Proceedings of the 36th Chinese Control Conference
July 26-28, 2017, Dalian, China

Adaptive Dynamic Programming Based Decentralized Tracking Control for Unknown Large-scale Systems

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Abstract: This paper investigates decentralized tracking control (DTC) problem for unknown large-scale systems via adaptive dynamic programming. The DTC consists of local desired control and local tracking error control. By replacing the actual states of coupled subsystems in interconnection terms with their desired states, the norm-boundedness assumption can be removed. Based on local neural network (NN) observers, unknown subsystems can be identified. It helps to obtain the local desired control with corresponding desired states. By establishing a proper local cost function, the local tracking error control is derived via local critic NN and the identified input gain matrix. Consider the entire error caused by replacement, identification and critic NN approximation, an adaptive robust term is added to construct the robust local cost function that reflects the entire error, regulation and control simultaneously. Therefore, the asymptotic stability can be guaranteed for the closed-loop tracking system via Lyapunov stability theorem. To demonstrate the effectiveness of the proposed DTC, simulation on a hard spring connected parallel inverted pendulum system is given.

Key Words: Adaptive dynamic programming, Unknown large-scale systems, Decentralized tracking control, Neural networks, Observer

1 Introduction

Decentralized control strategy is widely implemented to large-scale systems by using only the local information of corresponding subsystems. However, how to handle the interconnections is the major challenge in designing decentralized controllers. To solve this problem, considerable efforts have been made to design decentralized controllers for large-scale systems [1–5]. Despite of these methods have achieved excellent control performance, it is often desirable to design a controller which not only keeps systems stable, but also guarantees an adequate level of performance. It is worth pointing out that adaptive dynamic programming (ADP) [6] can solve Hamilton-Jacobi-Bellman (HJB) equations for optimal control of nonlinear systems.

Recently, ADP based decentralized control problems have been tackled extensively. For linear interconnected systems, Bian *et al*. [7] presented a decentralized control via robust ADP and policy iteration (PI) technique for complex systems with unknown parameters and dynamic uncertainties. Liu *et al*. [8] developed a decentralized stabilization by constructing the cost functions for isolated subsystems with the assumed known bounded interconnections. For unknown nonlinear interconnected systems, Liu *et al*. [9] investigated an actor-critic structure based decentralized control scheme via online model-free integral PI. For decentralized tracking control (DTC) problem, Mehraeen *et al*. [10] proposed a decentralized nearly optimal controller using online tuned action neural network (NN) and critic NN by assuming the input gain matrix was known and the unknown interconnection was weak. From the aforementioned literature, we can observe that the existing works on DTC via ADP mainly focused on the systems with available dynamics, rather than unknown dynamics.

In this paper, a DTC scheme via robust local cost function based ADP is proposed for unknown large-scale nonlinear systems. The superiorities of this approach lie in that: (i) The local NN observer identified subsystems can help to derive not only the local desired control, but also the local tracking error control. (ii) By replacing the actual states of coupled subsystems with their sharing desired states in interconnections, the common assumption on the boundedness of interconnections can be relaxed. (iii) Through considering the entire error caused by replacement, identification and critic NN approximation, a robust local cost function is established to guarantee the asymptotical stability of the closed-loop tracking system.

2 Problem Statement

Consider unknown large-scale systems that are composed of N interconnected subsystems, whose ith ($i = 1, 2, \ldots, N$) subsystem can be described by

$$\dot{x}_i(t) = f_i(x_i(t)) + g_i(x_i(t))u_i(x_i(t)) + h_i(x(t)), \quad (1)$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{i(n_i)}(t)]^\mathsf{T} \in \mathbb{R}^{n_i}, i = 1, \ldots, N$ and $u_i(x_i(t)) \in \mathbb{R}^{m_i}$ are the state vector and control input of the ith subsystem, respectively; $x(t) = [x_1(t), \ldots, x_N(t)]^\mathsf{T} \in \mathbb{R}^n$ with $n = \sum_{i=1}^N n_i$ is the overall system state vector; $f_i(x_i(t))$, $g_i(x_i(t))$ and $h_i(x(t))$ are

This work was supported in part by the National Natural Science Foundation of China under Grants U1501251, 61603387, 61533017, 61374051, 61374105 and 61503379, in part by the Scientific and Technological Development Plan Project in Jilin Province of China under Grants 20150520112JH and 20160414033GH, and in part by Beijing Natural Science Foundation under Grant 4162065.

unknown nonlinear internal dynamics, input gain matrix and interconnection term, respectively.

Assumption 1 *The nonlinear functions $f_i(x_i(t))$, $g_i(x_i(t))$ and $h_i(x(t))$ are Lipschitz and continuous in their arguments with $f_i(0) = 0$, and the subsystem (1) is controllable.*

To remove the assumption on the boundedness of interconnections, the actual states of coupled subsystems are replaced by their desired ones, so the interconnection term can be rewritten as

$$h_i(x) = h_i(x_i, x_{jd}) + \Delta h_i(x, x_{jd}), \tag{2}$$

where x_{jd} denotes the desired states of coupled subsystems with $j = 1, \ldots, i-1, i+1, \ldots, N$. $\Delta h_i(x, x_{jd}) = h_i(x) - h_i(x_i, x_{jd})$ denotes the replacement error. Thus, (1) becomes

$$\dot{x}_i = F_i(x_i, x_{jd}) + g_i(x_i)u_i(x_i) + \Delta h_i(x, x_{jd}), \tag{3}$$

where $F_i(x_i, x_{jd}) = f_i(x_i) + h_i(x_i, x_{jd})$, which is still Lipschitz continuous on a set $\Omega_i \in \mathbb{R}^{n_i}$ according to Assumption 1. Since the interconnection satisfies the global Lipschitz condition, which thus implies

$$\|\Delta h_i(x, x_{jd})\| \leq \sum_{j=1, j\neq i}^{n} d_{ij} E_j, \tag{4}$$

where $E_j = \|x_j - x_{jd}\|$, and $d_{ij} \geq 0$ is an unknown global Lipschitz constant.

