Data-based Robust Near-Optimal Decentralized Stabilization of Unknown Large-Scale Systems

Bo Zhao, Derong Liu, and Yuanchun Li

Abstract— **This paper investigates a data-based robust decentralized stabilizing control scheme for unknown large-scale systems via adaptive critic designs. The control consists of near-optimal stabilizing control and adaptive robustifying compensation. In order to avoid the common assumptions of boundedness and matched condition on interconnections, the actual states of interconnected subsystems are replaced with their desired ones. By using local input-output data, the subsystem dynamics can be obtained by neural network (NN) identifier. Then, with the help of local critic NN, the near-optimal decentralized control is derived via identifier-critic architecture for corresponding subsystems. Considering the replacement error, identification error and approximation error as overall error, an adaptive robustifying term is added to overcome it in real-time. Simulation example is given to verify the effectiveness of the present control scheme.**

I. Introduction

Modern systems become large-scale and complex to adapt the increasing demands of production efficiency and quality. Thus, the complexity of these systems inspires the development of decentralized control, which uses only the local information of corresponding subsystems. As is well known, how to deal with the interconnections is the major challenge in designing decentralized controllers for large-scale systems. The treatment quality affects the control performance directly, or even cause the system to be unstable. To solve this problem, considerable efforts have been made to design decentralized controllers for large-scale systems [1]–[4]. Although these methods have obtained excellent control performance, it is often desirable to design a controller which not only keeps systems stable, but also guarantees an adequate level of performance [5].

Optimal control problems of nonlinear systems can be addressed by solving the Hamilton-Jacobi-Bellman (HJB) equations, which can be solved by adaptive dynamic programming (ADP) [6], [7] with approximators, such as neural networks (NNs). In recent literature, ADP based decentralized control problems have been tackled extensively [8], [9]. For linear interconnected systems, Jiang *et al.* [10] and Bian *et al.* [11] presented robust ADP and policy iteration (PI) based decentralized control. Based on both PI and value iteration methods, Gao *et al.* [12] developed a data-driven output-feedback control scheme. Tlili *et al.* [13] formulated the control problem of linear systems with nonlinear interconnections as an optimization problem, and developed a decentralized observer and control approach by linear matrix inequality. Hioe *et al.* [14] addressed the robust nonlinear control problem which was transformed from the dissipativity shaping problem by employing linear partial differential Hamilton-Jacobi equation. Liu *et al.* [15] developed a decentralized control scheme by constructing the cost functions for the isolated subsystems by assuming the interconnections to be available bounded. Furthermore, Wang *et al.* [16] constructed a modified cost function for the overall plant, then developed the decentralized guaranteed cost control. For unknown nonlinear interconnected systems, Liu *et al.* [17] proposed a decentralized control scheme via online model-free integral PI algorithm based on actor-critic technique. Lu *et al.* [18] utilized direct heuristic dynamic programming (HDP) to solve the coordinated control for large power systems with uncertainties. Yang *et al.* [19] presented a direct HDP based decentralized tracking control scheme with filtered tracking error.

This paper establishes a data-based robust near-optimal decentralized stabilizing control scheme for large-scale nonlinear systems via adaptive critic design method. The contributions of this method lie in that: i) The assumptions of boundedness and matched condition on interconnections can be removed by replacing the actual states of the interconnection with their desired ones; ii) Consider the replacement error, identification error and approximation error of the local cost function as overall error, an adaptive robustifying term is added to guarantee the closed-loop system to be asymptotically stable, rather than ultimately uniformly bounded (UUB); iii) The local critic NN is trained online, it implies that the initial stabilizing control and persisting of excitation condition are not required any more.

II. Problem Statement

Consider unknown large-scale nonlinear systems that are composed of N interconnected subsystems, whose ith $(i = 1, 2, \dots, N)$ subsystem can be described by

$$\dot{x}_i(t) = f_i(x_i(t)) + g_i(x_i(t))u_i(x_i(t)) + h_i(x(t)), \quad (1)$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \dots, x_{i(n_i)}(t)]^{\mathsf{T}} \in \mathbb{R}^{n_i}, i = 1, \dots, N$ and $u_i(x_i(t)) \in \mathbb{R}^{m_i}$ are the state vector and

Bo Zhao is with the State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China. (email: zhaobo@ia.ac.cn).

Derong Liu is with the School of Automation, Guangdong University of Technology, Guangzhou 510006, China (email: derongliu@foxmail.com).

Yuanchun Li is with the Department of Control Science and Engineering, Changchun University of Technology, Changchun 130012, China. (email: liyc@ccut.edu.cn).

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control input of the ith subsystem, respectively; $x(t) = [x_1^\mathsf{T}(t), \ldots, x_N^\mathsf{T}(t)]^\mathsf{T} \in \mathbb{R}^n$ is the overall system state vector with $n = \sum_{i=1}^{N} n_i$; $f_i(x_i(t))$, $g_i(x_i(t))$ and $h_i(x(t))$ are unknown nonlinear internal dynamics, input gain matrix and mismatched interconnection term, respectively.

Assumption 1: The nonlinear functions $f_i(x_i(t))$, $g_i(x_i(t))$ and $h_i(x(t))$ are Lipschitz and continuous in their arguments with $f_i(0) = 0$, and the subsystem (1) is controllable.

