{\varepsilon_{1,k} : k \in M\}$, substituting (21–24) into (19), we have

$$LV_1 \le -\lambda_1 z_1^4 + \frac{z_2^4}{4} + \frac{3l_1^2}{4} + \frac{a_{1,\max}^2}{4} + \frac{\varepsilon_{1,\max}^4}{4} + \frac{\gamma_1}{r_1}\tilde{\theta}_1\hat{\theta}_1. \tag{25}$$

On the other hand, combining the following equality

$$\begin{aligned}\frac{\gamma_1}{r_1}\tilde{\theta}_1\hat{\theta}_1 &= \frac{\gamma_1}{r_1}\tilde{\theta}_1(\theta_1 - \tilde{\theta}_1)\\ &= \frac{\gamma_1}{r_1}\Big[-\tilde{\theta}_1^2 + \tilde{\theta}_1\theta_1\Big] \le \frac{\gamma_1}{r_1}\Big[-\tilde{\theta}_1^2 + \frac{\tilde{\theta}_1^2}{2} + \frac{\theta_1^2}{2}\Big]\\ &\le -\frac{\gamma_1}{2r_1}\tilde{\theta}_1^2 + \frac{\gamma_1}{2r_1}\theta_1^2,\end{aligned} \tag{26}$$

(25) can be expressed in the following form:

$$LV_1 \le -\lambda_1 z_1^4 - \frac{\gamma_1}{2r_1}\tilde{\theta}_1^2 + \varrho_1 + \frac{z_2^4}{4}, \tag{27}$$

where $\varrho_1 = \frac{3}{4}l_1^2 + \frac{1}{4}a_{1,\max}^2 + \frac{1}{4}\varepsilon_{1,\max}^4 + \frac{\gamma_1}{2r_1}\theta_1^2$.

Step $i\,(2 \le i \le n-1)$ According to the Itô formula and (13), we obtain

$$\mathrm{d}z_i = \big(f_{i,k}(\bar{x}_i) + x_{i+1} - L\alpha_{i-1}\big)\,\mathrm{d}t + \left(g_{i,k} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}g_{j,k}\right)^{\mathrm{T}}\mathrm{d}w, \tag{28}$$

where

$$L\alpha_{i-1} = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Big[f_{j,k}(\bar{x}_j) + x_{j+1}\Big] + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j + \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} + \frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p\partial x_q}g_{p,k}^T g_{q,k}. \tag{29}$$

Consider the following stochastic Lyapunov function

$$V_i = V_{i-1} + \frac{1}{4}z_i^4 + \frac{1}{2r_i}\tilde{\theta}_i^2. \tag{30}$$

where r_i is a positive design constant. Using the similar procedure as step 1, it follows

$$\begin{aligned}LV_i &= LV_{i-1} + z_i^3(z_{i+1} + \alpha_i + f_{i,k}(\bar{x}_i) - L\alpha_{i-1})\\ &\quad + \frac{3}{2}z_i^2\left(g_{i,k} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}g_{j,k}\right)^T\\ &\quad \times\left(g_{i,k} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}g_{j,k}\right) - \frac{1}{r_i}\tilde{\theta}_i\dot{\hat{\theta}}_i.\end{aligned} \tag{31}$$

By applying Young's inequality and the completion of squares, we have

$$\frac{3}{2}z_i^2\left\|g_{i,k} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}g_{j,k}\right\|^2 \le \frac{3}{4}l_i^2 + \frac{3}{4}l_i^{-2}z_i^4\left\|g_{i,k} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}g_{j,k}\right\|^4, \tag{32}$$

$$z_i^3 z_{i+1} \le \frac{3}{4}z_i^4 + \frac{z_{i+1}^4}{4} \tag{33}$$

where l_i represents a positive design constant. Substituting the above inequalities into (31), we get