The objective of this paper is to find a set of decentralized tracking control policies $u_1(x_1), \ldots, u_i(x_i), \ldots, u_N(x_N)$ such that the states of the entire unknown large-scale systems track the desired trajectories.

For the ith subsystem, define the tracking error as

$$e_i = x_i - x_{id}, \tag{5}$$

where x_{id} is the predefined desired trajectory.

Combining (5) with (2), the tracking error dynamics can be expressed as

$$\dot{e}_i = \dot{x}_i - \dot{x}_{id}. \tag{6}$$

Thus, associated with the tracking error dynamics (6), the local tracking error control policy should minimize the following local infinite horizon cost function

$$J_i(e_i(t)) = \int_t^{\infty} \left(\hat{\delta}_i \|e_i(\tau)\| + U_i\left(e_i(\tau), u_{ie}(\tau)\right) \right) \mathrm{d}\tau, \tag{7}$$

where $U_i(e_i(t), u_{ie}(t)) = e_i^{\mathsf{T}}(t) Q_i e_i(t) + u_{ie}^{\mathsf{T}}(t) R_i u_{ie}(t)$ is the utility function, $U_i(0,0) = 0$, and $U_i(e_i, u_{ie}) \geq 0$ for all e_i and u_{ie}, in which $Q_i \in \mathbb{R}^{2\times 2}$ and $R_i \in \mathbb{R}$ are positive definite matrices, $u_{ie} = u_i(x_i) - u_{id}(x_{id})$ is the local control input error, $u_{id}(x_{id})$ is the local desired control input, and $\hat{\delta}_i$ is the estimation of upper bound of the later defined entire error. It can be updated by

$$\dot{\hat{\delta}}_i = \Gamma_{i\delta} \|e_i\|. \tag{8}$$

From the so-called robust local cost function (7), we can see that it reflects the entire error, regulation and control simultaneously.

3 Decentralized tracking controller design

3.1 Local neural network observer based identifier

For the ith subsystem of the unknown large-scale system (1), it can be identified by a local NN observer as

$$\dot{\hat{x}}_i = \hat{F}_i(\hat{x}_i, x_{jd}) + \hat{g}_i(\hat{x}_i)u_i(x_i) + K_{io}(x_i - \hat{x}_i), \tag{9}$$

where $\hat{x}_i = [\hat{x}_{i1}, \ldots, \hat{x}_{i(n_i)}]^{\mathsf{T}} \in \mathbb{R}^{n_i}$ is the state vector of the developed local observer, $\hat{F}_i(\hat{x}_i, x_{jd})$ and $\hat{g}_i(\hat{x}_i)$ are the observation of nonlinear dynamics $F_i(x_i, x_{jd})$ and $g_i(x_i)$, respectively; $K_{io} = \mathrm{diag}[k_{i1o}, k_{i2o}]$ is a positive definite observation gain matrix.

Define the observation error vector as $e_{io} = x_i - \hat{x}_i$. Combining (3) with (9), the observation error dynamic is

$$\begin{aligned}\dot{e}_{io} = {} & F_i(x_i, x_{jd}) - \hat{F}_i(\hat{x}_i, x_{jd}) + \left(g_i(x_i) - \hat{g}_i(\hat{x}_i)\right) u_i(x_i) \\ & + \Delta h_i(x, x_{jd}) - K_{io} e_{io}.\end{aligned}$$

The nonlinear unknown terms $F_i(x_i, x_{jd})$ and $g_i(x_i)$ are approximated by two ideal radial basis function based NNs as

$$F_i(x_i, x_{jd}) = W_{if}^{\mathsf{T}} \sigma_{if}(x_i, x_{jd}) + \varepsilon_{if}, \ \ \|\varepsilon_{if}\| \leq \varepsilon_{i1}, \tag{10}$$

$$g_i(x_i) = W_{ig}^{\mathsf{T}} \sigma_{ig}(x_i) + \varepsilon_{ig}, \ \ \|\varepsilon_{ig}\| \leq \varepsilon_{i2}, \tag{11}$$

where W_{if} and W_{ig} are ideal weight vectors from the hidden layer to the output layer, $\sigma_{if}(x_i, x_{jd})$ and $\sigma_{ig}(x_i)$ are basis functions, ε_{if} and ε_{ig} are approximation errors, and ε_{i1} and ε_{i2} are unknown positive constants.

Let $\hat{W}_{if}$ and $\hat{W}_{ig}$ be the estimations of W_{if} and W_{ig}, respectively. We have

$$\hat{F}_i(\hat{x}_i, x_{jd}) = \hat{W}_{if}^{\mathsf{T}} \sigma_{if}(\hat{x}_i, x_{jd}), \tag{12}$$

$$\hat{g}_i(\hat{x}_i) = \hat{W}_{ig}^{\mathsf{T}} \sigma_{ig}(\hat{x}_i), \tag{13}$$

where $\hat{W}_{if}$ and $\hat{W}_{ig}$ can be updated by

$$\dot{\hat{W}}_{if} = \Gamma_{if} e_{io} \sigma_{if}(\hat{x}_i, x_{jd}), \tag{14}$$

$$\dot{\hat{W}}_{ig} = \Gamma_{ig} e_{io} \sigma_{ig}(\hat{x}_i) u_i, \tag{15}$$

where Γ_{if} and Γ_{ig} are positive constants.

