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Brief paper

Adaptive tracking control for switched stochastic nonlinear systems with unknown actuator dead-zone[☆]

Xudong Zhao [a,b], Peng Shi [c,d], Xiaolong Zheng [a], Lixian Zhang [e]

[a] College of Engineering, Bohai University, Jinzhou 121013, Liaoning, China
[b] Department of Mechanical Engineering, The University of Hong Kong, Hong Kong
[c] School of Electrical and Electronic Engineering, The University of Adelaide, Adelaide, SA 5005, Australia
[d] College of Engineering and Science, Victoria University, Melbourne, Vic. 8001, Australia
[e] School of Astronautics, Harbin Institute of Technology, Harbin, 150001, China

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ABSTRACT

This paper is concerned with the problem of adaptive tracking control for a class of switched stochastic nonlinear systems in nonstrict-feedback form with unknown nonsymmetric actuator dead-zone and arbitrary switchings. A variable separation approach is used to overcome the difficulty in dealing with the nonstrict-feedback structure. Furthermore, by combining radial basis function neural networks' universal approximation ability and adaptive backstepping technique with common stochastic Lyapunov function method, an adaptive control algorithm is proposed for the considered system. It is shown that the target signal can be almost surely tracked by the system output, and the tracking error is semi-globally uniformly ultimately bounded in 4^{th} moment. Finally, the simulation studies for a ship maneuvering system are presented to show the effectiveness of the proposed approach.

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1. Introduction

As a typical class of hybrid systems, switched systems have drawn considerable attention in the past decades since many physical systems can be mathematically modeled by such multiple-mode systems. Given their widespread applications, the studies on switched systems never cease and a number of excellent results have been reported, see for example, Briat (2014), Li, Wen, Soh, and Xie (2002), Veselỳ and Rosinová (2014), Xiang and Xiao (2014), Xie, Wen, and Li (2001), Zhang and Gao (2010), Zhang, Zhuang, and Shi (2015), Zhao, Yin, Li, and Niu (2015) and Zhao, Zhang, Shi, and Liu (2012) and the references therein. In Zhang and Gao (2010) and Zhao et al. (2012), the control problems for switched linear systems in both continuous-time and discrete-time contexts were solved under average dwell time switching and mode-dependent average dwell time switching, respectively. In Zhang et al. (2015), the authors considered H_∞ filtering for a class of switched linear discrete systems under their proposed persistent dwell-time switching. The problem of switching stabilization for slowly switched linear systems with mode-dependent average dwell time was investigated in Zhao et al. (2015) by using invariant subspace theory. Recent advances for switched systems in linear setting can be found in Briat (2014), Veselỳ and Rosinová (2014) and Xiang and Xiao (2014). In addition, for switched nonlinear systems, some sufficient conditions were derived in Xie et al. (2001) to ensure that the whole switched nonlinear system is input-to-state stabilizable (ISS) when each mode is ISS. A concept of generalized matrix measure for nonlinear systems was proposed in Li et al. (2002) to study the stability of switched nonlinear systems directly.

Note that during the most recent years, the adaptive backstepping-based control problems have been investigated for a class of switched nonlinear systems (Han, Ge, & Lee, 2009; Long & Zhao, 2015; Ma & Zhao, 2010; Wang, 2014). To list a few, the global stabilization problem for switched nonlinear systems in lower triangular form was investigated in both Long and Zhao (2015) and Ma and Zhao (2010) by using common Lyapunov function and multiple Lyapunov functions, respectively; Han et al. (2009) presented an adaptive neural control design for a class of switched nonlinear systems with uncertain switching signals. However, the

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E-mail addresses: xdzhaohit@gmail.com (X. Zhao), peng.shi@adelaide.edu.au (P. Shi), zxldwlsj@gmail.com (X. Zheng), lixianzhang@hit.edu.cn (L. Zhang).

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systems in these studies are of a strict-feedback form which greatly limits the applicability in practice.

On the other hand, it is well known that stochastic disturbance frequently exists in engineering applications. The systems subject to the stochastic disturbance are usually termed as stochastic systems that have also been widely probed, with switching dynamics (Hou, Fu, & Duan, 2013; Wu, Cui, Shi, & Karimi, 2013; Wu, Yang, & Shi, 2010; Wu, Zheng, & Gao, 2013; Zhang, Wu, & Xia, 2014) or without (Wu, Xie, & Zhang, 2007; Xie & Duan, 2010; Xie, Duan, & Yu, 2011; Xie, Duan, & Zhao, 2014; Xie & Liu, 2012). To mention a few, the dissipativity-based sliding mode control was adopted in Wu, Zheng et al. (2013) for switched stochastic linear systems. Stabilization problems for stochastic nonlinear systems with Markovian switching were studied in Wu et al. (2010). By using a novel homogeneous domination approach, the output feedback stabilization problem was studied in Xie and Liu (2012) for stochastic high-order nonlinear systems with time-varying delay. The global stabilization was considered in Xie et al. (2011) for high-order stochastic nonlinear systems with stochastic integral input-to-state stability inverse dynamics. The concept of input-to-state practical stability is extended to stochastic nonlinear systems in Wu et al. (2007), upon which they considered the control problem for a class of stochastic nonlinear systems with unmodeled dynamics by using stochastic small-gain theorem. However, most of these control strategies require that the nonlinear stochastic systems are known precisely or the unknown parameters appear linearly with respect to the known nonlinear functions. Furthermore, existing results on switched stochastic systems also suppose that the systems are of a strict-feedback form. Clearly, these ideal requirements cannot be satisfied in many practical situations.

Moreover, dead-zone characteristics are encountered in many physical components of control systems. They are particularly common in actuators, such as hydraulic servo-valve, electric servomotors, and biomedical systems. It is well known that a system will become oscillating, and the regulation quality will decline if some undesired dead-zone nonlinearities occur in the system. Thus, it is more realistic and reliable to design controllers based on the system model where the dead-zone nonlinearities are taken into consideration. However, up to now, few results on adaptive control have been developed for switched stochastic nonlinear systems with dead-zone characteristics.

It is seen from the above observations that it is of both practical and theoretical significance to investigate the problem of adaptive tracking control for nonstrict-feedback switched stochastic systems with actuator dead-zone, which is however challenging and has not been studied so far. This motivates us to carry out the present study. In this paper, a new approach of constructing common virtual control functions is proposed for the studied system, and a backstepping-based adaptive control methodology is systematically developed with low computation burden since the controller only has two parameters need to be modulated. The contributions of the paper lie in that: (i) our considered switched system model is of a nonstrict-feedback form; (ii) the uncertainty can be completely unknown; (iii) the unknown nonsymmetric actuator dead-zone is taken into account; (iv) the stochastic disturbance is considered in the switched system model; and (v) fewer parameters need to be designed which is more efficient in practice.

Notation R^n denotes the n-dimensional space, R^+ is the set of all nonnegative real numbers. $\mathcal{C}^i$ stands for a set of functions with continuous ith partial derivatives. For a given matrix A (or vector v), A^T (or v^T) denotes its transpose, and $Tr\{A\}$ denotes its trace when A is a square. $\mathcal{K}$ represents the set of functions: $R^+ \to R^+$, which are continuous, strictly increasing and vanishing at zero; $\mathcal{K}_\infty$ denotes a set of functions which is of class $\mathcal{K}$ and unbounded. In addition, $\|\cdot\|$ refers to the Euclidean vector norm.

2. Preliminaries and problem formulation

Consider the following switched stochastic nonlinear system in nonstrict-feedback form:

$$\begin{aligned} dx_i &= (g_{i,\sigma(t)}x_{i+1} + f_{i,\sigma(t)}(x))dt + \psi^T_{i,\sigma(t)}(x)dw, \\ &\quad 1 \le i \le n-1, \\ dx_n &= (g_{n,\sigma(t)}v_{\sigma(t)} + f_{n,\sigma(t)}(x))dt + \psi^T_{n,\sigma(t)}(x)dw, \\ v_{\sigma(t)} &= D_{\sigma(t)}(u_{\sigma(t)}), \\ y &= x_1, \end{aligned} \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)^T \in R^n$ is the system state, w is an r-dimensional independent standard Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge0}, P)$ with Ω being a sample space, $\mathcal{F}$ a σ-field, $\{\mathcal{F}_t\}_{t\ge0}$ a filtration, P a probability measure, and y is the system output; $\sigma(t) : [0, \infty) \to M = \{1, 2, \ldots, m\}$ represents the switching signal; $v_{\sigma(t)}$, $u_{\sigma(t)} \in R$ are the actuator output and input. For any $i = 1, 2, \ldots, n$ and $k \in M$, $f_{i,k}(x)$: $R^n \to R$, $\psi_{i,k} : R^n \to R^r$ are locally Lipschitz unknown nonlinear functions and $g_{i,k}$ are positive known constants.