Unlike assuming $h_i(x)$ to be bounded and satisfying the matching conditions [15], the desired states of the interconnected subsystems are employed to replace their actual ones, so the interconnection term can be expressed as

$$h_i(x) = h_i(x_i, x_{jd}) + \Delta h_i(x, x_{jd}), \tag{2}$$

where x_{jd} denotes the desired states of the interconnected subsystems with $j = 1, \ldots, i-1, i+1, \ldots N$. $\Delta h_i(x, x_{jd}) = h_i(x) - h_i(x_i, x_{jd})$ denotes the replacement error. Thus, (1) becomes

$$\dot{x}_i = F_i(x_i, x_{jd}) + g_i(x_i)u_i(x_i) + \Delta h_i(x, x_{jd}), \tag{3}$$

where $F_i(x_i, x_{jd}) = f_i(x_i) + h_i(x_i, x_{jd})$, which is still Lipschitz continuous on a set $\Omega_i \in \mathbb{R}^{n_i}$. According to Assumption 1, the interconnection satisfies the global Lipschitz condition, which implies

$$\|\Delta h_i(x, x_{jd})\| \leq \sum_{j=1, j\neq i}^{n} d_{ij} E_j, \tag{4}$$

where $E_j = \|x_j - x_{jd}\|$, and $d_{ij} \geq 0$ is an unknown global Lipschitz constant.

The objective of this paper is to find a set of decentralized control policies $u_1(x_1), \ldots, u_i(x_i), \ldots, u_N(x_N)$ to stabilize the states of the overall unknown large-scale nonlinear system.

Remark 1: The interconnection term is approximated by local subsystem states and desired states of interconnected subsystems. It is worth mentioning that the desired states, which are shared to each subsystem before operating the controlled system, depend on the control objective.

III. Robust Decentralized Stabilization

A. Neural network based subsystem identification

For the unknown subsystem dynamics (1), it can be identified by input-output data and local NN as

$$\dot{\hat{x}}_i = \hat{F}_i(\hat{x}_i, x_{jd}) + \hat{g}_i(\hat{x}_i)u_i(x_i) + K_{io}(x_i - \hat{x}_i), \tag{5}$$

where $\hat{x}_i = [\hat{x}_{i1}, \ldots, \hat{x}_{i(n_i)}]^\mathsf{T} \in \mathbb{R}^{n_i}$ is the state vector of the developed identifier, $\hat{F}_i(\hat{x}_i, x_{jd})$ and $\hat{g}_i(\hat{x}_i)$ are the estimations of nonlinear dynamics $F_i(x_i, x_{jd})$ and $g_i(x_i)$, respectively; $K_{io} = \operatorname{diag}[k_{i1o}, k_{i2o}]$ is a positive definite identification gain matrix.

Define the identification error vector as $e_{io} = x_i - \hat{x}_i$. Combining (3) with (5), the identification error dynamics can be described as

$$\begin{aligned}\dot{e}_{io} = {} & F_i(x_i, x_{jd}) - \hat{F}_i(\hat{x}_i, x_{jd}) + (g_i(x_i) - \hat{g}_i(\hat{x}_i))\, u_i(x_i) \\ & + \Delta h_i(x, x_{jd}) - K_{io}e_{io}.\end{aligned}$$

The nonlinear unknown terms $F_i(x_i, x_{jd})$ and $g_i(x_i)$ are approximated by two ideal radial basis function (RBF) NNs as

$$F_i(x_i, x_{jd}) = W_{if}^\mathsf{T}\sigma_{if}(x_i, x_{jd}) + \varepsilon_{if}, \;\; \|\varepsilon_{if}\| \leq \varepsilon_{i1}, \tag{6}$$

$$g_i(x_i) = W_{ig}^\mathsf{T}\sigma_{ig}(x_i) + \varepsilon_{ig}, \;\; \|\varepsilon_{ig}\| \leq \varepsilon_{i2}, \tag{7}$$

where W_{if} and W_{ig} are ideal weight vectors from the hidden layer to the output layer, $\sigma_{if}(x_i, x_{jd})$ and $\sigma_{ig}(x_i)$ are basis functions, ε_{if} and ε_{ig} are approximation errors, and ε_{i1} and ε_{i2} are unknown positive constants.

Let $\hat{W}_{if}$ and $\hat{W}_{ig}$ be the estimations of W_{if} and W_{ig}, respectively. We have

$$\hat{F}_i(\hat{x}_i, x_{jd}) = \hat{W}_{if}^\mathsf{T}\sigma_{if}(\hat{x}_i, x_{jd}), \tag{8}$$

$$\hat{g}_i(\hat{x}_i) = \hat{W}_{ig}^\mathsf{T}\sigma_{ig}(\hat{x}_i), \tag{9}$$

where $\hat{W}_{if}$ and $\hat{W}_{ig}$ can be updated by the adaptive laws as

$$\dot{\hat{W}}_{if} = \Gamma_{if}e_{io}\hat{\sigma}_{if}(\hat{x}_i, x_{jd}), \tag{10}$$

$$\dot{\hat{W}}_{ig} = \Gamma_{ig}e_{io}\hat{\sigma}_{ig}(\hat{x}_i)u_i, \tag{11}$$

where Γ_{if} and Γ_{ig} are positive constants, $\hat{\sigma}_{if}$ and $\hat{\sigma}_{ig}$ are estimations of basis functions σ_{if} and σ_{ig}, respectively.

