Adaptive Curved Feature Detection Based on Ridgelet

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Abstract. Feature detection always is an important problem in image processing. Ridgelet performs very well for objects with linear singularities. Based on the idea of ridgelet, this paper presents an adaptive algorithm for detecting curved feature in anisotropic images. The curve is adaptively partitioned into fragments with different length, and these fragments are nearly straight at fine scales, then it can be detected by using ridgelet transform. Experimental results prove the efficiency of this algorithm.

1 Introduction

Edge detection is always an important problem in image processing. Recently, several methods based on wavelets had been proposed for edge detection. Wavelets perform very well for objects with point singularities and are shown to be optimal basis for representing discontinuous functions in one dimension and functions with point-like phenomena in higher dimensions. However, edges always represent 1-dimensional singularities and wavelets are not the optimal basis for representing them. To resolve this problem, Candès introduce a new analysis tool named ridgelets in his Ph.D. Thesis [1]. The bivariate ridgelt function is defined as follow:

$$\psi_{a,b,\theta}(x) = a^{-1/2}\psi\left(\left(x_1\cos\theta + x_2\sin\theta - b\right)/a\right). \tag{1}$$

$a > 0$, $b \in R$, $\theta \in [0, 2\pi)$. Given an integrable bivariate function $f(x)$, its ridgelet coefficients are defined by [1], [4]:

$$R_f(a,b,\theta) = \int \psi_{a,b,\theta}(x) f(x) dx. \tag{2}$$

Ridgelets can effectively deal with linelike phenomena in dimension 2. But to objects with curved singularities, the approach performance of ridgelet is equal to wavelet and not the optimal basis. Candès present a method named monoscale ridgelets analysis that we can smoothly partition the image into many blocks with same size and each fragment of the curve in the block are nearly straight at fine scales [2]. This is a non-adaptive method for representing the image. It is difficult for us to decide the size of the partitioned block. The size of the block being too large would produce errors after detection and too small would increase the cost of the computation.

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This paper advances a novel adaptive algorithm based on ridgelet transform for detecting curved feature in an image in the frame of ridgelet analysis. We apply this method to sar images and results prove the efficiency of this algorithm.

We firstly outline an implementation strategy of discrete ridgelet transform and next introduce the basic ideas and the implementation process of our curved feature detection algorithm in detail. We present results of several experiments in section 3 and make an analysis. Finally we propose the conclusion and possibilities for future work.

2 Adaptive Curved Feature Detection Based on Ridgelet

2.1 Discrete Ridgelet Transform

Ridgelet analysis can be construed as wavelet analysis in the Radon domain and the ridgelet transform is precisely the application of a 1-D wavelet transform to the slices of the Radon transform [6]. We have

$$R_f(a,b,\theta) = \int Rf(\theta,t)\, a^{-1/2} \psi\left((t-b)/a\right) dt\,, \tag{3}$$

where $Rf(\theta,t)$ is the Radon transform of the function which is given by $Rf(\theta,t) = \int f(x_1,x_2)\delta(x_1\cos\theta + x_2\sin\theta - t)\,dx_1dx_2$ where $(\theta,t)\in[0,2\pi)\times R$ and δ is the Dirac distribution. $\psi((t-b)/a)$ is a 1-D wavelet function. So linear singularities in the image can be send to point singularities by Radon transform, and then wavelets are fully efficient at dealing with point-like singularities, which is equal to that ridgelet perform well to linear singularities.

So the key step of ridgelet transform is accurate Radon transform. To a digital image, a widely used approach of the Radon transform is applying the 1-D inverse Fourier transform to the 2-D Fourier transform restricted to radial lines going through the origin [3]. It can obtained by the following steps (let $(f(i_1,i_2))$, $1\le i_1,i_2\le n$ be a digital image):

①. 2-D FFT. Compute the 2-D FFT of f giving the array $(\hat{f}(k_1,k_2))$, $-n\le k_1,k_2\le n-1$ after padding the array f $n\times n$ to be $2n\times 2n$ by adding extra rows and columns of zeros in every tow rows and columns respectively.

②. Using the interpolation scheme and regarding the center of the image $\hat{f}(k_1,k_2)$ as the coordinate origin, we obtain the Cartesian-to-polar conversion and the radial slice of the Fourier transform. There are in total $2n$ direction angles, each direction corresponding to a radial array composed of $2n$ points. We used trigonometric interpolation rather than nearest-neighbor interpolation used by Starck in [5]. Fig. 1 shows the geometry of the polar grid, and each line crossing the origin denote a direction. (where n =8, so there are in total 16 directions).