The nonsymmetric dead-zone nonlinearity is considered in the paper, which is defined as the form in Tao and Kokotović (1994):

$$v_k = D_k(u_k) = \begin{cases} m_{r_k}(u_k - b_{r_k}), & u_k \ge b_{r_k} \\ 0, & -b_{l_k} < u_k < b_{r_k} \\ m_{l_k}(u_k + b_{l_k}), & u_k \le -b_{l_k}. \end{cases} \tag{2}$$

Here, $m_{r_k} > 0$ and $m_{l_k} > 0$ stand for the right and the left slopes of the dead-zone characteristic, respectively. $b_{r_k} > 0$ and $b_{l_k} > 0$ represent the breakpoints of the input nonlinearity. It should be noticed that, in this paper, m_{r_k}, m_{l_k}, b_{r_k} and b_{l_k} are unknown.

In order to facilitate the analysis and design, it is assumed that the nonsymmetric dead-zone nonlinearity can be reformulated as:

$$v_k = D'_k(u_k) + \iota_k, \tag{3}$$

where $D'_k(u_k)$ is an unknown smooth function, ι_k is the error between $D_k(u_k)$ and $D'_k(u_k)$ with $|\iota_k| \le \bar{\iota}_k$.

Moreover, we have

$$\begin{aligned} v_k &= u_k + (D'_k(u_k) - u_k + \iota_k) \\ &= u_k + \eta'_k(u_k) + \iota_k, \end{aligned} \tag{4}$$

where $\eta'_k(u_k) = D'_k(u_k) - u_k$ is an unknown function. The controller can be designed as

$$u_k = u_{c_k} - u_{\phi_k}. \tag{5}$$

Then (4) can be rewritten as

$$v_k = u_{c_k} + \eta'_k(u_k) - u_{\phi_k} + \iota_k. \tag{6}$$

where u_{ϕ_k} is the compensator of dead-zone nonlinearity and u_{c_k} is a main controller of system (1).

Our control objective is to design state-feedback controllers such that a given time-varying signal $y_d(t)$ can be tracked by the output of system (1) under arbitrary switching, while overcoming the problem of actuator dead-zone. In this paper, we also assume that the following assumptions hold:

Assumption 1. The tracking target $y_d(t)$ and its time derivatives up to nth order $y_d^{(n)}(t)$ are continuous and bounded. Also, it is assumed that $|y_d(t)| \le d$, where $d > 0$ is a constant known a priori.

Assumption 2. There exist strictly increasing smooth functions $\phi_{i,k}(\cdot)$, $\rho_{i,k}(\cdot) : R^+ \to R^+$ with $\phi_{i,k}(0) = \rho_{i,k}(0) = 0$ such that for $i = 1, 2, \ldots, n$ and $k \in M$,

$$|f_{i,k}(x)| \le \phi_{i,k}(\|x\|). \tag{7}$$

$$\left|\psi_{i,k}(x)\right| \le \rho_{i,k}(\|x\|). \tag{8}$$

Remark 1. The increasing properties of $\phi_{i,k}(\cdot),\ \rho_{i,k}(\cdot)$ imply that if $a_i, b_i \geq 0$, for $i = 1, 2, \ldots, n$, then $\phi_{i,k}(\sum_{i=1}^{n} a_i) \leq \sum_{i=1}^{n} \phi_{i,k}(na_i)$, $\rho_{i,k}(\sum_{i=1}^{n} b_i) \leq \sum_{i=1}^{n} \rho_{i,k}(nb_i)$. Notice that $\phi_{i,k}(s),\ \rho_{i,k}(s)$ are smooth functions, and $\phi_{i,k}(0) = \rho_{i,k}(0) = 0$. Therefore, it follows that there exist smooth functions $h_{i,k}(s), \eta_{i,k}(s)$ such that $\phi_{i,k}(s) = sh_{i,k}(s),\ \rho_{i,k}(s) = s\eta_{i,k}(s)$ which results in

$$\phi_{i,k}\left(\sum_{j=1}^{n} a_j\right) \leq \sum_{j=1}^{n} na_j h_{i,k}(na_j). \tag{9}$$

$$\rho_{i,k}\left(\sum_{j=1}^{n} b_j\right) \leq \sum_{j=1}^{n} nb_j \eta_{i,k}(nb_j). \tag{10}$$

In Sanner and Slotine (1992), it has been proved that the radial basis function (RBF) neural networks can approximate any continuous real function $f(Z)$ over a compact set $\Omega_Z \subset R^q$. Specifically, for arbitrary $\bar{\varepsilon} > 0$, there exists a neural network $W^T S(Z)$ such that

$$f(Z) = W^T S(Z) + \varepsilon(Z),\ \varepsilon(Z) \leq \bar{\varepsilon}, \tag{11}$$

where $Z \in \Omega_Z \subset R^q,\ W = [w_1, w_2, \ldots, w_l]^T$ is the ideal constant weight vector, and $S(Z) = [s_1(Z), s_2(Z), \ldots, s_l(Z)]^T$ is the basis function vector, with $l > 1$ being the number of the neural network nodes and $s_i(Z)$ being chosen as Gaussian functions, i.e., for $i = 1, 2, \ldots, l$

$$s_i(Z) = \exp\left[\frac{-(Z-\mu_i)^T(Z-\mu_i)}{\zeta_i^2}\right], \tag{12}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]^T$ is the center vector, and ζ_i is the width of the Gaussian function.

Definition 1. Consider the stochastic system $dx = f(x,t)dt + h(x,t)dw$. For any given $V(x,t) \in \mathcal{C}^{2,1}$, define the differential operator $\mathcal{L}$ as follows:

$$\mathcal{L}V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x,t) + \frac{1}{2} Tr\left\{h^T \frac{\partial^2 V}{\partial x^2} h\right\}. \tag{13}$$

Definition 2. The trajectory $\{x(t), t \geq 0\}$ of switched stochastic system (1) is said to be semi-globally uniformly ultimately bounded (SGUUB) in pth moment, if for any compact subset $\Sigma \subset R^n$ and all $x(t_0) = x_0 \in \Sigma$, there exist a constant $\varepsilon > 0$ and a time constant $T = T(\varepsilon, x_0)$ such that $E(|x(t)|^p) < \varepsilon$, for all $t > t_0 + T$. In particular, when $p = 2$, it is usually called SGUUB in mean square.

Lemma 1 (*Krstić & Hua, 1998*). *Suppose that there exist a $C^{2,1}$ function $V(x,t): R^n \times R^+ \to R^+$, two constants $c_1 > 0$ and $c_2 > 0$, class $\mathcal{K}_\infty$ functions $\bar{\alpha}_1$ and $\bar{\alpha}_2$ such that*

$$\begin{cases} \bar{\alpha}_1(|x|) \leq V(x,t) \leq \bar{\alpha}_2(|x|) \\ \mathcal{L}V \leq -c_1 V(x,t) + c_2 \end{cases}$$

for all $x \in R^n$ and $t > t_0$. Then, there is an unique strong solution of system (1) for each $x_0 \in R^n$, which satisfies

$$E[V(x,t)] \leq V(x_0, t_0)e^{-c_1 t} + \frac{c_2}{c_1}, \quad \forall t > t_0.$$

Lemma 2 (*Polycarpou & Ioannou, 1996*). *For any $\xi \in R$ and $\varpi > 0$, the following inequality holds:*

$$0 \leq |\xi| - \xi \tanh\left(\frac{\xi}{\varpi}\right) \leq \delta\varpi, \tag{14}$$

with $\delta = 0.2785$.