Combining (6) and (8), (7) and (9), we have

$$\begin{aligned}F_i(x_i, x_{jd}) - \hat{F}_i(\hat{x}_i, x_{jd}) = {} & W_{if}^\mathsf{T}\tilde{\sigma}_{if}(x_i, \hat{x}_i, x_{jd}) \\ & + \tilde{W}_{if}^\mathsf{T}\sigma_{if}(x_i, x_{jd}) + \varepsilon_{if},\end{aligned} \tag{12}$$

$$g_i(x_i) - \hat{g}_i(\hat{x}_i) = W_{ig}^\mathsf{T}\tilde{\sigma}_{ig}(x_i, \hat{x}_i) + \tilde{W}_{ig}^\mathsf{T}\sigma_{ig}(x_i) + \varepsilon_{ig}, \tag{13}$$

where $\tilde{W}_{if} = W_{if} - \hat{W}_{if}$ and $\tilde{W}_{ig} = W_{ig} - \hat{W}_{ig}$ are the weight estimation errors, $\tilde{\sigma}_{if}(x_i, \hat{x}_i, x_{jd}) = \sigma_{if}(x_i, x_{jd}) - \hat{\sigma}_{if}(\hat{x}_i, x_{jd})$ and $\tilde{\sigma}_{ig}(x_i, \hat{x}_i) = \sigma_{ig}(x_i) - \sigma_{ig}(\hat{x}_i)$ are the estimation errors, respectively.

Theorem 1: For interconnected subsystem (1), the developed local NN identifier can guarantee the identification error e_{io} to be UUB online with the updating laws (10)–(11).

Proof: Choose a Lyapunov function candidate as

$$L_{i1} = \frac{1}{2}e_{io}^\mathsf{T}e_{io} + \frac{1}{2}\tilde{W}_{if}^\mathsf{T}\Gamma_{if}^{-1}\tilde{W}_{if} + \frac{1}{2}\tilde{W}_{ig}^\mathsf{T}\Gamma_{ig}^{-1}\tilde{W}_{ig}. \tag{14}$$

The time derivative of (14) is

$$\begin{aligned}\dot{L}_{i1} = {} & e_{io}^\mathsf{T}\dot{e}_{io} - \tilde{W}_{if}^\mathsf{T}\Gamma_{if}^{-1}\dot{\hat{W}}_{if} - \tilde{W}_{ig}^\mathsf{T}\Gamma_{ig}^{-1}\dot{\hat{W}}_{ig} \\ = {} & e_{io}^\mathsf{T}(F_i(x_i, x_{jd}) - \hat{F}_i(\hat{x}_i, x_{jd}) + \Delta h_i(x, x_{jd}) \\ & + (g_i(x_i) - \hat{g}_i(\hat{x}_i))\, u_i(x_i) - K_{io}e_{io}) \\ & - \tilde{W}_{if}^\mathsf{T}\Gamma_{if}^{-1}\dot{\hat{W}}_{if} - \tilde{W}_{ig}^\mathsf{T}\Gamma_{ig}^{-1}\dot{\hat{W}}_{ig}.\end{aligned} \tag{15}$$

Combining (15) with (12) and (13), we have

$$\begin{aligned}\dot{L}_{i1} = {} & e_{io}^\mathsf{T}(\tilde{W}_{if}^\mathsf{T}\sigma_{if}(\hat{x}_i, x_{jd}) + \tilde{W}_{ig}^\mathsf{T}\sigma_{ig}(\hat{x}_i)u_i(x_i) \\ & + w_{i1} + \Delta h_i(x, x_{jd})) - e_{io}^\mathsf{T}K_{io}e_{io} \\ & - \tilde{W}_{if}^\mathsf{T}\Gamma_{if}^{-1}\dot{\hat{W}}_{if} - \tilde{W}_{ig}^\mathsf{T}\Gamma_{ig}^{-1}\dot{\hat{W}}_{ig},\end{aligned} \tag{16}$$

where $w_{i1} = W_{if}^{\mathsf{T}}\tilde{\sigma}_{if}(x_i, \hat{x}_i, x_{jd}) + \varepsilon_{if} + (W_{ig}^{\mathsf{T}}\tilde{\sigma}_{ig}(x_i, \hat{x}_i) + \varepsilon_{ig})u_i$ denotes the overall NN approximation error.

Substituting (10) and (11) into (16), we have

$$\dot{L}_{i1} = e_{io}^{\mathsf{T}}\left(w_{i1} + \Delta h_i(x, x_{jd})\right) - e_{io}^{\mathsf{T}}K_{io}e_{io}. \qquad (17)$$

Assumption 2: The defined approximation w_{i1} is norm-bounded, i.e., $\|w_{i1}\| \leq \eta_{i1}$, where η_{i1} is an unknown positive constant.