③. 1-D IFFT. Compute the 1-DIFFT along each line. We denote it by $R(u_1, u_2)$, $1 \le u_1, u_2 \le 2n$, where u_1 denote the distance between the center of the block and the line, u_2 denote the angle of the line in the block.

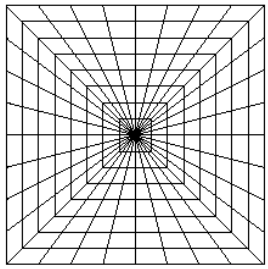

Fig. 1. The geometry of the polar grid *(n=8)*

Because of our using 1-D IFFT of length $2n$ on $2n$ lines, the total work takes $O(N \log N)$, where $N = n^2$ .To complete the ridgelet transform, we must take a 1-D wavelet transform along the radial variable in Radon domain. We choose the dyadic wavelets transform [7], defined by:

$$Wf(u, 2^j) = \int_{-\infty}^{+\infty} f(t) \frac{1}{\sqrt{2^j}} \phi\left(\frac{t-u}{2^j}\right) dt = f * \overline{\phi}_{2^j}(u), \tag{4}$$

where $\overline{\phi}_{2^j}(t) = \phi_{2^j}(-t) = \frac{1}{\sqrt{2^j}} \phi\left(-\frac{t}{2^j}\right)$. Because of its undecimated property it can capture as many as possible characteristics of a signal or an image and make us can measure the position and the magnitude of the point-like singularities. We use 3-order B-spline wavelet which is wildly used in edge detections.

2.2 The Basic Idea of the Proposed Algorithm

Ridgelet only performs very well to detect linear features in an image. However, edges are typically curved rather than straight. The object with curved singularities is still curvilineal one and not a point after Radon transform. So in the Radon space, its wavelet coefficients are not sparse and the ridgelet alone can not yield efficient representations. Candès introduce monoscale ridgelets transform that the image is partitioned into several congruent blocks with fixed side-length, and at sufficiently fine scales, a curved edge is almost straight then it can be detected by using ridgelet transform [2].

We promise only one line exist in each block. However, because of the limitation of the pels, we can not partition the image infinitely. To a n by n image, we have $2n$ directions while we make randon transform to it. With the size of the block smaller, the number of the directions would decrease more and more. Then it would produce errors when we detected the direction of the line and the computation cost would increase. If the size of the block were too large, we can not detect the position and the length of the curved features accurately. Figure 2 give out four cases which maybe produce errors after detection because of too large block.

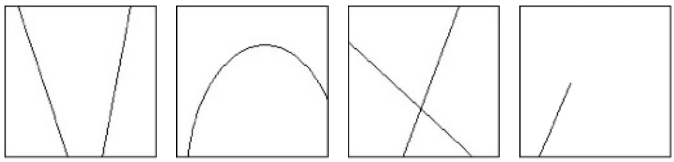

Fig. 2. The cases which the size of the block is too large and maybe produce errors after detection

2.3 Adaptive Algorithm Based on Ridgelet for Detecting Curved Features

We present an adaptive algorithm based on ridgelet transform for detecting curved features here. The size of the block can be changed adaptively. An image is partitioned into several congruent blocks with initial side-length. When each of four cases shown above came forth, that block should be partitioned into four parts with the same size. Then in each part, we use ridgelet to detect it again.

Firstly, we make ridgelet transform to each block partitioned using proposed method. We get ridgelets coefficients array denoted as $W_R f(u_1, u_2)$.

Because of Radon transform, linear singularities are be sent into point singularities, wavelets coefficients of these points are local maximum values. So we search for the maximum value whose absolute value is the biggest in $W_R f(u_1, u_2)$, and write it as Mmax. Then search for the maximum absolute value denoted by Mmax2 in this block except the small region with Mmax being the center. The size of this small region is set to be 5×5 in experiments. Let T be the threshold.

- While Mmax>T and Mmax2>T, it is corresponding to that two lines or one curve which radian value is large exist in one block. Then we partition the block into four parts with the same size, and each part will be deal with again.
- While Mmax $\leq$ T, it shows that no lines or curves exist in this block. Then it wouldn't be detected.
- While Mmax>T and Mmax2 $\leq$ T, it represent that only one line exists in the block, and there are two cases:

  ①. While Mmax>kT, it is corresponding to that the line cross the whole block. Then this block should be detected immediately. We define k=1.5 in experiments.