Combining (10) and (12), (11) and (13), we have

$$\begin{aligned}F_i(x_i, x_{jd}) - \hat{F}_i(\hat{x}_i, x_{jd}) = {} & W_{if}^{\mathsf{T}} \tilde{\sigma}_{if}(x_i, \hat{x}_i, x_{jd}) \\ & + \tilde{W}_{if}^{\mathsf{T}} \sigma_{if}(\hat{x}_i, x_{jd}) + \varepsilon_{if},\end{aligned} \tag{16}$$

$$\begin{aligned}& g_i(x_i) - \hat{g}_i(\hat{x}_i) \\ & \quad = W_{ig}^{\mathsf{T}} \tilde{\sigma}_{ig}(x_i, \hat{x}_i) + \tilde{W}_{ig}^{\mathsf{T}} \sigma_{ig}(\hat{x}_i) + \varepsilon_{ig},\end{aligned} \tag{17}$$

where $\tilde{W}_{if} = W_{if} - \hat{W}_{if}$ and $\tilde{W}_{ig} = W_{ig} - \hat{W}_{ig}$ are the weight estimation errors, $\tilde{\sigma}_{if}(x_i, \hat{x}_i, x_{jd}) = \sigma_{if}(x_i, x_{jd}) - \hat{\sigma}_{if}(\hat{x}_i, x_{jd})$ and $\tilde{\sigma}_{ig}(x_i, \hat{x}_i) = \sigma_{ig}(x_i) - \sigma_{ig}(\hat{x}_i)$ are the estimation errors of radial basis functions, respectively.

Theorem 1 *For the ith subsystem of unknown large-scale systems (1), the developed local NN observer can guarantee the observation error e_{io} to be ultimately uniformly bounded (UUB) with the updated laws (14)–(15).*

Proof Select a Lyapunov function candidate as

$$L_{i1} = \frac{1}{2}e_{io}^{\mathsf{T}}e_{io} + \frac{1}{2}\tilde{W}_{if}^{\mathsf{T}}\Gamma_{if}^{-1}\tilde{W}_{if} + \frac{1}{2}\tilde{W}_{ig}^{\mathsf{T}}\Gamma_{ig}^{-1}\tilde{W}_{ig}. \quad (18)$$

Combining (19) with (16) and (17), the time derivative of (18) is

$$\begin{aligned}\dot{L}_{i1} &= e_{io}^{\mathsf{T}}\dot{e}_{io} - \tilde{W}_{if}^{\mathsf{T}}\Gamma_{if}^{-1}\dot{\hat{W}}_{if} - \tilde{W}_{ig}^{\mathsf{T}}\Gamma_{ig}^{-1}\dot{\hat{W}}_{ig}\\ &= e_{io}^{\mathsf{T}}\Big(\tilde{W}_{if}^{\mathsf{T}}\sigma_{if}(\hat{x}_i, x_{jd}) + \tilde{W}_{ig}^{\mathsf{T}}\sigma_{ig}(\hat{x}_i)u_i(x_i)\\ &\quad + w_{i1} + \Delta h_i(x, x_{jd})) - e_{io}^{\mathsf{T}}K_{io}e_{io}\\ &\quad - \tilde{W}_{if}^{\mathsf{T}}\Gamma_{if}^{-1}\dot{\hat{W}}_{if} - \tilde{W}_{ig}^{\mathsf{T}}\Gamma_{ig}^{-1}\dot{\hat{W}}_{ig} \end{aligned}\quad (19)$$

where

$$w_{i1} = W_{if}^{\mathsf{T}}\tilde{\sigma}_{if}(x_i, \hat{x}_i, x_{jd}) + \varepsilon_{if} + (W_{ig}^{\mathsf{T}}\tilde{\sigma}_{ig}(x_i, \hat{x}_i) + \varepsilon_{ig})u_i$$

denotes the entire approximation error.

Substituting (14) and (15) into (19), we have

$$\dot{L}_{i1} = e_{io}^{\mathsf{T}}\left(w_{i1} + \Delta h_i(x, x_{jd})\right) - e_{io}^{\mathsf{T}}K_{io}e_{io}. \quad (20)$$

Assumption 2 *The defined entire approximation error w_{i1} is norm-bounded, i.e., $\|w_{i1}\| \le \eta_{i1}$, where η_{i1} is an unknown positive constant.*

Letting $\eta_{i2} = \sum_{j=1, j\neq i}^{N} d_{ij}E_j$. According to (4), (20) becomes

$$\begin{aligned}\dot{L}_{i1} &\le \|e_{io}\|\left(\eta_{i1} + \eta_{i2}\right) - \lambda_{\min}(K_{io})\|e_{io}\|^2\\ &= -\|e_{io}\|\left(\lambda_{\min}(K_{io})\|e_{io}\| - (\eta_{i1} + \eta_{i2})\right),\end{aligned}$$

where $\lambda_{\min}(K_{io})$ denotes the minimum eigenvalue of K_{io}. We can observe that $\dot{L}_{i1} \le 0$ when e_{io} lies outside of the compact set $\Omega_{e_{io}} = \left\{e_{io} : \|e_{io}\| \le \frac{\eta_{i1}+\eta_{i2}}{\lambda_{\min}(K_{io})}\right\}$. Therefore, according to Lyapunov's direct method, the observation error e_{io} is UUB. This completes the proof.

3.2 The decentralized tracking controller design

The optimal tracking control is generally composed of the feedforward control and the feedback control [11]. Thus, the local desired control as feedforward control should be designed as follows.