Letting $\eta_{i2} = \sum_{j=1, j\neq i}^{N} d_{ij}E_j$, according to (4), (17) becomes

$$\begin{aligned}\dot{L}_{i1} &\leq \|e_{io}\|\left(\eta_{i1} + \eta_{i2}\right) - \lambda_{\min}(K_{io})\|e_{io}\|^2 \\ &= -\|e_{io}\|\left(\lambda_{\min}(K_{io})\|e_{io}\| - (\eta_{i1} + \eta_{i2})\right),\end{aligned}$$

where $\lambda_{\min}(K_{io})$ denotes the minimum eigenvalue of K_{io}. We can observe that $\dot{L}_{i1} \leq 0$ when e_{io} lies outside of the compact set $\Omega_{e_{io}} = \left\{e_{io}\colon \|e_{io}\| \leq \frac{\eta_{i1}+\eta_{i2}}{\lambda_{\min}(K_{io})}\right\}$. Therefore, according to Lyapunov stability theorem, the identification error e_{io} is UUB. This completes the proof.

B. Robust decentralized controller design

Associated with the subsystem dynamics (3), the decentralized control policy should minimize the following infinite horizon local cost function

$$V_i(x_i(t)) = \int_t^{\infty} U_i\left(x_i(\tau), u_i(\tau)\right) \mathrm{d}\tau, \qquad (18)$$

where $U_i(x_i(t), u_i(x_i)) = x_i^{\mathsf{T}}(t)Q_ix_i(t) + u_i^{\mathsf{T}}(x_i)R_iu_i(x_i)$ is the local utility function, $U_i(0,0) = 0$, and $U_i(x_i, u_i) \geq 0$ for all x_i and u_i, in which $Q_i \in \mathbb{R}^{n_i\times n_i}$ and $R_i \in \mathbb{R}^{m_i}$ are positive definite matrices.

In order to obtain a feedback control policy, which can guarantee the closed-loop system to be stable for nonlinear subsystem (1), a stabilizing control $\mu_i(x_i) \in \psi_i(\Omega_i)$ will be derived by minimizing the continuously differentiable infinite local cost function

$$V_i(x_i(t)) = \int_t^{\infty} U_i\left(x_i(\tau), \mu_i(\tau)\right) \mathrm{d}\tau. \qquad (19)$$

Then, the infinitesimal version of (19) is the so-called Lyapunov equation

$$0 = U_i\left(x_i, \mu_i\right) + \left(\nabla V_i(x_i)\right)^{\mathsf{T}}\dot{x}_i \qquad (20)$$

with $V_i(0) = 0$, and the term $\nabla V_i(x_i)$ denotes the partial derivative of $V_i(x_i)$ with respect to the subsystem state x_i, i.e., $\nabla V_i(x_i) = \partial V_i(x_i)/\partial x_i$.

The Hamiltonian of the optimal control problem and the optimal cost function can be formulated as

$$H_i\left(x_i, \mu_i, \nabla V_i(x_i)\right) = U_i\left(x_i, \mu_i\right) + \left(\nabla V_i(x_i)\right)^{\mathsf{T}}\dot{x}_i,$$

and

$$V_i^*(x_i) = \min_{\mu_i\in\psi_i(x_i)} \int_t^{\infty} U_i\left(x_i(\tau), \mu_i(\tau)\right) \mathrm{d}\tau.$$

Thus,

$$0 = \min_{\mu_i\in\psi_i(x_i)} H_i\left(x_i, \mu_i, \nabla V_i^*(x_i)\right),$$

where $\nabla J_i^*(x_i) = \partial J_i^*(x_i)/\partial x_i$. If the solution $V_i^*(x_i)$ exists and is continuously differentiable, the ideal decentralized control can be described as

$$\mu_i^*(x_i) = -\frac{1}{2}R_i^{-1}g_i^{\mathsf{T}}(x_i)\nabla V_i^*(x_i). \qquad (21)$$

Since the local cost function is highly nonlinear and nonanalytic, it can be approximated by NNs, which are powerful tools for approximating nonlinear functions. For the ith subsystem, a critic NN is employed to approximate the corresponding assumed continuous local cost function on the compact set Ω_i as

$$V_i(x_i) = W_{ic}^{\mathsf{T}}\sigma_{ic}(x_i) + \varepsilon_{ic}(x_i), \qquad (22)$$

where $W_{ic} \in \mathbb{R}^{l_i\times n_i}$ is the ideal weight vector, $\sigma_{ic}(x_i) \in \mathbb{R}^{l_i}$ is the activation function, l_i is the number of neurons in the hidden layer, and $\varepsilon_{ic}(x_i)$ is the approximation error of NN. Then, the gradient of $V_i(x_i)$ with respect to x_i is

$$\nabla V_i(x_i) = \left(\nabla\sigma_{ic}(x_i)\right)^{\mathsf{T}}W_{ic} + \nabla\varepsilon_{ic}(x_i), \qquad (23)$$

where $\nabla\sigma_{ic}(x_i) = \partial\sigma_{ic}(x_i)/\partial x_i \in \mathbb{R}^{l_i}$ and $\nabla\varepsilon_{ic}(x_i)$ are the gradients of the activation function and the approximation error, respectively.

Combining (20) with (23), we have

$$0 = U_i\left(x_i, \mu_i\right) + \left(\left(\nabla\sigma_{ic}(x_i)\right)^{\mathsf{T}}W_{ic} + \nabla\varepsilon_{ic}(x_i)\right)^{\mathsf{T}}\dot{x}_i.$$

Therefore, the local Hamiltonian can be expressed as

$$\begin{aligned}H_i\left(x_i, \mu_i, W_{ic}\right) &= U_i\left(x_i, \mu_i\right) + W_{ic}^{\mathsf{T}}\nabla\sigma_i(x_i)\dot{x}_i \\ &= -\nabla\varepsilon_{ic}(x_i)\dot{x}_i = e_{icH},\end{aligned} \qquad (24)$$

where e_{icH} is the NN approximation error.