$$\begin{aligned}LV_i &\le -\sum_{j=1}^{i-1}\left(\lambda_j z_j^4 + \frac{\gamma_j}{2r_j}\tilde{\theta}_j^2\right) + \sum_{j=1}^{i-1}\varrho_j + \frac{3}{4}l_i^2\\ &\quad + z_i^3\bar{f}_{i,k} + z_i^3\alpha_i + \frac{1}{4}z_{i+1}^4 - \frac{1}{r_i}\tilde{\theta}_i\dot{\hat{\theta}}_i,\end{aligned} \tag{34}$$

where

$$\bar{f}_{i,k} = f_{i,k}(\bar{x}_i) - L\alpha_{i-1} + \frac{3}{4}l_i^{-2}z_i\left\|g_{i,k} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}g_{j,k}\right\|^4 + z_i. \tag{35}$$

According to Lemma 4, for a given $\varepsilon_{i,k} > 0$, there exists a fuzzy logic system $\Phi_{i,k}^{\mathrm{T}}\xi_{i,k}(X_i)$ such that

$$\bar{f}_{i,k} = \Phi_{i,k}^{\mathrm{T}}\xi_{i,k}(X_i) + \delta_{i,k}(X_i), \quad |\delta_{i,k}(X_i)| \le \varepsilon_{i,k} \tag{36}$$

where $X_i = [\bar{x}_i^T, \bar{\hat{\theta}}_{i-1}^T, \bar{y}_d^{(i)T}]^T \in \Omega_{Z_i} \subset R^{3i}$ with $\bar{\hat{\theta}}_{i-1} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{i-1}]^T$. Then, applying the same method in (21), we have

$$z_i^3 \bar{f}_{i,k} \le \frac{z_i^6 \theta_i}{2a_{i,k}^2} + \frac{1}{2}a_{i,k}^2 + \frac{3}{4}z_i^4 + \frac{1}{4}\varepsilon_{i,k}^4. \tag{37}$$

where $a_{i,k}$ represents a positive design parameter. Define $a_{i,\min} = \max\{a_{i,k} : k \in M\}$, $a_{i,\max} = \max\{a_{i,k} : k \in M\}$ and $\varepsilon_{i,\max} = \max\{\varepsilon_{i,k} : k \in M\}$ and choose the virtual control signal and the adaptation law as

$$\alpha_i = -\left(\lambda_i + \frac{3}{4}\right) z_i - \frac{\hat{\theta}_i z_i^3}{2a_{i,\min}^2}, \tag{38}$$

$$\dot{\hat{\theta}}_i = \frac{r_i z_i^6}{2a_{i,\min}^2} - \gamma_i \hat{\theta}_i, \hat{\theta}_i(0) \ge 0 \tag{39}$$

where λ_i and γ_i are positive design constants. Similar to (24) and (26), the following equalities hold.

$$\frac{\hat{\theta}_i z_i^6}{2a_{i,k}^2} \le \frac{\hat{\theta}_i z_i^6}{2a_{i,\min}^2}, \tag{40}$$

$$\frac{\gamma_i}{r_i}\tilde{\theta}_i \hat{\theta}_i \le -\frac{\gamma_i}{2r_i}\tilde{\theta}_i^2 + \frac{\gamma_i}{2r_i}\theta_i^2. \tag{41}$$

Substituting (37)–(41) into (34) yields

$$LV_i \le -\sum_{j=1}^{i}\left(\lambda_j z_j^4 + \frac{\gamma_j}{2r_j}\tilde{\theta}_j^2\right) + \sum_{j=1}^{i}\varrho_j + \frac{1}{4}z_{i+1}^4, \tag{42}$$

where

$$\varrho_j = \frac{3}{4}l_j^2 + \frac{1}{4}a_{j,\max}^2 + \frac{1}{4}\varepsilon_{j,\max}^4 + \frac{\gamma_j}{2r_j}\theta_j^2. \tag{43}$$

Step n According to Itô formula and (13), we have

$$\mathrm{d}z_n = (f_{n,k}(\bar{x}_n) + u_k - L\alpha_{n-1})\mathrm{d}t + \left(g_{n,k} - \sum_{j=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_j}g_{j,k}\right)^{\mathrm{T}} \mathrm{d}w, \tag{44}$$

where $L\alpha_{n-1}$ is defined in (29) with $i = n$.