According to the local NN observer (9), the identified dynamics of subsystem can be described as

$$\dot{\hat{x}}_i = \hat{F}_i(\hat{x}_i, x_{jd}) + \hat{g}_i(\hat{x}_i)u_i(\hat{x}_i).$$

By using the desired states x_{id} of the ith subsystem, the local desired control can be obtained as

$$u_{id}(x_{id}) = \hat{g}_i^{+}(x_{id})\left(\dot{x}_{id} - \hat{F}_i(x_d)\right), \quad (21)$$

where $\hat{g}_i^{+}(x_{id})\hat{g}_i(x_{id}) = I$, $I \in \mathbb{R}^{m_i\times m_i}$ is identity matrix.

To obtain a optimal tracking control performance, the designed local tracking error control policy should be admissible [12, 13].

Definition 1 *For local tracking error dynamics (6), a local tracking error control policy $\mu_{ie}(e_i)$ is defined to be admissible if $\mu_{ie}(e_i)$ is continuous on a set Ω_i with $\mu_{ie}(0) = 0$, $\mu_{ie}(e_i)$ ensures the convergence of the ith subsystem (6) on Ω_i, and $J_i(e_i(t))$ is finite for all $e_i \in \Omega_i$.*

For any admissible control policy $\mu_i(e_i) \in \psi_i(\Omega_i)$ of subsystem (6), where $\psi_i(\Omega_i)$ is the set of admissible control, if the robust local cost function

$$V_i(e_i(t)) = \int_t^{\infty}\left(\hat{\delta}_i\|e_i(\tau)\| + U_i\left(e_i(\tau), \mu_{ie}(\tau)\right)\right)\mathrm{d}\tau \quad (22)$$

is continuously differentiable, then the infinitesimal version of (20) is the so-called local Lyapunov equation

$$0 = \hat{\delta}_i\|e_i(t)\| + U_i\left(e_i, \mu_{ie}\right) + \left(\nabla V_i(e_i)\right)^{\mathsf{T}}\dot{e}_i \quad (23)$$

with $V_i(0) = 0$, and the term $\nabla V_i(e_i)$ denotes the partial derivative of $V_i(e_i)$ with respect to the local tracking error e_i, i.e., $\nabla V_i(e_i) = \partial V_i(e_i)/\partial e_i$.

The Hamiltonian of the optimal problem and the optimal robust local cost function can be formulated as

$$\begin{aligned}H_i\left(e_i, \mu_{ie}, \nabla V_i(e_i), \hat{\delta}_i\right) = \;&\hat{\delta}_i\|e_i(t)\| + U_i\left(e_i, \mu_{ie}\right)\\ &+ \left(\nabla V_i(e_i)\right)^{\mathsf{T}}\dot{e}_i,\end{aligned}$$

and

$$J_i^*(e_i) = \min_{\mu_{ie}\in\psi_i(e_i)}\int_t^{\infty}\left(\hat{\delta}_i\|e_i(\tau)\| + U_i\left(e_i(\tau), \mu_{ie}(\tau)\right)\right)\mathrm{d}\tau.$$

Thus,

$$0 = \min_{\mu_{ie}\in\psi_i(e_i)} H_i\left(e_i, \mu_{ie}, \nabla J_i^*(e_i), \hat{\delta}_i\right),$$

where $\nabla J_i^*(e_i) = \partial J_i^*(e_i)/\partial e_i$. If the solution $J_i^*(e_i)$ exists and is continuously differentiable, the ideal local optimal tracking error control can be described as

$$u_{ie}^*(e_i) = -\frac{1}{2}R_i^{-1}\hat{g}_i^{\mathsf{T}}(x_i)\nabla J_i^*(e_i). \quad (24)$$

In (24), we can see that $\nabla J_i^*(e_i)$ is difficult to obtain. Here, we employ a local critic NN to approximate the robust local cost function on the compact set Ω_i as

$$V_i(e_i) = W_{ic}^{\mathsf{T}}\sigma_{ic}(e_i) + \varepsilon_{ic}(e_i),$$

where $W_{ic} \in \mathbb{R}^{l_i}$ is the ideal weight vector, $\sigma_{ic}(e_i) \in \mathbb{R}^{l_i}$ is the activation function, l_i is the number of neurons in the hidden layer, and $\varepsilon_{ic}(e_i)$ is the approximation error. Then, the gradient of $V_i(e_i)$ with respect to e_i is

$$\nabla V_i(e_i) = \left(\nabla\sigma_{ic}(e_i)\right)^{\mathsf{T}}W_{ic} + \nabla\varepsilon_{ic}(e_i), \quad (25)$$

where $\nabla\sigma_{ic}(e_i) = \partial\sigma_{ic}(e_i)/\partial e_i \in \mathbb{R}^{l_i\times n_i}$ and $\nabla\varepsilon_{ic}(e_i)$ are the gradients of the activation function and the approximation error, respectively.