3. Main result

In this section a systemic control design and stability analysis procedure will be presented by using adaptive backstepping technique (the readers may refer to Krstić and Kokotović (1995) and Krstić, Kokotović, and Kanellakopoulos (1995) for more details about adaptive backstepping technique). For $i = 1, 2, \ldots, n-1$, let us define a common virtual control function α_i as

$$\alpha_i = \frac{1}{g_{i,\min}}\left[-\left(\lambda_i + \frac{3}{4}\right) z_i - \frac{1}{2a_i^2} z_i^3 \hat{\theta} S_i^T S_i\right], \tag{15}$$

where $\lambda_i, a_i > 0$ are design parameters, $g_{i,\min} = \min\{g_{i,k} : k \in M\}$, z_i represents the new state after the coordinate transformation: $z_i = x_i - \alpha_{i-1},\ \alpha_0 = y_d$. $\hat{\theta}$ is an unknown constant that will be specified later. $S_i = S_i(X_i)$ is the basis function vector. $X_i = [\bar{x}_i^T, \bar{\hat{\theta}}_i^T, \bar{y}_d^{(i)T}]^T$ with $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T$, $\bar{\hat{\theta}}_i = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_i]^T$, $\bar{y}_d^{(i)} = [y_d, \dot{y}_d, \ldots, y_d^{(i)}]^T$. The z-system can be obtained as follows:

$$\begin{aligned} dz_i &= (g_{i,k}x_{i+1} + f_{i,k} - \mathcal{L}\alpha_{i-1})dt \\ &\quad + \left(\psi_{i,k} - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\psi_{j,k}\right)^T dw, \quad 1 \leq i \leq n-1 \\ dz_n &= (g_{n,k}v_k + f_{n,k} - \mathcal{L}\alpha_{n-1})dt \\ &\quad + \left(\psi_{n,k} - \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}\psi_{j,k}\right)^T dw, \end{aligned} \tag{16}$$

where the differential operator $\mathcal{L}$ is defined in Definition 1, then $\mathcal{L}\alpha_{i-1}$ is given by:

$$\begin{aligned} \mathcal{L}\alpha_{i-1} &= \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} + \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s}(f_{s,k} + g_{s,k}x_{s+1}) \\ &\quad + \sum_{s=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(s)}} y_d^{(s+1)} + \frac{1}{2}\sum_{p,q=1}^{i-1} \frac{\partial^2 \alpha_{i-1}}{\partial x_p \partial x_q}\psi_{p,k}^T \psi_{q,k}. \end{aligned} \tag{17}$$

Consider the following common stochastic Lyapunov function candidate

$$V = \sum_{i=1}^{n} \frac{1}{4} z_i^4 + \frac{1}{2r_1}\tilde{\theta}^2 + \frac{1}{2r_2}\tilde{\vartheta}^2, \tag{18}$$

where $r_1,\ r_2 > 0$ are design parameters; $\tilde{\theta} = \theta - \hat{\theta},\ \tilde{\vartheta} = \vartheta - \hat{\vartheta}$ with θ and ϑ will be specified later; $\hat{\theta}$ and $\hat{\vartheta}$ represent the estimation of θ and ϑ respectively.

Lemma 3. *From the coordinate transformations $z_i = x_i - \alpha_{i-1},\ i = 1, 2, \ldots, n,\ \alpha_0 = y_d$, the following inequality holds*

$$\|x\| \leq \sum_{i=1}^{n} |z_i|\, \varphi_i(z_i, \hat{\theta}) + d, \tag{19}$$

where $\varphi_i(z_i, \hat{\theta}) = \frac{1}{g_{i,\min}}[(\lambda_i + \frac{3}{4}) + \frac{1}{2a_i^2} z_i^2 \hat{\theta} S_i^T S_i] + 1$, for $i = 1, 2, \ldots, n-1$, and $\varphi_n = 1$.

Proof. From Assumption 1 and (15), one gets

$$\begin{aligned} \|x\| &\leq \sum_{i=1}^{n} |x_i| \leq \sum_{i=1}^{n} (|z_i| + |\alpha_{i-1}|) \leq \sum_{i=1}^{n} |z_i| + y_d \\ &\quad + \sum_{i=1}^{n-1}\left(\frac{1}{g_{i,\min}}\left[\left(\lambda_i + \frac{3}{4}\right) + \frac{1}{2a_i^2} z_i^2 \hat{\theta} S_i^T S_i\right]\right)|z_i| \\ &\leq \sum_{i=1}^{n} |z_i|\, \varphi_i(z_i, \hat{\theta}) + d. \end{aligned}$$

The proof of Lemma 3 is completed here. ■

By using Definition 1, (16) and (17), $\mathcal{L}V$ can be given by

$$
\begin{aligned}
\mathcal{L}V =& \sum_{i=1}^{n-1}\Bigg\{z_i^3\Bigg(f_{i,k}+g_{i,k}x_{i+1}-\sum_{s=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(s)}}y_d^{(s+1)}\\
&-\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}-\sum_{s=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_s}(f_{s,k}+g_{s,k}x_{s+1})\\
&-\frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p\partial x_q}\psi_{p,k}^T\psi_{q,k}\Bigg)\\
&+\frac{3}{2}z_i^2\left\|\psi_{i,k}-\sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_{j,k}\right\|^2\Bigg\}\\
&+z_n^3\Bigg(f_{n,k}+g_{n,k}v_k-\sum_{s=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(s)}}y_d^{(s+1)}\\
&-\sum_{s=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_s}(f_{s,k}+g_{s,k}x_{s+1})-\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\\
&-\frac{1}{2}\sum_{p,q=1}^{n-1}\frac{\partial^2\alpha_{n-1}}{\partial x_p\partial x_q}\psi_{p,k}^T\psi_{q,k}\Bigg)-\frac{1}{r_1}\tilde{\theta}\dot{\hat{\theta}}\\
&+\frac{3}{2}z_n^2\left\|\psi_{n,k}-\sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\psi_{n,k}\right\|^2-\frac{1}{r_2}\tilde{\vartheta}\dot{\hat{\vartheta}}\\
=& \sum_{i=1}^{n}\Bigg\{z_i^3\Bigg(f_{i,k}-\sum_{s=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_s}f_{s,k}\\
&-\sum_{s=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(s)}}y_d^{(s+1)}-\sum_{s=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_s}g_{s,k}x_{s+1}\\
&-\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}-\frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p\partial x_q}\psi_{p,k}^T\psi_{q,k}\Bigg)\\
&+\frac{3}{2}z_i^2\left\|\psi_{i,k}-\sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_{j,k}\right\|^2\Bigg\}-\frac{1}{r_1}\tilde{\theta}\dot{\hat{\theta}}\\
&-\frac{1}{r_2}\tilde{\vartheta}\dot{\hat{\vartheta}}+\sum_{i=1}^{n-1}z_i^3g_{i,k}x_{i+1}+z_n^3g_{n,k}v_k. \qquad (20)
\end{aligned}
$$

By using Assumption 2 and Lemma 3, one has

$$
\begin{aligned}
&z_i^3\left(f_{i,k}-\sum_{s=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_s}f_{s,k}(x)\right)=-z_i^3\sum_{s=1}^{i}\sigma_{i-1,s}f_{s,k}(x)\\
&\le\frac{3}{4}nz_i^4\sum_{s=1}^{i}(\sigma_{i-1,s})^{\frac{4}{3}}+\sum_{s=1}^{i}\sum_{l=1}^{n}z_l^4\bar{\phi}_{s,k}^4(z_l,\hat{\theta})\\
&+\left|z_i^3\right|\sum_{s=1}^{i}\left|\sigma_{i-1,s}\right|\phi_{s,k}((n+1)d), \qquad (21)
\end{aligned}
$$

where $\bar{\phi}_{s,k}^4(z_l,\hat{\theta})=\frac{1}{4}(n+1)^4\varphi_l^4(z_l,\hat{\theta})h_{s,k}^4((n+1)|z_l|\varphi_l(z_l,\hat{\theta}))$, $\frac{\partial\alpha_0}{\partial x_s}=0$, and $\sigma_{i-1,s}$ is defined as $\sigma_{i-1,s}=\frac{\partial\alpha_{i-1}}{\partial x_s}$, $s=1,2,\ldots,i-1$, $\sigma_{i-1,i}=-1$.