The local critic NN (22) can be estimated as

$$\hat{V}_i(x_i) = \hat{W}_{ic}^{\mathsf{T}}\sigma_{ic}(x_i), \qquad (25)$$

where $\hat{W}_{ic} \in \mathbb{R}^{l_i\times n_i}$ is the weight estimation.

Then, the gradient of (25) with respect to x_i is

$$\nabla\hat{V}_i(x_i) = \left(\nabla\sigma_{ic}(x_i)\right)^{\mathsf{T}}\hat{W}_{ic}.$$

Hence, the approximate local Hamiltonian is

$$H_i\left(x_i, \mu_i, \hat{W}_{ic}\right) = U_i\left(x_i, \mu_i\right) + \hat{W}_{ic}^{\mathsf{T}}\nabla\sigma_i(x_i)\dot{x}_i = e_{ic}. \qquad (26)$$

Let $\theta_i = \nabla\sigma_i(x_i)\dot{x}_i$. From (24) and (26), we have

$$e_{ic} = e_{icH} - \tilde{W}_{ic}^{\mathsf{T}}\theta_i,$$

where $\tilde{W}_{ic} = W_{ic} - \hat{W}_{ic}$, and it can be updated as

$$\dot{\tilde{W}}_{ic} = -\dot{\hat{W}}_{ic} = l_{i1}(e_{icH} - \tilde{W}_{icH}^{\mathsf{T}}\theta_i)\theta_i, \qquad (27)$$

where $l_{i1} > 0$ is the learning rate of the local critic NN.

To obtain the updating rule of the critic NN weight vector $\hat{W}_{ic}$, the local objective function $E_{ic} = \frac{1}{2}e_{ic}^{\mathsf{T}}e_{ic}$ should be minimized with the steepest decent algorithm as

$$\dot{\hat{W}}_{ic} = -\dot{\tilde{W}}_{ic} = -l_{i1}e_{ic}\theta_i. \qquad (28)$$

Therefore, the ideal decentralized control can be derived as

$$\mu_i(x_i) = -\frac{1}{2} R_i^{-1} g_i^{\mathsf{T}}(x_i) \left(\left(\nabla \sigma_{ic}(x_i) \right)^{\mathsf{T}} W_{ic} + \nabla \varepsilon_{ic}(x_i) \right).$$

Consider the nonlinear system is unknown, together with the approximate critic NN (25), the decentralized control can be expressed as

$$\hat{\mu}_i(x_i) = -\frac{1}{2} R_i^{-1} \hat{g}_i^{\mathsf{T}}(x_i) \left(\nabla \sigma_{ic}(x_i) \right)^{\mathsf{T}} \hat{W}_{ic}. \tag{29}$$

Theorem 2: For ith interconnected subsystem (1), the weight approximation error $\tilde{W}_{ic}$ can be guaranteed to be UUB as long as the weights of the local critic NN are updated by (28).

Proof: Choose the Lyapunov function candidate as

$$L_{i2} = \frac{1}{2 l_{i1}} \tilde{W}_{ic}^{\mathsf{T}} \tilde{W}_{ic}. \tag{30}$$

Along the solutions of (27), the time derivative of (30) is

$$\begin{aligned} \dot{L}_{i2} &= \frac{1}{l_{i1}} \tilde{W}_{ic}^{\mathsf{T}} \dot{\tilde{W}}_{ic} \\ &= \tilde{W}_{ic}^{\mathsf{T}} e_{icH} \theta_i - \left\| \tilde{W}_{ic} \theta_i \right\|^2 \\ &\leq \frac{1}{2} e_{icH}^2 - \frac{1}{2} \left\| \tilde{W}_{ic} \theta_i \right\|^2. \end{aligned}$$

Assume $\|\theta_i\| \leq \theta_{iM}$. Hence, $\dot{L}_{i2} < 0$ whenever the approximation error of the critic NN $\tilde{W}_{ic}$ lies outside of the compact set $\Omega_{\tilde{W}_{ic}} = \left\{ \tilde{W}_{ic} \colon \left\| \tilde{W}_{ic} \right\| \leq \left\| \frac{e_{icH}}{\theta_{iM}} \right\| \right\}$. According to Lyapunov's direct method, the weight approximation error is UUB. This completes the proof.

Remark 2: Different from our previous work [20], we can see from (28) that the weights of critic NN $\hat{W}_{ic}$ can be obtained online, it implies that the initial stabilizing control and persisting of excitation condition are no longer required.

Taking the replacement error $\Delta h_i(x_i)$, identification error as well as the approximation error between (21) and (29) into account, they may cause the system performance degradation or even destroy the system stability. Thus, they should be compensated by an adaptive robustifying term as

$$u_{ic} = -\hat{g}_i^{+}(x_i) \operatorname{sgn}(x_i) \hat{w}_i, \tag{31}$$

where $g_i^{+}(\cdot)$ is the Moore-Penrose pseudo-inverse of $g_i(\cdot)$, $\operatorname{sgn}(x_i) = [\operatorname{sgn}(x_{i1}), \ldots, \operatorname{sgn}(x_{i(n_i)})]^{\mathsf{T}}$, $\hat{w}_i$ is the estimation of overall error w_i, which will be defined later. It can be updated by the following adaptive law

$$\dot{\hat{w}}_i = \Gamma_{iw} \sum_{k=1}^{n_i} |x_{ik}|, \tag{32}$$

where Γ_{iw} is a positive constant.