Thus, according to (24), the ideal local optimal tracking error control can be derived as

$$\mu_{ie}(e_i) = -\frac{1}{2}R_i^{-1}\hat{g}_i^{\mathsf{T}}(x_i)\left(\left(\nabla\sigma_{ic}(e_i)\right)^{\mathsf{T}}W_{ic} + \nabla\varepsilon_{ic}(e_i)\right). \quad (26)$$

According to the framework of ADP-based approximate optimal control design, an approximate critic NN is established to estimate the infinite horizon robust local cost function as

$$\hat{V}_i(e_i) = \hat{W}_{ic}^{\mathsf{T}}\sigma_{ic}(e_i), \quad (27)$$

where $\hat{W}_{ic} \in \mathbb{R}^{l_i \times n_i}$ is the weight estimation. Similarly, the gradient of (27) with respect to e_i is

$$\nabla \hat{V}_i(e_i) = (\nabla \sigma_{ic}(e_i))^{\mathsf{T}} \hat{W}_{ic}. \tag{28}$$

Thus, using (26) and (28), the local tracking error control can be obtained as

$$\hat{\mu}_{ie}(e_i) = -\frac{1}{2} R_i^{-1} \hat{g}_i^{\mathsf{T}}(x_i) (\nabla \sigma_{ic}(e_i))^{\mathsf{T}} \hat{W}_{ic}. \tag{29}$$

Considering (23) and (25), one can obtain

$$\begin{aligned} 0 = &\Big((\nabla \sigma_{ic}(e_i))^{\mathsf{T}} W_{ic} + \nabla \varepsilon_{ic}(e_i) \Big) \dot{e}_i \\ &+ \hat{\delta}_i \|e_i\| + U_i(e_i, \mu_{ie}). \end{aligned}$$

Therefore, the Hamiltonian can be expressed as

$$\begin{aligned} &H_i(e_i, \mu_{ie}, W_{ic}, \hat{\delta}_i) \\ &\quad = \hat{\delta}_i \|e_i\| + U_i(e_i, \mu_{ie}) + W_{ic}^{\mathsf{T}} \nabla \sigma_i(e_i) \dot{e}_i \\ &\quad = -\nabla \varepsilon_{ic}(x_i) \dot{e}_i = e_{icH}, \end{aligned} \tag{30}$$

where e_{icH} is the residual error.

Hence, the approximate Hamiltonian can be formulated by

$$\begin{aligned} \hat{H}_i(e_i, \mu_{ie}, \hat{W}_{ic}, \hat{\delta}_i) &= \hat{\delta}_i \|e_i\| + U_i(e_i, \mu_{ie}) + \hat{W}_{ic}^{\mathsf{T}} \nabla \sigma_i(e_i) \dot{e}_i \\ &= e_{ic}. \end{aligned} \tag{31}$$

Let $\theta_i = \nabla \sigma_i(e_i) \dot{e}_i$. From (30) and (31), we have

$$e_{ic} = e_{icH} - \tilde{W}_{ic}^{\mathsf{T}} \theta_i,$$

where $\tilde{W}_{ic} = W_{ic} - \hat{W}_{ic}$, and it can be updated as

$$\dot{\tilde{W}}_{ic} = -\dot{\hat{W}}_{ic} = l_{i1}(e_{icH} - \tilde{W}_{icH}^{\mathsf{T}} \theta_i) \theta_i, \tag{32}$$

where $l_{i1} > 0$ is the learning rate of the local critic NN.

To obtain the update rule of the critic NN weight vector $\hat{W}_{ic}$, the objective function $E_{ic} = \frac{1}{2} e_{ic}^{\mathsf{T}} e_{ic}$ should be minimized with the steepest decent algorithm as

$$\dot{\hat{W}}_{ic} = -\dot{\tilde{W}}_{ic} = -l_{i1} e_{ic} \theta_i. \tag{33}$$

Theorem 2 *For the ith subsystem of unknown large-scale system (1), the local critic NN weight approximation error $\tilde{W}_{ic}$ can be guaranteed to be UUB as long as the weights of the local critic NN are updated by (32).*

Proof Select the Lyapunov function candidate as

$$L_{i2} = \frac{1}{2 l_{i1}} \tilde{W}_{ic}^{\mathsf{T}} \tilde{W}_{ic}. \tag{34}$$

Considering (32), the time derivative of (34) is

$$\begin{aligned} \dot{L}_{i2} &= \frac{1}{l_{i1}} \tilde{W}_{ic}^{\mathsf{T}} \dot{\tilde{W}}_{ic} \\ &= \tilde{W}_{ic}^{\mathsf{T}} e_{icH} \theta_i - \left\| \tilde{W}_{ic} \theta_i \right\|^2 \\ &\le \frac{1}{2} e_{icH}^2 - \frac{1}{2} \left\| \tilde{W}_{ic} \theta_i \right\|^2. \end{aligned}$$

Assume $\|\theta_i\| \le \theta_{iM}$, Hence, $\dot{L}_{i2} < 0$ whenever the approximation error of the local critic NN $\tilde{W}_{ic}$ lies outside of the compact set $\Omega_{\tilde{W}_{ic}} = \left\{ \tilde{W}_{ic} : \left\| \tilde{W}_{ic} \right\| \le \left\| \frac{e_{icH}}{\theta_{iM}} \right\| \right\}$. According to Lyapunov stability theorem, the weight approximate error of local critic NN is UUB. This completes the proof.

Now, the DTC can be obtained as

$$u_i = u_{id} + \hat{\mu}_{ie}. \tag{35}$$

3.3 Stability analysis

Theorem 3 *Consider the ith subsystem of unknown large-scale system (1) and the robust local cost function (7). The proposed DTC scheme (35) can guarantee the closed-loop unknown large-scale system to be asymptotically stable.*