Consider the following stochastic Lyapunov function

$$V_n = V_{n-1} + \frac{1}{4}z_n^4 + \frac{1}{2r_n}\tilde{\theta}_n^2, \tag{45}$$

where r_n represents a positive design constant. From (2) and (7), we have

$$LV_n = LV_{n-1} + z_n^3(f_{n,k}(\bar{x}_n) + m_k v_k + \bar{d}_k - L\alpha_{n-1}) - \frac{\tilde{\theta}_n}{r_n}\dot{\hat{\theta}}_n + \frac{3}{2}z_n^2\left(g_{n,k} - \sum_{j=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_j}g_{j,k}\right)^T\left(g_{n,k} - \sum_{j=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_j}g_{j,k}\right). \tag{46}$$

According to (10), we have

$$z_n^3 \bar{d}_k \le \frac{3}{4}z_n^4 + \frac{1}{4}\bar{d}^{*4}. \tag{47}$$

Furthermore, applying the results of (32), (35)–(37) with $(i = n)$ and the result of (42) with $(i = n-1)$, (46) can be rewritten as

$$LV_n \le -\sum_{j=1}^{n-1}\left(\lambda_j z_j^4 + \frac{\gamma_j}{2r_j}\tilde{\theta}_j^2\right) + \sum_{j=1}^{n-1}\varrho_j + \frac{z_n^6\theta_n}{2a_{n,k}^2} + \frac{1}{2}a_{n,k}^2 + \frac{3}{2}z_n^4 + \frac{1}{4}\varepsilon_{n,k}^4 + z_n^3 m_k v_k + \frac{3}{4}l_n^2 + \frac{1}{4}\bar{d}^{*4}. \tag{48}$$

Define $a_{n,\min} = \max\{a_{n,k} : k \in M\}$, $a_{n,\max} = \max\{a_{n,k} : k \in M\}$ and $\varepsilon_{n,\max} = \max\{\varepsilon_{n,k} : k \in M\}$ and choose the control signal and the adaptation law as

$$v_k = -\frac{1}{b_m}\left[\left(\lambda_n + \frac{3}{2}\right) z_n - \frac{\hat{\theta}_n z_n^3}{2a_{n,\min}^2}\right], \tag{49}$$

$$\dot{\hat{\theta}}_n = \frac{r_n z_n^6}{2a_{n,\min}^2} - \gamma_n \hat{\theta}_n, \hat{\theta}_n(0) \ge 0 \tag{50}$$

where λ_n and γ_n are positive design constants.

According to (40)–(41) and Assumption 1, we have

$$LV_n \le -\sum_{j=1}^{n}\left(\lambda_j z_j^4 + \frac{\gamma_j}{2r_j}\tilde{\theta}_j^2\right) + \sum_{j=1}^{n}\varrho_j + \frac{1}{4}\bar{d}^{*4}, \tag{51}$$

where $\varrho_j (1 \le j \le n)$ is defined in (43).

Define $\lambda = \min\{4c_j, \gamma_j, j = 1, 2, \ldots, n.\}$ and $c = \sum_{j=1}^{n}\varrho_j + \frac{1}{4}\bar{d}^{*4}$, (51) can be rewritten as

$$LV_n \le -\lambda V_n + c, t \ge 0. \tag{52}$$

From the definition of V_n and Lemma 1, z_j and $\tilde{\theta}_j$ are bounded in probability. Furthermore, according to [25], we can get

$$\frac{dE(V_n(t))}{dt} \le -\lambda E(V_n(t) + c, t \ge 0. \tag{53}$$

where $E(\cdot)$ indicates an expectation operator. From (53), we have

$$0 \le E[V_n(t)] \le \left(V_n(0) - \frac{c}{\lambda}\right)e^{-\lambda t} + \frac{c}{\lambda}, \tag{54}$$

which means that

$$E[V_n(t)] \le \frac{c}{\lambda}, \quad t \to \infty. \tag{55}$$

From (54) and (55), we can obtain

$$E\left(\sum_{j=1}^{n} z_j^4\right) \leq 4E[V_n(t)] \leq \frac{4c}{\lambda}, \quad t \to \infty. \tag{56}$$