In summary, the overall data-based robust near-optimal decentralized control can be developed as

$$u_i = \hat{\mu}_i + u_{ic}. \tag{33}$$

Remark 3: From the decentralizing stabilizing control design procedure, we can see that it is designed in a local manner. It implies that each of subsystem can achieve optimal control performance, rather than the entire system. Therefore, the proposed scheme is a near-optimal approach.

C. Stability analysis

Theorem 3: Consider the unknown large-scale nonlinear systems which are composed of N subsystems as in (1) with the local cost function (18). The developed data-based robust near-optimal decentralized control (33) can guarantee the closed-loop system to converge to zero asymptotically.

Proof: Choose the Lyapunov function candidate as

$$L_{i3} = \frac{1}{2} x_i^{\mathsf{T}} x_i + V_i(x_i) + \Gamma_{iw}^{-1} \tilde{w}_i^2. \tag{34}$$

As $F_i(\cdot)$ is locally Lipschitz, there exists a positive constant η_{if} such that $\|F_i(x_i, x_{jd})\| \leq \eta_{if} \|x_i\|$. Assuming that $\|g_i(x_i)\| \leq \eta_{ig}$ and denoting $\hat{\mu}_i = \mu_i - \tilde{\mu}_i$, the time derivative of (34) becomes

$$\begin{aligned} \dot{L}_{i3} &= x_i^{\mathsf{T}} \dot{x}_i + \nabla V_i(x_i) \dot{x}_i - \Gamma_{iw}^{-1} \dot{\hat{w}}_i \tilde{w}_i \\ &= x_i^{\mathsf{T}} \left(F_i(x_i, x_{jd}) + \Delta h_i(x, x_{jd}) \right) - U_i(x_i, \mu_i) \\ &\quad + x_i^{\mathsf{T}} g_i(x_i) \left(\mu_i - \tilde{\mu}_i + u_{ic} \right) - \Gamma_{iw}^{-1} \dot{\hat{w}}_i \tilde{w}_i \\ &\leq x_i^{\mathsf{T}} \Delta h_i(x, x_{jd}) - \left(\lambda_{\min}(Q_i) - \eta_{if} - \frac{1}{2} \right) \|x_i\|^2 \\ &\quad + x_i^{\mathsf{T}} (\hat{g}_i(x_i) + \tilde{g}_i(x_i)) u_{ic} - x_i^{\mathsf{T}} g_i(x_i) \tilde{\mu}_i \\ &\quad - \left(\lambda_{\min}(R_i) - \frac{1}{2} \eta_{ig}^2 \right) \|\mu_i\|^2 - \Gamma_{iw}^{-1} \tilde{w}_i \dot{\hat{w}}_i. \end{aligned} \tag{35}$$

Denoting $\delta_i = \tilde{g}_i u_{ic} + g_i(x_i) \tilde{\mu}_i(x_i) + \Delta h_i(x, x_{jd})$ as the overall error, where $\tilde{g}_i = g_i(x_i) - \hat{g}_i(\hat{x}_i)$, and δ_i is assumed to be the upper bounded, i.e., $\|\delta_i\| \leq w_i$, we have

$$\begin{aligned} \dot{L}_{i3} =& - \left(\lambda_{\min}(Q_i) - \eta_{if} - \frac{1}{2} \right) \|x_i\|^2 + x_i^{\mathsf{T}} g_i(x_i) u_{ic} \\ & + |x_i| \, w_i - \left(\lambda_{\min}(R_i) - \frac{1}{2} \eta_{ig}^2 \right) \|\mu_i\|^2 - \Gamma_{iw}^{-1} \dot{\hat{w}}_i \tilde{w}_i \\ \leq & - \left(\lambda_{\min}(Q_i) - \eta_{if} - \frac{1}{2} \right) \|x_i\|^2 + x_i^{\mathsf{T}} \hat{g}_i(x_{id}) u_{ic} \\ & + \sum_{k=1}^{n_i} |x_{ik}| \, w_i - \left(\lambda_{\min}(R_i) - \frac{1}{2} \eta_{ig}^2 \right) \|\mu_i\|^2 \\ & - \Gamma_{iw}^{-1} \dot{\hat{w}}_i \tilde{w}_i. \end{aligned} \tag{36}$$

Substituting (31) into (36), and combining with (32), we have

$$\begin{aligned} \dot{L}_{i3} =& - \left(\lambda_{\min}(Q_i) - \eta_{if} - \frac{1}{2} \right) \|x_i\|^2 + \sum_{k=1}^{n_i} |x_{ik}| \, \tilde{w}_i \\ & - \left(\lambda_{\min}(R_i) - \frac{1}{2} \eta_{ig}^2 \right) \|\mu_i\|^2 - \Gamma_{iw}^{-1} \dot{\hat{w}}_i \tilde{w}_i \\ =& - \left(\lambda_{\min}(Q_i) - \eta_{if} - \frac{1}{2} \right) \|x_i\|^2 \\ & - \left(\lambda_{\min}(R_i) - \frac{1}{2} \eta_{ig}^2 \right) \|\mu_i\|^2. \end{aligned}$$

We can observe that $\dot{L}_{i3} \leq 0$ whenever the following conditions hold

$$\begin{cases} \lambda_{\min}(Q_i) \geq \eta_{if} + \dfrac{1}{2}, \\ \lambda_{\min}(R_i) \geq \frac{1}{2} \eta_{ig}^2. \end{cases}$$

It implies that the developed data-based robust near-optimal decentralized control (33) ensures the unknown large-scale closed-loop system states converge to zero asymptotically. This completes the proof.