  ②. While T<Mmax $\leq$ kT, it shows that the line don't cross the whole block. Then we should partition it into four parts and each part should be judged or detected again.

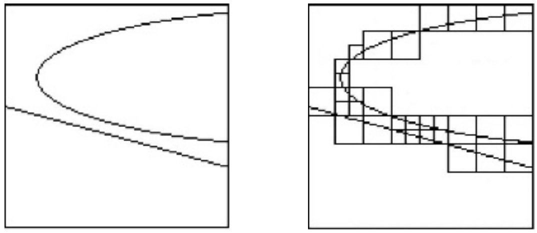

Fig. 3. (*Left*) original image with curved features, (*Right*) partitioned image using adaptive method

Fig. 3 shows the sketch map after partitioned using our method. Now we present the detailed process of the adaptive algorithm based on ridgelet:

Step1. Apply the method introduced by Hou Biao to a digital image $f(i_1, i_2)$, $1 \le i_1, i_2 \le n$ to form ridgelet subbands [8], and denote it by $f_i(i_1, i_2), (i = 1, 2)$. And partition $f_2(i_1, i_2)$ into non-overlapping blocks with size $L \times L$, denoting it by $G_{i,j}(k,l)$ $(1 \le k, l \le L \quad 1 \le i, j \le n/L)$. Initialize $L = L_{\max}$, the smallest side-length of the block is $L_{\min}$.

Step2. Make Radon transform mentioned in sec.2.1 to each block. Results are given by $R_{i,j}(u_1, u_2)$, $(1 \le u_1, u_2 \le 2L \quad 1 \le i, j \le n/L)$.

Step3. Take dyadic wavelet transform to each column of $R_{i,j}(u_1, u_2)$ and results are written as $W^0_{i,j} f(u_1, u_2)$, $W^m_{i,j} f(u_1, u_2)$ $(m = 1, 2, 3)$. Then find out the value Mmax and Mmax2 in $W^1_{i,j} f(u_1, u_2)$ using the method mentioned above. Set T be the threshold.

Step4. While $L = L_{\min}$, this block can not be partitioned into four parts and while Mmax>T, go to step 5, otherwise don't detect it.

While $L > L_{\min}$, do:

If Mmax $\le T$, this block would not be detected and move to the next block. Go back to step2.

If T <Mmax $\le kT$ and Mmax2 $\le T$, then partition this block to four parts and repeat processes mentioned above from step 2.

If Mmax2> T, then partition this block to four parts and repeat processes mentioned above from step2.

If Mmax2 $\le T$ and Mmax> kT, then detect this block immediately. Go to step 5.

Step5. Search for the location of the maximum absolute value of ridgelet coefficients array $W^1_{i,j} f(u_1, u_2)$ and record the corresponding coordinate,

$(u_{1\max}, u_{2\max})$. From $(u_{1\max}, u_{2\max})$ we can obtain the distance form the line to the center of the block and angel of the line, writing them as (t, θ).

Step6. Define an array which is a zero matrix with the same size of the block. Regard the center of this block as the coordinate origin. On the line across the origin and of direction angle $\theta + \pi / 2$, we find out the point the distance from which to the origin is equal to t. Then the line across this point and of the slope θ is the desired one. We find out the coordinates of the two points on the line that intersect the borderlines of the block, and then computer all the coordinates of the points on the line between two points using linear interpolation.

Step7. Go to step2 and detect the next block. Finally synthesize a binary edge image composed of several blocks obtained above.

Because the block after partitioning are non-overlapping, while one curve is across the corner of this block, the part of the curve in the block is weak and it is difficult to detect it accurately on the effect of the noises. In experiments, we can see that the curve in the result have several broken parts (shown in Fig. 4). To resolve it, we search for broken parts and use linear interpolation to connect the adjacent line segments. Because these broken parts are always small, the result after interpolation is not likely to change the original result too much.

Applying the above steps to an image, we can detect curved singularities efficiently, each of which is composed of many linear segments with different length. It also can detect the length of curves or lines accurately. It is hard to do for classical Radon transform and Hough transform.