Proof Select the Lyapunov function candidate as

$$L_{i3} = \frac{1}{2} e_i^{\mathsf{T}} e_i + V_i(e_i) + \Gamma_{i\delta}^{-1} \tilde{\delta}_i^2, \tag{36}$$

where $\tilde{\delta}_i = \delta_i - \hat{\delta}_i$ denotes the estimation error of δ_i. The time derivative of (36) is

$$\begin{aligned} \dot{L}_{i3} &= e_i^{\mathsf{T}} \dot{e}_i + \nabla V_i^{\mathsf{T}}(e_i) \dot{e}_i - \Gamma_{i\delta}^{-1} \tilde{\delta}_i \dot{\hat{\delta}}_i \\ &= e_i^{\mathsf{T}} \Big(F_i(x_i, x_{jd}) - F_i(x_d) + F_i(x_d) - \hat{F}_i(x_d) \\ &\quad + \Delta h_i(x, x_{jd}) \Big) + e_i^{\mathsf{T}} \big((g_i(x_i) - g_i(x_{id}) + g_i(x_{id}) \\ &\quad - \hat{g}_i(x_{id}) + \hat{g}_i(x_{id})) u_i(x_i) - \hat{g}_i(x_{id}) u_{id}(x_{id}) \big) \\ &\quad - \hat{\delta}_i \|e_i\| - U_i(e_i, \mu_{ie}) - \Gamma_{i\delta}^{-1} \tilde{\delta}_i \dot{\hat{\delta}}_i. \end{aligned} \tag{37}$$

As $F_i(\cdot)$ is locally Lipschitz, there exists a positive constant η_{if} such that $\|F_i(x_i, x_{jd}) - F_i(x_d)\| \le \eta_{if} \|e_i\|$. Assuming that $\|\hat{g}_i(x_{id})\| \le \eta_{ig}$, and introducing (35), (37) becomes

$$\begin{aligned} \dot{L}_{i3} &\le \eta_{if} \|e_i\|^2 + e_i^{\mathsf{T}} w_{i2} + e_i^{\mathsf{T}} \Delta h_i(x, x_{jd}) + \eta_{ig} \|e_i\| \|\mu_{ie}\| \\ &\quad - \hat{\delta}_i \|e_i\| - U_i(e_i, \mu_{ie}) - \Gamma_{i\delta}^{-1} \tilde{\delta}_i \dot{\hat{\delta}}_i \\ &\le \eta_{if} \|e_i\|^2 + e_i^{\mathsf{T}} w_{i2} + e_i^{\mathsf{T}} \Delta h_i(x, x_{jd}) + \frac{1}{2} \|e_i\|^2 \\ &\quad - \hat{\delta}_i \|e_i\| - \lambda_{\min}(Q_i) \|e_i\|^2 \\ &\quad - \left(\lambda_{\min}(R_i) - \eta_{ig}^2 \right) \|\mu_{ie}\|^2 - \Gamma_{i\delta}^{-1} \tilde{\delta}_i \dot{\hat{\delta}}_i, \end{aligned}$$

where $w_{i2} = \tilde{F}_i(x_d) + \tilde{g}_i(x_i, x_{id}) u_i(x_i) - \tilde{\mu}_{ie}$ with $\tilde{g}_i(x_i, x_{id}) = g_i(x_i) - \hat{g}_i(x_{id})$, which can be assumed to be boundedness, i.e., $\|w_{i2}\| \le \delta_{i2}$.

Thus, according to (7), we have

$$\begin{aligned} \dot{L}_3 &= \sum_{i=1}^{n} \dot{L}_{i3} \\ &\le \sum_{i=1}^{n} \Big(\eta_{if} \|e_i\|^2 + e_i^T \delta_{i2} + \frac{1}{2} \|e_i\|^2 - \lambda_{\min}(Q_i) \|e_i\|^2 \\ &\quad - \hat{\delta}_i \|e_i\| - \left(\lambda_{\min}(R_i) - \eta_{ig}^2 \right) \|\mu_{ie}\|^2 - \Gamma_{i\delta}^{-1} \tilde{\delta}_i \dot{\hat{\delta}}_i \Big) \\ &\quad + \max_{ij}\{d_{ij}\} \sum_{i=1}^{n} \|e_i\| \sum_{j=1}^{n} E_j. \end{aligned} \tag{38}$$

Noticing that $\|e_i\| \le \|e_j\| \Leftrightarrow E_i \le E_j$. Using Chebyshev inequality, we have

$$\sum_{i=1}^{n} \|e_i\| \sum_{j=1}^{n} E_j \le n \sum_{i=1}^{n} \|e_i\| E_j. \tag{39}$$

Combining (38) and (39), we have

$$\begin{aligned} \dot{L}_3 &\le \sum_{i=1}^{n} \Big((\eta_{if} - \lambda_{\min}(Q_i)) \|e_i\|^2 + \|e_i\| \delta_{i2} + \frac{1}{2} \|e_i\|^2 \\ &\quad - \hat{\delta}_i \|e_i\| - \left(\lambda_{\min}(R_i) - \eta_{ig}^2 \right) \|\mu_{ie}\|^2 - \Gamma_{i\delta}^{-1} \tilde{\delta}_i \dot{\hat{\delta}}_i \Big) \\ &\quad + n \max_{ij}\{d_{ij}\} \sum_{i=1}^{n} \|e_i\| E_i. \end{aligned} \tag{40}$$

Defining $\delta_i = \delta_{i2} + n \max_{ij}\{d_{ij}\}$, and we have

$$\begin{aligned}\dot{L}_3 \leq & \sum_{i=1}^{n} \Big((\eta_{if} - \lambda_{\min}(Q_i)) \|e_i\|^2 + \|e_i\| \delta_i + \frac{1}{2} \|e_i\|^2 \\ & -\hat{\delta}_i \|e_i\| - \left(\lambda_{\min}(R_i) - \eta_{ig}^2\right) \|\mu_{ie}\|^2 - \Gamma_{i\delta}^{-1} \tilde{\delta}_i \dot{\hat{\delta}}_i \Big) \\ = & \sum_{i=1}^{n} \left(- \left(\lambda_{\min}(Q_i) - \eta_{if} - \frac{1}{2} \right) \|e_i\|^2 + \|e_i\| \tilde{\delta}_i \right. \\ & \left. - \left(\lambda_{\min}(R_i) - \eta_{ig}^2\right) \|\mu_{ie}\|^2 - \Gamma_{i\delta}^{-1} \tilde{\delta}_i \dot{\hat{\delta}}_i \right). \qquad (41)\end{aligned}$$