Remark 4: Different from our previous work [20], the local cost function in this method is constructed in a more simpler manner, which takes the replacement error, identification error and approximation error of the decentralized control into account. It implies that the proposed data-based robust near-optimal decentralized control can guarantee the closed-loop system to be asymptotically stable.

IV. SIMULATION STUDY

Reconfigurable manipulators [3] that consist of standard links and joint modules can be considered as a set of subsystems interconnected by coupling torques. Therefore, in this simulation, the effectiveness of the developed data-based robust near-optimal decentralized control is proved by a 2-DOF (degree of freedom) reconfigurable manipulator, whose entire dynamics can be expressed as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u,$$

where $q \in \mathbb{R}^2$ is the vector of joint displacements, $M(q) \in \mathbb{R}^{2\times 2}$ is the inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^2$ is the Coriolis and centripetal force, $G(q) \in \mathbb{R}^2$ is the gravity term, and $u \in \mathbb{R}^2$ is the applied joint torque. The system matrices are

$$M(q) = \begin{bmatrix} 0.36\cos(q_2) + 0.6066 & 0.18\cos(q_2) + 0.1233 \\ 0.18\cos(q_2) + 0.1233 & 0.1233 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} -0.36\sin(q_2)\dot{q}_2 & -0.18\sin(q_2)\dot{q}_2 \\ 0.18\sin(q_2)(\dot{q}_1 - \dot{q}_2) & 0.18\sin(q_2)\dot{q}_1 \end{bmatrix},$$

$$G(q) = \begin{bmatrix} -5.88\sin(q_1+q_2) - 17.64\sin(q_1) \\ -5.88\sin(q_1+q_2) \end{bmatrix}.$$

For the development of decentralized control, each joint is considered as a subsystem of the entire manipulator system interconnected by coupling torque. By separating terms only depending on local variables $(q_i, \dot{q}_i, \ddot{q}_i)$ from those terms of other joint variables, each subsystem dynamic model can be formulated in joint space as

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) + Z_i(q,\dot{q},\ddot{q}) = u_i \quad (37)$$

with

$$\begin{aligned} Z_i(q,\dot{q},\ddot{q}) = &\left\{ \sum_{j=1, j\neq i}^{n} M_{ij}(q)\ddot{q}_j + [M_{ii}(q) - M_i(q_i)]\ddot{q}_i \right\} \\ &+ \left\{ \sum_{j=1, j\neq i}^{n} C_{ij}(q,\dot{q})\dot{q}_j + [C_{ii}(q,\dot{q}) - C_i(q_i,\dot{q}_i)]\dot{q}_i \right\} \\ &+ \left[\bar{G}_i(q) - G_i(q_i)\right], \end{aligned}$$

where $q_i, \dot{q}_i, \ddot{q}_i, \overline{G}_i(q)$ and $\bar{u}_i$ are the ith element of the vectors $q, \dot{q}, \ddot{q}, G(q)$ and u, $M_{ij}(q)$ and $C_{ij}(q,\dot{q})$ are the *ij*th element of the matrices $M(q)$ and $C(q,\dot{q})$, respectively.

Let $x_i = [x_{i1}, x_{i2}]^{\mathsf{T}} = [q_i, \dot{q}_i]^{\mathsf{T}}$, (37) can be expressed as

$$\begin{cases} \dot{x}_{i1} = x_{i2}, \\ \dot{x}_{i2} = f_i(q_i,\dot{q}_i) + g_i(q_i)u_i + h_i(q,\dot{q},\ddot{q}), \end{cases} \quad (38)$$

where x_i is the state of the ith subsystem, and

$$\begin{aligned} f_i(q_i,\dot{q}_i) &= M_i^{-1}(q_i)\left[-C_i(q_i,\dot{q}_i)\dot{q}_i - G_i(q_i)\right], \\ g_i(q_i) &= M_i^{-1}(q_i), \\ h_i(q,\dot{q},\ddot{q}) &= -M_i^{-1}(q_i)Z_i(q,\dot{q},\ddot{q}). \end{aligned}$$

Let the initial states of the subsystems be $x_{10} = x_{20} = [1, 0]^{\mathsf{T}}$, the initial states of the identifiers be $\hat{x}_{10} = [2, -1]^{\mathsf{T}}$, $\hat{x}_{20} = [1.5, -0.5]^{\mathsf{T}}$, the identifier gain matrix be $K_{io} = \mathrm{diag}[10, 100]$, the RBFNN weights learning rate of the identifier be $\Gamma_{if} = 500$ and $\Gamma_{ig} = 1$, the desired states of interconnected subsystems to be zero since the stabilizing control problem is solved in this paper. The local cost function (18) is approximated by critic NN, whose structure is chosen as 2–3–1 with 2 input neurons, 3 hidden neurons and 1 output neuron, and the initial values of weight vector as $\hat{W}_{1c} = [0.2, 1.5, 1.1]^{\mathsf{T}}$ and $\hat{W}_{2c} = [1.2, 0.8, 0.9]^{\mathsf{T}}$, the weight learning rates of the critic NN be $l_{i1} = 0.0002$, the gain of the compensator (32) be $\Gamma_{iw} = 5$, $Q_i = I_2$, $R_i = 0.001I$.