Therefore, z_j eventually is convergent to the compact set Ω_Z which is defined as

$$\Omega_Z = \left\{ z_j \middle| \sum_{j=1}^{n} E[|z_j|^4] \leq \frac{4c}{\lambda} \right\}. \tag{57}$$

So far, based on the backstepping technique, an adaptive fuzzy control design has been completed. We have the following main result.

Theorem 1 *Consider the switched stochastic nonlinear systems with actuator dead zone (5) under Assumptions 1–2. For bounded initial conditions, the controller (49), together with the intermediate control signals (38) and parameter adaptive laws (39), guarantees that all the signals in the closed-loop system remain bounded in probability, and the tracking error is convergent to a neighborhood of the origin.*

4 Simulation example

In this section, a numerical example is presented to demonstrate the feasibility and the control performance of the proposed adaptive fuzzy scheme.

Fig. 1 Trajectories of y (*solid line*) and y_d (*dashed line*)

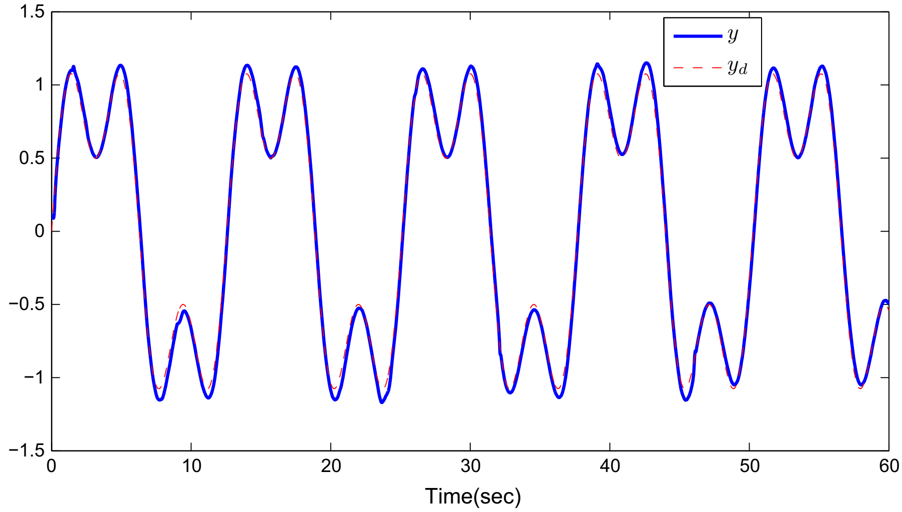
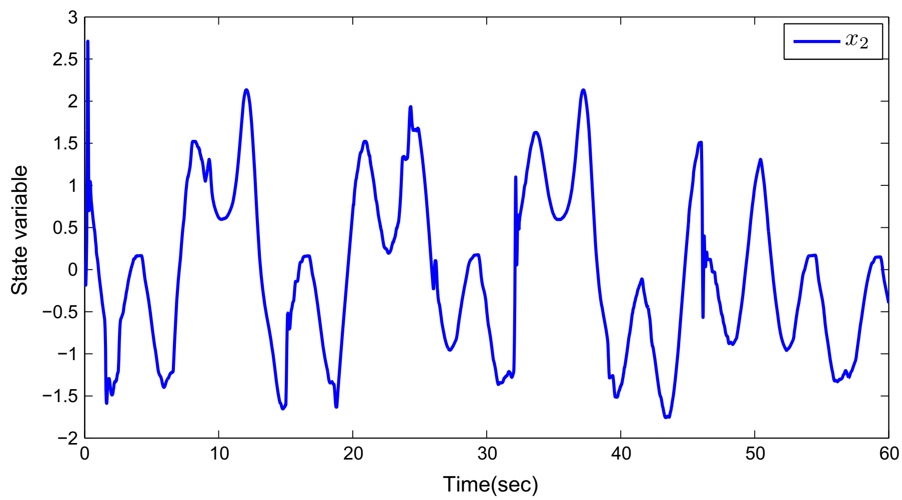