Then, the following inequality can be obtained

$$
\begin{aligned}
&\frac{3}{2}z_i^2\left\|\psi_{i,k}-\sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_{j,k}\right\|^2\\
&\le\frac{9}{8}i^2(n+1)^2nz_i^4+\sum_{l=1}^{n}z_l^4\bar{\rho}_{i,k}^4(z_l,\hat{\theta})+\sum_{j=1}^{i-1}\sum_{l=1}^{n}z_l^4\bar{\rho}_{j,k}^4(z_l,\hat{\theta})\\
&+\frac{9}{8}i^2(n+1)^2nz_i^4\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^4\\
&+\frac{9}{8}i^2(n+1)^2z_i^4l_{ii}^{-2}\rho_{i,k}^4((n+1)d)+\sum_{j=1}^{i}l_{ij}^2\\
&+\frac{9}{8}i^2(n+1)^2z_i^4\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^4l_{ij}^{-2}\rho_{j,k}^4((n+1)d), \qquad (22)
\end{aligned}
$$

where l_{ij} is a positive constant, and $\frac{\partial\alpha_0}{\partial x_j}=0$ since $\alpha_0=y_d$, and

$$
\begin{aligned}
&-\frac{1}{2}z_i^3\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p\partial x_q}\psi_{p,k}^T\psi_{q,k}\le(i-1)\sum_{s=1}^{i-1}\sum_{l=1}^{n}z_l^4\bar{\rho}_{s,k}^4(z_l,\hat{\theta})\\
&+\frac{1}{8}(n+1)^2nz_i^6\sum_{s=1}^{i-1}\sum_{j=1}^{i-1}\left(\frac{\partial^2\alpha_{i-1}}{\partial x_s\partial x_j}\right)^2\\
&+\frac{1}{2}(n+1)\left|z_i^3\right|\sum_{s=1}^{i-1}\sum_{j=1}^{i-1}\left|\frac{\partial^2\alpha_{i-1}}{\partial x_s\partial x_j}\right|\rho_{s,k}^2((n+1)d), \qquad (23)
\end{aligned}
$$

where $\bar{\rho}_{s,k}^4(z_l,\hat{\theta})=\frac{1}{2}(n+1)^4\varphi_l^4(z_l,\hat{\theta})\eta_{s,k}^4((n+1)|z_l|\varphi_l(z_l,\hat{\theta}))$, $s=1,2,\ldots,i-1$.

Substituting (21)–(23) into (20) gives

$$
\begin{aligned}
\mathcal{L}V\le&\sum_{i=1}^{n}\frac{3}{4}nz_i^4\sum_{s=1}^{i}(\sigma_{i-1,s})^{\frac{4}{3}}+\sum_{i=1}^{n}\sum_{s=1}^{i}\sum_{l=1}^{n}z_l^4\bar{\phi}_{s,k}^4(z_l,\hat{\theta})\\
&+\sum_{i=1}^{n}\left|z_i^3\right|\sum_{s=1}^{i}\left|\sigma_{i-1,s}\right|\phi_{s,k}((n+1)d)\\
&+\sum_{i=1}^{n}\sum_{s=1}^{i-1}\sum_{l=1}^{n}(i-1)z_l^4\bar{\rho}_{s,k}^4(z_l,\hat{\theta})\\
&+\sum_{i=1}^{n}\sum_{s=1}^{i-1}\sum_{j=1}^{i-1}\frac{1}{8}(n+1)^2nz_i^6\left(\frac{\partial^2\alpha_{i-1}}{\partial x_s\partial x_j}\right)^2\\
&+\sum_{i=1}^{n}\sum_{s=1}^{i-1}\sum_{j=1}^{i-1}\frac{1}{2}(n+1)\left|z_i^3\right|\rho_{s,k}^2((n+1)d)\left|\frac{\partial^2\alpha_{i-1}}{\partial x_s\partial x_j}\right|\\
&+\sum_{i=1}^{n}\Bigg\{\frac{9}{8}i^2(n+1)^2nz_i^4+\sum_{l=1}^{n}z_l^4\bar{\rho}_{i,k}^4(z_l,\hat{\theta})\\
&+\sum_{j=1}^{i-1}\sum_{l=1}^{n}z_l^4\bar{\rho}_{j,k}^4(z_l,\hat{\theta})\\
&+\frac{9}{8}i^2(n+1)^2nz_i^4\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^4\\
&+\frac{9}{8}i^2(n+1)^2z_i^4l_{ii}^{-2}\rho_{i,k}^4((n+1)d)+\sum_{j=1}^{i}l_{ij}^2\\
&+\frac{9}{8}i^2(n+1)^2z_i^4\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^4l_{ij}^{-2}\rho_{j,k}^4((n+1)d)\Bigg\}\\
&+\sum_{i=1}^{n}z_i^3\Bigg(-\sum_{s=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(s)}}y_d^{(s+1)}-\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\\
&-\sum_{s=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_s}g_{s,k}x_{s+1}\Bigg)+\sum_{i=1}^{n-1}z_i^3g_{i,k}x_{i+1}+z_n^3g_{n,k}v_k\\
&-\frac{1}{r_1}\tilde{\theta}\dot{\hat{\theta}}-\frac{1}{r_2}\tilde{\vartheta}\dot{\hat{\vartheta}}. \qquad (24)
\end{aligned}
$$

Define $U_{i,k}$ as

$$U_{i,k} = \sum_{s=1}^{i} |\sigma_{i-1,s}|\phi_{s,k}((n+1)d) + \frac{1}{2}(n+1)\sum_{s=1}^{i-1}\sum_{j=1}^{i-1}\left|\frac{\partial^2\alpha_{i-1}}{\partial x_s \partial x_j}\right|\rho_{s,k}^2((n+1)d). \tag{25}$$

By using Lemma 2 one has

$$\left|z_i^3\right| U_{i,k} \le z_i^3 U_{i,k}\tanh\left(\frac{z_i^3 U_{i,k}}{\varpi_{i,k}}\right) + \delta\varpi_{i,k}. \tag{26}$$

It can be noticed in (24) that

$$\sum_{i=1}^{n-1} z_i^3 g_{i,k}x_{i+1} = \sum_{i=1}^{n-1} z_i^3 g_{i,k}z_{i+1} + \sum_{i=1}^{n-1} g_{i,k}z_i^3\alpha_i, \tag{27}$$

and

$$\sum_{i=1}^{n}\sum_{s=1}^{i}\sum_{l=1}^{n} z_l^4\bar{\phi}_{s,k}^4(z_l,\hat{\theta}) = \sum_{i=1}^{n} z_i^4 \sum_{s=1}^{n}(n-s+1)\bar{\phi}_{s,k}^4(z_i,\hat{\theta}),$$

$$\sum_{i=1}^{n}(i-1)\sum_{s=1}^{i-1}\sum_{l=1}^{n} z_l^4\bar{\rho}_{s,k}^4(z_l,\hat{\theta}) = \sum_{i=1}^{n} z_i^4 \sum_{s=1}^{n-1}(n-s)(i-1)\bar{\rho}_{s,k}^4(z_i,\hat{\theta}),$$

$$\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{l=1}^{n} z_l^4\bar{\rho}_{j,k}^4(z_l,\hat{\theta}) = \sum_{i=1}^{n} z_i^4 \sum_{j=1}^{n}(n-j+1)\bar{\rho}_{j,k}^4(z_i,\hat{\theta}).$$