3 Experiments

Based on the algorithm mentioned above, three images: a basic curve, two noisy circularities (standard deviation Sigma=40, PSNR=16.0973) and a sar image are worked out. And experiment results is shown in fig. 4, fig. 5 and fig. 6 respectively. In the experiment we initialize the size of blocks to be 16×16, i.e. $L_{\max}$=16. And the size of the smallest partitioned block is 8×8. The smallest distance between two circles (see fig.5) is no more than 16. To the sar image (see fig.6), we detect the river after filtering because this can decrease the effect of the speckle noises, where we use median filter. Then we apply wavelet and our method to detect river edges respectively. We choose 3-order B-spline wavelet basis and use uniform threshold method [7] in experiments.

In fig. 4, we can see that our method not only performs well for detecting general curved singularities in an image, but also the part whose curvature is large. Though the result has many broken parts which are small, the direction of the curve has been detected accurately. After we search for broken parts and fill them using linear interpolation method, we can obtain the exactly full detected results (see fig.5 and fig.6). From results of SAR image after detection in fig. 6, we can see that our method is better than the method based on wavelets for restraining the effect of speckle noises on edges of this image. The whole contour of the river has been detected accurately, and we can locate the positions of the curves and compute the length of the curves.

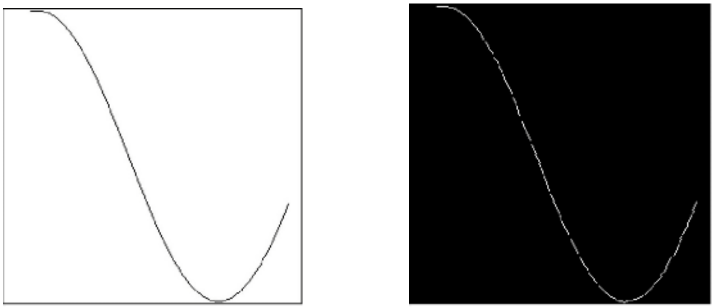

Fig. 4. (*Left*) a basic curve image, (*Right*) the result after detection

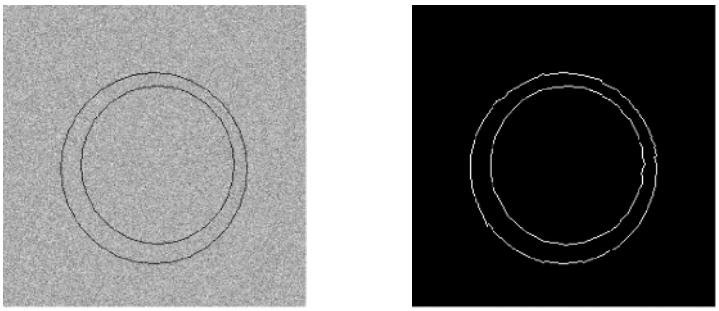

Fig. 5. (*Left*) an original image with two noisy circularities (*standard deviation Sigma=40, PSNR=16.0973*), (*Right*) the result after detection

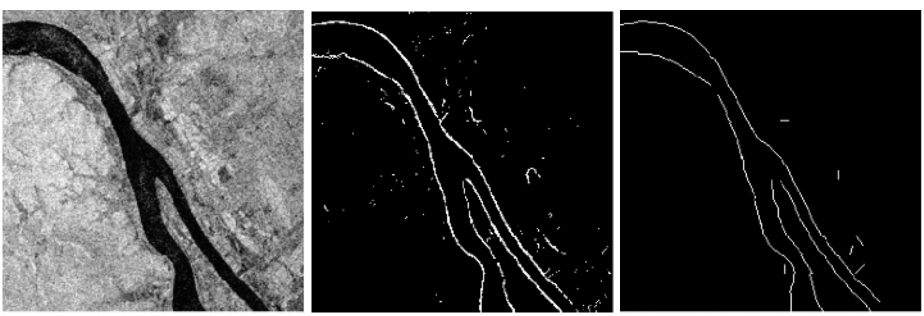

Fig. 6. (*Left*) a SAR image with speckle noises, (*Middle*) the result after detection using wavelets, (*Right*) the result after detection using our method

4 Conclusion

Ridgelets send linear singularities into point singularities by using its capability of reducing dimension. It can capture the linear singularities in the image rapidly and this is hard for wavelet to do. Based on ridgelet transform we change problem of curved singularities detection to the problem of linear singularities detection by using the idea of adaptively partitioning the image. We can locate the position of each linear segment and its length. The results of the experiments prove the efficiency and advantage of our algorithm. However, because the blocks partitioned are non-overlapping, we can see several broken parts in results. How to avoid these? Can we partition the image into many blocks which are overlapping? But the overlapping blocks must lead the cost of the computation to increase. These give us a next question to discuss.

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