Fig. 2 State variable x_2



Example Consider the following second-order stochastic nonlinear switched systems with dead-zone input:

$$\begin{aligned} \mathrm{d}x_1 &= (x_2 + f_{1,\sigma(t)}(\bar{x}_1))\mathrm{d}t + g_{1,\sigma(t)}(\bar{x}_1)\mathrm{d}w, \\ \mathrm{d}x_2 &= u_{\sigma(t)} + f_{2,\sigma(t)}(\bar{x}_2))\mathrm{d}t + g_{2,\sigma(t)}(\bar{x}_2)\mathrm{d}w, \\ y &= x_1, \end{aligned}$$

where $\sigma(t) : [0, \infty] \to \{1, 2, 3\}$, $f_{11} = x_1$, $f_{12} = \frac{x_1^2}{1+x_1^2}$, $f_{13} = 2x_1 \cos(x_1)$, $f_{21} = (x_1)^2 \cos^2(x_2)$, $f_{22} = \frac{x_2^2}{1+x_1^2+x_2^2}$, $f_{23} = 2\sin^2(x_1)x_2^2$, $g_{11} = \frac{0.1x_1^2}{1+x_1^2}$, $g_{12} = 0.05\cos(x_1)$, $g_{13} = \frac{0.03x_1^2}{1+x_1^2}$, $g_{21} = 0.6\sin(2x_1x_2)$, $g_{22} = 0.05\cos(x_1)$, $g_{23} = \frac{0.05x_2^2}{1+x_1^2}$, $u_{\sigma(t)} \in R$ denotes dead-zone output defined in (6). The dead-zone parameters are chosen as $m_{r,\sigma(t)} = m_{l,\sigma(t)} = 1.5$, $b_{l,\sigma(t)} = b_{r,\sigma(t)} = 1$.

The objective is to design an adaptive fuzzy controller v_k such that all the signals are bounded in probability and y follows a desired reference signal y_d under arbitrary switchings, where $y_d = \sin(0.5t) + 0.5\sin(1.5t)$.

In the simulation, the design parameters are chosen as $\lambda_1 = 4$, $\lambda_2 = 6$, $a_{1,1} = 1, a_{1,2} = a_{1,3} = 2, a_{2,1} = 3, a_{2,2} = 2.5, a_{2,3} = 2, r_1 = 3, r_2 = 2$, $\gamma_1 = 0.2$, $\gamma_2 = 0.2$. The initial conditions are chosen as $[x_1(0), x_2(0)]^T = [0.1, 0.1]^T$, and $[\hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0.25, 0.1]^T$.

According to Theorem 1, the virtual control signal, the actual controller and the adaptive laws are, respectively, designed as