For any $i = 1, 2, \ldots, n$ and $k \in M$, define $\bar{f}_{i,k}$ as

$$\begin{aligned}
\bar{f}_{i,k} = {} & \frac{3}{4}nz_i\sum_{s=1}^{i}(\sigma_{i-1,s})^{\frac{4}{3}} + z_i\sum_{s=1}^{n}(n-s+1)\bar{\phi}_{s,k}^4(z_i,\hat{\theta}) \\
& + z_i\sum_{s=1}^{n-1}(n-s)(i-1)\bar{\rho}_{s,k}^4(z_i,\hat{\theta}) \\
& + \sum_{s=1}^{i-1}\sum_{j=1}^{i-1}\frac{1}{8}(n+1)^2 n z_i^3\left(\frac{\partial^2\alpha_{i-1}}{\partial x_s\partial x_j}\right)^2 \\
& + \frac{9}{8}i^2(n+1)^2 z_i\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^4 l_{ij}^{-2}\rho_{j,k}^4((n+1)d) \\
& + z_i\sum_{j=1}^{n}(n-j+1)\bar{\rho}_{j,k}^4(z_i,\hat{\theta}) + \frac{9}{8}i^2(n+1)^2 n z_i \\
& + \frac{9}{8}i^2(n+1)^2 z_i l_{ii}^{-2}\rho_{i,k}^4((n+1)d) \\
& + \frac{9}{8}i^2(n+1)^2 n z_i\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^4 \\
& - \sum_{s=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(s)}}y_d^{(s+1)} - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} - \sum_{s=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_s}g_{s,k}x_{s+1} \\
& + U_{i,k}\tanh\left(\frac{z_i^3 U_{i,k}}{\varpi_{i,k}}\right) + g_{i,k}z_{i+1},
\end{aligned} \tag{28}$$

with $z_{n+1} = 0$.

Substituting (6) and (26)–(28) into (24) yields

$$\begin{aligned}
\mathcal{L}V \le {} & \sum_{i=1}^{n-1} z_i^3(\bar{f}_{i,k} + g_{i,k}\alpha_i) + z_n^3\bar{f}_{n,k} + z_n^3 g_{n,k}(u_{c_k} + \eta_k' - u_{\phi_k} + \iota_k) \\
& - \frac{1}{r_1}\tilde{\theta}\dot{\hat{\theta}} - \frac{1}{r_2}\tilde{\vartheta}\dot{\hat{\vartheta}} + \sum_{i=1}^{n}\left(\delta\varpi_{i,k} + \sum_{j=1}^{i} l_{ij}^2\right).
\end{aligned} \tag{29}$$

By using neural networks' approximation ability and Young's inequality, the following inequalities can be obtained.

$$\begin{aligned}
z_i^3\bar{f}_{i,k} = {} & z_i^3 W_{i,k}^T S_{i,k} + z_i^3\varepsilon_{i,k} \\
\le {} & \frac{1}{2a_i^2}z_i^6\left\|W_{i,k}\right\|^2 S_{i,k}^T S_{i,k} + \frac{a_i^2}{2} + \frac{3}{4}z_n^4 + \frac{\bar{\varepsilon}_{i,k}^4}{4}, \\
\le {} & \frac{1}{2a_i^2}z_i^6\theta_i S_i^T S_i + \frac{a_i^2}{2} + \frac{3}{4}z_i^4 + \frac{\bar{\varepsilon}_i^4}{4},
\end{aligned} \tag{30}$$

$$\begin{aligned}
z_n^3(\eta_k' + \iota_k) = {} & z_n^3 W_{\eta,k}^T S_{\eta,k} + z_n^3(\varepsilon_{\eta,k} + \iota_k) \\
\le {} & \frac{1}{2a_\eta^2}z_n^6\vartheta_\eta S_\eta^T S_\eta + \frac{a_\eta^2}{2} + \frac{3z_n^4 + \bar{\varepsilon}_\eta^4}{4},
\end{aligned} \tag{31}$$

where $\theta_{i,k} = \left\|W_{i,k}\right\|^2$, $\vartheta_{\eta,k} = \left\|W_{\eta,k}\right\|^2$, $\theta_i = \max\{\theta_{i,k} : k \in M\}$, $\vartheta_\eta = \max\{\vartheta_{\eta,k} : k \in M\}$, $\left|\varepsilon_{i,k}\right| \le \bar{\varepsilon}_i$, $\left|\varepsilon_{\eta,k} + \iota_k\right| \le \bar{\varepsilon}_\eta$.

Substituting (30) and (31) into (29) gives

$$\begin{aligned}
\mathcal{L}V \le {} & \sum_{i=1}^{n-1} z_i^3\left(\frac{z_i^3\theta_i}{2a_i^2}S_i^T S_i + g_{i,k}\alpha_i\right) + z_n^3\left(\frac{z_n^3\theta_n}{2a_n^2}S_n^T S_n + g_{n,k}u_{c_k}\right) \\
& + z_n^3 g_{n,k}\left(\frac{1}{2a_\eta^2}z_n^3\theta_\eta S_\eta^T S_\eta - u_{\phi_k}\right) + g_{n,k}\left(\frac{a_\eta^2}{2} + \frac{3}{4}z_n^4 + \frac{\bar{\varepsilon}_\eta^4}{4}\right) \\
& + \sum_{i=1}^{n}\left(\frac{2a_i^2 + 3z_i^4 + \bar{\varepsilon}_i^4}{4}\right) - \frac{1}{r_1}\tilde{\theta}\dot{\hat{\theta}} \\
& - \frac{1}{r_2}\tilde{\vartheta}\dot{\hat{\vartheta}} + \sum_{i=1}^{n}\left(\delta\varpi_i + \sum_{j=1}^{i} l_{ij}^2\right),
\end{aligned} \tag{32}$$

where $\varpi_i = \max\{\varpi_{i,k},\ k \in M\}$.

Design the virtual control function as

$$\alpha_i = \frac{1}{g_{i,\min}}\left[-\left(\lambda_i + \frac{3}{4}\right)z_i - \frac{1}{2a_i^2}z_i^3\hat{\theta}S_i^T S_i\right], \tag{33}$$

where $\hat{\theta} = \sum_{i=1}^{n}\hat{\theta}_i$ is the estimation of θ, and $\lambda_i > 0$ is a design parameter.

The true actuator input is given as

$$u_k = u_{c_k} - u_{\phi_k}, \tag{34}$$

where

$$u_{c_k} = \frac{1}{g_{n,k}}\left[-\left(\lambda_n + \frac{3}{4}\right)z_n - \frac{1}{2a_n^2}z_n^3\hat{\theta}S_n^T S_n\right], \tag{35}$$

$$u_{\phi_k} = \left(\lambda_\eta + \frac{3}{4}\right)z_n + \frac{g_{n,\max}}{2a_\eta^2 g_{n,k}}z_n^3\hat{\vartheta}S_\eta^T S_\eta, \tag{36}$$

with $\lambda_n, \lambda_\eta, a_n, a_\eta > 0$ being the design parameters, $g_{n,\max} = \max\{g_{n,k},\ k \in M\}$, $g_{n,\min} = \min\{g_{n,k},\ k \in M\}$, and $\hat{\vartheta}$ the estimation of ϑ. The adaptive laws can be designed as

$$\dot{\hat{\theta}} = \sum_{i=1}^{n}\frac{r_1}{2a_{i,\min}^2}z_i^6 S_i^T S_i - \beta_1\hat{\theta}, \tag{37}$$

$$\dot{\hat{\vartheta}} = \frac{g_{n,\max}r_2}{2a_{\eta,\min}^2}z_n^6 S_\eta^T S_\eta - \beta_2\hat{\vartheta}. \tag{38}$$

Then, one can get from (32)–(38) that

$$\begin{aligned}
\mathcal{L}V \le {} & -\sum_{i=1}^{n}\lambda_i z_i^4 - \lambda_\eta z_n^4 + g_{n,k}\left(\frac{a_\eta^2}{2} + \frac{\bar{\varepsilon}_\eta^4}{4}\right) + \sum_{i=1}^{n}\left(\frac{a_i^2}{2} + \frac{\bar{\varepsilon}_i^4}{4}\right) \\
& + \frac{\beta_1}{r_1}\tilde{\theta}\hat{\theta} + \frac{\beta_2}{r_2}\tilde{\vartheta}\hat{\vartheta} + \sum_{i=1}^{n}\left(\delta\varpi_i + \sum_{j=1}^{i} l_{ij}^2\right).
\end{aligned} \tag{39}$$