Substituting (8) into (41), we have

$$\begin{aligned}\dot{L}_3 \leq & \sum_{i=1}^{n} \Big(\eta_{if} \|e_i\|^2 + \|e_i\| \delta_i + \frac{1}{2} \|e_i\|^2 - \lambda_{\min}(Q_i) \|e_i\|^2 \\ & -\hat{\delta}_i \|e_i\| - \left(\lambda_{\min}(R_i) - \eta_{ig}^2\right) \|\mu_{ie}\|^2 - \Gamma_{i\delta}^{-1} \tilde{\delta}_i \dot{\hat{\delta}}_i \Big) \\ = & \sum_{i=1}^{n} \left(- \left(\lambda_{\min}(Q_i) - \eta_{if} - \frac{1}{2} \right) \|e_i\|^2 \right. \\ & \left. - \left(\lambda_{\min}(R_i) - \eta_{ig}^2\right) \|\mu_{ie}\|^2 \right).\end{aligned}$$

We can observe that $\dot{L}_{i3} \leq 0$ as long as

$$\begin{cases} \lambda_{\min}(Q_i) \geq \eta_{if} + \frac{1}{2}, \\ \lambda_{\min}(R_i) \geq \eta_{ig}^2. \end{cases}$$

It implies that the developed DTC scheme (35) ensures the closed-loop unknown large-scale system to be asymptotically stable based on Lyapunov stability theorem. This completes the proof.

4 Simulation study

Consider a hard spring connected parallel inverted pendulum system [14, 15] for simulation.

Let $x_i = [x_{i1}, x_{i2}]^{\mathsf{T}} = [\theta_{i1}, \dot{\theta}_{i1}]^{\mathsf{T}} \in \mathbb{R}^2$, the dynamic model can be expressed as

$$\dot{x}_i = f_i(x_i) + g_i(x_i) u_i + h_i(x),$$

where $f_i(x_i) = \begin{bmatrix} x_{i2} \\ 5.88 \sin x_{i1} - 0.036 x_{i2} \end{bmatrix}$, $g_i(x_i) = \begin{bmatrix} 0 \\ \delta_i \end{bmatrix}$, $h_i(x) = \begin{bmatrix} 0 \\ 4F a_i \cos(x_{i1} - \beta) \end{bmatrix}$.

The desired trajectories are given as

$$\begin{cases} x_{11d} = 0.5 \cos(0.5t), \\ x_{21d} = 0.8 \sin(0.3t + \pi/6). \end{cases}$$

In the simulation, the NNs of local observers are chosen as radial basis functions. Let the initial states of the subsystems be $x_{10} = x_{20} = [1, 0]^{\mathsf{T}}$, the initial states of the observers be $\hat{x}_{10} = [2, -1]^{\mathsf{T}}$, $\hat{x}_{20} = [1.5, -0.5]^{\mathsf{T}}$, the observer gain matrix be $K_{io} = \mathrm{diag}[400, 1200]$, the weights learning rate of the observer be $\Gamma_{if} = \Gamma_{ig} = 0.002$. The robust local cost function (7) is approximated by local critic NN, whose structure is chosen as 2–3–1 with 2 input neurons, 3 hidden neurons and 1 output neuron, and the weight vector as $\hat{W}_{ic} = [\hat{W}_{ic1}, \hat{W}_{ic2}, \hat{W}_{ic3}]^{\mathsf{T}}$ with initial values $\hat{W}_{1c} = [0.4, 1.8, 1.2]^{\mathsf{T}}$ and $\hat{W}_{2c} = [0.2, 0.4, 0.2]^{\mathsf{T}}$. The

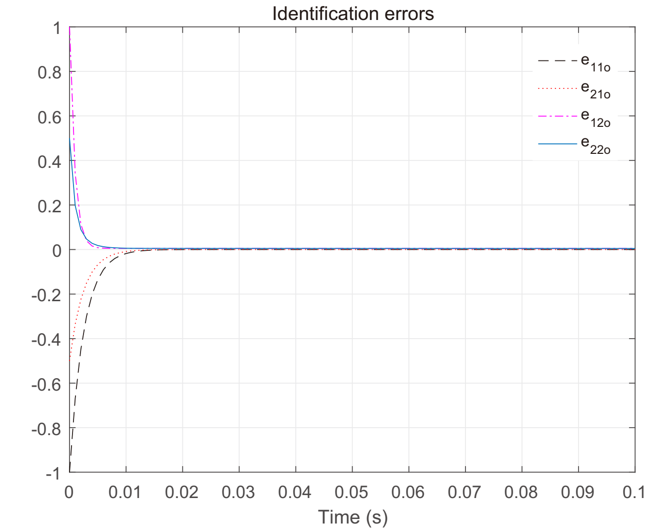

Fig. 1: The identification errors by using the local NN observer based identifier