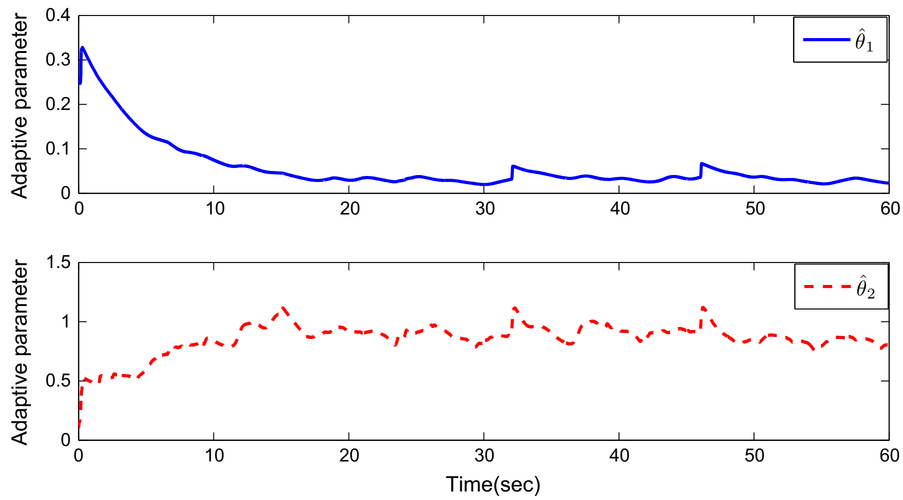


Fig. 3 The adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$

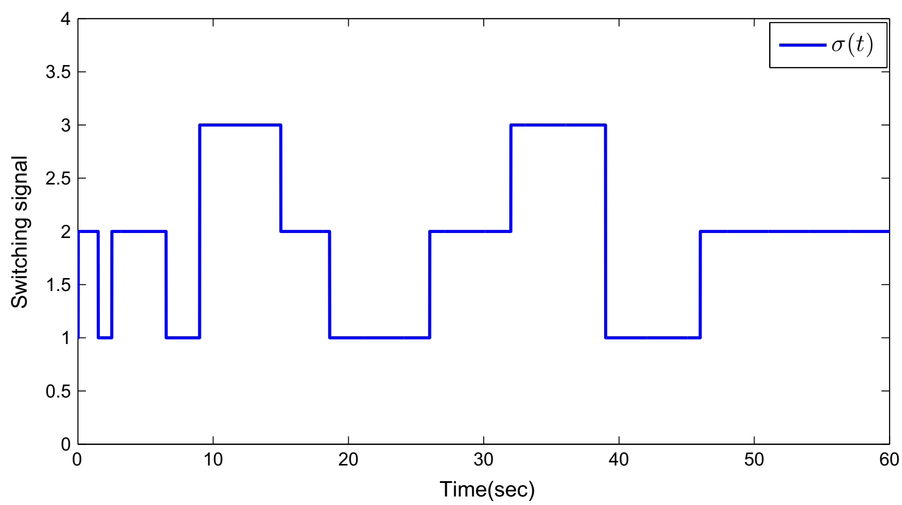


Fig. 4 The switching signal $\sigma(t)$

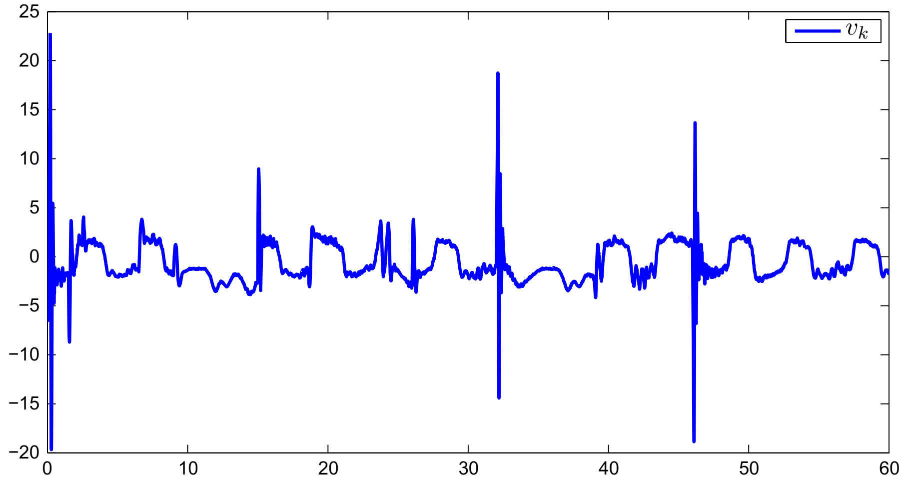


Fig. 5 The control input signal v_k

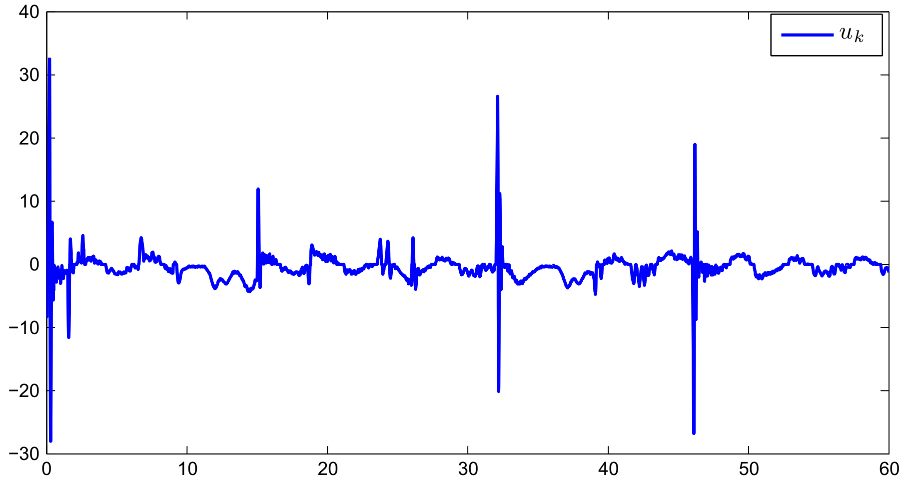


Fig. 6 The system input signal u_k

$$\alpha_1 = -\left(\lambda_1 + \frac{3}{4}\right) z_1 - \frac{\hat{\theta}_1 z_1^3}{2a_{1,1}^2}, \tag{58}$$

$$v_k = -\frac{2}{3}\left[\left(\lambda_2 + \frac{3}{2}\right) z_2 - \frac{\hat{\theta}_2 z_2^3}{2a_{2,3}^2}\right], \tag{59}$$

$$\dot{\hat{\theta}}_1 = \frac{r_1 z_1^6}{2a_{1,1}^2} - \gamma_1 \hat{\theta}_1, \tag{60}$$

$$\dot{\hat{\theta}}_2 = \frac{r_2 z_2^6}{2a_{2,3}^2} - \gamma_n \hat{\theta}_2, \tag{61}$$

where $z_1 = x_1 - y_d, z_2 = x_2 - \alpha_1$. The simulation results are displayed in Figs. 1, 2, 3, 4, 5 and 6. Figure 1 displays the system output y and the reference signal y_d. Figure 2 exhibits the trajectory of the state variable x_2. Figure 3 depicts the trajectories of adaptive parameters $\hat{\theta}_1$, $\hat{\theta}_2$. Figure 4 illustrates the evolution of switching signal. Figure 5 demonstrates the trajectory of the control signal v_k. Figure 6 displays the response curves of dead-zone input u_k of the switched system. From the simulation results, it is seen that the output y converges to a small neighborhood of the reference signal y_d and all the closed-loop signals are bounded.

5 Conclusion

This paper investigates an tracking control problem of a class of stochastic switched uncertain systems in strict-feedback form with dead-zone input. In this paper, stochastic disturbances and nonlinear functions of the system are completely unknown. By employing the fuzzy logic systems' universal approximation property, the technical difficulty of the Itô stochastic differentiation and the unknown nonlinear functions is dealt with. By applying the adaptive backstepping technique, a common state feedback controller independent of switching signals is designed. Even under arbitrary switching conditions, the proposed controller can guarantee that the system output converges to a small neighborhood of the reference signal and all the signals in the closed-loop system remain bounded in probability. Finally, simulation results exhibit the effectiveness of the main result.

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