It is true that

$$\tilde{\theta}\hat{\theta} = \tilde{\theta}(\theta - \tilde{\theta}) \le -\frac{1}{2}\tilde{\theta}^2 + \frac{1}{2}\theta^2, \tag{40}$$

$$\tilde{\vartheta}\hat{\vartheta} = \tilde{\vartheta}(\vartheta - \tilde{\vartheta}) \le -\frac{1}{2}\tilde{\vartheta}^2 + \frac{1}{2}\vartheta^2. \tag{41}$$

Combining (39) with (40) and (41) gives

$$\begin{aligned}\mathcal{L}V \le & -\sum_{i=1}^{n}\lambda_i z_i^4 - \frac{\beta_1}{2r_1}\tilde{\theta}^2 - \frac{\beta_2}{2r_2}\tilde{\vartheta}^2 \\ & + g_{n,k}\left(\frac{a_\eta^2}{2} + \frac{\bar{\varepsilon}_\eta^4}{4}\right) + \sum_{i=1}^{n}\left(\frac{a_i^2}{2} + \frac{\bar{\varepsilon}_i^4}{4}\right) \\ & + \sum_{i=1}^{n}\left(\delta\varpi_i + \sum_{j=1}^{i} l_{ij}^2\right) + \frac{\beta_1\theta^2}{2r_1} + \frac{\beta_2\vartheta^2}{2r_2} \\ \le & -p_0 V + q_0,\end{aligned} \tag{42}$$

where $\lambda_n := \lambda_n + \lambda_\eta$, $p_0 = \min\{4\lambda_i, \beta_1, \beta_2 : 1 \le i \le n\}$, $q_0 = \sum_{i=1}^{n}(\frac{a_i^2}{2} + \frac{\bar{\varepsilon}_i^4}{4}) + \sum_{i=1}^{n}\left(\delta\varpi_i + \sum_{j=1}^{i} l_{ij}^2\right) + \frac{\beta_1\theta^2}{2r_1} + \frac{\beta_2\vartheta^2}{2r_2} + g_{n,k}(\frac{a_\eta^2}{2} + \frac{\bar{\varepsilon}_\eta^4}{4})$. By using Lemma 1, we have

$$\frac{dE[V(t)]}{dt} \le -p_0 E[V(t)] + q_0, \tag{43}$$

Then, the following inequality holds

$$0 \le E[V(t)] \le V(0)e^{-p_0 t} + \frac{q_0}{p_0}, \tag{44}$$

where $V(0) = \sum_{j=1}^{n}\frac{z_j^2(0)}{4} + \frac{1}{2r_1}\tilde{\theta}(0)^2 + \frac{1}{2r_2}\tilde{\vartheta}(0)^2$. Eq. (44) means that all the signals in the closed-loop system are bounded in probability. It follows from (44) that

$$E[|z_i|^4] \le \frac{4q_0}{p_0}, \quad t \to \infty. \tag{45}$$

Now, we are ready to present our main result in the following theorem.

Theorem 1. *Consider the closed-loop system (1) with unknown non-symmetric actuator dead-zone (2). Suppose that for $1 \le i \le n$, $k \in M$, the unknown functions $\bar{f}_{i,k}$ can be approximated by neural networks in the sense that the approximation error $\varepsilon_{i,k}$ are bounded. Under the state feedback controller (34) and the adaptive laws (37), (38), the following statements hold:*

(i) *All the signals of the closed-loop z-system (16) under arbitrary switching are SGUUB in 4th moment and*

$$P\left\{\lim_{t\to\infty}\sum_{i=1}^{n} E[|z_i|^4] \le \frac{4q_0}{p_0}\right\} = 1.$$

(ii) *The output y of the closed-loop system (1) under arbitrary switching can be almost surely regulated to a small neighborhood of the target signal.*

Proof. It is not difficult to complete the proof by the above derivations. ■

Remark 2. From (37) and (38), one can see that there are only two adaptive parameters that need to be modulated in our results. Hence, the problem of over parameterization can be avoided by our approach, and the computation burden can be greatly reduced.

4. An illustrative example

In this section, the simulation studies for a ship maneuvering system are used to illustrate the effectiveness of our results.

The ship maneuvering system can be described by the following Norrbin nonlinear model (Lim & Forsythe, 1983).

$$T_{\sigma(v_s)}\dot{h} + h + \alpha_{\sigma(v_s)}h^3 = K_{\sigma(v_s)}\delta + \phi_{\sigma(v_s)}^T(\psi, h, \delta)w,$$

where $T_{\sigma(v_s)}$ is the time constant, $h = \dot{\psi}$ denotes the yaw rate, ψ stands for the heading angle, $\alpha_{\sigma(v_s)}$ is Norrbin coefficient, $K_{\sigma(v_s)}$ represents the rudder gain, δ is the rudder angle and w stands for an r-dimensional independent standard Brownian motion, $\phi_{\sigma(v_s)}(\psi, h, \delta) : R^3 \to R^{3\times r}$ is an unknown function, and $\sigma(v_s)$ is the switching signal which satisfies:

$$\sigma(v_s) = \begin{cases} 1, & 0 < v_s \le v_L \\ 2, & v_L < v_s \le v_M \\ 3, & v_M < v_s \le v_T \end{cases}$$

v_L, v_M, v_T represent the value of low speed, medium speed and top speed, respectively.

A simplified mathematical model of the rudder system can be described as follows:

$$T_{E,\sigma(v_s)}\dot{\delta} + \delta = K_{E,\sigma(v_s)}\delta_{E,\sigma(v_s)},$$

where $T_{E,\sigma(v_s)}$ represents the rudder time constant, δ stands for the actual rudder angle, $K_{E,\sigma(v_s)}$ denotes the rudder control gain and $\delta_{E,\sigma(v_s)}$ is the rudder order.

Let $x_1 = \psi$, $x_2 = h$, $x_3 = \delta$, $v_{\sigma(v_s)} = \delta_{E,\sigma(v_s)}$, we have the following switched nonlinear system model with actuator dead-zone to describe the dynamic behavior of the ship with low speed, medium speed and high speed respectively.

$$\begin{aligned} dx_1 &= x_2 dt, \\ dx_2 &= (f_{\sigma(v_s)} + b_{\sigma(v_s)}x_3)dt + \phi_{\sigma(v_s)}^T d\omega, \\ dx_3 &= \left(-\frac{1}{T_{E,\sigma(v_s)}}x_3 + \frac{K_{E,\sigma(t)}}{T_{E,\sigma(v_s)}}v_{\sigma(v_s)}\right)dt, \\ v_{\sigma(v_s)} &= D(u_{\sigma(v_s)}) \end{aligned}$$

where $f_{\sigma(v_s)} = -\frac{1}{T_{\sigma(v_s)}}x_2 - \frac{\tau_{\sigma(v_s)}}{T_{\sigma(v_s)}}x_2^3$, $b_{\sigma(v_s)} = \frac{K_{\sigma(v_s)}}{T_{\sigma(v_s)}}$.

The vessel data comes from a ship which has a length overall of 160.9 m. $v_L = 3.7$m/s, $K_1 = 32$ s^{-1}, $T_1 = 30$ s, $\tau_1 = 40$ s^2, $T_{E,1} = 4$ s, $K_{E,1} = 2$; $v_M = 7.5$ m/s, $K_2 = 11.4$ s^{-1}, $T_2 = 63.69$ s, $\tau_2 = 30$ s^2, $T_{E,2} = 2.5$ s, $K_{E,2} = 1$; $v_T = 15.3$ m/s, $K_3 = 5.1$ s^{-1}, $T_3 = 80.47$ s, $\tau_3 = 25$ s^2, $T_{E,3} = 1$ s, $K_{E,3} = 0.72$; The initial conditions are $x_1(0) = 2$, $x_2(0) = -0.05$, $x_3(0) = 0.03$, $\hat{\theta}(0) = 10$, $\hat{\vartheta}(0) = 1$. We construct the basis function vectors S_1, S_2, S_3 and S_η using 11, 15, 21 and 48 nodes, the centers $\mu_1, \mu_2, \mu_3, \mu_\eta$ evenly spaced on $[-1.5, 4.5]\times[-3, 4]\times[-10, 8]$, $[-5, 4] \times [-30, 20] \times [-0.5, 5.5]$, $[-5.5, 8] \times [-12, 25] \times [-0.1, 2]$ and $[-10, 2]\times[-60, 2]\times[-0.2, 10.5]$, the widths $\zeta_1 = 1.2$, $\zeta_2 = 2.2$, $\zeta_3 = 2$, $\zeta_\eta = 1.8$. The design parameters are $a_1 = a_2 = a_3 = a_\eta = 10$, $r_1 = 2$, $r_2 = 10$, $\beta_1 = 0.5$, $\beta_2 = 0.1$, $\lambda_1 = \lambda_2 = \lambda_3 = 5$, $\lambda_\eta = 3$. The desired trajectory is $y_d = 10\sin 0.08t$.