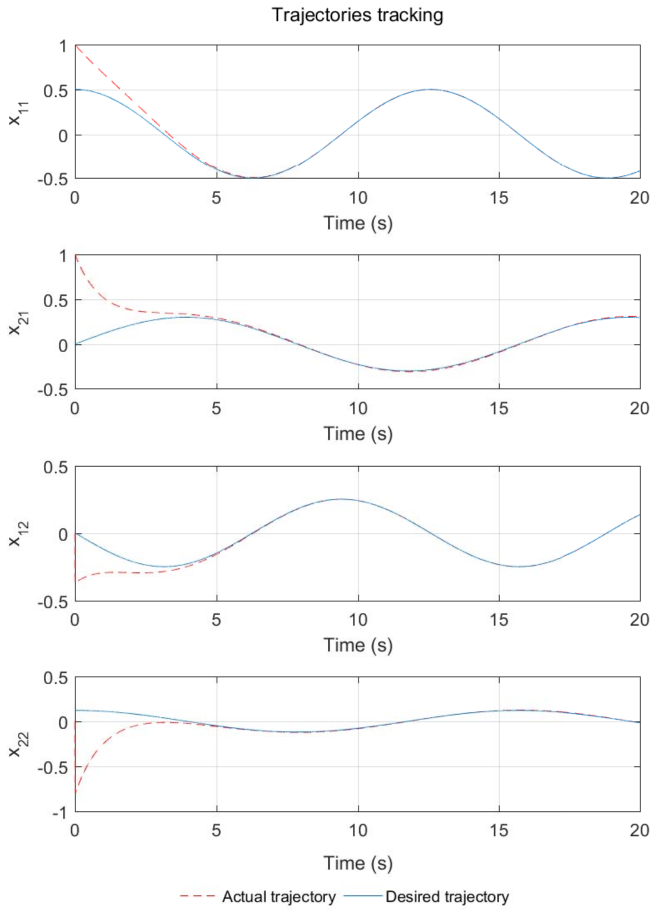

Fig. 2: The trajectories tracking performance

activation function of the critic NN is chosen as $\sigma_{ic}(e_i) = [e_{i1}^2, e_{i1}e_{i2}, e_{i2}^2]$. Let the weight learning rates of the critic NN be $l_{i1} = 0.1$, the gain of the robust term in (7) be $\Gamma_{i\delta} = 15$, and $Q_i = 2I_2$, $R_1 = 0.001I$, $R_2 = 0.0005I$, where I_n denotes the identity matrix with n dimensions, respectively.

The simulation results are shown as Figs. 1–4. By using

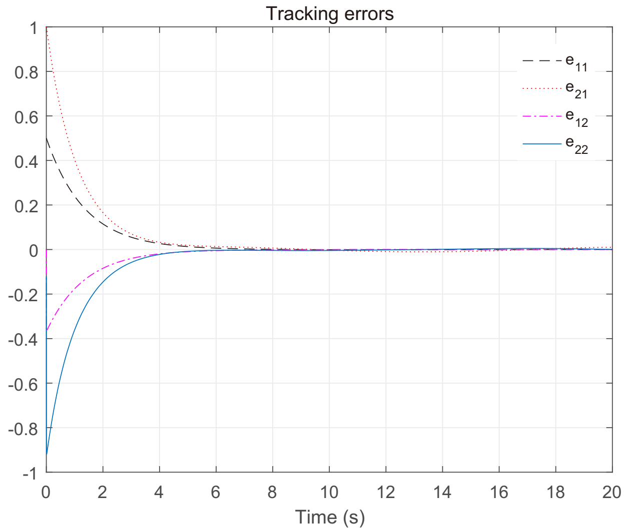

Fig. 3: The tracking errors

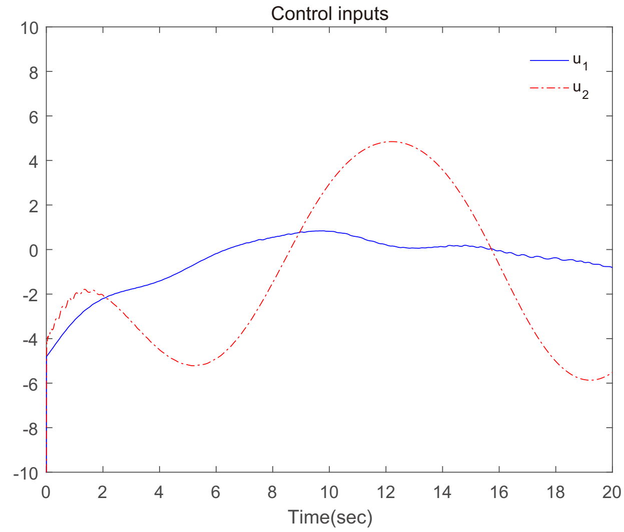

Fig. 4: The control inputs

local NN observers (9), the identification errors which are illustrated in Fig. 1 demonstrate that the unknown subsystem can be identified successfully. From Fig. 2 and Fig. 3, we can see that the actual trajectories follow their desired ones after a short time of operation with the developed DTC (37). Fig. 4 gives the curves of control inputs. From these figures, the closed-loop system can be guaranteed to be asymptotically stable. Therefore, the simulation results demonstrate the effectiveness of the proposed DTC scheme.

5 Conclusion

This paper addresses the DTC problems with ADP algorithm for unknown large-scale systems. The common boundedness assumption on interconnections is removed by substituting the actual states of the coupled subsystems with their desired states. Then, with the help of local NN observers, the unknown dynamics of subsystems can be identified. Furthermore, the local desired control can be derived with the desired trajectories of corresponding subsystems. By constructing robust local cost function which reflects the entire error, regulation and control simultaneously, the local tracking error control can be obtained. Thus, the DTC can be constructed with the combination of the local desired control and local tracking error control. The effectiveness of the proposed DTC scheme is demonstrated by simulations on a hard spring connected parallel inverted pendulum system.

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