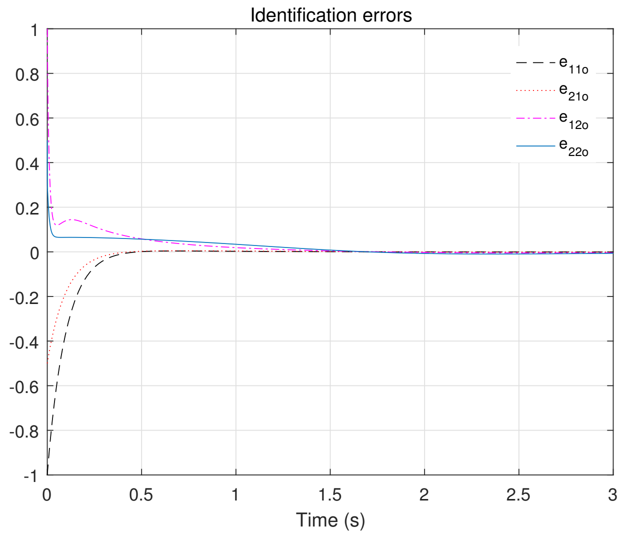

Fig. 1. The identification errors

The simulation results are shown as Figs. 1–3. Fig. 1 describes that the identification errors converge to a small region by using the local NN identifiers. It implies that the unknown subsystems are identified online successfully. The curves in Fig. 2 show that the system states can converge to zero after the system runs for a short time by using the developed data-based robust near-optimal decentralized control scheme (33). Fig. 3 gives the curves of control inputs. From these figures, the closed-loop system of the reconfigurable manipulator can be guaranteed to be asymptotically stable. Therefore, the simulation result demonstrates the effectiveness of the proposed stabilizing scheme.

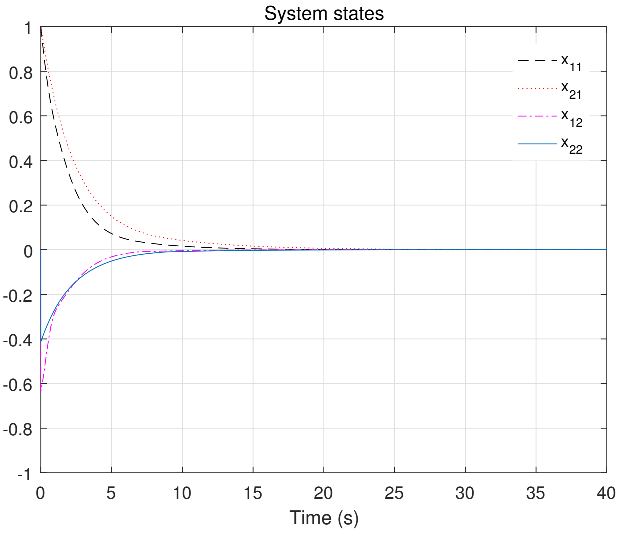

Fig. 2. The system states

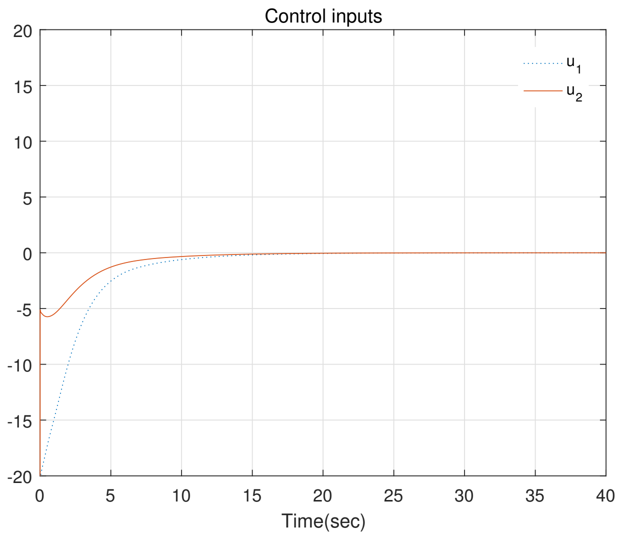

Fig. 3. The control inputs

V. Conclusion

In this paper, we develop a data-based robust near-optimal decentralized stabilization scheme for unknown large-scale nonlinear systems via adaptive critic designs. Local input-output data and NN are employed to identify the unknown subsystem. Hereafter, a critic NN is constructed to approximate the local cost function, and the desired decentralized control can be obtained directly. Then, the overall error caused by the replacement, identification and approximation of the local critic NN can be compensated by an adaptive robustifying term. Therefore, the overall decentralized control can guarantee the closed-loop system to be asymptotically stable by Lyapunov's direct method. A 2-DOF reconfigurable manipulator is employed in simulation to verify the effectiveness of the proposed scheme.

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