According to Theorem 1, the adaptive laws $\dot{\hat{\theta}}$, $\dot{\hat{\vartheta}}$ and the control laws u_{c_k}, u_{ϕ_k} are chosen, respectively, as

$$\dot{\hat{\theta}} = \sum_{i=1}^{3} 0.01 z_i^6 S_i^T S_i - 0.5\hat{\theta},$$

$$\dot{\hat{\vartheta}} = 0.036 z_3^6 S_\eta^T S_\eta - 0.1\hat{\vartheta},$$

$$u_{c_k} = \frac{1}{g_{3,k}}[-5.75 z_3 - 0.005 z_3^3\hat{\theta}S_3^T S_3],$$

$$u_{\phi_k} = 3.75 z_3 + \frac{0.00057}{g_{3,k}} z_3^3\hat{\vartheta}S_\eta^T S_\eta,$$

where $u_k = u_{c_k} - u_{\phi_k}$, $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$, $z_3 = x_3 - \alpha_2$ and α_1, α_2 are given by

$$\alpha_1 = -5.75 z_1 - 0.005 z_1^3\hat{\theta}S_1^T S_1,$$

$$\alpha_2 = -92 z_2 - 0.08 z_2^3\hat{\theta}S_2^T S_2.$$

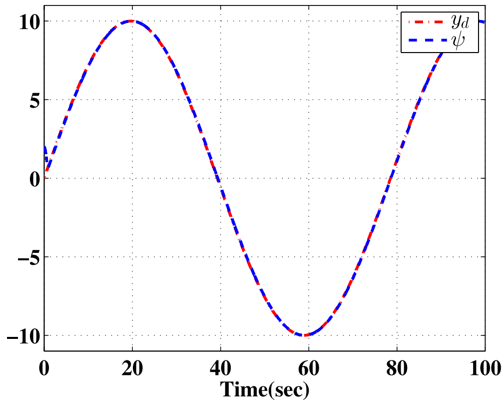

Fig. 1. Tracking performance.

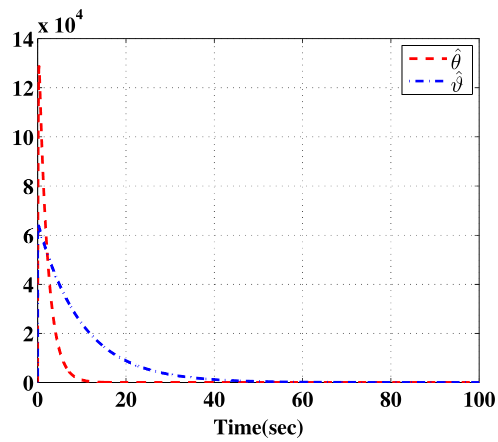

Fig. 2. The responses of adaptive laws.

In order to give the simulation results, we assume that

$$v_k = D(u_k) = \begin{cases} 10(u_k - 50), & u_k \geq 50 \\ 0, & -60 < u_k < 50 \\ 20(u_k + 60), & u_k \leq -60 \end{cases}$$

and $\phi_1 = 0.5x_1 \sin x_2 x_3$, $\phi_2 = 0.25x_1^2 x_2 \cos x_2$, $\phi_3 = 0.1x_1 x_3$. The simulation results are shown in Figs. 1–4, where Fig. 1 presents the system output ψ and target signal y_d, Fig. 2 shows the trajectories of adaptive laws, Fig. 3 demonstrates the responses of $D(u_{c_k})$ (without dead-zone compensation controller) and $D(u_{c_k} - u_{\phi_k})$ (with dead-zone compensation controller), and Fig. 4 illustrates the evolution of switching signal. From Fig. 1, it can be seen that the output ψ can track the target signal y_d within a small bounded error. On the other hand, Fig. 3 verifies that the dead-zone nonlinearity can be compensated by u_{ϕ_k}.

5. Conclusions

This paper has investigated the tracking control problem for a class of switched stochastic nonlinear systems in nonstrict-feedback form under arbitrary switchings, where the unknown nonsymmetric actuator dead-zone is taken into account. Adaptive state feedback controllers are designed for the considered systems. It is shown that the target signal can be almost surely tracked by the system output within a small bounded error, and the tracking error is SGUUB in 4th moment. In our future works, we will pay attention to the control problems of more complicated systems such as high-order switched stochastic systems by using efficient adaptive algorithms.

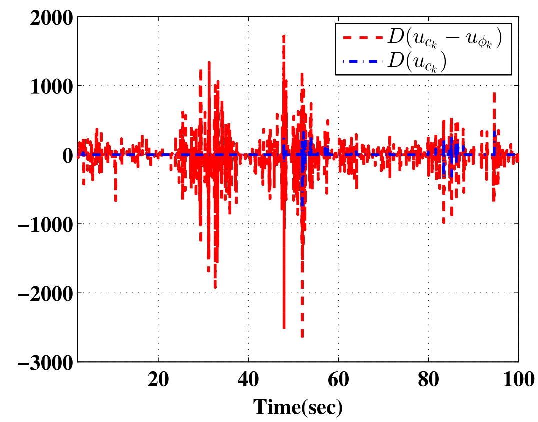

Fig. 3. The responses of $D(u_{c_k} - u_{\phi_k})$ and $D(u_{c_k})$.

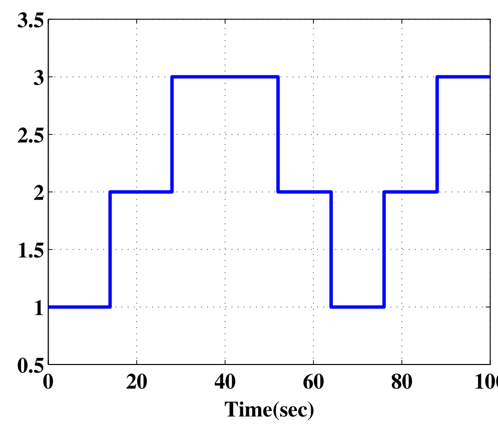

Fig. 4. The response of switching signal.

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References

Briat, C. (2014). Convex lifted conditions for robust l_2-stability analysis and l_2-stabilization of linear discrete-time switched systems with minimum dwell-time constraint. *Automatica*, *50*(3), 976–983.

Han, T. T., Ge, S. S., & Lee, T. H. (2009). Adaptive neural control for a class of switched nonlinear systems. *Systems & Control Letters*, *58*(2), 109–118.

Hou, M., Fu, F., & Duan, G. (2013). Global stabilization of switched stochastic non-linear systems in strict-feedback form under arbitrary switchings. *Automatica*, *49*(8), 2571–2575.

Krstić, M., & Hua, D. (1998). *Stabilization of nonlinear uncertain systems*. London: Springer-Verlag.

Krstić, M., & Kokotović, P. V. (1995). Control Lyapunov functions for adaptive nonlinear stabilization. *Systems & Control Letters*, *26*(1), 17–23.

Krstić, M., Kokotović, P. V., & Kanellakopoulos, I. (1995). *Nonlinear and adaptive control design*. John Wiley & Sons, Inc..

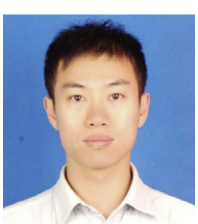
Li, Z., Wen, C., Soh, Y., & Xie, W. (2002). Generalized matrix measure of switched nonlinear systems. *IEEE Transactions on Automatic Control*, *47*(1), 178–183.

Lim, C., & Forsythe, W. (1983). Autopilot for ship control, part 1: Theoretical design. *Control Theory and Applications, IEE Proceedings D*, *130*, 281–287.

Long, L., & Zhao, J. (2015). Adaptive output-feedback neural control of switched uncertain nonlinear systems with average dwell time. *IEEE Transactions on Neural Networks and Learning Systems*, *26*(7), 1350–1362.

Ma, R., & Zhao, J. (2010). Backstepping design for global stabilization of switched nonlinear systems in lower triangular form under arbitrary switchings. *Automatica*, *46*, 1819–1823.

Polycarpou, M., & Ioannou, P. (1996). A robust adaptive nonlinear control design. *Automatica*, *32*(3), 423–427.
Sanner, R., & Slotine, J.-J. (1992). Gaussian networks for direct adaptive control. *IEEE Transactions on Neural Networks*, *3*(6), 837–863.
Tao, G., & Kokotović, P. (1994). Adaptive control of plants with unknown dead-zones. *IEEE Transactions on Automatic Control*, *39*(1), 59–68.
Veselỳ, V., & Rosinová, D. (2014). Robust MPC controller design for switched systems using multi-parameter dependent Lyapunov function. *International Journal of Innovative Computing, Information and Control*, *10*(1), 269–280.
Wang, C. (2014). Adaptive tracking control of uncertain MIMO switched nonlinear systems. *International Journal of Innovative Computing, Information and Control*, *10*(3), 1149–1159.
Wu, Z., Cui, M., Shi, P., & Karimi, H. R. (2013). Stability of stochastic nonlinear systems with state-dependent switching. *IEEE Transactions on Automatic Control*, *58*(8), 1904–1918.
Wu, Z., Xie, X., & Zhang, S. (2007). Adaptive backstepping controller design using stochastic small-gain theorem. *Automatica*, *43*(4), 608–620.
Wu, Z., Yang, J., & Shi, P. (2010). Adaptive tracking for stochastic nonlinear systems with Markovian switching. *IEEE Transactions on Automatic Control*, *55*(9), 2135–2141.
Wu, L., Zheng, W. X., & Gao, H. (2013). Dissipativity-based sliding mode control of switched stochastic systems. *IEEE Transactions on Automatic Control*, *58*(3), 785–791.
Xiang, W., & Xiao, J. (2014). Stabilization of switched continuous-time systems with all modes unstable via dwell time switching. *Automatica*, *50*(3), 940–945.
Xie, X., & Duan, N. (2010). Output tracking of high-order stochastic nonlinear systems with application to benchmark mechanical system. *IEEE Transactions on Automatic Control*, *55*(5), 1197–1202.
Xie, X., Duan, N., & Yu, X. (2011). State-feedback control of high-order stochastic nonlinear systems with SiISS inverse dynamics. *IEEE Transactions on Automatic Control*, *56*(8), 1921–1926.
Xie, X., Duan, N., & Zhao, C. (2014). A combined homogeneous domination and sign function approach to output-feedback stabilization of stochastic high-order nonlinear systems. *IEEE Transactions on Automatic Control*, *5*(59), 1303–1309.
Xie, X., & Liu, L. (2012). Further results on output feedback stabilization for stochastic high-order nonlinear systems with time-varying delay. *Automatica*, *48*(10), 2577–2586.
Xie, W., Wen, C., & Li, Z. (2001). Input-to-state stabilization of switched nonlinear systems. *IEEE Transactions on Automatic Control*, *46*(7), 1111–1116.
Zhang, L., & Gao, H. (2010). Asynchronously switched control of switched linear systems with average dwell time. *Automatica*, *46*(5), 953–958.
Zhang, H., Wu, Z., & Xia, Y. (2014). Exponential stability of stochastic systems with hysteresis switching. *Automatica*, *50*(2), 599–606.
Zhang, L., Zhuang, S., & Shi, P. (2015). Non-weighted quasi-time-dependent H_∞ filtering for switched linear systems with persistent dwell-time. *Automatica*, *54*(0), 201–209.
Zhao, X., Yin, S., Li, H., & Niu, B. (2015). Switching stabilization for a class of slowly switched systems. *IEEE Transactions on Automatic Control*, *60*(1), 221–226.
Zhao, X., Zhang, L., Shi, P., & Liu, M. (2012). Stability and stabilization of switched linear systems with mode-dependent average dwell time. *IEEE Transactions on Automatic Control*, *57*, 1809–1815.



Xudong Zhao was born in Harbin, China, on July. 7. 1982. He received the B.S. degree in Automation from Harbin Institute of Technology in 2005 and the Ph.D. degree from Control Science and Engineering from Space Control and Inertial Technology Center, Harbin Institute of Technology in 2010. Dr. Zhao was a lecturer and an associate professor at the China University of Petroleum, China. Since 2013, he joined Bohai University, China, where he is currently a professor. He is also a Postdoctoral Fellow in The University of Hong Kong from January 2014. Dr. Zhao serves as associate editor for Neurocomputing and International Journal of General Systems, and reviewer for various peer-reviewed journals, including Automatica and IEEE Transactions on Automatic Control. In addition, he has been granted as the outstanding reviewer for Automatica, IET Control Theory & Applications and Journal of the Franklin Institute. His research interests include hybrid systems, positive systems, multi-agent systems, fuzzy systems, Hinf control, filtering and their applications. His works have been widely published in international journals and conferences.

Peng Shi (M95-SM98-F15) received the B.Sc. degree in Mathematics from Harbin Institute of Technology, China; the M.E. degree in Systems Engineering from Harbin Engineering University, China; the Ph.D. degree in Electrical Engineering from the University of Newcastle, Australia; the Ph.D. degree in Mathematics from the University of South Australia; and the D.Sc. degree from the University of Glamorgan, UK.

Dr. Shi was a post-doctorate and lecturer at the University of South Australia; a senior scientist in the Defence Science and Technology Organisation, Australia; and a professor at the University of Glamorgan, UK. Now, he is currently a professor at The University of Adelaide, and Victoria University, Australia. His research interests include system and control theory, computational intelligence, and operational research; and he has published widely in these areas.

Dr. Shi has actively served in the editorial board of a number of journals, including Automatica, IEEE Transactions on Automatic Control; IEEE Transactions on Fuzzy Systems; IEEE Transactions on Cybernetics; IEEE Transactions on Circuits and Systems-I; and IEEE Access. He is a Fellow of the Institute of Electrical and Electronic Engineers, the Institution of Engineering and Technology, and the Institute of Mathematics and its Applications. He is a Member of College of Expert, Australian Research Council, and was the Chair of Chapter on Control and Aerospace Electronic Systems at the IEEE South Australia Section.

Xiaolong Zheng was born in Hubei Province, China on October 8, 1990. He received the B.S. degree in automation from Yangtze University College of Technology and Engineering, Jingzhou, China, in 2013. He is studying for the M.S. degree in control theory and control engineering in Bohai University, Jinzhou, China. His research interests include adaptive control, switched nonlinear systems, intelligent control, stochastic nonlinear systems and their applications.

Lixian Zhang received the Ph.D. degree in control science and engineering from Harbin Institute of Technology, China, in 2006. From Jan. 2007 to Sep. 2008, he worked as a postdoctoral fellow in the Dept. Mechanical Engineering at Ecole Polytechnique de Montreal, Canada. He was a visiting professor at Process Systems Engineering Laboratory, Massachusetts Institute of Technology (MIT) during Feb. 2012 to March 2013. Since Jan. 2009, he has been with the Harbin Institute of Technology, China, where he is currently a Professor in the Research Institute of Intelligent Control and Systems.

Dr. Zhang's research interests include nondeterministic and stochastic switched systems, networked control systems, model predictive control and their applications. He serves as Associated Editor for various peer-reviewed journals including IEEE Transactions on Automatic Control, IEEE Transactions on Cybernetics, etc., and was a leading Guest Editor for the Special Section of "Advances in Theories and Industrial Applications of Networked Control Systems" in IEEE Transactions on Industrial Informatics.